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Research Article

Long-Range Scalar Forces in Five-Dimensional General Relativity

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We present new results regarding the long-range scalar field that emerges from the classical Kaluza unification of general relativity and electromagnetism. The Kaluza framework reproduces known physics exactly when the scalar field goes to one, so we studied perturbations of the scalar field around unity, as is done for gravity in the Newtonian limit of general relativity. A suite of interesting phenomena unknown to the Kaluza literature is revealed: planetary masses are clothed in scalar field, which contributes 25% of the mass-energy of the clothed mass; the scalar potential around a planet is positive, compared with the negative gravitational potential; at laboratory scales, the scalar charge which couples to the scalar field is quadratic in electric charge; a new length scale of physics is encountered for the static scalar field around an electrically-charged mass, $L_s = \mu_0 Q^2/M$; the scalar charge of elementary particles is proportional to the electric charge, making the scalar force indistinguishable from the atomic electric force. An unduly strong electrogravitic buoyancy force is predicted for electrically-charged objects in the planetary scalar field, and this calculation appears to be the first quantitative falsification of the Kaluza unification. Since the simplest classical field, a long-range scalar field, is expected in nature, and since the Kaluza scalar field is as weak as gravity, we suggest that if there is an error in this calculation, it is likely to be in the magnitude of the coupling to the scalar field, not in the existence or magnitude of the scalar field itself.

1. Introduction

In 1919, Einstein received a paper from Kaluza [1] showing that the field equations of general relativity, and the field equations of electromagnetism, behave as if the gravitational tensor field $g_{\mu\nu}$, and the electromagnetic vector field, A_μ, are components of a 5-dimensional (5D) tensor gravitational field $\tilde{g}_{ab}$. Since a 5D metric tensor has 15 components, an additional long-range scalar field potential ϕ is implied. Standard 4D physics is recovered when this scalar potential goes to one, and that was Kaluza's original assumption.

The 5D picture holds not only for the field equations, but for the equations of motion too, as Kaluza originally showed. When the geodesic equation is written in 5 dimensions, it is found to contain the standard 4D geodesic equation, along with the Lorentz force law of electromagnetism; there is an additional term for the scalar force that is not identified in nature. As in the field equations, the 4D limit of the equations of motion is obtained when the scalar field goes to one.

Here, we build on the equations developed in a previous work [2] to obtain expressions for the associated scalar force under a range of conditions. Let us summarize first the context of these considerations.

We are careful not to confuse the "Kaluza-Klein" theory of compact dimensions with the strictly classical Kaluza theory of 5D general relativity and its long-range scalar field addressed here. After Kaluza's paper, classical field equations that properly incorporated the long-range scalar field were developed to various degrees by independent research groups: led by Einstein at Princeton (1930s-1940s), by Lichnerowicz in France (1940s), Scherrer in Switzerland (1940s), by Jordan in Germany (1950s), and by Dicke at Princeton (1960s) [3, 4].

The review by Gonner [3] shows that the equations of the classical Kaluza scalar field were developed by multiple European research groups in the mid-20th century, but none of those results were published in English language journals. The war caused further disruptions in the dissemination of results obtained by the European groups.

Progress on analysis of the classical field equations accelerated in the 1980s with the availability of English translations [5] of work by Thiry [6] under Lichnerowicz. During the 1980s and 1990s, significant work was done on the classical Kaluza theory, including a general solution for the 5D metric of a charged object [7], analogous to the Schwarzschild and Reissner-Nordstrom solutions.

A notable feature of the 5D ansatz introduced by Kaluza was the cylinder condition, that no field component depends on the fifth coordinate, and so its derivatives vanish. This was seen by Kaluza as the mathematical expression of the absence of a detectable fifth coordinate.

In 1926, Klein [8] adapted the Kaluza 5D ansatz to quantum considerations and hypothesized a compact, microscopic fifth dimension, thereby bringing Planck's constant and quantum considerations into a framework that is typically known as "Kaluza-Klein." Klein's suggestion turned out to be a forced marriage of classical and quantum theory that did not satisfactorily describe 4D physics, but it profoundly influenced the direction of quantum field theory, making compact small dimensions an accepted part of physics, in the Kaluza theory [9], and in higher-dimensional theories [5]. Today, many researchers reflexively think of the Kaluza fifth dimension as compact and microscopic.

It is important to bear in mind that this work is entirely classical, a treatment of general relativity in 5 dimensions. The entirety of the mathematics is simply to write general relativity in 5 dimensions instead of 4, and to set to zero derivatives with respect to the 5th dimension. When this is done, 4D general relativity and classical electrodynamics are reproduced perfectly. The debate ensues about what this mathematics "means." Does it mean that the 5th dimension is "real"? Is it compact? Is it microscopic? Fortunately, since we have force equations in this theory, the answer to these philosophical questions are irrelevant to testable predictions. Some of those predictions are provided here.

The results here cannot be constrained by particle accelerator measurements of subatomic structure, just as such experiments cannot constrain general relativity or the Maxwell equations. Indeed, the validity of a classical theory of charged particles is confined only to results independent of elementary particle structure [10]. Certainly, the Maxwell equations and the Einstein equations describe macroscopic fields whose underlying reality is quantum, yet still yield testable predictions. So, too, do we apply the classical 5D theory, in the hopes that it can still yield testable predictions independent of atomic structure. Rohrlich [10] assures us that classical theories can yield valid descriptions of atomic systems in those cases where the result does not depend on assumptions of the atomic structure, and where the results are convergent.

Even so, many of the references in the Kaluza literature adopt the traditional classical field equations and the cylinder condition, while opining in the text of a microscopic, compact fifth dimension. The math is still classical, but classical thinking is abandoned. What are naturally interpreted as classical long-range gravitational, electromagnetic, and scalar fields, are reinterpreted to be the "$n = 0$" modes of a Fourier expansion of fields, on a manifold with a compact 5th dimension [5]. Ad hoc quantum considerations are laid on the classical theory, and Planck's constant emerges amid new free parameters.

This seems to violate the spirit of a consistent classical theory of charged particles, as enunciated by Rohrlich. Correspondingly, we make no assumption that the fifth dimension is compact, since that is not necessitated by the Kaluza field equations or by the cylinder condition [11]. This work is not the "Kaluza-Klein" theory, but the strictly classical Kaluza theory of 5D general relativity. The conceptual framework we adopt for the mathematics is that the fifth dimension is open and macroscopic, like the other four of spacetime. No macroscopic classical experiment contradicts that assumption, and classical theories can only be tested in classical experiments. As we will see, the cylinder condition produces a nontrivial constant of the motion irrelevant to compact dimensions.

The unique nature of the fields in 5D general relativity and their mutual coupling are revealed in the Kaluza Lagrangian:

$$\mathscr{L} = g^{1/2}\left[\frac{c^4\phi}{16\pi G}g^{\alpha\beta}R_{\alpha\beta} - \frac{\phi^3}{4\mu_0}g^{\alpha\mu}g^{\beta\nu}F_{\alpha\beta}F_{\mu\nu}\right], \quad (1)$$

where G is the gravitational constant, μ_0 is the permeability of free space, c is the speed of light, $R_{\mu\nu}$ is the Ricci tensor, $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field strength tensor, g is the determinant of $g_{\mu\nu}$, the metric tensor, and the gravitational field equations are obtained from variation with respect to $g^{\mu\nu}$. ϕ is a new scalar field necessitated by the 5D metric hypothesis. We can call this the "long-range Kaluza scalar field." It is clear that the 4D limit of the theory occurs when $\phi \longrightarrow 1$.

The English language Kaluza literature of the late 20th century (see Ref. [11] for a review) contains variations in the field equations and Kaluza Lagrangian. The correct form of the Kaluza Lagrangian was obtained by Refs. [12, 13], and their result was verified using tensor algebra software [14]. Ref. [13] also obtains the correct curvature tensors, while ref. [12] has some minor errors in the 5D Ricci tensors.

The scalar-electromagnetic couplings make the Kaluza theory unique among scalar-tensor theories and markedly different from the Brans-Dicke theory. The Kaluza Lagrangian contains aspects of the Bekenstein scalar-electromagnetic theory [15], with a variable vacuum permittivity, although Bekenstein did not start from a Lagrangian, and wrote in terms of a variable fine structure constant. The Kaluza Lagrangian (1) also contains aspects of the Brans-Dicke scalar-tensor Lagrangian [16, 17] and appears formally identical when the BD parameter $\omega = 0$. In each case, the scalar field can be viewed as a variable gravitational constant.

The correspondence between the BD theory and neutral-matter 5-dimensional Kaluza general relativity cannot be framed in terms of the BD free parameter ω. The BD ω corresponds to a scalar field kinetic term in the Lagrangian, which is obviously not present in (1). It arises in the BD theory to generalize conformal transformations of (1), written in the "Jordan frame", to a coordinate system in which the scalar field vanishes from the Ricci tensor term, the "Einstein frame." Yet, this transformation does not change the physics, because test particles still move on geodesics of the Jordan frame. Formally, the BD ω term seems only to add an awkward, *second* scalar-field energy-momentum tensor to the BD gravitational field equations.

More importantly, a 4D conformal transformation of the 5D metric would impact the other terms in the Kaluza Lagrangian. This is because the identification of the 4D metric and electromagnetic potentials relies on the presence of a scalar field. A transformation of the scalar field implicates a transformation of the other fields in 5D. Put another way, the identification of the electromagnetic field in 5D is tantamount to the Jordan frame. We are free to work in the 5D Einstein frame, as in Ref. [12], but we find the Jordan frame more intuitive.

Treatment of the 5D sources in the field equations varies in the literature. Kaluza and subsequent authors typically assumed weak specific charges in the source terms, to avoid conceptual issues or deviations from standard physics at high specific charges. By specific charge, we mean the ratio of electric charge to mass of a body. In classical theory, charge is a specific quantity that can go smoothly to zero, and the charge carriers are not quantized, consistent with the demand that the classical predictions be independent of the particle structure. The prior work [2] established a comprehensive treatment of source terms for all specific charges.

Although the 5D geodesic equation has been long known and well-studied [7, 13, 18, 19], the corresponding energy-momentum tensor was obtained only recently [2]. General covariance of the 5D field equations, and the requirement that the source term be a 5D tensor to match the 5D Einstein tensor, is essential to establishing the correct 5D energy-momentum tensor corresponding to the 5D geodesic equation. When 5D covariance of the source terms is established, unique "saturation" effects emerge in the source terms. The saturation effects are such that the coupling of a test body to either the gravitational, electric, or scalar forces can vary with its specific charge.

In this work, we examine the field equations and source terms developed in [2] and the coupled equations of gravity, the electric field, and the scalar field. We recover the intriguing result, originally described by Dicke [20, 21], that the scalar field coexists with the Newtonian gravitational field, but may masquerade as gravity. Furthermore, the mass-energy of the Kaluza scalar field contributes to the total effective mass we measure through Kepler's laws, so that planetary gravitating mass comprises rest mass "clothed" by scalar field.

We provide an identification of scalar charge and relate it to electric charge and rest mass. We encounter a new physical length scale of the scalar field of a body of mass M and electric charge Q, $L_s = \mu_0 Q^2/M$, that is also new to the Kaluza literature.

We find that the Kaluza theory implies an electrogravitic buoyancy force around planet-sized masses for electrically-charged test bodies. This is because the planetary scalar field is positive and acts uniformly outward on scalar charges, while the scalar charge is itself quadratic in the electric charge. The predicted magnitude for the effect is large, perhaps too large to be true. Yet, if so, it would provide the first experimental falsification of a prediction stemming from the Kaluza hypothesis. Until now, the theory has always reproduced known classical physics in the limit of a constant scalar field, and the Kaluza theory has limited freedom of parameters.

We find that the saturation effects in the source terms introduced in [2] act to alter the nature of the scalar coupling in high specific charge environments, so that the Kaluza scalar field may masquerade as the electric force in the parameter regimes of atomic systems. Furthermore, it appears that the scalar potential goes to zero for point particles, suppressing the scalar force altogether for atomic systems.

In the following development, we consider solutions to the field equations obtained in ref. [2] and identify gravitational, electric, and scalar charges. We provide static, spherically-symmetric solutions for the scalar, electric, and gravitational potentials around charged, massive bodies. Three limits in the solutions are encountered for different values of specific charge of the sources: neutral, weak charge states, and strong charge states. From the potentials and the charges, the scalar, electric, and gravitational forces between charged bodies are established. An electrogravitic buoyancy force is identified.

2. Overview of Field Equations with Sources

Let us summarize some key general results from [2] that establish the 5D field equations in the presence of sources. Then, we will obtain solutions for successive cases of neutral, weakly-charged, and strongly-charged matter.

The gravitational field equations for $g_{\mu\nu}$ in the presence of electromagnetic and scalar fields, and matter, are ([2] equation (21)):

$$G_{\mu\nu} = \phi^{-1} T^{\phi}_{\mu\nu} + \frac{8\pi G}{\mu_0 c^4}\phi^2 T^{EM}_{\mu\nu} + \frac{8\pi G}{c^3\phi}\frac{d\tau}{ds}\frac{\rho}{g^{1/2}}g_{\mu\alpha}\frac{dx^\alpha}{dt}U_\nu, \quad (2)$$

where ρ is the mass density of the sources, U_ν is the covariant 4-velocity of the sources, ds is the 5D length element, $d\tau$ is the 4D proper time, and t is the ordinary time coordinate. Also,

$$T^{\phi}_{\mu\nu} \equiv \nabla_\mu \nabla_\nu \phi - g_{\mu\nu}\nabla_\alpha \nabla^\alpha \phi \quad (3)$$

is the Kaluza scalar field energy-momentum tensor, and

$$T^{EM}_{\mu\nu} \equiv g^{\alpha\beta}F_{\mu\alpha}F_{\nu\beta} - \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta} \quad (4)$$

is the electromagnetic energy-momentum tensor. Greek indices range over the 4 coordinates of spacetime.

The Kaluza modification (2) to the Einstein equations comes in the scalar field energy-momentum, in the scalar field coupling to electromagnetic energy momentum, and in the term in $d\tau/ds$ in the material energy momentum. The Kaluza scalar field behaves like a variable gravitational constant, as in the Brans-Dicke scalar-tensor theory, *except* with respect to its coupling to electromagnetic energy momentum.

The term in $d\tau/ds$ relates the 4D and 5D length elements of a body; a key result of this work will be our expression for it in terms of ϕ.

The electromagnetic field equations for A_μ in the presence of gravitational and scalar fields, and electrically charged matter, are ([2] equation (25)):

$$\nabla_\nu\left(\phi^3 F^{\nu\mu}\right)=\mu_0 \frac{\rho}{g^{1/2}} kc\tilde{U}_5 \frac{dx^\mu}{dt}. \tag{5}$$

The quantity $\rho kc\tilde{U}_5$ is identified with the electric charge density, reproducing the expected source for the Maxwell equations. We shall define $\tilde{U}_5$ shortly. The Kaluza modification to the Maxwell equations comes in the scalar field, which acts as a variable dielectric constant, similar to the Bekenstein theory [15]. As an aside, the quantity $\phi^3 F_{\mu\nu}$ emerges as an invariant under conformal transformations [12].

The constant k is the characteristic electrogravitic scale parameter of the Kaluza theory, given in MKS units as [2]:

$$kc \equiv \sqrt{16\pi G\varepsilon_0}=\sqrt{16\pi G/\mu_0 c^2} \simeq 1.7\times 10^{-10} C/kg. \tag{6}$$

It is closely related to the ADM mass [22].

The scalar field equation for ϕ in the presence of gravitational and electromagnetic fields, and electrically charged and neutral matter, is ([2], Eqs. (24) and (26))

$$-3\nabla_\alpha\nabla^\alpha\phi=\mu_0 k^2 c\frac{\rho}{g^{1/2}}\frac{\tilde{U}_5^2}{\phi^2}\frac{ds}{dt}-\frac{8\pi G}{c}\frac{d\tau}{ds}\frac{\rho}{g^{1/2}}\frac{d\tau}{dt} -\frac{3}{4}\phi^3 k^2 F_{\alpha\beta}F^{\alpha\beta}. \tag{7}$$

This equation is new to physics. Therefore, our inferences about it rely on its mathematical emergence alongside the electromagnetic and gravitational fields, which we can constrain with known physics. Note that equation (7) for ϕ is dynamical, even though ϕ enters only algebraically in the Lagrangian (1). Charged matter and neutral matter enter as sources for ϕ with the opposite sign.

Note also that (7) contains the trace of the field equations (2). In the absence of electric charge and electromagnetic fields, the Kaluza scalar field will act to neutralize the scalar curvature $R=R_{\mu\nu}g^{\mu\nu}$ by enforcing $R=0$ against whatever sources of matter exist in spacetime. The scalar field will play a role in the total energy budget of spacetime, and in this way, masquerade as gravity [16], as we will see shortly.

Now let us turn to the 5D interval ds. The 5D proper velocity is defined by

$$\tilde{U}^a \equiv \frac{dx^a}{ds}, \tag{8}$$

where x^a is a 5-vector of coordinates and where small roman indices range over the 5 coordinates of spacetime plus the fifth coordinate x^5.

We now consider anew the 5D length element given by equation (8) of ref. [2]:

$$\varepsilon_a\tilde{a}^2 ds^2=\tilde{g}_{ab}dx^a dx^b=g_{\mu\nu}dx^\mu dx^\nu+\varepsilon_\phi\phi^2\left(dx^5+kA_\nu dx^\nu\right)^2, \tag{9}$$

where $\tilde{a}$ is a constant. In the 5D length element (9), the parameters $\varepsilon_a, \varepsilon_\phi=\pm 1$. They represent that the 5D hypothesis does not fix the sign, timelike, or spacelike of the 5D length element, ε_a, or of the fifth metric component, ε_ϕ.

Variation of the ε_a and ε_ϕ can lead to differing effects in the electric charge and mass compared to what we will report here. We find that a nonimaginary mass requires that $\varepsilon_a=+1$ and $\varepsilon_\phi=+1$, implying that both the 5D length element and the signature of the fifth dimension in the metric are timelike. We avoid imaginary-mass solutions because of their absence from planetary and laboratory physics.

Results in the Kaluza literature often indicate the signature of the fifth dimension is spacelike. However, the form of the Kaluza Lagrangian (1) seems to suggest that the signature of the fifth coordinate is timelike [14]. An experimental investigation [23] also lends support to that conclusion, by testing for the existence of a rest frame for motion along the fifth coordinate. We therefore adopt $\varepsilon_a=\varepsilon_\phi=+1$ in the following, but the alternative results can to some extent be inferred from results reported here.

The free parameter $\tilde{a}$ in the 5D length element must be fixed by correspondence to known physics. In fact, this is the only free numerical parameter in 5D general relativity. As with the ε_a and ε_ϕ, differing choices of $\tilde{a}$ can lead to different effects in the couplings than reported here.

We recall the cylinder condition, that no fields depend on the fifth coordinate. Applying a standard result of general relativity, we see that the absence of a dependence of the metric on the fifth coordinate, $\partial_5\tilde{g}_{ab}=0$, implies that that the fifth covariant component of $\tilde{U}^a$ is a 5D constant of the motion ([2] equation (9)):

$$\tilde{U}_5=\tilde{g}_{5b}\tilde{U}^b=\phi^2\left(\tilde{U}^5+kA_\nu\tilde{U}^\nu\right)=\text{constant}. \tag{10}$$

This is a standard result of the Kaluza theory. The cylinder condition implies a nontrivial constant of the motion, and this reinforces our choice to adopt a classical perspective and treat the cylinder condition as an imposed boundary condition, somewhat akin to a time-independent boundary condition that would imply a conserved energy. There is no suggestion here of a compact fifth dimension.

The constant (10) is related to $\tilde{a}$ through (9):

$$\tilde{a}^2=\left(\frac{cd\tau}{ds}\right)^2+\frac{\tilde{U}_5^2}{\phi^2}, \tag{11}$$

where the proper time τ is defined as $g_{\mu\nu}\equiv c^2 d\tau^2$.

At this point, we turn toward new results by noting first from (1) that $\phi\longrightarrow 1$ in the 4D limit. Therefore, we contemplate a Newtonian-style perturbation expansion of ϕ such that

$$\phi\simeq 1+\xi+O\left(\xi^2\right), \quad \xi\ll 1. \tag{12}$$

We will verify at the end that $\xi\ll 1$.

Let us now fix the constant $\tilde{a}$ from (11). In the limit that $\tilde{U}_5 \longrightarrow 0$, then $cd\tau/ds \longrightarrow 1$. Similarly, we know that asymptotically from (12), $\phi \longrightarrow 1$. Therefore,

$$\tilde{a}^2 \equiv 1 + \tilde{U}_5^2. \tag{13}$$

We see that $(1 + \tilde{U}_5^2)ds^2$ plays the same role in 5D as $c^2 d\tau$ does in 4D, an invariant length element, with a characteristic velocity.

Now let us use (12) and (13) to rewrite (11):

$$\left(\frac{cd\tau}{ds}\right)^2 = 1 + \tilde{U}_5^2\left(1 - \phi^{-2}\right) = 1 + 2\xi\tilde{U}_5^2 + O\left(\xi^2\right) \simeq 1 + 2\xi\tilde{U}_5^2. \tag{14}$$

This is a new expression in the Kaluza theory and one whose implications we will pursue here. It acts as a coupling coefficient and manifests interesting saturation effects in the coupling to fields. For neutral bodies, $\tilde{U}_5 = 0$, and $d\tau/ds = 1$. Yet, for electrically charged matter, the coupling becomes a function of the scalar field perturbation ξ.

The 5D geodesic equation yields a force equation in terms of 4D quantities ([2] equation (12))

$$\frac{d\tau}{ds}\left(\frac{dU^\nu}{d\tau} + \Gamma^\nu_{\alpha\beta}U^\alpha U^\beta\right) = k\tilde{U}_5 g^{\nu\mu}F_{\mu\alpha}U^\alpha + \tilde{U}_5^2\left(\frac{ds}{d\tau}\right)\frac{(\partial_\alpha\phi)}{\phi^3}\left[g^{\nu\alpha} - \frac{U^\nu U^\alpha}{c^2}\right], \tag{15}$$

where the 4D proper velocity of a particle is

$$U^\mu \equiv \frac{dx^\mu}{d\tau}. \tag{16}$$

We see in (15) the 3 terms corresponding to the 3 forces: gravitational, electromagnetic, and scalar. The scalar force operates orthogonal to the 4-velocity of a test body, accounting for Dicke's statement that scalar forces accelerate at constant energy.

This completes our summary of the governing equations for scalar fields and forces. In subsequent sections, we obtain solutions for 3 different parameter regimes.

3. Cosmological Scalar Field

Early in development of the Kaluza scalar field theory, it was realized that Kaluza's original assumption that $\phi \longrightarrow 1$ was incompatible with the scalar field equation (7). When sources $\rho \longrightarrow 0$, an unnatural constraint is implied for the electromagnetic field, $F^{\alpha\beta}F_{\alpha\beta} = 0$. Conversely, the existence of ambient electromagnetic fields will influence ϕ, driving it away from 1. Yet, we may ask how $\phi \longrightarrow 1$ in a self-consistent fashion.

Our analysis of ϕ must recognize the hierarchy of length scales at issue. It is similar in this respect to gravity. On laboratory length scales and locally in any gravitational field, the gravitational field is the Minkowski metric. On cosmological length scales, the gravitational field is the Robertson-Walker metric, and on noncosmological timescales, the Robertson-Walker metric of the universe approximates the Minkowski metric. Yet, around massive objects, we encounter other gravitational fields, such as the Schwarzschild metric or the Kerr metric, on much shorter length scales.

All of these gravitational fields exist simultaneously, and they overlap in space and time. How are they distinguished? By the length scale or timescale under consideration. Just as the gravitational field can be viewed as either strongly curved or flat, depending on the length scale under consideration, so it is with the Kaluza scalar field. That is, regions of local scalar field variation where $F^{\alpha\beta}F_{\alpha\beta} \neq 0$ can coexist with scalar fields on different length scales, for which $F^{\alpha\beta}F_{\alpha\beta} = 0$.

Indeed, this is the case when we consider scalar field cosmology. We recall that long-range scalar field research was founded on the study of a variable gravitational constant, which is cosmological by definition. Therefore, if $\phi \longrightarrow 1$ cosmologically, if it is to be identified with the gravitational constant in the Lagrangian (1), then the value of $F_{\mu\nu}$ in that limit must also be cosmological.

We find that the scalar field equation (7) for the cosmological conditions of a Lambda-Cold-Dark-Matter universe implies a cosmological scalar field ϕ_c:

$$\phi_c^3 = \frac{\mu_0\rho_c c^2}{3B_c^2}, \tag{17}$$

where $\rho_c c^2$ is the cosmological energy density and B_c is a cosmological magnetic field. Clearly, this involves the ratio of a matter energy density to a bulk cosmological magnetic energy density.

The scalar field equation does allow a consistent, constant scalar field solution that can be identified cosmologically with the gravitational constant. Yet, it also seems to imply and require a cosmological magnetic field to support the Kaluza scalar field. The implied values of B_c, above 10^{-10} T, seem consistent with intergalactic or primordial magnetic field values. At these levels, B_c is negligible in the cosmological energy budget, and it will be lost in the cosmic background radiation component. Yet, the cubic dependence of ϕ_c on those parameters results in a very weak dependence, and stability of ϕ around 1 for a range of cosmological parameters.

Therefore, ϕ is set to ~ 1 by cosmological parameters. It can be approximately constant over large length scales. Variations from the flat space value are observed around planetary masses and local bulk electromagnetic fields, just as with gravity. That is how the scalar field will be approached here.

4. Identification of the Scalar Charge

The equation of motion (15) has been studied by various researchers, including [13, 18, 19]. The term in brackets on the RHS of (15) that is quadratic in U^α arises from the transformation of derivatives with respect to s, to derivatives with respect to τ, and using (10). This is indeed the form expected for a scalar field force [24].

The term linear in $\tilde{U}_5$ in (15) must be identified with the electric charge, Q, of a body of rest mass M, to correspond with the Lorentz force law; this identification is standard in the Kaluza literature. The coefficient of the gravitational terms in (15) is identified with mass, and the coefficient of the scalar field term is identified as the Kaluza scalar charge.

We therefore identify the three charges associated with the three forces: mass for gravity, electric charge for electromagnetism, and a new scalar charge for the scalar force. Multiplying (15) through by Mc and using (14):

$$M\frac{cd\tau}{ds} \simeq M\sqrt{1+2\xi\tilde{U}_5^{\,2}} \equiv \tilde{M} \longrightarrow \text{mass}, \tag{18}$$

$$Mck\tilde{U}_5 \equiv Q \longrightarrow \text{electric charge}, \tag{19}$$

$$Mc\tilde{U}_5^2\frac{ds}{d\tau} \simeq \frac{Mc^2\tilde{U}_5^2}{\sqrt{1+2\xi\tilde{U}_5^2}} \equiv \tilde{S} = \frac{Q^2}{\tilde{M}k^2} \longrightarrow \text{scalar charge}. \tag{20}$$

The assignments of mass (18) and charge (19) are common in the Kaluza literature, e.g., [13, 18]. The expression for the scalar charge is more variable in the Kaluza literature; (20) is a new result, as is the ξ dependence of (18).

The expression for mass (18) implied by (19) is seen to have a peculiar dependence on the electric charge and the long-range scalar field ϕ. This is to be expected. It is characteristic of long-range scalar fields that, if they interact with a particle, the particle mass must be a function of the scalar field [20, 21]. In this theory, the scalar field brings to life a mass variation for charged bodies. However, since $\xi \ll 1$, the variation is small for all charged systems. No macroscopic experiment should show this effect.

An effective mass of the form (18) is common in the Kaluza literature, but with variation in form. The form used in (18) matches closely ref. [18] and is similar in nature to that in ref. [13]. Ref. [12] uses an entirely different form. The work presented here is unique in its parameterization of the Kaluza scalar field in terms of ξ, via (12), and will lead to unique results. Previous research on the Kaluza scalar field has not considered the implications of a Newtonian-like perturbation.

The form of (18) is positive-definite, due to our choice of ε_a in (9). It ascribes an increase in mass to interaction with the scalar field. Choosing the opposite sign of ε_a would lead to a decrease in mass, but also potentially to imaginary mass. The concept of imaginary mass has a place in physis, but we wish to avoid it here on the grounds that planetary masses are understood to be real numbers, and there is no accepted interpretation of the gravitational field of an imaginary mass.

Electric charge is identified in (19) with a 5D constant of the motion, $\tilde{U}_5$, and is functionally invariant for all charge states. The constant ck (6) forms a characteristic charge-to-mass ratio. It is better known as the ADM mass [22] when combined with the quantum of electric charge. The preceding analysis may indicate why no particle with the ADM mass exists. It is not a breakdown of the classical theory, but is a misappropriation of an intrinsic quantity, a universal charge-to-mass ratio.

Note from (10) that a term of $\tilde{U}_5$ depends on particle 4-velocity. Therefore, the 5D invariant electric charge $Q \propto \tilde{U}_5$ can be understood as a sort of canonical electric charge, analogous to the canonical momentum of a particle in an electromagnetic field. However, the motional charge is proportional to kA_μ, which is typically very small, perhaps too small to measure.

The scalar charge expression (20) takes more variable forms in the literature, because there is no mapping to known physics of the Kaluza scalar force. Its form was very much an open question in the monopole solution by ref. [7]. The Kaluza scalar field perturbation $\xi \ll 1$, and so for many electrically charged objects, the scalar charge is $\propto Q^2/M$.

For highly charged objects, when $\tilde{U}_5 \gg \xi^{-1}$, the scalar charge saturates to a value linear in Q:

$$\text{saturated scalar charge} \longrightarrow \frac{Mc^2\tilde{U}_5}{\sqrt{2\xi}}, \quad \tilde{U}_5 \gg \xi^{-1}. \tag{21}$$

That is, the scalar coupling becomes proportional to the electric charge for high-specific-charge objects such as elementary particles. This means that the scalar force merges with the electric force, and the scalar force can masquerade as the electric force. A similar electrical-masquerading effect is also seen for the saturated gravitational force.

$$\text{saturated mass} \longrightarrow M\tilde{U}_5\sqrt{2\xi}, \quad \tilde{U}_5 \gg \xi^{-1}. \tag{22}$$

The implication of these results will be developed in greater detail below.

These charges, and the associated source terms in the gravitational, electromagnetic, and scalar field equations, can also be understood in terms of fluxes of an energy-momentum-charge 5-vector. It is analogous to how the energy-momentum tensor for 4D dust can be understood as the flux of an energy-momentum 4-vector. In that case, the energy corresponds to a momentum in time, in that it arises from a change in the time coordinate, just as momentum arises from a change in the space coordinate.

The spacetime flux of the 5th component of the 5-velocity corresponds to the electric current 4-vector. The change along the 5th coordinate of the 5th component of the 5-velocity corresponds to the scalar charge. The 5th component of the 5-velocity corresponds to the energy bound into charge, just as the time component of the 5-velocity describes the energy bound into the rest mass. So, we can rephrase the identifications (18), (19) and (20):

(i) Gravitational source current $\propto \tilde{M}U_\mu U_\nu$

(ii) Electromagnetic source current $\propto QU_\mu$

(iii) Scalar source current $\propto Q^2/\tilde{M}$

5. Calculation of Scalar Fields

In this section, we solve the general field equations (2), (5) and (7) for time-independent, spherically-symmetric solutions. The Kaluza scalar field is treated as a small perturbation, like the gravitational field. The gravitational, electric, and scalar fields are obtained for nonrelativistic sources. Due to the dependence of the scalar charge (20) and mass (18) on the electric charge, 3 successive limits of specific charge are investigated: neutral matter, achievable laboratory specific charge, and specific charge characteristic of atomic systems.

The general field equations given above for gravitational, electromagnetic, and scalar field are solved now under a series of typical constraints, which we enumerate here for convenience:

(1) Spherical symmetry: fields depend spatially only on a radial coordinate, r

(2) Time independent: time derivatives vanish

(3) Magnetic fields vanish, so $F^{\alpha\beta}F_{\alpha\beta} = -2E^2/c^2$ when electric charges are present

(4) Sources are at rest

(5) Test particle speeds $v \ll c$

(6) Consider weak perturbations of the gravitational field, $g_{\mu\nu} \simeq \eta_{\mu\nu} + h_{\mu\nu}, \quad h_{\mu\nu} \ll 1$. With regard to gravity, this is a Newtonian limit

(7) Consider weak perturbations of the scalar field, such that $\phi \simeq 1 + \xi, \quad \xi \ll 1$

(8) The metric signature is $(+, -, -, -)$

Constraint no. 5 implies $dt/d\tau \simeq 1$. It also implies that the gravitational force term in the geodesic equation (15) is dominated by Γ^{ν}_{tt}, the time-time components of the connection. These are of course the components of relativistic gravity that accounts for the Newtonian limit.

Constraints nos. 2, 6, and 8 imply $\nabla_\mu \nabla^\mu = -\nabla^2$, where ∇^2 is the ordinary 3-space Laplacian operator.

Under the assumptions above, our task to solve the coupled gravitational, electromagnetic, and scalar field equations reduce to solving coupled equations for 3 scalar potentials:

(i) The scalar perturbation ψ of the time-time component of the metric, $g_{tt} \simeq 1 + \psi$

(ii) A radial electric field $E(r)$, given by the radial gradient of the Coulomb potential

(iii) The perturbation ξ of the Kaluza scalar field, $\phi \simeq 1 + \xi$

We will conduct our analysis by investigating first integrals of the field equations. We will apply the field equations at planetary and atomic scales. Our application to atomic scales will follow the prescription for classical predictions which do not rely on atomic structure, and which are well behaved when an artifical size parameter goes to zero [10]. We examine volume integrals of the sources and relate our findings back to Newton's law of gravity and Coulomb's law.

Let us first establish a general result which will be of use throughout these calculations. We write the Ricci tensor suggestively in terms of a divergence:

$$\begin{aligned} -R_{\mu\nu} \equiv -R^{\alpha}_{\mu\alpha\nu} &= \partial_\nu \Gamma^{\alpha}_{\mu\alpha} - \partial_\alpha \Gamma^{\alpha}_{\mu\nu} + \Gamma^{\beta}_{\mu\alpha}\Gamma^{\alpha}_{\beta\nu} - \Gamma^{\beta}_{\mu\nu}\Gamma^{\alpha}_{\alpha\beta} \\ &= -g^{-1/2}\partial_\alpha\left(g^{1/2}\Gamma^{\alpha}_{\mu\nu}\right) + \partial_\mu\partial_\nu \ln g^{1/2} + \Gamma^{\beta}_{\mu\alpha}\Gamma^{\alpha}_{\beta\nu}, \end{aligned} \tag{23}$$

where the second equality follows from a common identity for $\Gamma^{\alpha}_{\alpha\beta}$ in terms of the determinant g of the metric. The first term is then the covariant divergence of $\Gamma^{\alpha}_{\mu\nu}$, as if it were a set of ten 4-vectors, each of index $\mu\nu$.

5.1. Scalar Field of a Planet. Now we consider the simplest case of an electrically-neutral body of mass M, anticipated to be of planetary magnitude. This corresponds to the case $\tilde{U}_5 = 0$. Electric charge and electric fields vanish, so the problem reduces to two unknowns: the metric perturbation, ψ, and the scalar field perturbation, ξ. In this case, the mass (18) reduces to the rest mass M, and the electric and scalar charges (19) and (20) are zero.

We make a typical linearized expansion about the Minkowski metric, but in spherical coordinates. Recall that the diagonal components of the Minkowski metric $\eta_{\mu\nu}$ are $(c^2, -1, -r^2, -r^2\sin^2\theta)$, so that in this linearization, $\eta_{\mu\nu}$ is not constant.

As usual, the time-time component of the metric, $g_{tt} \simeq 1 + \psi$, where $\psi \ll 1$. The other components of the metric will also have perturbations, and we choose to work in standard form of the static, isotropic metric. In this form, there is no perturbation to the angular components of the Minkowski metric; the perturbations are only to the components η_{tt} and η_{rr} [25]. Without solving the field equations for the rr perturbation, we can safely assume it will be of the same order of magnitude as ψ. Therefore, we can write the perturbed determinant of the metric:

$$g^{1/2} = r^2 \sin\theta(1 + \psi) + \mathcal{O}(\psi^2). \tag{24}$$

Let us start with the scalar field equation (7) under the foregoing assumptions. The Kaluza scalar field has the peculiar quality of acting to neutralize the scalar curvature R in spacetime arising from the presence of neutral matter. Recall that we are linearizing such that $\phi \simeq 1 + \xi$:

$$\nabla^2\xi = \frac{1}{g^{1/2}}\partial_i\left(g^{1/2}\partial^i\xi\right) = -\frac{8\pi G}{3c^2}\frac{\rho}{g^{1/2}} + \mathcal{O}\left(\xi^2\right). \tag{25}$$

Now, we are in a position to integrate (25) for the scalar perturbation ξ. As is usual in a linear treatment, the metric and scalar potential are only carried to zeroth order in the source terms [27]. Therefore, in this calculation, we can define the total rest mass M in these coordinates simply as

$$M \equiv \int \rho d^3 x. \tag{26}$$

Using the definition (26), let us now integrate (25) over all space and apply the Stokes theorem to obtain

$$\left.\frac{\partial \xi}{\partial r}\right|_{\text{planet}} = -\frac{2}{3}\frac{GM}{r^2 c^2}. \tag{27}$$

Here is the interesting result that the Kaluza scalar field perturbation is the same size as the metric perturbation. As Dicke [20, 21] noted, the strength of the scalar field is as *weak* as gravity, in that $\xi \ll 1$ like $\psi \ll 1$. For the purposes of a bouyancy concept, we realize that the scalar potential is as *large* as the gravitational potential. We shall see if the forces are as well.

The magnitude of ξ in (27) is consistent with the assumption (12). It is perhaps counter-intuitive that the dimensionless Newtonian gravitational potential $\psi_N \sim GM_{\oplus}/R_{\oplus}c^2$ at the surface of the earth is $\sim 10^{-10}$, a vanishingly small number. Yet, its gradients create forces of engineering significance because of a coupling into mass energy Mc^2, which is a large number. We are contemplating something similar for the Kaluza scalar field.

Equation (27) is a main result of this work, but is familiar from Brans and Dicke (BD) [16] for $\omega = 0$. The correspondence is because the BD theory has essentially the same equation for the scalar field, (7), based on their observation that the simplest covariant equation for the BD scalar field involves the D'Alembertian equated to the trace of the matter energy-momentum tensor.

Now let us consider the gravitational field equations (2). The Kaluza scalar field in the absence of electric sources acts to maintain the scalar curvature R at zero, per (7). Then, we can write the gravitational field equation for G_{tt} in (2), under the listed constraints:

$$G_{tt} = R_{tt} = \nabla^2 \xi + \frac{8\pi G}{c^2}\rho + \mathcal{O}\left(\psi^2\right) = \frac{16\pi G}{3c^2}\rho + \mathcal{O}\left(\psi^2\right) + \mathcal{O}\left(\xi^2\right), \tag{28}$$

where we used the scalar field equation (25) in the last step and where we are now carrying terms linear in both ψ and ξ.

Note that only the components Γ^{μ}_{tt} enter Newtonian dynamics. Therefore from (23), we need only to evaluate R_{tt} to get an equation for ψ. For R_{tt}, the last two terms on the RHS of (23) vanish. The term in $g^{1/2}$ vanishes with the time derivatives. We assert, without showing here, that the term quadratic in the connections is of second order in the metric perturbations, $\Gamma^{\beta}_{t\alpha}\Gamma^{\alpha}_{\beta t} = 0 + \mathcal{O}(\psi^2)$. Therefore,

$$R_{tt} = g^{-1/2}\partial_i\left(g^{1/2}\Gamma^{i}_{tt}\right) + \mathcal{O}\left(\psi^2\right). \tag{29}$$

It is clear, then, that a spherical volume integral of R_{tt} will pick off only the radial component Γ^{r}_{tt} of Γ^{i}_{tt}. Therefore,

$$\int R_{tt} g^{1/2} d^3 x \simeq \int \partial_i\left(g^{1/2}\Gamma^{i}_{tt}\right) d^3 x = \oint \Gamma^{r}_{tt} r^2 d\Omega. \tag{30}$$

We evaluate this component for the perturbed metric to find

$$\Gamma^{r}_{tt} = \frac{1}{2}\partial_r \psi + \mathcal{O}\left(\overline{\psi^2}\right). \tag{31}$$

The gravitational field equation (28) can now be readily integrated over all space, using (30), (31) and (26), to obtain

$$\left.\frac{\partial \psi}{\partial r}\right|_{planet} = \frac{8}{3}\frac{GM}{r^2 c^2}. \tag{32}$$

Let us now compare key features of the two potentials, ψ (32) and ξ (27). One is that the magnitude of the scalar potential is 1/4 the gravitational potential. Another is that the signs are opposite, with $\xi > 0$, while $\psi < 0$, as usual. This means that the scalar potential associated with the mass M is repulsive, and this is the origin of the electrogravitic bouyancy concept derived below.

A third feature is that the expression (32) for the gravitational field in the presence of a scalar field differs from the usual Newtonian result, $\psi_N = 2GM/rc^2$. The difference between these two is that in the Kaluza picture, the effective gravitational mass-energy of a planet includes a contribution of mass-energy from the Kaluza scalar field bound to the mass. There is, therefore, a mass "clothed" by the scalar field, and which accounts for Keplerian dynamics, and a "bare" mass. Therefore, we can make the identification

$$(GM)_{clothed} = \frac{4}{3}(GM)_{bare}. \tag{33}$$

The Kaluza scalar field stores 1/3 the mass-energy as the matter it is bound to, and the total gravitating mass-energy is increased above the bare mass by this amount. We keep the factor G because a redefinition of M in these terms is indistinguishable from a redefinition of G.

In his analysis of long range scalar fields [20, 21], Dicke discussed how the scalar force would "masquerade" as gravity. He specifically meant that the action of the scalar force would be indistinguishable from gravity. The Kaluza scalar field masquerades here as gravity, although in a different sense than Dicke meant. Here, in terms of the potentials, the action of scalar field mass energy is indistinguishable from the action of material mass energy, insofar as Keplerian dynamics is determined by the g_{tt} component of the metric.

The similarity to Dicke's conception of a long-range scalar field breaks down when we consider the nature of matter coupling to that field.

5.2. Scalar Fields in the Lab. Now we consider the Kaluza long-range scalar field around a body of mass M and electric charge Q, anticipated to both be of magnitudes that are accessible in laboratory experiments.

This corresponds to the case $\tilde{U}_5 = Q/Mck \ll \xi^{-1}$. As in the neutral matter case, $cd\tau/ds \simeq 1$, and the mass (18) reduces to the rest mass M. There is now an electric charge Q and a scalar charge $S = Q^2/Mk^2$.

Let us start with the Maxwell equations, (5):

$$g^{-1/2}\partial_r\left(g^{1/2}\phi^3 E^r/c\right) = \mu_0 \rho ck\tilde{U}_5, \quad (34)$$

where E^r is the radial component of the electric field. Recall that $\phi^3 g^{1/2} = r^2 \sin\theta + \mathcal{O}(\psi) + \mathcal{O}(\xi)$. For the electric field equation, we will write explicit terms to zeroth order in the perturbations, but still keep track of the ordering.

Let us introduce the total charge integral Q, analogous to the mass integral in (26):

$$Q \equiv \int \rho ck\tilde{U}_5 d^3x. \quad (35)$$

Let us use this definition of the electric charge to integrate (34) over all space and use the Stokes theorem to obtain

$$E^r = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} + \mathcal{O}(\psi) + \mathcal{O}(\xi). \quad (36)$$

This is clearly the typical Coulomb expression. The perturbation in ψ is a normal Newtonian perturbation to the relativistic Maxwell equations.

A perturbation effect from the Kaluza scalar field would be new. Yet, because of the presumed small size of ξ, its effects are likely to be minor and masked as an effective dielectric constant, as in the Bekenstein theory of scalar-electromagnetic coupling [15]. Or they might be difficult to distinguish also from curvature effects in ψ, which are of the same magnitude.

Let us now consider the Kaluza scalar field perturbation described by (7) for this case:

$$\frac{3}{g^{1/2}}\partial_r\left(g^{1/2}\partial^r\xi\right) = \mu_0 k^2 c^2 \frac{\rho}{g^{1/2}}\frac{\tilde{U}_5^2}{\phi^2} - \frac{8\pi G}{c^2}\frac{\rho}{g^{1/2}} + \frac{3k^2}{2c^2}\phi^3 E^2 + \mathcal{O}\left(\xi^2\right). \quad (37)$$

We can use the zeroth order expression for the electric field from (36) in (37). We see that it is essentially the usual electrostatic mass, M_E, defined as the integral of the electrostatic energy density over all space:

$$M_E c^2 \equiv \int \frac{1}{2}\varepsilon_0 E^2 d^3x. \quad (38)$$

Now let us define the scalar charge integral S, as we did for mass (26) and electric charge (35).

$$S \equiv c^2 \int \rho\tilde{U}_5^2 d^3x. \quad (39)$$

Such an expression for the scalar charge is unique and has no analog in conventional physics. Its units are energy. It can be understood as the integral of the specific charge squared per unit rest mass of the source. For a uniform source, we can write

$$S = \frac{Q^2}{Mk^2} \quad (40)$$

consistent with (20). We also see that this implies the coupling coefficient for the scalar charge in (37) is $\mu_0 k^2 = 16\pi G/c^4$.

Now, we can use these expressions to integrate the Kaluza scalar field equation (37) over all space, to find

$$\frac{\partial\xi}{\partial r} = \frac{\mu_0}{12\pi}\frac{Q^2/M}{r^2} - \frac{2G}{3c^2}\frac{M}{r^2} + \frac{4G}{c^2}\frac{M_E}{r^2} + \mathcal{O}\left(\xi^2\right). \quad (41)$$

The expression (41) for the scalar field perturbation has some interesting features. One is that neutral matter sources have the opposite sign as charged matter and as electric fields. That is, electric charge and electric fields are attractive sources of the Kaluza scalar field, while neutral matter is a repulsive source of the scalar field, much like positive and negative electric charges can act as attractive or repulsive sources of the electric field.

Here also is a key result of this work, that a third electrogravitic length scale, $\mu_0 Q^2/M$, appears alongside the two length scales that characterize the Reissner-Nordstrom metric, GM/c^2 and $\left(Q^2 G/\varepsilon_0 c^4\right)^{1/2}$. A third electrogravitic length scale would seem to implicate new physics not anticipated in electrodynamics and general relativity.

The term in Q^2 in (41) will dominate laboratory scalar fields. A typical laboratory capacitance might be 10^{-10} farad, and typical lab voltages are 10^3 volts. Therefore, laboratory charges $Q_{lab} \sim 10^{-7}$ coulombs. For sizes of order 1 meter and masses of order 1 kilogram, the term in the scalar charge is of order 10^{-22}. This is much smaller than the planetary scalar potential, of order 10^{-10}, yet much larger than the mass term in (41), of order 10^{-27}. The term in the electrostatic mass M_E is even smaller. Therefore, we can approximate the laboratory Kaluza long-range scalar field of a charged, massive object:

$$\left.\frac{\partial\xi}{\partial r}\right|_{\text{lab}} \simeq \frac{\mu_0}{12\pi}\frac{Q^2/M}{r^2}. \quad (42)$$

The scalar potential (42) can be compared with the weak field solution of Chodos and Detweiler [7], which is similar quantitatively to findings reported here, but is the opposite sign because those authors choose the 5th coordinate signature to be spacelike. In that case, the scalar field behaves like gravity and is attractive. That is also true of Ferrari's sign choice [13].

Let us now consider the gravitational field equation (2), specifically the R_{tt} component. It has sources in matter, electromagnetic field, and scalar field energy.

$$R_{tt} - \frac{1}{2}\eta_{tt}R \simeq T^{\phi}_{tt} + \frac{8\pi G}{\mu_0 c^4}T^{EM}_{tt} + \frac{8\pi G}{c^2\phi}\frac{\rho}{g^{1/2}}. \quad (43)$$

The scalar field will act to maintain the scalar curvature R according to (7):

$$R = -\mu_0 k^2 c^2 \frac{\rho}{g^{1/2}\phi}\frac{\tilde{U}_5^2}{\phi^2} - \frac{3k^2}{2c^2}\phi^2 E^2. \quad (44)$$

The tt component of the scalar field energy-momentum tensor is

$$T^{\phi}_{tt} = \nabla^2\xi + \mathcal{O}\left(\xi^2\right) = \frac{\mu_0}{3}k^2c^2\frac{\rho}{g^{1/2}}\frac{\tilde{U}_5^2}{\phi^2} - \frac{8\pi G}{3c^2}\frac{\rho}{g^{1/2}} + \frac{k^2\phi^3E^2}{2c^2} + \mathcal{O}\left(\xi^2\right), \quad (45)$$

where the second equality follows from (37).

The term in (2) in the tt component of the electromagnetic energy-momentum tensor is

$$\frac{8\pi G}{\mu_0 c^4}\phi^2 T^{EM}_{tt} = \frac{8\pi G}{c^4}\phi^2\frac{1}{2}\varepsilon_0 E^2. \quad (46)$$

The field equation for ψ is obtained when we combine (2), (44), (45), (46), (29) and (31):

$$\frac{1}{g^{1/2}}\partial_r\left(g^{1/2}\partial_r\psi/2\right) = \frac{\rho}{g^{1/2}\phi}\left(\frac{16\pi G}{3c^2} - \frac{\mu_0}{6}k^2c^2\frac{\tilde{U}_5^2}{\phi^2}\right). \quad (47)$$

The terms in the electric field have canceled.

Now, integrate (47) over all space and use Stokes theorem with (26) and (39) to obtain

$$\left.\frac{\partial\psi}{\partial r}\right|_{lab} = \frac{8}{3}\frac{GM}{r^2c^2} - \frac{\mu_0}{12\pi}\frac{S}{r^2} \simeq -\frac{\mu_0}{12\pi}\frac{S}{r^2}. \quad (48)$$

The mass expression in (48) is seen in the neutral matter case, (32), but here, the M is understood to be of laboratory dimensions. As discussed for (41), the term in M is of order 10^{-27} and negligible in the laboratory, as we expect for laboratory gravitational effects. Since the term in S is larger, it seems to imply the scalar charge can swamp the matter charge in the sourcing of the gravitational field. Yet, these considerations are for laboratory scale only. It still appears no laboratory charge could outweigh the Kaluza scalar field of a planet.

The scalar charge acts negatively as a source of gravitational field in (48). The terms in M and S are also in the expression for ξ, (41). In that expression, matter creates a repulsive potential as we saw previously, and the scalar charge is an attractive potential. Therefore, the scalar charge and mass act oppositely as sources of the gravitational and Kaluza scalar fields. A scalar charge source creates equal and opposite perturbations of the metric and of the scalar field, as seen by comparing (48) and (42).

Note that (41) and (48) sum to

$$\frac{\partial\psi}{\partial r} + \frac{\partial\xi}{\partial r} \simeq \frac{2GM}{r^2c^2} + \mathcal{O}\left(\xi^2\right) + \mathcal{O}\left(\psi^2\right) \quad (49)$$

which is the ordinary Newtonian potential in terms of the bare mass.

5.3. Atomic Scalar Fields. We have seen that planetary $\xi \sim 1$ 0^{-10}, consistent with the perturbation expansion (12). Laboratory-generated $\xi \sim 10^{-22}$, much smaller still. Achievable laboratory values of $\tilde{U}_5 \sim 10^3$, as defined in (19). The terms in $\xi\tilde{U}_5^2$ in (18) and (20) are therefore negligible for all values of $\tilde{U}_5 < 10^5$. This means that the saturated limits (22) and (21) of (18) and (20) are achieved only for very high charge-to-mass ratios. Such ratios are only found in atomic systems: the electron $\tilde{U}_5|_e \sim 10^{21}$ and the proton $\tilde{U}_5|_p \sim 10^{18}$. It appears the saturated limits (21) and (22) are appropriate for atomic systems.

Following Rohrlich, we direct the classical theory at elementary charged particles only insofar as we can make predictions that are independent of particle structure, since that is outside the domain of validity of the classical theory. Yet, the classical theory can be used to address atomic systems in a structure-independent way. In practical terms, it means assuming a structure of characteristic size r_0 and then letting $r_0 \longrightarrow 0$. If the resultant quantity is finite and well behaved, it is a valid calculation. If the result is infinite, then it indicates a failure in the application of the theory. One example of such failure, as pointed out by Rohrlich, is the model of a point particle. If the classical model is a charged sphere, then the electric field energy goes to infinity as the sphere size goes to zero, and therefore, the point particle is not a valid classical model of a charged particle.

In the limit that $\tilde{U}_5^2 \gg \xi^{-1}$, mass (18) $\longrightarrow M\tilde{U}_5\sqrt{2\xi}$, (22), and scalar charge (20) $\longrightarrow Mc^2\tilde{U}_5/\sqrt{2\xi}$, (21). Now there is a formal convergence in that the mass, electric charge, and scalar charge are all proportional to $\tilde{U}_5$ and to the electric charge. The 3 forces masquerade as the Coulomb electric force at ultra-high specific charge states. We will calculate ξ and then double check that our assumptions are satisfied, and the saturation limit is correct.

The Coulomb electric force in this limit is the same as the previous case, given by (36) to zeroth order in the perturbations. There is no saturation effect present in the Maxwell equations (5) or in the electric charge (19).

Consider now the Kaluza long-range scalar field equation (7) once more, similar to (37), but with saturated charges:

$$\frac{3}{g^{1/2}}\partial_r\left(g^{1/2}\partial^r\xi\right) = \mu_0 k^2c^2\frac{\rho}{g^{1/2}}\frac{\tilde{U}_5^2}{\sqrt{2\xi}} - \frac{8\pi G}{c^2}\frac{\rho}{g^{1/2}}\sqrt{2\xi} + \frac{3k^2}{2c^2}\phi^3E^2 + \mathcal{O}\left(\xi^2\right). \quad (50)$$

The second term on the RHS is of first order in ξ relative to the first term, and so can be ignored, consistent with our approximation of the sources of perturbations to only zeroth order in those perturbations [27].

Now let us integrate (50) from a minimum radius r_0, which we nominally take to be 10^{-10} meters. The first term on the RHS of (50) has a dependence on ξ, which we take to be the value of ξ evaluated at the particle. We assume no structure inside r_0 and so take ξ_0 to be the constant value of ξ inside r_0. It therefore forms a boundary value on the source term in the integral. Using (35) and (36), we find

$$\frac{\partial\xi}{\partial r} \simeq \frac{\mu_0}{12\pi}\frac{kc}{\sqrt{2\xi_0}}\frac{Q}{r^2} + \frac{\mu_0}{2\pi}\frac{G}{c^2 r_0}\frac{Q^2}{r^2} \simeq \frac{\mu_0}{12\pi}\frac{kc}{\sqrt{2\xi_0}}\frac{Q}{r^2}. \tag{51}$$

We find that the term in Q^2 is of order 10^{-20} smaller than the term linear in Q at $r_0 \sim 10^{-10}$ m, and so we ignore it. Eventually, the Coulomb term diverges as r_0 goes to zero, but that is not particular to the Kaluza theory.

Now solve for ξ_0 by evaluating (51) at r_0 to find

$$\xi_0 = \left(\frac{\mu_0 Qkc}{12\sqrt{2}\pi r_0}\right)^{2/3}. \tag{52}$$

For typical atomic parameters, $\xi_0 \sim 10^{-17}$, still $\ll 1$ in the high charge states of atomic systems. Therefore, the atomic value of the Kaluza scalar field from an elementary particle is given by

$$\left.\frac{\partial\xi}{\partial r}\right|_{\text{atomic}} \simeq \left(\frac{\mu_0 Qkc}{12\sqrt{2}\pi}\right)^{2/3}\frac{r_0^{1/3}}{r^2}. \tag{53}$$

Note this scalar potential behaves gravitationally in that like charges attract, but it appears opposite charges repel, even as it is apparently proportional to electric charge in this limit.

We see from this analysis that, as $r_0 \longrightarrow 0$, $\xi_{\text{atomic}} \longrightarrow 0$. Therefore, the theory seems to imply a structure-independent stability of the Kaluza scalar field for point particles. This is because the scalar field enters in the denominator of the source term in (50). As the scalar field strength increases, it reduces the magnitude of its own source.

Now let us consider the gravitational field equation for ψ and the saturated mass (22). We consider the time-time component of the gravitational field equations (2), the time-time component of the scalar field energy-momentum tensor (3) given by (7), and the time-time component of the electromagnetic energy-momentum given by (46), to obtain an equation very similar to the low-charge equation for ψ (47):

$$\frac{1}{g^{1/2}}\partial_r\left(g^{1/2}\partial_r\psi/2\right) = \frac{\rho}{g^{1/2}\phi}\left(\frac{16\pi G}{3c^2}\tilde{U}_5\sqrt{2\xi} - \frac{\mu_0}{6}\frac{k^2c^2}{\sqrt{2\xi}}\frac{\tilde{U}_5}{\phi^2}\right), \tag{54}$$

where the electric terms again have cancelled. The term in G from the usual matter source is order ξ smaller than the term in μ_0, the scalar source term in ψ. Therefore, we can drop the term in G to this approximation and keep only the term in μ_0. Then, the gravitational potential at atomic scale is given by

$$\left.\frac{\partial\psi}{\partial r}\right|_{\text{atomic}} \simeq -\frac{\mu_0}{12\pi}\frac{Q}{r^2}\frac{kc}{\sqrt{2\xi_0}}. \tag{55}$$

Once more, we have set ξ_0 to be the value of ξ at the boundary of the source r_0, and then a limit is taken as $r_0 \longrightarrow 0$. We see that the gravitational field is actually repulsive in this limit and opposite to the scalar potential, which is attractive. However, the scaling of the two potentials ξ and ψ is the same in the saturated limit, and both obey (53). It appears that the saturated gravitational potential goes to zero as $r_0 \longrightarrow 0$, along with the saturated scalar potential.

Having completed an evaluation of the scalar, electric, and gravitational fields for charged and neutral sources, let us turn to the implications for those fields on the motion of test bodies.

6. Scalar Forces

In this section, we combine the previous expressions for charges and potentials to obtain the forces between massive, charged objects. As done for the potentials, we consider the 3 cases of neutral, weakly charged, and strongly charged systems. A unique electrogravitic, buoyant force is discovered.

6.1. Lift in the Planetary Scalar Field. Consider now the motion of an electrically-charged test particle of mass m and charge Q, where $\tilde{U}_5^{\,2} \ll \xi^{-1}$, obeying the equation of motion (15), moving in the planetary, neutral-matter potentials described in (27) and (32). The particle will of course couple to the gravitational field irrespective of its charge or mass, according to the equivalence principle, and there is no ambient electric field to couple with. The scalar charge (20) is given by (40), but now with the rest mass as m. The mass (18) is just m.

Consider then the radial component of (15):

$$m\frac{dU^r}{dt} + \frac{4}{3}\frac{GMm}{r^2} = \frac{(Q/m)^2}{16\pi G\varepsilon_0}\frac{2}{3}\frac{GMm}{r^2} + \mathcal{O}\left(\frac{U^2}{c^2}\right) + \mathcal{O}\left(\psi^2\right) + \mathcal{O}\left(\xi^2\right). \tag{56}$$

The quantity M in the preceding analysis is understood to be the bare mass of the planet. Let us convert to the clothed mass M_{cl} from (33), corresponding to the mass measured in gravitational experiments, and now ignore the second-order terms:

$$m\frac{dU^r}{dt} + \frac{GmM_{cl}}{r^2} \simeq \frac{(Q/m)^2}{16\pi G\varepsilon_0}\frac{GmM_{cl}}{2r^2}. \tag{57}$$

The scalar force in (57) acts counter to the Newtonian gravity and in proportion to the gravitational weight of the test body. Yet, its coupling is quasi-electric. Therefore, we consider this an “electrogravitic” bouyancy effect. Against the gravitational weight, the electric charge appears as an opposing weight. In fact, we can define an electrogravitic mass, whose upward force is equal to the weight of the mass:

$$m_{\mathrm{EG}} = \frac{Q^2/m}{32\pi G\varepsilon_0}. \tag{58}$$

The electrogravitic constant (6) sets a charge-to-mass scale of 10^{-10} C/kg. In laboratory settings, charges are conveniently expressed in terms of capacitance C and electric potential V, $Q = CV$. Typical capacitances for meter-sized objects are 10^{-10} farad. An object of mass 10 kg, charged to 1000 V, would feel a scalar force of 10^4 times its weight.

This scalar force is quite different than the one contemplated by Dicke, because the Kaluza scalar coupling is electrostatic. The field equations for the scalar field are the same, but the scalar charge is different. Indeed, the Brans-Dicke scalar field is explicitly barred from the force equations and enters the field equations only. Therefore, the Kaluza scalar force would not masquerade as gravity as Dicke anticipated. Dicke viewed the scalar field as attractive to mass, like gravity and coupling to mass, like gravity. The electric coupling of the Kaluza theory leads to vastly larger scalar forces than Dicke anticipated based on his field equations, and opposite in direction.

6.2. Forces between Neutral Masses. Now let us consider the forces between 2 neutral massive bodies of bare rest mass M_1 and M_2, separated by a distance r. Their electric charge is zero, so there is no electric interaction. However, there are both gravitational and scalar forces.

Except for the effect of the clothed mass discussed above, the gravitational force between massive bodies abides the conventional Newtonian limit. The gravitational potential ψ_M of a bare mass M is given by (32)

$$\psi_M = -\frac{8}{3}\frac{GM}{rc^2} = -\frac{2GM_{cl}}{rc^2}. \tag{59}$$

The mass (18) reduces to the bare rest mass M. Therefore, the gravitational force between the two bodies is

$$F_g = \frac{4}{3}\frac{GM_1M_2}{r^2} = \frac{GM_{1cl}M_2}{r^2}. \tag{60}$$

This illustrates that the potential depends on the clothed mass, but the coupling depends on the bare mass.

The scalar potential of bare mass M is given by (27):

$$\xi_M = \frac{2}{3}\frac{GM}{rc^2}. \tag{61}$$

In this case, the scalar charge (20) is zero, because $\tilde{U}_5 = 0$. Therefore, the scalar force between neutral masses is zero.

The Kaluza scalar field contributes indirectly to the gravitational interaction through its own energy density, but no scalar force manifests between neutral masses. This is similar to the Brans-Dicke theory, except they posit from the outset that the scalar field can have no effect on the motion of material bodies.

6.3. Forces between Weak Charges. Consider now two charges, Q_1 and Q_2, of mass M_1 and M_2, and such that $\tilde{U}_5^2 \ll \xi^{-1}$ for both. They will experience mutual gravitational, electric, and scalar forces.

The scalar potential generated by Q_1 is given by (42)

$$\left.\frac{\partial\xi}{\partial r}\right|_1 \simeq \frac{\mu_0}{12\pi}\frac{Q_1^2/M_1}{r^2}. \tag{62}$$

Meanwhile, the scalar charge of Q_2 is given by (40), Q_2^2/M_2k^2. Therefore, the scalar force between Q_1 and Q_2 is

$$F_S^r = \frac{-1}{16\pi G\varepsilon_0^2}\frac{Q_1^2}{M_1}\frac{Q_2^2}{M_2}\frac{1}{r^2}. \tag{63}$$

This means that the scalar force is attractive between charges, irrespective of sign. In this way, it is like gravity. Let us compare it to the usual Coulomb force:

$$F_E^r = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r^2}. \tag{64}$$

The ratio of the two forces is seen to be

$$\frac{F_S^r}{F_E^r} = \frac{1}{3}\frac{(Q_1/M_1)(Q_2/M_2)}{16\pi G\varepsilon_0}. \tag{65}$$

For achievable laboratory charge-to-mass ratios of order 10^{-7} C/kg, the ratio of forces can be of order 10^5, which is extremely large. Here is a verifiable difference from Coulomb's law, in that the additional scalar force can be large in laboratory environments.

Ferrari [13] also examined deviations from Coulomb's law in the Kaluza picture. He considered time-independent scalar and electric forces between charged objects, and found a significant variation from the Lorentz force law, as we are finding in (65). However, Ferrari made significantly different assumptions than used here, and his results may have been compromised by them.

Ferrari obtained his solution by assuming all fields could be expanded in powers of the two length scales he identified in the system: GM/c^2 and $Q\sqrt{G/4\pi\varepsilon_0c^4}$. Yet, the solution seen in (42) indicates there is a third length scale, μ_0Q^2/M. Ferrari also seems to miss the scalar field contribution to gravitational mass seen in (33). Finally, Ferrari notes some discrepancies between his results and conventional limits. Therefore, while the general prediction of deviations from the Coulomb force between charges due to a scalar interaction was predicted by Ferrari, the magnitudes and mathematical scaling of these results appear to be different.

The general result of scalar-induced deviations from Coulomb's law seen by various researchers seems to invite an experimental investigation. The experimentalist should note this is not a deviation in the $1/r^2$ geometric part of Coulomb's law, but a deviation in the magnitude of the force.

6.4. Forces between Strong Charges. We previously recognized that only atomic particles satisfy the requirement for strong charges, $\tilde{U}_5 \gg \xi^{-1}$. Let us then consider bodies of charge Q_1 and Q_2 and masses M_1 and M_2.

For atomic sources of mass and electric charge, the electric field and electric force have their usual form as in (36), and no modifications need be discussed.

The saturated gravitational (55) and scalar (51) potentials are equal and opposite. The saturated gravitational charge is a factor ξ smaller than the saturated scalar charge. Therefore, the attraction between charges is approximately the scalar attraction:

$$F^r_{S+} \propto \frac{Q_1 Q_2}{\varepsilon_0 r^2} \frac{1}{\sqrt{\xi_1 \xi_2}}. \quad (66)$$

In the saturated regime characteristic of atomic systems, the scalar force masquerades as the electric force, similar to the way Dicke anticipated it would masquerade as the gravitational force. Yet, as shown in (53), the scalar potential would appear to go to zero as the source goes to a point particle, so that the scalar interaction is strongly suppressed for atomic systems.

As we pass from outer atomic scales of 10^{-10} m and approach point particles, the electrostatic energy must be considered. Since the Kaluza scalar field is attractive for like charges, akin to gravity, then it might provide a stabilizing influence in the energy budget of the point particle, as shown already by ref. [22] for Coulomb electric fields.

For the purposes of a classical theory, $F^r_{S+} \longrightarrow 0$ as $r_0 \longrightarrow 0$. The vanishing of the scalar force at atomic scales is accompanied by a masquerading of the electric force in that regime, in that the scalar charge could become linear in electric charge at certain high specific charge states.

7. Tuning the Scalar Coupling to Zero

We have encountered forces from the Kaluza scalar field that are apparently large, according to the mathematics, yet the field equations otherwise reproduces 4D physics. It seems unlikely that effects of the magnitude described here will be validated in the laboratory, for they should have already been discovered by now, if they exist. The reason is the large relative size of the scalar charge compared to the gravitational and electromagnetic charge in the force equation (15).

We might therefore ask whether we can somehow tune the theory to zero out the scalar interaction. We have seen that the atomic scalar interaction appears to vanish by virtue of the preceding analysis, leaving only macroscopic and laboratory-scale effects. To zero out the scalar force, either the field must be zero or its coupling to charge must be zero.

It is very difficult to hide the effects of the Kaluza long-range scalar field in the field equations. A conformal transformation to the Einstein frame would eliminate explicit force terms from the equations of motion, but particle geodesics in the Einstein frame still reflect the scalar influence, compared to its absence in the Jordan frame. And we are already investigating tiny perturbations of the scalar field around 1. Also, the early arguments by Dicke, and the results here, show that the Kaluza scalar field can masquerade as gravity in the absence of other couplings. Therefore, we consider it more likely that the scalar charge is somehow zeroed out in a way our analysis has not grasped so far.

The only free parameters in the Kaluza picture are in the invariant length element (9), $\tilde{a}$, and the choices of the 5D signature. The fundamental relation is given by (11), and we assigned the value $\tilde{a}^2 = 1 + \tilde{U}_5^2$ as the only natural choice, given that 4D physics is recovered in the limit that $\phi \longrightarrow 1$, and given that we desire the scalar mass function (18) $c\,d\tau/ds$ to be positive.

Chodos and Detweiler [7] also discuss tuning the scalar interaction to zero by an appropriate choice of $\tilde{a}^2$, so that the difference between the matter terms in (7) goes to zero. They find no good explanation for why that should be so, nor do we. But if that does account for the observed absence of the scalar interaction, the Kaluza scalar field should still clothe planetary masses.

Instead of choosing (13), we might instead set $\tilde{a}^2$ in (11) such that $\tilde{U}_5^2/\phi^2 \sim 0$ in (11), but this is tantamount to setting $\tilde{U}_5 = 0$, if a 4D limit is to be obtained in (1) as $\phi \longrightarrow 1$. And then correspondence with the Lorentz force law is lost. In other words, it is difficult to tune $\tilde{a}$ to make the scalar charge (20) go to zero without also driving the electric charge (19) to zero. If such large force effects are not validated, this may be the first testable classical falsification of the 5D hypothesis that electromagnetism and gravity are aspects of a 5D metric.

We might also consider the signature in the metric of the fifth coordinate, ε_ϕ in (9). If we flip the sign on ϕ^2, then we might be able to convert the planetary scalar field to a negative value, like gravity, and the electrogravitic force would be attractive. Yet, it would not change the magnitude of the effect, and such an effect should still be seen if it exists.

8. Energy Considerations

Let us consider a feature of the Kaluza scalar field that was anticipated by Dicke: the action of the scalar field on a body produces acceleration at constant energy. The energy is also constant for the Newtonian approximation to gravity. Although the Newtonian gravitational field can accelerate test particles, terms quadratic in speed are ignored, and the energy including rest energy is essentially constant by approximation.

Energy under action of the Kaluza scalar field is constant for a more fundamental reason. The equation of motion (15) shows that if $\partial_t \phi = 0$, then $dU^t/d\tau = 0$ from the scalar field, and therefore, the energy is constant. More generally, the scalar field gradient must be orthogonal to the particle 4-velocity. However, the scalar field is conservative like the gravitational field. Dicke anticipated the energy gained by acceleration under the scalar field would be offset by a loss of rest mass energy from interaction with the scalar field. Here, we find the energy of acceleration is offset by a loss of scalar field potential energy.

Table 1: Limiting values of scalar, electric, and gravitational potentials and charges, for bodies of electric charge Q and bare mass M; for static fields, $\psi \ll 1$, $\xi \ll 1$, nonrelativistic matter and spherical symmetry. The clothed mass is $4M/3$. Only leading terms in potentials are shown. The term in electric field energy, $M_E c^2$, is large for $\tilde{U}_5^2 \gg \xi^{-1}$, but is omitted for clarity. $\xi_0 \equiv \xi(r_0)$, $\xi \longrightarrow 0$ as $r_0 \longrightarrow 0$.

Charge state	Scalar		Electric		Gravitational	
(neutral, lab, atomic)	Charge	Potential, ξ	Charge	Potential	Charge	Potential, ψ
$Q = 0$	0	$\frac{2}{3}\frac{GM}{rc^2}$	0	0	Mc^2	$-\frac{8}{3}\frac{GM}{rc^2}$
$\frac{Q^2/M^2}{G\varepsilon_0} \ll \xi^{-1}$	$\frac{c^2 Q^2/M}{16\pi G\varepsilon_0}$	$-\frac{\mu_0}{12\pi}\frac{Q^2}{Mr}$	Q	$\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$	Mc^2	$\frac{\mu_0}{12\pi}\frac{Q^2}{Mr}$
$\frac{Q^2/M^2}{G\varepsilon_0} \gg \xi^{-1}$	$\frac{c^2 Q/\sqrt{2\xi}}{\sqrt{16\pi G\varepsilon_0}}$	$-\frac{\mu_0}{12\pi}\frac{Q}{r}\sqrt{\frac{8\pi G\varepsilon_0}{\xi_0}}$	Q	$\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$	$\frac{c^2 Q\sqrt{2\xi}}{\sqrt{16\pi G\varepsilon_0}}$	$\frac{\mu_0}{12\pi}\frac{Q}{r}\sqrt{\frac{8\pi G\varepsilon_0}{\xi_0}}$

Let us consider the 5D energy for the time-independent case. Then, the covariant time component $\tilde{U}_t$ is constant and given by

$$\begin{aligned}\tilde{U}_t &= \text{constant} = g_{tt}\tilde{U}^t + kA_t\tilde{U}_5 + g_{ti}\tilde{U}^i \\ &= g_{tt}\frac{cd\tau}{ds}\frac{cdt}{d\tau}, \simeq \left(1 - \frac{GM_{cl}}{rc^2} + \frac{GM_{cl}}{rc^2}\frac{m_{EG}}{m}\right) \\ &\quad + \mathcal{O}\left(\psi^2\right) + \mathcal{O}\left(\xi^2\right),\end{aligned} \tag{67}$$

where we are using the clothed mass M_{cl} of the planet, and where we used (14), (32), (33), and (58), with $cdt/d\tau \simeq 1$ as usual.

The Kaluza scalar field potential is an additional term in the test particle energy budget, along with rest energy and gravitational potential energy:

$$\text{total energy} \longrightarrow mc^2 - \frac{GM_{cl}m}{r} + \frac{Gm_{EG}M_{cl}}{r}. \tag{68}$$

An ADM-like analysis [22] of the Kaluza scalar field potential energy might be suggested, to investigate any stabilizing effect on a point particle, such as that found for the electric field. However, we note that (68) only applies to macroscopic bodies. Behavior at strong charge states characteristic of elementary particles was considered above, and these are the charge states that would be subjected to an ADM-like analysis.

Another feature of the scalar field anticipated by Dicke is that any interaction with a scalar field must result in a variation in rest mass. This point was further investigated by ref. [26]. They found that conformal transformations, interactions with a scalar, can be interpreted as "apparent" gravitational fields because the rest mass includes the potential energy in such a field, just as given in (67). The rest mass of a particle is relative, then. Yet, a special value can be singled out in the same way we single out the rest mass associated with the Minkowski metric.

9. Conclusions

The tensor gravitational potential $g_{\mu\nu}$ and the vector electromagnetic potential A_μ behave mathematically as if they are components of a 5D gravitational potential $\tilde{g}_{ab}$, but that implies the existence of a third field, a scalar potential ϕ in 4D. The Kaluza scalar field is a long-range scalar field, presumably associated with a spin 0 massless boson, as the gravitational field is presumably associated with a spin 2 massless graviton. The gravitational, electric, and scalar charges and potentials for the 3 cases of neutral, weakly-charged, and strongly-charged, sources are tabulated for convenience in Table 1.

The absence of a detectable 5th dimension is enforced as a boundary condition on the fields such that derivatives $\partial_5 \tilde{g}_{ab} = 0$. Far from being an unnatural or ad hoc simplification, this reveals a nontrivial constant of the motion corresponding to electric charge. With this identification, the geodesic equation in 5D provides the 4D gravitational and electromagnetic forces, augmented with a scalar force that couples to scalar charge. In new results here, we find the scalar charge of a body of electric charge Q and mass M is proportional to Q^2/M. That the scalar charge can be expressed in terms of the gravitational charge (mass) and electric charge is because the 5D length element of a particle is a constant of the motion.

The expression (42) for the scalar potential when $\tilde{U}_5 \ll \xi^{-1}$ can be compared with the monopole solution found by Ferrari [13]. Ferrari found a clever solution by assuming that there were only 2 length scales characterizing an electrically-charged mass: GM/c^2 and $(Q^2 G/4\pi\varepsilon_0 c^4)^{1/2}$. Yet, the solution seen in (42) indicates there is a third length scale, $\mu_0 Q^2/M$, that is not anticipated by the other two length scales from gravity or electric forces. The third length scale is an electrogravitic length scale characteristic of the scalar charge (20). Therefore, the solution by Ferrari, while predicting a significant force from the scalar interaction, does not capture the unique scaling of the scalar interaction. Nor was this length scale discerned by Chodos and Detweiler [7] in their static monopole solution.

When these considerations are applied to massive charged and neutral bodies, for the simplified cases of static, spherically-symmetric fields and nonrelativistic sources, it is found that neutral mass is clothed in the Kaluza scalar field. The mass of planets determined from Kepler's laws, and the component g_{tt} of the metric, is a clothed mass that includes contributions from the bare mass and its associated scalar field. The magnitude of this scalar field is the same as calculated in the Brans-Dicke theory for $\omega = 0$, but the nature of the Kaluza scalar field is much different, in both its couplings and its field equation.

The force equations imply that electrically-charged objects immersed in the Kaluza scalar field of planets should experience an electrogravitic lift that is effectively a buoyancy force. Dicke noted that long-range scalar fields have the peculiar property of providing acceleration at constant energy. He anticipated that the energy would come from variation of rest mass. We find acceleration at constant energy here, except the variation in rest mass can be understood as a variation of potential energy in the scalar field, as described by Rohrlich and Witten. The upward momentum provided by the buoyant lifting force is compensated by the ambient scalar field and by recoil of the field that ultimately couples to recoil of the earth.

A saturation effect exists in the gravitational and electric charges at the high specific charge states characteristic of elementary particles. We find that this saturation effect alters the gravitational and scalar charges such that they go over to dependence on electric charge, possibly masquerading as the electric force in such regimes, before going to zero in the limit of point particles.

Some of the scalar forces predicted for laboratory charges are quite large and should have been seen already if they exist. Yet, it is difficult to tune the theory to drive the scalar forces to zero, without compromising identification with standard physics in other areas. This should therefore be considered the first testable classical falsification or verification of the hypothesis of five-dimensional general relativity.

Yet, there are reasons to expect a long-range scalar field in nature, and such fields can go undetected or masquerade as gravity. The Kaluza scalar field is appropriately weak, and its cosmological prediction of the gravitational constant variation seems to accord with measurement. Therefore, we might consider that some part of our analysis in the coupling of matter to scalar fields is in error. The absence of an electrogravitic buoyancy effect would not falsify the existence of a scalar field, but perhaps only our treatment here of the coupling to it.

Data Availability

This is a theoretical mathematical paper that derives mathematical results from a set of assumed equations. Therefore, there is no data analysis associated with this article.

Conflicts of Interest

The author declares no conflicts of interest.

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