

# Isoscalar and isovector spin responses in *p*-shell and *sd*-shell nuclei

Hiroyuki Sagawa<sup>1,2,\*</sup> and Toshio Suzuki<sup>3,4,\*\*</sup>

<sup>1</sup>RIKEN, Nishina Center, Wako, 351-0198, Japan

<sup>2</sup>Center for Mathematics and Physics, University of Aizu, Aizu-Wakamatsu, Fukushima 965-8560, Japan

<sup>3</sup>Department of Physics, College of Humanities and Science, Nihon University, Sakurajosui 3, Setagaya-ku, Tokyo 156-8550, Japan

<sup>4</sup>National Astronomical Observatory of Japan, Mitaka, Tokyo 181-8588, Japan

**Abstract.** We study the spin magnetic dipole transitions in *sd*-shell even-even nuclei with  $N=Z$  and a *p*-shell nucleus  $^{12}\text{C}$  by modern shell model calculations. The shell model wave functions take into account enhanced isoscalar (IS) spin-triplet pairing as well as the effective spin operators. We point out that the IS pairing and the effective spin operators give a large quenching effect on the isovector (IV) spin transitions to be consistent with observed data by  $(p, p')$  experiments. On the other hand, the observed IS spin strengths do not show large quenching effect suggested by the calculated investigation. The IS pairing gives a substantial quenching effect on the spin magnetic dipole transitions, especially on the IV ones.

## 1 Introduction

The spin-isospin response and spin-isospin dependent interactions in nuclei are fundamental important subjects in nuclear physics and astrophysics. The quenching phenomena of magnetic moments and Gamow-Teller (GT) strengths have been extensively studied by taking into account the mixings of particle-hole (p-h) configurations, meson exchange currents and the coupling to the  $\Delta$  resonances [1–5]. On the quenching of the GT sum rule,  $3(N-Z)$  [6], there have been a serious question whether the effect of many-particle many hole states or the coupling to  $\Delta$ -isobar states is a dominant contribution [4, 5]. After a long debate [7], experimental investigations by charge-exchange  $(p, n)$  and  $(n, p)$  reactions on  $^{90}\text{Zr}$  have revealed that about 90% of the GT sum rule strength exists in the energy region up to  $E_x=50$  MeV [8, 9]. This demonstrates the importance of higher order effect beyond the mean-field approximations; mostly, the two-particle two-hole (2p-2h) configuration mixings due to the central and tensor forces [3], although the coupling to  $\Delta$  is not completely excluded.

The isovector (IV) spin excitations have been measured by several experimental probes; GT states by charge exchange reactions  $(p, n)$  and  $(^3\text{He}, t)$ ,  $M1$  transitions by  $\gamma$ -ray and electron scattering experiments and also magnetic moment measurements. On the other hand, the empirical information of isoscalar (IS)  $M1$  transitions is very rare. This is due to the fact that the IS spin *g*-factor is much smaller than the IV one, i.e.,  $(g_s^{\text{IS}}/g_s^{\text{IV}})^2 \sim 1/30$  for the electromagnetic process. For the hadronic process, the IS spin coupling is also much smaller than the IV one,  $V_\sigma^{\text{IS}}/V_\sigma^{\text{IV}} < 1/6$  at the intermediate reaction energy.

One clear observation of IS spin excitation is  $1^+$  state at 12.71 MeV in  $^{12}\text{C}$  by electron scattering. This observation shows no quenching in IS  $M1$  strength compared with the shell model calculation, but indicating a large enhancement. Recently, both IS and IV spin  $M1$  transitions have been investigated by high-resolution proton inelastic scattering measurements at  $E_p=295$  MeV [10]. The IV spin  $M1$  transitions induced by the operator  $\vec{\sigma} \cdot \vec{t}_z$  can be regarded as analogous to GT transitions, while the IS spin  $M1$  transitions are free from the coupling to  $\Delta$  and the quenching of IS strength should be due to the couplings to higher order p-h configurations such as 2p-2h, 3p-3h and so on. In this aspect, the measurement of IS spin excitation may provide an important information on the quenching mechanism of spin-dependent excitations in nuclei.

The IS spin-triplet pairing correlations have been reported to play an important role in enhancing the GT strength near the ground states of daughter nuclei with mass  $N \sim Z$  [11–14]. At the same time, the total sum rule of the GT strength is quenched by ground state correlations due to the IS pairing [15]. The IS paring is also found to be important to reduce  $0\nu\beta\beta$  decay matrix elements [16]. In this report, we investigate the IS and IV spin  $M1$  responses based on large-scale shell model calculations with enhanced IS pairing correlations for the  $N=Z$  *p*-shell and *sd*-shell nuclei. Simultaneous calculations of the IS and IV responses within the same nuclear model may be advantageous to distinguish the effect of the higher order configurations from the  $\Delta$ -hole coupling.

The spin  $M1$  operators are introduced in Sec. 2 and their sum rules are also defined. Section 3 is devoted to the shell model calculations of  $N=Z$  even-even nuclei in comparisons with available experimental data by  $(p, p')$  reactions. The accumulated sum rule values of IS and IV spin transitions are extracted in Sec. 4. The proton-neutron

\*e-mail: sagawa@ribf.riken.jp

\*\*e-mail: suzuki@chs.nihon-u.ac.jp

spin-spin correlations are also discussed in terms of the IS spin-triplet pairing correlations. The summary is given in Sec. 5.

## 2 Spin $M1$ operators and sum rules

The bare IS and IV spin  $M1$  operators are written to be  $\hat{O}_{\text{IS}} = \sum_k \vec{\sigma}(k)$  and  $\hat{O}_{\text{IV}} = \sum_k \vec{\sigma}(k) \tau_z(k)$ , respectively, while the GT charge exchange excitation operators are expressed as  $\hat{O}_{\text{GT}\pm} = \sum_k \vec{\sigma}(k) t_{\pm}(k)$ . The sum rule values for the  $M1$  spin transitions are defined by

$$S(\text{IS}) = \sum_f \frac{1}{2J_f+1} |\langle J_f | \hat{O}_{\text{IS}} | J_i \rangle|^2, \quad (1)$$

$$S(\text{IV}) = \sum_f \frac{1}{2J_f+1} |\langle J_f | \hat{O}_{\text{IV}} | J_i \rangle|^2. \quad (2)$$

For the GT transition, the sum rule value is also defined as

$$S(\text{GT}\pm) = \sum_f \frac{1}{2J_f+1} |\langle J_f | \hat{O}_{\text{GT}\pm} | J_i \rangle|^2, \quad (3)$$

which gives the well-known model independent sum rule,

$$S(\text{GT}_-) - S(\text{GT}_+) = 3(N - Z). \quad (4)$$

According to Ref. [10], the proton-neutron spin-spin correlation is obtained from the sum rule values, Eqs. (1) and (2), as

$$\Delta_{\text{spin}} = \frac{1}{16} [S(\text{IS}) - S(\text{IV})] = \langle J_i | \vec{S}_p \cdot \vec{S}_n | J_i \rangle, \quad (5)$$

where  $\vec{S}_p = \sum_{k \in p} \vec{s}(\vec{k})$  and  $\vec{S}_n = \sum_{k \in n} \vec{s}(\vec{k})$ . The correlation value is 0.25 and -0.75 for a pure spin-triplet and a spin-singlet proton-neutron pair, respectively. The former corresponds to the ferromagnet limit of the spin alignment, while the latter is the antiferromagnetic one.

## 3 Shell model calculations with effective operators and IS pairing correlations

The shell model calculations are performed in full  $p$ -shell and  $sd$ -shell model space with the CKPOT and USDB interactions [17], respectively. Here, we present results based on USDB interaction. To take into account the effects of higher order configuration mixings as well as meson exchange currents and  $\Delta$ -isobar effect, effective operators are commonly adopted in the study of magnetic moments, GT transitions and spin and spin-isospin dependent  $\beta$  decays.

For the spin operators, the effective operators read for the IS operator

$$\hat{O}_{\text{IS}}^{\text{eff}} = f_s^{\text{IS}} \vec{\sigma} + f_l^{\text{IS}} \vec{l} + f_p^{\text{IS}} \sqrt{8\pi} [Y_2 \times \vec{\sigma}]^{(\lambda=1)}, \quad (6)$$

and also for the IV spin operator,

$$\hat{O}_{\text{IV}}^{\text{eff}} = f_s^{\text{IV}} \vec{\sigma} \tau_z + f_l^{\text{IV}} \vec{l} \tau_z + f_p^{\text{IV}} \sqrt{8\pi} [Y_2 \times \vec{\sigma}]^{(\lambda=1)} \tau_z, \quad (7)$$

where  $f_{\alpha}^{\text{IS(IV)}}$  ( $\alpha = s, l, p$ ) are the effective coefficients of IS (IV) spin, orbital and spin-tensor operators. The effective

coefficients for the IS spin operator obtained by Towner are  $f_s^{\text{IS}} = 0.745$ ,  $f_l^{\text{IS}} = 0.0526$  and  $f_p^{\text{IS}} = -0.0157$ . For the IV part, Towner obtained the corrections for the spin, orbital and the spin-tensor operators of GT transitions of  $1d$ -orbit as

$$\hat{O}_{\text{GT}}^{\text{eff}} = (1 + \delta g_s) \vec{\sigma} t_{\pm} + \delta g_l \vec{l} t_{\pm} + \delta g_p \sqrt{8\pi} [Y_2 \times \vec{\sigma}]^{(\lambda=1)} t_{\pm}, \quad (8)$$

with

$$\delta g_s = -0.139, \quad \delta g_l = 0.0103, \quad \delta g_p = 0.0283, \quad (9)$$

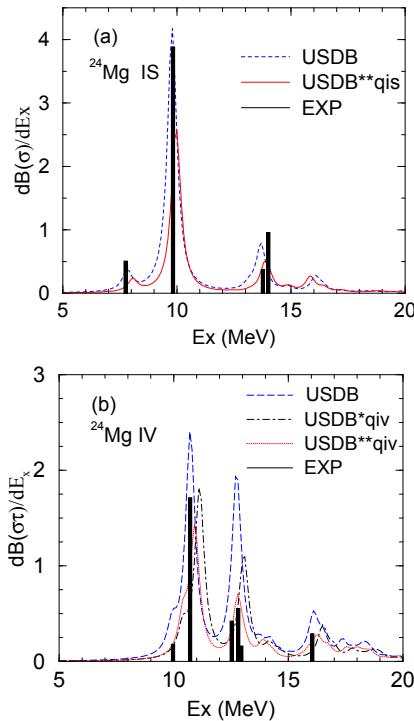
due to the various higher order effects. In the shell model calculations with USD interaction, the IV spin and charge exchange GT excitations are the same features since no isospin breaking interaction such as Coulomb interaction and charge symmetry breaking forces is included. We adopt the GT effective operators for IV spin transitions. For the IS part, the quenching factor for spin operator is introduced to check the sensitivity of transition strength on the effective operator. The effective operators for IS orbital and spin-tensor are not introduced in the present study.

In the following calculations for  $sd$ -shell nuclei, we introduce effective interactions with enhanced IS spin-triplet pairing matrices on top of USDB interaction. The interactions USDB\* and USDB\*\* denote the effective ones whose IS pairing matrices are enhanced by multiplying a factor 1.1 and 1.2, respectively. Results with effective operators are marked by “qis” or “qiv” for the IS and IV transitions, respectively. For the IS case, we perform four different versions of calculations:

- (1) USDB: the original interaction with the bare spin operator.
- (2) USDB\*: the IS spin-triplet pairing matrices are enhanced multiplying by a factor 1.1 on the relevant matrix elements of USDB interaction. The bare spin operator is adopted.
- (3) USDB\*qis: the same as USDB\* except that the IS spin operator is 10% quenched:  $f_s^{\text{IS}} = 0.9$ .
- (4) USDB\*\*qis: the IS spin-triplet pairing matrices are enhanced multiplying by a factor 1.2. The IS spin operator is 10% quenched:  $f_s^{\text{IS}} = 0.9$ .

For the IV case, we perform three different calculations:

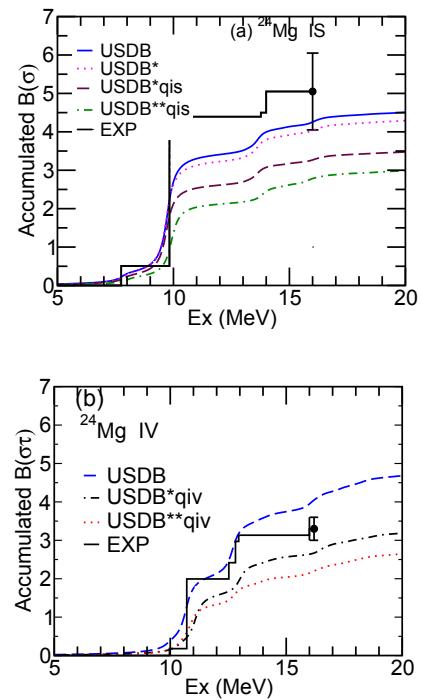
- (1) USDB: the original interaction with the bare spin operator.
- (2) USDB\*qiv: the IS spin-triplet pairing matrices are enhanced multiplying by a factor 1.1. The effective IV spin operator Eq. (7) is adopted.
- (3) USDB\*\*qiv: the same as USDB\*qiv except that the effective IV spin operator Eq. (7) is adopted.



**Figure 1.** (Color online) IS (top) and IV spin- $M1$  (bottom) transition strengths in  $^{24}\text{Mg}$ . Shell model calculations are performed in the full  $sd$ -shell model space with an USDB effective interaction as well as (a) USDB\*\* qis and (b) USDB\* qiv and USDB\*\* qiv. Calculated results are smoothed by taking a Lorentzian weighting factor with the width of 0.5 MeV, while the experimental data are shown in the units of  $B(\sigma)$  for the IS excitations and  $B(\sigma\tau)$  for the IV excitations. Experimental data are taken from Ref. [10].

### 3.1 $p$ -shell nuclei

For a  $p$ -shell nucleus, we study the IS and IV  $1^+$  states in  $^{12}\text{C}$  in which the IS and IV  $1^+$  states are observed at  $E_x=12.71$  and 15.11 MeV, respectively. The  $B(M1)$  values are extracted from  $(e, e')$  scattering experiments to be  $B(M1)=0.0402$  and 2.679 in terms of nuclear magneton  $(e\hbar/2mc)^2$  [18]. The shell model calculations with CK-POT interaction give  $B(M1)=0.01434$  and 2.314 in units of nuclear magneton at  $E_x=12.45$  and 15.09 MeV, respectively, with the bare magnetic transition operators. The calculations with a larger model space were performed recently with the  $p$ - $sd$  shell Hamiltonian by Suzuki-Fujimoto-Otsuka (SFO) [19]. The model space of SFO is  $p$ - $sd$  shell and the excitations from  $p$ -shell to  $sd$ -shell are included up to  $2\hbar\omega$ . These calculations give  $B(M1)=0.0131$  and  $2.515 \mu_N^2$  for the IS and IV transitions, respectively. It is noticed that the experimental value is about three times larger than the calculated value for the IS  $1^+$  state at  $E_x=12.71$  MeV, while the calculated value for the IV state is close to the experimental value. The  $(p, p')$  data was reported for the two  $1^+$  states to be  $B(\sigma)=3.174 \pm 0.842$  at  $E_x=12.71$  MeV and  $B(\sigma\tau)=1.909 \pm 0.094$  at  $E_x=15.11$  MeV, respectively. The shell model results with SFO are  $B(\sigma)=1.516$  and  $B(\sigma\tau)=1.937$ , respectively. The proton inelastic scattering data of the state at  $E_x=12.71$  MeV show also a factor



**Figure 2.** (Color online) Cumulative sum of the IS spin- $M1$  strength (top) and the IV spin- $M1$  strength (bottom) as a function of the excitation energy in  $^{24}\text{Mg}$ . Dot with a vertical error bar denotes the experimental accumulated sum of the strengths. See the text and the captions to Fig. 1 for details.

two larger value than the shell model results. The isospin mixing between the two  $1^+$  states have been discussed as an origin of the enhancement of IS spin matrix element. A large isospin mixing was claimed to enhance IS magnetic transition observed by the electron scattering.

The same effect is expected for  $B(\sigma)$ . When the IS  $1^+$  state at 12.71 MeV and IV  $1^+$  state at 15.11 MeV states are mixed by the isospin-mixing effect,

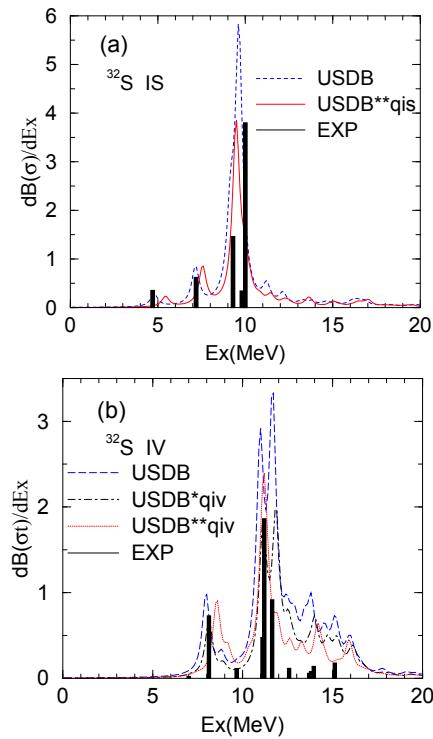
$$|1^+, 12.71 \text{ MeV}\rangle = \sqrt{1-a^2} |1^+, T=0\rangle + a |1^+, T=1\rangle, \quad (10)$$

$$|1^+, 15.11 \text{ MeV}\rangle = \sqrt{1-a^2} |1^+, T=1\rangle - a |1^+, T=0\rangle.$$

We get an enhancement of  $B(\sigma)$  as well as a reduction of  $B(\sigma\tau)$ .  $B(\sigma)$  is enhanced from 1.516 to 1.714 while  $B(\sigma\tau)$  is reduced from 1.937 to 1.750 and the mixing amplitude  $a=0.056$  [20]. The proton-neutron spin-spin correlation  $\Delta_{\text{spin}}$  in Eq. (8) is found to be enhanced by 0.024. Though the isospin-mixing gives rise to favorable effects, it is still not enough to reproduce the experimental value of  $B(\sigma)$ .

### 3.2 $sd$ -shell nuclei

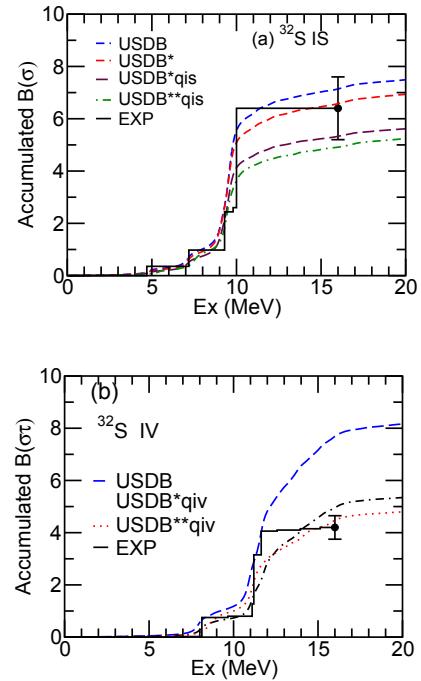
We study detailed characteristics of spin excitations of  $^{24}\text{Mg}$  and  $^{32}\text{S}$  as representatives of  $sd$ -shell nuclei. The full study of  $N=Z$   $sd$ -shell nuclei can be found in Ref. [21]. Figures 1 and 2 show the energy spectra of the spin excitations and their cumulative sums, respectively, in  $^{24}\text{Mg}$ . For the IS case, the experimental data



**Figure 3.** (Color online) IS (top) and IV spin- $M1$  (bottom) transition strengths in  $^{32}\text{S}$ . See the captions to Fig. 1 for details.

give the strong spin  $M1$  strength at  $E_x=9.828$  MeV with  $B(\sigma)=3.886\pm 1.102$ . The shell model results with USDB give IS  $1^+$  state at  $E_x=9.818$  MeV with  $B(\sigma)=3.278$ . In the  $(p, p')$  data, other IS strengths are also found at 7.748 MeV with  $B(\sigma)=0.508$  and at  $E_x\sim 14$  MeV with  $B(\sigma)\sim 1.2$ . The calculated results reproduce strong  $M1$  states at very similar energies  $E_x=7.82$  and 13.7 MeV with  $B(\sigma)=0.24$  and 0.59, respectively. The calculations with USDB show also the same amount of  $B(\sigma)$  value as the experimental data around  $E_x=14$  MeV. The summed strength up to  $E_x=16$  MeV is  $B_{\text{exp}}(\sigma; E_x \leq 16 \text{ MeV})=5.061\pm 1.166$ , while the calculated sum is  $B_{\text{cal}}(\sigma; E_x \leq 16 \text{ MeV})=4.256$ . The calculated results of  $\text{USDB}^{**}\text{qis}$  changes only slightly the excitation energies of  $1^+$  states by about  $100\div 200$  keV, while the summed  $B(\sigma)$  value is decreased by 30%.

The experimental analysis shows a strong IV spin strength at  $E_x=10.71$  MeV with  $B(\sigma\tau)=1.714$ . The calculation gives at  $E_x=10.723$  MeV with  $B(\sigma\tau)=1.854$ . Experimental data show also substantial strength around  $E_x=12.8$  MeV with  $B(\sigma\tau)\sim 1$  and at  $E_x=9.968$  and 16.046 MeV with  $B(\sigma\tau)=0.18$  and 0.29, respectively. The calculated results give large strengths at  $E_x=9.939$  MeV and 10.75 MeV with  $B(\sigma\tau)=0.238$  and 1.521, respectively. The experimental summed strength is  $B_{\text{exp}}(\sigma\tau; E_x \leq 16 \text{ MeV})=3.180\pm 0.236$ , while the calculated value is  $B_{\text{cal}}(\sigma\tau; E_x \leq 16 \text{ MeV})=3.855$ . There are about 20% quenching in the empirical sum rule strength of IV spin excitations below  $E_x=16$  MeV compared with USDB results with the bare spin operator. The  $\text{USDB}^*\text{qiv}$  and  $\text{USDB}^{**}\text{qiv}$  results with the effective spin operator



**Figure 4.** (Color online) Cumulative sum of the IS spin- $M1$  strength (top) and the IV spin- $M1$  strength (bottom) as a function of the excitation energy in  $^{32}\text{S}$ . See the captions to Fig. 2 for details.

show about 30 and 40% quenching of the accumulated strength up to  $E_x=16$  MeV, respectively.

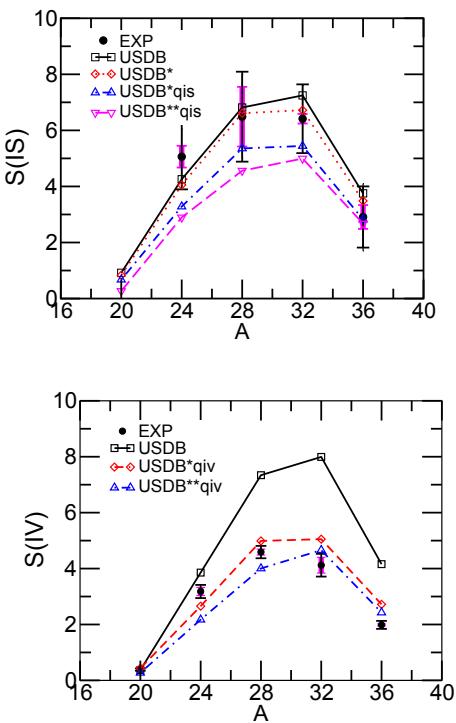
We study the effects of the isospin-mixing in  $^{24}\text{Mg}$ . The  $1^+, T=0$  states at  $E_x=7.747$  MeV and 9.827 MeV are mixed with the  $1^+, T=1$  state at  $E_x=9.966$  MeV by the two-body Coulomb interaction  $V_{\text{CD}}$  [22]. Using the mixing amplitudes obtained by [22],

$$\frac{\langle T=1|V_{\text{CD}}|T=0 \rangle}{\Delta E} \quad \text{with} \quad \langle T=1|V_{\text{CD}}|T=0 \rangle = 49 \text{ keV},$$

an enhancement is obtained for  $S(\text{IS})$  from 4.256 to 4.492, and a reduction is shown for  $S(\text{IV})$  from 3.856 to 3.652. These changes give an enhancement of  $\Delta_{\text{spin}}$  by 0.026 for USDB. These effects are favorable and consistent with the experimental data though their magnitudes are not so large.

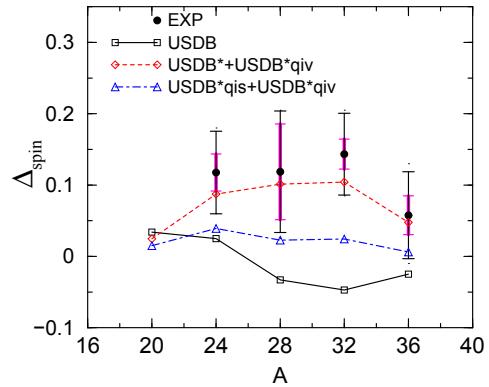
The IS excitation of  $^{32}\text{S}$  is shown in Fig. 3(a). The experimental data show a strong state at  $E_x=9.956$  MeV with  $B(\sigma)=3.810\pm 1.118$ . The corresponding state is found in the calculated results at  $E_x=9.632$  MeV with  $B(\sigma)=4.312$ . Another strong IS transition was found at  $E_x=9.297$  MeV with  $B(\sigma)=1.461\pm 0.436$ , while the calculations show a state at  $E_x=9.154$  MeV with  $B(\sigma)=1.293$  MeV. The observed IS sum rule strength is  $B_{\text{exp}}(\sigma; E_x \leq 16 \text{ MeV})=6.414\pm 1.227$ , while theoretically  $B_{\text{cal}}(\sigma; E_x \leq 16 \text{ MeV})=7.623$  in Fig. 4(a). We can see a small quenching effect corresponding to  $f_s^{\text{IS}}(\text{eff})=0.92$  for the sum rule strength below  $E_x=16$  MeV.

The IV response in  $^{32}\text{S}$  is shown Fig. 3(b). The IV spin strength is concentrated at  $E_x\sim 11.3$  MeV having 80% of the total strength below  $E_x=16$  MeV. The calculated results show also very large fraction of the to-



**Figure 5.** (Color online) Accumulated spin- $M1$  transition strengths of IS channel (top) and IV channel (bottom). Experimental and theoretical data are summed up to  $E_x=16$  MeV. Shell model calculations are performed with the USDB effective interaction: (top) In the results of USDB\* and USDB\*qis, the IS spin-triplet pairing interaction is enhanced by multiplying the relevant matrix elements by a factor of 1.1 compared with the original USDB interactions and the quenching factor  $f_s^{IS}=1.0$  and 0.9 for IS spin operator, respectively. For USDB\*\*qis, the IS pairing interaction is enhanced by a factor of 1.2 and a quenching factor  $f_s^{IS}=0.9$  is introduced for the IS spin operator. Experimental data are taken from Ref. [10]. Long thin error bars indicate the total experimental uncertainty, while the short thick error bars denote the partial uncertainty from the spin assignment. (bottom) The effective IV operators are adopted for spin, orbital and spin-tensor operators in the case of USDB\*qiv and USDB\*\*qiv. The effective operators are taken from Ref. [2]. For the results of USDB\*qiv and USDB\*\*qiv, the IS pairing interaction is enhanced by a factor of 1.1 and 1.2, respectively, with the effective operators.

tal strength of about 87% of the total strength. Another strong state is found experimentally at  $E_x=8.125$  MeV with  $B(\sigma\tau)=0.730\pm0.040$ , while the calculations show a state at  $E_x=7.959$  MeV with  $B(\sigma\tau)=0.743$ . The agreement between theory and experiment is quite satisfactory as far as the gross feature of IV spin response is concerned. The accumulated strength of IV transitions is  $B_{\text{exp}}(\sigma\tau; E_x \leq 16 \text{ MeV})=4.120\pm0.407$ , while the calculated results are  $B_{\text{cal}}(\sigma\tau; E_x \leq 16 \text{ MeV})=7.993$  in Fig. 4(b). We see a large quenching for IV case with  $f_s^{IV}(\text{eff})=0.72$ . The results USDB\*\*qiv with the enhanced IS pairing and the effective IV spin operator give good account of the accumulated strength.



**Figure 6.** (Color online) Experimental and calculated proton-neutron spin-spin correlation  $\Delta_{\text{spin}}$  in Eq. (8). Spin  $M1$  transition strengths are summed up to  $E_x=16$  MeV. Shell model calculations are performed with an effective interaction USDB. In the results of USDB\* and USDB\*qis for the IS channel, the IS spin-triplet interaction is enhanced multiplying the relevant matrix elements by a factor of 1.1 compared with the original USDB, and the IS quenching factor is  $f_s^{IS}=1.0$  and 0.9, respectively. The effective spin operators are used for the USDB\*qiv for the IV transitions. Experimental data are taken from Ref. [10]. See the captions to Fig. 5 for the explanation of the experimental error bars.

#### 4 Accumulated strength of IS and IV spin $M1$ excitations

Figure 5 shows the sum rule values of  $S(\text{IS})$  and  $S(\text{IV})$  for the variations of interactions, respectively. The 10% enhanced IS pairing in USDB\* give a small quenching effect on the accumulated IS sum rule value; about 5% in average and at most 7% in  $^{32}\text{S}$  and  $^{36}\text{Ar}$ . With the quenching factor  $f_s^{IS}=0.9$  in USDB\*qis, the IS accumulated strength is further decreased by 22÷25% compared with the original value by USDB interaction. The decrease of the accumulated IS value is going down further to be 29÷33% in the case of USDB\*\*qis with the 20% enhanced IS pairing. Compared with USDB calculations, the empirical accumulated IS values are 20% enhanced in  $^{24}\text{Mg}$  and gradually quenched from  $A=28$  to 36; 0.95, 0.88 and 0.77 in  $^{28}\text{Si}$ ,  $^{32}\text{S}$  and  $^{36}\text{Ar}$ , respectively. Thus, the quenching effect in the experimental data is rather small and at most 23% of the USDB calculations with the bare spin operator.

The IV accumulated sum rule values up to  $E_x=16$  MeV are shown in Fig. 5. The IS pairing interactions are enhanced by factors of 1.1 for USDB\*qiv and 1.2 for USDB\*\*qiv, respectively, with the effective operators from Ref. [2]. The results of USDB\*qiv give 31÷36% quenched sum rule values, while the stronger IS pairing in USDB\*\*qiv gives additional quenching of the strength, i.e., 41÷45% quenching of the summed strength. The empirical values show also large quenching; 33% in  $^{20}\text{Ne}$ , 15% in  $^{24}\text{Mg}$ , 27% in  $^{28}\text{Si}$ , 47% in  $^{32}\text{S}$  and 52% in  $^{36}\text{Ar}$ , respectively, compared with the USDB calculations.

Figure 6 shows the experimental and the calculated proton-neutron spin-spin correlations (5). Although the experimental data still have large error bars, the calculated

results with the USDB interaction show poor agreement with the experimental data. The results with an enhanced IS spin-triplet pairing improve the agreement appreciably. The effective operator gives a smaller spin-spin correlation  $\Delta_{\text{spin}}$  than the case of bare IS spin operator. The positive value of the correlation indicates that the population of spin triplet pairs in the ground state is larger than that of the spin singlet pairs. We should remind that the spin and the spin-isospin  $M1$  strengths may exist in the energy region above  $E_x=16$  MeV. In the analysis of Fig. 6, these higher energy contributions are assumed to be the same for both the IS and IV channels.

## 5 Summary

In summary, we studied the IS and IV spin  $M1$  transitions in even-even  $N=Z$   $p$ -shell and  $sd$ -shell nuclei using shell model calculations with CKPOT and USDB interactions, respectively. The model space are taken to be the full  $p$ -shell and  $sd$ -shell model space. We introduced the effective operators for the spin and spin-isospin  $M1$  operators in Eqs. (6) and (7) as well as the enhanced IS spin-triplet pairing for the calculations of  $sd$ -shell nuclei, while the bare operator and the original interaction were adopted in the  $p$ -shell calculations. In general, the calculated results show good agreement with the experimental energy spectra in  $N=Z$  nuclei as far as the excitation energies are concerned. It was pointed out that the  $M1$  strength to IS  $1^+$  state in  $^{12}\text{C}$  shows a large enhancement compared with the shell model value. Compared with the experimental  $M1$  results of  $sd$ -shell nuclei, the accumulated IS spin strengths up to 16 MeV show small quenching effect, corresponding to the effective quenched operator  $q^{\text{IS}}(\text{eff})\sim 0.9$ , while a large quenching  $q^{\text{IV}}(\text{eff})\sim 0.7$  is extracted for the IV channel. The similar quenching on the IS spin  $M1$  transitions is obtained by the 20% enhanced IS spin-triplet pairing correlations with the bare spin operator. Positive contributions for the spin-spin correlations are found by the enhanced IS spin-triplet pairing interaction in these  $sd$ -shell nuclei.

The Towner's effective spin operators work well to reproduce the accumulated experimental IV spin strength of  $sd$ -shell nuclei, while the quenching of the effective operators is much larger than the observed one in the IS spin channel. In the past, a large quenching of IS magnetic transition strength was suggested in the literature. However, the electron scattering data in  $^{12}\text{C}$  and the  $(p, p')$  data in Ref. [10] do not show any sign of the large quenching effect on the IS spin transitions. This point should be examined further in other  $N=Z$  nuclei in  $pf$ -shell region by the  $(p, p')$  experiments. Other possible IS probes such as  $(d, d')$  reactions [23] would be also useful to entangle IS spin transitions.

We would like to thank H. Matsubara for providing the experimental data. This work was supported in part by JSPS KAKENHI Grants No. JP16K05367 and No. JP15K05090.

## References

- [1] A. Arima, K. Shimizu, W. Bentz, H. Hyuga, *Advances in Nuclear Physics*, eds. J.W. Negele, E. Vogt (Plenum, New York, 1987), Vol. **18**, p. 1.
- [2] I.S. Towner, Phys. Rep. **155**, 263 (1987)
- [3] G. F. Bertsch, I. Hamamoto, Phys. Rev. C **26**, 1323 (1982)
- [4] A. Bohr, B. R. Mottelson, Phys. Lett. B **100**, 10 (1981)
- [5] M. Rho, Nucl. Phys. A **231**, 493 (1974); E. Oset, M. Rho, Phys. Rev. Lett. **42**, 47 (1979)
- [6] C. Gaarde *et al.*, Nucl. Phys. A **369**, 258 (1981)
- [7] A. Arima, *Proc. of Int. Symposium on New Facet of Spin Giant Resonances in Nuclei*, eds. H. Sakai, H. Okamura, T. Wakasa (University of Tokyo, 1997), p. 3.
- [8] K. Yako *et al.*, Phys. Lett. B **615**, 193 (2005)
- [9] M. Ichimura, H. Sakai, T. Wakasa, Prog. Part. Nucl. Phys. **56**, 446 (2006)
- [10] H. Matsubara *et al.*, Phys. Rev. Lett. **115**, 102501 (2015); and private communications
- [11] C.L. Bai, H. Sagawa, M. Sasano, T. Uesaka, K. Hagino, H.Q. Zhang, X.Z. Zhang, F.R. Xu, Phys. Lett. B **719**, 116 (2013)
- [12] H. Sagawa, C.L. Bai, G. Colò, Phys. Scr. **91**, 083011 (2016)
- [13] Y. Fujita *et al.*, Phys. Rev. Lett. **112**, 112502 (2014)
- [14] Y. Tanimura, H. Sagawa, K. Hagino, Prog. Theor. Exp. Phys. **2014**, 053D02 (2014)
- [15] S.J.Q. Robinson, L. Zamick, Phys. Rev. C **66**, 034303 (2002)
- [16] P. Van Isacker, J. Engel, K. Nomura, Phys. Rev. C **96**, 064305 (2017)
- [17] B.A. Brown, W.A. Richter, Phys. Rev. C **74**, 034315 (2006)
- [18] P. von Neumann-Cosel *et al.*, Nucl. Phys. A **669**, 3 (2000)
- [19] T. Suzuki, R. Fujimoto, T. Otsuka, Phys. Rev. C **67**, 044302 (2003)
- [20] J. B. Flanz *et al.*, Phys. Rev. Lett. **43**, 1922 (1979)
- [21] H. Sagawa, T. Suzuki, M. Sasano, Phys. Rev. C **94**, 041303(R) (2016); H. Sagawa, T. Suzuki, Phys. Rev. C **97**, 054333 (2018)
- [22] C.D. Hoyle *et al.*, Phys. Rev. C **27**, 1244 (1983)
- [23] T. Kawabata *et al.*, Phys. Rev. C **70**, 034308 (2004)