INPOP planetary ephemerides: Recent results in testing gravity

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In this paper, are given numerical estimations of the sensitivity of the latest version of the INPOP planetary ephemerides (INPOP13c) to GR parameters: the PPN parameters β, γ, the flattening of the sun $J_2^{\odot}$ but also to time variations of the gravitational mass of the sun μ. A first estimation is obtained by fitting these parameters with the classic method of least squares to planetary observations together with other parameters used for planetary ephemeris construction. A second approach is investigated by a new method of construction of alternative ephemerides based on the same dynamical modeling and observational samples but in a non-GR framework by considering non-zero or non-unity GR parameters. Some alternative ephemerides are found to be close to INPOP13c and acceptable intervals of GR parameters are then defined at the light of the present INPOP13c accuracy. These intervals are compared with the one obtained with the direct least square estimations and with those extracted from the literature. Based on these results and comparisons, no violation of GR is at this point noticeable.

1 INPOP13c

1.1 Description

By the use of the tracking data of the MESSENGER mission, INPOP13a becomes an interesting tool for testing GR close to the Sun [1]. The MESSENGER mission was indeed the first mission dedicated to the study of Mercury. The spacecraft orbits the smallest and closest to the sun planet of the solar system since 2011. In [1] are described the methods and procedures used for the analysis of the MESSENGER Doppler and range data included in the construction of the Mercury improved ephemerides, INPOP13a as well as the determination of acceptable intervals of non-unity values for the PPN β and γ.

INPOP13c [5] is an upgraded version of INPOP13a, fitted to LLR observations, and including new observables of Mars and Venus deduced from MEX, Mars Odyssey and VEX tracking data[2,3,4].

Together with the eight planets and the Moon initial conditions, the INPOP13c adjustment also includes the gravitational mass of the sun as recommended by the IAU resolution B2 as well as the sun oblateness ($J_2^{\odot}$), the ratio between the mass of the earth and the mass of the moon (EMRAT) and the mass of the Earth-Moon barycenter. Perturbations of 290 individual asteroids are taken into account in the dynamical modeling as well as perturbations induced by an asteroid ring at 3.15 AU. The mass of this ring as well as 290 individual asteroid masses are also fitted to observations.

1.2 Comparisons to other planetary ephemerides

A classical approach to estimate planetary ephemerides uncertainties is to make comparisons between different ephemerides: the JPL DE430 [6], the IAA EPM2011[7] and INPOP13c. These three ephemerides differ in their dynamical modeling mainly in the modeling of the asteroid perturbations but are adjusted with approximatively the same sample of observations. DE430 fits an important number (343) of objects with a priori values and uncertainties when EPM2011 fits a more limited number of objects (21) in association with more global estimations such as main belt and TNO rings and mean taxonomic densities. INPOP13c is an intermediate approach combining numerous individual fit (290) with a global main belt ring. The orbit differences between the ephemerides do not only picture the differences in the modeling and fitting strategy. They also include differences in the weighting scheme used for the construction of the ephemerides. Comparisons between the orbits of the planets provided by these ephemerides give the present uncertainties on the planetary orbits.

Knowing these uncertainties, it is then possible to consider alternative planetary orbits built on the basis of the INPOP13c ephemerides with different values of GR parameters and to compare these ephemerides to INPOP13c, DE430 and EPM2011. An acceptable alternative theory will be the one with differences to INPOP13c smaller or comparable to differences between INPOP13c, DE430 and EPM2011. These figures will be used as limits for considering an alternative ephemerides as acceptable at the light of the present ephemeris differences. Important differences for Mercury and Saturn are induced by indepandant spacecraft navigation analysis done by JPL and INPOP teams. Such thresholds can be scaled by the maximum residuals of INPOP13c for the fit dataset and besides the Messenger case, the maximum differences between INPOP13c and DE430 are below or about 50% of the maximum residuals of INPOP13c. EPM2011 does not include Messenger and Cassini tracking data. This explains the important differences for Mercury and Saturn. Furthermore, other data for Mars and Venus are also not included in EPM2011 but in DE430 and INPOP13c. Here again, for INPOP13c and EPM2011 common periods (before 2011), the differences stay below 50 % of the maximum postfit residuals of INPOP13c. The threshold of 50 % of the maximum INPOP13c postfit residuals for the maximum differences between ephemerides is adopted as a possible threshold for defining *close enough* ephemerides.

2 GR tests with INPOP

2.1 Implementation

Since INPOP10a, regular estimations of possible non-unity values for PPN parameters β and γ are regularly done with INPOP. For this work, we add to the INPOP dynamical modeling the possibility of constraining variations of the gravitational mass of the sun, μ, considering a variation of the mass of the sun noted $\dot{M}_\odot$ and a variation of the gravitational constant $\dot{G}$. At each step t of the numerical integration of the INPOP equations of motion, we then estimate :

$$M_\odot(t) = M_\odot(J2000) + (t - J2000) \times \dot{M}_\odot \quad (1)$$

$$G(t) = G(J2000) + (t - J2000) \times \dot{G} \quad (2)$$

$$\mu(t) = G(t) \times M_\odot(t) \quad (3)$$

$\dot{\mu}/\mu$ is also updated in the computation of the Shapiro delay of the observables (see [8]). We use for $M_\odot(J2000)$ the fitted mass of the sun and for $G(J2000)$ the Newtonian gravitation constant as defined by the IAU [9]. We then deduce values of $\dot{G}/G$ by considering a fixed value for the Sun total mass loss. We choose for this work the [10] total solar mass loss updated with the [11] mean mass loss from wind emission of charged particules during the 11-year solar cycle. This update leads to an interval of values of about $\frac{\dot{M}}{M_\odot} = (-0.92 \pm 0.46) \times 10^{-13}\,\mathrm{yr}^{-1}$. This value is used in the following section for deducing $\dot{G}/G$ from the estimated $\dot{\mu}/\mu$ gathered in Table 1.

Table 1: Results compared to values found in the literature. For least square determinations (LS), uncertainties are given at 3-σ. Each line gives the results obtained after the fit including 60 (*Limited*) or 290 asteroid masses (*Full*), spacecraft bias (SC) and observational station bias (DSN), defined in section 2.2. The Monte Carlo uncertainties give the length of the acceptable interval of violation as defined in the text. For the MC and LS estimations of $\dot{G}/G$, the values are deduced from the estimated values of $\dot{\mu}/\mu$ and in considering $\frac{\dot{M_\odot}}{M_\odot} = (-0.92 \pm 0.46) \times 10^{-13}\,\mathrm{yr}^{-1}$.

Method	PPN $\beta - 1$ $\times 10^5$	PPN $\gamma - 1$ $\times 10^5$	$\dot{G}/G$ $\times 10^{13}$ yr^{-1}	$J_2^\odot$ $\times 10^7$
Least squares (LS)				
Limited	-12.8 ± 6.7	10.2 ± 0.8	1.12 ± 0.47	2.23 ± 0.2
Limited + SC + DSN	-2.3 ± 8.4	3.1 ± 2.2	0.94 ± 0.48	2.23 ± 0.2
Full	-4.9 ± 6.4	-2.0 ± 6.4	-0.58± 0.63	2.27 ± 0.3
Full + SC + DSN	-6.7 ± 6.9	-0.81 ± 5.7	0.42 ± 0.75	2.27 ± 0.25
Monte Carlo (MC)				
MC + GA 50 %	-0.49 ± 6.31	-1.19 ± 4.43	0.36 ±1.22	2.26 ± 0.11
MC + GA 25 %	-1.06 ± 4.46	-0.75 ± 3.23	0.41 ±1.00	2.28 ± 0.08
MC + GA χ^2 H_{iter}	0.34 ± 6.91	-1.67 ± 5.12	0.51 ± 1.18	2.218 ± 0.135
MC + GA χ^2 H1	0.11 ± 7.07	-1.62 ± 5.10	0.52± 1.18	2.220 ± 0.135
MC + GA χ^2 H2	0.05 ± 7.12	-1.62 ± 5.17	0.53 ± 1.20	2.221 ± 0.137
MC + GA χ^2 H3	-0.01 ± 7.10	-1.67 ± 5.25	0.55 ±1.22	2.220 ± 0.14
MC + GA (50 % + χ^2)	0.0 ± 6.90	-1.55 ± 5.01	0.494 ±1.20	2.224 ± 0.131
Planetary ephemerides				
DE [27]	4 ± 24	fixed to (2.1 ± 2.3)	0.0	fixed to 1.8
	fixed	18 ± 26	0.0	fixed to 1.8
	0.0	0.0	1.02 ± 2.06*	fixed to 1.8
DE [13]	0.0	0.0	0.0	2.1 ± 0.70
EPM [14]	-2 ± 3	4 ± 6	0.29 ±0.89*	2.0 ± 0.2
INPOP13a [1]	0.2 ± 2.5	-0.3 ± 2.5	0.0	2.40 ± 0.20
INPOP10a [15]	-4.1 ± 7.8	-6.2 ± 8.1	0.0	2.40 ± 0.25
INPOP08 [16]	7.5 ± 12.5	0.0	0.0	1.82 ± 0.47
LLR				
[17]	12 ± 11	fixed to (2.1 ± 2.3)		
[18]	0.0		± 3	
[19]	0.0	0.0	-0.7 ± 3.8	
	3 ± 13	fixed to (2.1 ± 2.3)	0.0	
Other technics				
Cassini [20]	0.0	2.1 ± 2.3	0.0	NC
VLBI [21]	0.0	-8 ± 12	0.0	fixed
Planck + Brans-Dicke [22]			-1.315± 2.375	
Binary pulsar [24]			40± 50	
Big Bang nucleosynthesis [23]			0 ± 4	
Heliosismo. [25,26]				2.206 ± 0.05

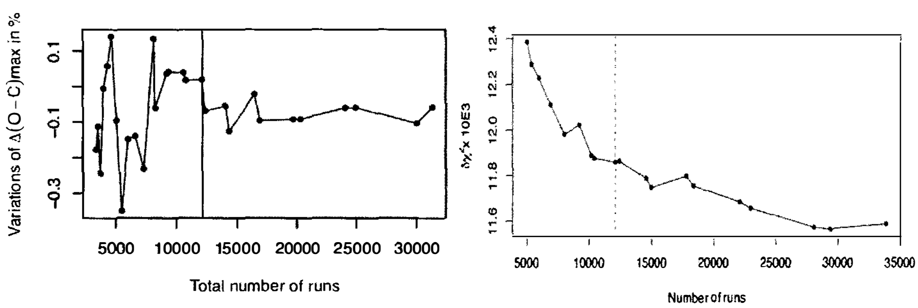

Figure 1 – Evolution of the number of selected ephemerides based on the two $\Delta(O-C)_{max}$ criteria (left) and the four H3 χ^2 criteria (right) described in the text.

2.2 Direct Least square estimations

In using INPOP13c as a reference ephemeris, least square adjustments of GR parameters together with regular planetary ephemeris adjusted parameters were done considering i) the impact of the asteroid perturbations on the dynamical modeling with two different fits (one with a limited number of fitted asteroid masses (60) and one with 290 fitted asteroid masses) and ii) the observational bias induced either by systematic effects related to the spacecraft itself (electronic degradation, mismodeling of the spacecraft macromodel) either by calibration uncertainties at the DSN stations. Table 1 gathers the fitted GR parameter values and uncertainties deduced from the different least squares.

These results are globally consistent with previous analysis done by [27] or [28] which stress the limitation due to the asteroid perturbations on the determination of $\dot{\mu}/\mu$, $J_2^{\odot}$ and β. [29] also point out the importance of asteroid perturbations for $J_2^{\odot}$ determinations. Observational bias play also a role especially in the case of the limited modeling, stressing the importance of the asteroid perturbations on the GR tests which can be done with planetary ephemerides.

Finally, the χ^2 obtained with the full modeling fit including the estimation of the spacecraft bias is still very close to the one without bias χ^2 : the difference between the two χ^2 is below 1 %. For the limited modeling, the differences between the χ^2 obtained with and without observational bias are more important indicating again a better robustness of the full modeling in comparison to the limited one.

2.3 Monte Carlo optimized estimations

Besides such computations, theoreticians often ask if some GR violations can be possible in the frame of some specific modeling of the solar system. In order to answer to this type of questions, one can introduce possible violations of GR through PPN parameters and time variation of G in the planetary ephemerides and to fit such ephemerides by comparison to observations. Acceptable intervals of GR violations can then be defined such as inducing fitted planetary ephemerides with small differences (relative to planetary ephemeris uncertainties) in comparison to a GR planetary ephemeris, in our case INPOP13c, built with $\beta = \gamma = 1$, $\dot{\mu}/\mu = 0$ and $J_2^{\odot} = 2.3 \times 10^{-7}$.

In order to investigate a wide range of possible values for GR parameters, we set up an algorithm based on a genetic combination of PPN β, γ, $J_2^{\odot}$ and $\dot{\mu}/\mu$. For each combination of GR paramcters, we built a fitted planetary ephemeris that we test by considering two criteria: one based on the maximum differences in postfit residuals relative to INPOP13c and one based on the χ^2 differences. These tests say if the planetary ephemeris is *close enough* to INPOP13c by limiting the differences in $\Delta(O-C)_{max}$ (criteria 1) or the differences in $\delta\chi^2$ (criteria 2).

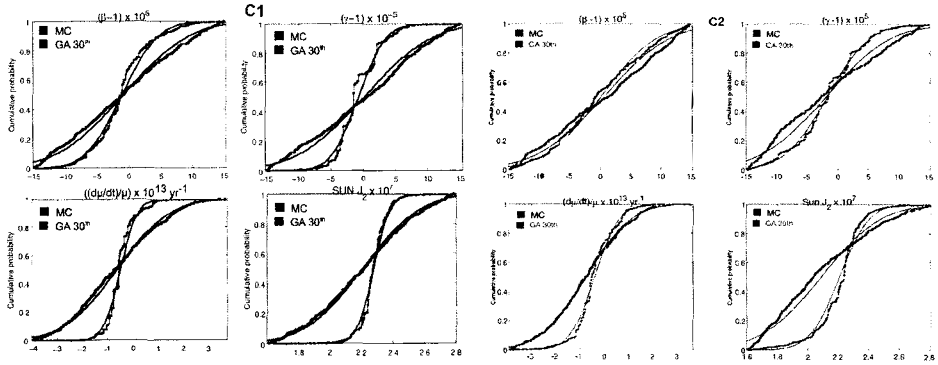

Figure 2 – Cumulative histogram of (PPN β, PPN γ, $J_2^{\odot}$, $\dot{\mu}/\mu$) for the generation 0 of ephemerides selected with the $\Delta(O-C)_{max}$ criteria (left) and the H3 χ^2 criteria (right, noted MC and colored in black) and for the final generation noted GA 30^{th} and colored in red also selected with the $\Delta(O-C)_{max}$ criteria (left) and the H3 χ^2 criteria(right). The full lines are the corresponding cumulative histograms for the normal distribution fitted on the distributions of the first generation and the 30th generation.

In order to test the sensitivity of the algorithm to these thresholds, we consider two values for the $\Delta(O-C)_{max}$ limits (25% and 50% as noticed by the INPOP13c, DE430 and EMP2011 differences) and four values of the $\delta\chi^2$ criteria: H_{iter} for ephemerides with $\delta\chi^2 < 0.5\%$, H1 for ephemerides with $\delta\chi^2 < 1\%$, H2 for ephemerides with $\delta\chi^2 < 2\%$ and finally H3 with $\delta\chi^2 < 3\%$. These limits are consistent with $\delta\chi^2$ values experienced during the least squares estimations of GR parameters presented in section 2.2.

If two fitted planetary ephemerides are selected by one of the two criteria (both being selected by the same criteria), the corresponding GR parameters are recombined using a two crossover algorithm associated with a mutation probability of 10%. A new set of GR parameters of then obtained and a new planetary ephemeris is created and fitted and the same testing procedure is applied and new sets of GR parameters are selected (constituting a new generation of parameters) before a new recombination is done.

A total of 35800 runs spread over 30 generations were computed on the MesoPSL computer center of Paris Science et Lettres (`www.mesopsl.fr`). We stop the generational process until the average change in the maximum differences of postfit residuals $\Delta(O-C)_{max}$ or in $\delta\chi^2$ are stable. As one can see on Figure 1, these differences stabilized at about 12 000 runs corresponding to the 18th generation. The selected samples of GR parameters with which produced ephemerides are selected based on the $\Delta(O-C)_{max}$ or on the χ^2 criteria constitute gaussian samples from which one can define mean and 1-σ. The gaussianity of the selected samples is improving with the number of runs as one can see on Figure 2 as well as the dispersion of the selected parameters is decreasing with the number of runs (see Figure 3).

3 Discussion

Table 1 gathers the results obtained with this work as well as very diverse estimations found in the literature. The first values presented in Table 1 are those estimated by direct least square procedures described in section 2.2. As discussed in this section, GR parameters estimated directly from a global fit of planetary ephemerides are sensitive to the dynamical modeling but also to observational bias. However, the full modeling adjustment presents a better robustness in comparison to the limited modeling. The values of GR parameters deduced from the full modeling tend to have consistent values at 3-σ with or without observational bias when higher variations are noticeable for the limited modeling. Greater variations in the χ^2 values are also present for the limited modeling. Considering results obtained with the MC simulations and more specifically the χ^2 criteria, one can note the consistency of the deduce intervals for the four criteria of selections, stressing the robustness of the method. Intervals deduced from the $\Delta(O-C)_{max}$ criteria appear to have greater variations but always in keeping consistent intervalles. In this context, in order to exhibit one single set of values of acceptable intervals for the four parameters randomly modified in this work, one can consider the mean values of the most numerous MC+GA selection presented in Table 1, gathering values of (PPN β, PPN γ, $J_2^{\odot}$, $\dot{\mu}/\mu$) inducing ephemerides with $\Delta(O-C)_{max} < 50\%$ and ephemerides selected with the four χ^2 criteria. We then obtain the values labeled MC + GA (50 % + χ^2) in Table 1.

As noticed in [1] the interval of possible violations for the PPN parameters β and γ with no time variation of the Newtonian gravitational constant G and in fixing the value of the Sun flattening is as accurate as the reference values obtained with the Cassini experiment [20]. However by adding the variations of $\dot{\mu}/\mu$ and $J_2^{\odot}$, we have enlarged the possible interval of violations for the four parameters as given in line MC + GA (50 % + χ^2) of Table 1.

Furthermore, by selecting non GR ephemerides with $\Delta(O-C)_{max}$ smaller than the present uncertainty of planetary ephemerides (see section 1.2) and with $\Delta\chi^2$ compatible with the differences in χ^2 of the same order as the increase of the number of fitted asteroid masses or by the addition of observational bias (see section 2.2), we have obtained a selection of ephemerides compatible with the actual estimations of uncertainties induced either by the dynamical modeling (differences between DE430, EPM2011 and INPOP13c, various numbers of fitted asteroid masses) or the observational accuracy (addition of observational bias). From this selection are deduced the intervals of parameters presented in Table 1 which by construction include uncertainties induced by the differences in dynamical modeling and adjustment procedures (through $\Delta(O-C)_{max}$ and $\Delta\chi^2$ thresholds). An example is the estimation of the acceptable interval of $\dot{\mu}/\mu$ and $\dot{G}$/G obtained with MC simulations which is twice larger than the one obtained by LS. This increase of the interval is consistent with the important variability of the $\dot{\mu}/\mu$ LS determination due to significant correlations with asteroid masses. More generally, in comparisons to other values found in the literature, the LS uncertainties are compatible with those given by[27], LLR [19] and VLBI [21] estimations. When these comparable values are estimated with one or more GR parameters kept fixed in the fit, but only EPM values are obtained in a global fit as the one done with INPOP. The EPM uncertainties are generally smaller than the LS or MC ones. They are also not balanced as for INPOP LS or MC determinations: EPM determinations show smaller uncertainties for β and greater error bars for γ when LS and MC values face the opposite. LS and EPM uncertainties on $\dot{\mu}/\mu$ are quite compatible. MC interval of $\dot{\mu}/\mu$ is larger than the EPM values but still compatible at 3-σ. One can also note the differences in the Sun flattening determinations between EPM and LS, MC estimations, the EPM value being smaller (2.0 ± 0.2) than the MC and LS mean value ($(2.255 \pm 0.146) \times 10^{-7}$). The Lense-Thirring effect, inducing variations in $J_2^{\odot}$ up to 10%, was not included in the LS and MC estimations, but it is not clear if it was taken into account in the EPM determinations. However the determinations of the Sun flattening by LS or MC without Lense-Thirring effect give values very close to the one obtained with helioseismology. Finally, values of $\dot{G}$/G obtained by astrophysical technics such as pulsar timing analysis give larger intervals than those obtained in the solar system.

4 Conclusions

In this work we have estimated in using two methods possible violations of general relativity with the PPN parameters β, γ in considering in the same time time variations of the gravitational constant G and values of the sun flattening.

We first made an global adjustment of the GR parameters together with parameters usually considered for the construction of planetary ephemerides such as INPOP. Important variations

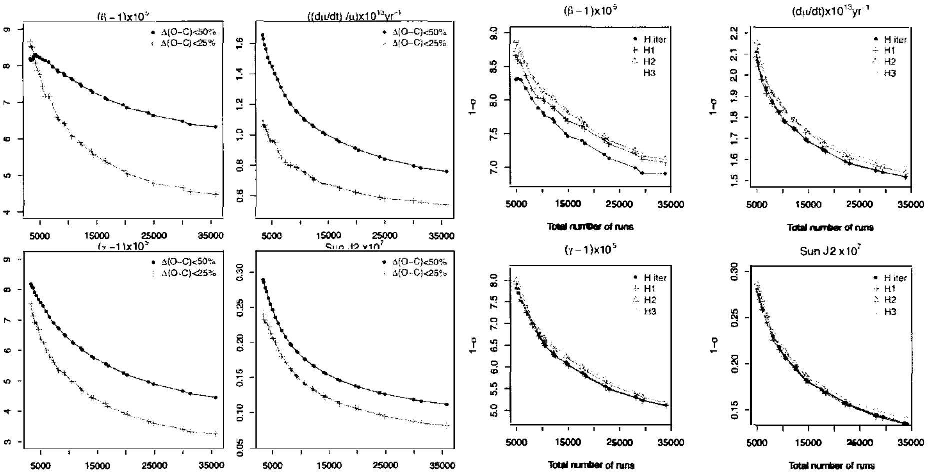

Figure 3 – Evolution with the number of selected runs of the 1-σ of the gaussian distribution of the PPN β, PPN γ, $J_2^{\odot}$, and $\dot{\mu}/\mu$ parameters corresponding to the ephemerides selected with the $\Delta(O-C)_{max}$ criteria (left) and χ^2 criteria (right).

(up to the factor 3) in the fitted values but also in the obtained uncertainties were then obtained depending the number of fitted asteroid masses (from 60 to 190) and the observational bias considered in the fit (s/c delay and station calibration bias). However, in considering the most complete modeling, no violation of GR is then statistically detectable at the level of 1×10^{-4} for β and γ, and $2\times10^{-13}\text{yr}^{-1}$ for $\dot{G}/G$ and the sun flattening is found to be compatible with other values found in the literature with an uncertainty of about 4×10^{-8}.

Such variability of the least square results leads us to consider another approach based on random selections of GR parameters. We then used Monte Carlo simulations and genetic algorithm procedures for producing more than 35000 planetary ephemerides fitted to observations and compared to INPOP13c. Using a χ_2 selection and postfit residual procedures based on planetary ephemerides uncertainty analysis (see section 1.2), we have identified intervals of parameters inducing the smallest modifications to the planetary dynamics relative to their current uncertainty (estimated for INPOP13c in section 1.2).

We have reduced the uncertainty of the estimation of the sun flattening by a factor 2 (up to 1.2×10^{-8}) in comparison to the previous estimations so far and we have explored a wide range of possible combination of parameters (35000 runs). Considering all the given figures of Table 1 one should conclude that no deviation to general relativity is noticeable for the four GR parameters modified simultaneously at the level of 7×10^{-5} for β, 5×10^{-5} for γ, and $2\times10^{-13}\text{yr}^{-1}$ for $\dot{G}/G$.

New tests will be implemented such as the addition of supplementary terms in the equation of motions of the planets as proposed by alternative theories [31], [32], [33]. Tests of the equivalence principal can also be proposed for Monte Carlo simulations and genetic algorithm procedures. In the case of the planetary orbits, one would have to consider one ratio of gravitational and inertial masses for each planet which would multiply the number of runs by an important scale.

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