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CANONICAL ANOMALIES AND BROKEN SCALE INVARIANCE*

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ABSTRACT

Canonical behavior of strong interactions at short distances causes an anomaly in the trace identity involving two electromagnetic currents and the energy-momentum tensor. The anomaly is connected with the high energy behavior of $e^+e^- \to \gamma \to$ hadrons and the $\epsilon(700)\gamma\gamma$ coupling constant.

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The ideas of broken scale invariance[1,2] have been applied in three areas of high energy physics. Operator product expansions at short distances and near the light cone have been developed[2] as a phenomenological theory of deep inelastic weak and electromagnetic interactions. Scalar meson dominance of the trace of the energy-momentum tensor,[3,4,5,6] referred to as Partially Conserved Dilatation Current (PCDC)[3] or Partially Zero (0) Trace (POT),[4] has been used to study couplings of the $\epsilon(700)$ meson. Finally, anomalies[7,8] in current algebra calculations, such as the low energy theorem for $\pi^0 \to \gamma\gamma$, have been understood[9] in terms of the canonical singularity structure of multiple products of hadronic currents at short distances.

Here we discuss a result which forges an amusing link between these three applications of scale invariance. We show that if the time-ordered product $T^*(\theta_{\alpha\beta}(0) J_\mu(x) J_\nu(y))$ ($\theta_{\mu\nu}$ and J_μ are respectively the energy momentum tensor and electromagnetic current of the hadrons) has the canonical ϵ^{-10} singularity when $x \sim y \sim \epsilon \to 0$, then there is an anomaly in the trace identity relating

$$\Delta_{\mu\nu}(p) \equiv \int d^4x\, d^4y\, e^{ip\cdot(x-y)} \langle T^* \theta_\lambda{}^\lambda(0) J_\mu(x) J_\nu(y) \rangle_\Omega \qquad (1)$$

to the current propagator

$$\Pi_{\mu\nu}(p) \equiv i \int d^4x\, e^{ip\cdot x} \langle T^* J_\mu(x) J_\nu(0) \rangle_\Omega \quad . \qquad (2)$$

In the trace identity, this canonical anomaly is directly responsible for giving $e^+e^- \to \gamma \to$ hadrons its expected[10] s^{-1} high energy behavior. It is also directly responsible (using POT[4] or PCDC[3]) for a nonvanishing but small $\epsilon\gamma\gamma$ coupling. <u>If the assumptions of canonical short distance behavior and scalar meson</u>

dominance of $\theta_\mu{}^\mu$ are both correct, then we do not expect to see a significant ϵ signal in $\gamma\gamma \to \pi\pi$. This is in contrast to analyses based on finite energy sum rules,[11] according to which there should be a prominent ϵ signal.

Wilson[1] has observed that the anomaly responsible for the $\pi^0 \to \gamma\gamma$ decay can be understood if the hadronic currents have canonical short distance singularities. The standard current algebra derivation[12] of the relevant Ward identity requires partial integrations in configuration space. If $T^*(J_\mu(x) J_\nu(y) A_\lambda(0)) = C_{\mu\nu\lambda}(x,y)\, I + \cdots$ as $x,y \to 0$, where $C_{\mu\nu\lambda}(x,y) \sim \epsilon^{-9}$ for $\epsilon \sim x \sim y \to 0$, then these partial integrations include surface terms which give rise to the anomaly and are determined by the leading singularity of $C_{\mu\nu\lambda}(x,y)$. In analyzing deep inelastic scattering, a model for light cone and short distance singularities derived from basic spin $\frac{1}{2}$ fields has proved successful.[2,13] One may also apply this model to calculate the rate for $\pi^0 \to \gamma\gamma$: the configuration space calculation must then agree with the lowest order perturbation theory calculation performed in momentum space.[7] According to Adler and Bardeen,[8] the result is unaffected by higher order strong and electromagnetic effects, which is not true of the singularities relevant for deep inelastic processes. This is a clue that the $\pi^0 \to \gamma\gamma$ anomaly may be an especially reliable probe of the basic field (or parton) structure of hadrons. The $\pi^0 \to \gamma\gamma$ rate is larger by a factor of about 9 than that calculated using standard fractionally charged quarks. The rate agrees with that calculated from a model with three triplets of fractionally charged quarks, or one triplet of integrally charged quarks. Since deep inelastic data seem to favor fractional charges,[13] the three triplet model seems to be preferred. In this model, the sum of the square of the charges of the fundamental spin $\frac{1}{2}$ fields is 2.

Are there other anomalies besides those related to $\pi^0 \to \gamma\gamma$? We have investigated the trace identity which relates $\Delta_{\mu\nu}(1)$ to $\Pi_{\mu\nu}(2)$, and is naively

$$\Delta_{\mu\nu}(p) = \left(2 - p \cdot \frac{\partial}{\partial p}\right) \Pi_{\mu\nu}(p) . \tag{3}$$

This Ward identity has an anomaly if

$$\int d^4x\, d^4y\, (y_\alpha y_\beta) \frac{\partial}{\partial x_\lambda} \langle T^* D_\lambda(x) J_\alpha(y) J_\beta(-y)\rangle_\Omega \neq 0,$$

where $D_\lambda(x) \equiv x^\rho\, \theta_{\rho\lambda}(x)$ is the scale current. Broken scale invariance[1] suggests that this integral may well be nonvanishing, because for $x \sim y \sim \epsilon \to 0$ we expect

$$\langle T^* \theta_{\lambda\rho}(x) J_\alpha(y) J_\beta(-y)\rangle_\Omega \propto \epsilon^{-10}. \tag{4}$$

We calculate the anomaly using the constituent model for hadronic short distance behavior discussed earlier. It suffices to calculate $\Delta^{(2)}_{\mu\nu}$ and $\Pi^{(2)}_{\mu\nu}$, the two and three point functions given by lowest order perturbation theory (c.f., Figs. 1 and 2), since by assumption $\Delta^{(2)}_{\mu\nu}$ and $\Pi^{(2)}_{\mu\nu}$ have the same short distance behavior as the corresponding hadronic amplitudes, $\Delta_{\mu\nu}$ and $\Pi_{\mu\nu}$. By explicit calculation then,[†,††]

$$\Delta_{\mu\nu}(p) = \left(2 - p \cdot \frac{\partial}{\partial p}\right) \Pi_{\mu\nu}(p) - \frac{R}{6\pi^2} (p_\mu p_\nu - g_{\mu\nu} p^2) \tag{5}$$

where

$$R = \sum_{\substack{i \\ \text{spin } \frac{1}{2}}} Q_i^2 + \frac{1}{4} \sum_{\substack{j \\ \text{spin } 0}} Q_j^2 ,$$

i.e., R is the sum of the square of the charges of spin $\frac{1}{2}$ current constituents plus one fourth of the same quantity for spin 0 constituents. We emphasize that although we have used perturbation theory to evaluate the anomaly, the result is just a consequence of the coefficient of the ϵ^{-10} singularity in (4) and follows from canonical behavior of strong interactions at short distances. The possible existence of such an anomaly has been argued on general grounds by Coleman and Jackiw,[15] and a similar anomaly exists in quantum electrodynamics.[16]

The trace identity (5) could in principle have Callan-Symanzik[16,17] anomalies, as found in higher orders of perturbation theory in renormalizable models of strong interactions. Such anomalies reflect a breakdown of canonical short distance behavior, leading in general to logarithmic violations of scaling. Accordingly we assume they are absent.

The canonical anomaly (5) has intriguing applications both at low and high energies. The naive trace identity (3) has been used to show that if the $\epsilon(700)$ meson dominates $\theta_\mu{}^\mu$ at low momenta, then $g_{\epsilon\gamma\gamma}$ defined by

$$\mathscr{L}_{\epsilon\gamma\gamma} = - \frac{e^2}{2} g_{\epsilon\gamma\gamma} \phi_\epsilon F_{\mu\nu} F^{\mu\nu}$$

should vanish.[5] The anomaly (5) means that the coupling does not vanish:

$$g_{\epsilon\gamma\gamma} = \frac{R}{12\pi^2} \frac{1}{F_\epsilon} \tag{6}$$

where F_ϵ is defined by

$$\langle\Omega|\theta_\mu{}^\mu(0) \,|\, \epsilon(p)\rangle = m_\epsilon^2 F_\epsilon .$$

Essentially the same result (with R = 1) was obtained in 1851 by Schwinger,[18] who evaluated the fermion loop for an $\epsilon\gamma\gamma$ coupling. As we have already remarked, the estimate (6) is much smaller[†††] than that obtained by other methods, and suggests that in the two photon process $ee \rightarrow ee\pi\pi$ the $\epsilon(700)$ signal will be at least $1\frac{1}{2}$ orders of magnitude smaller than the Born term background.

Also interesting are the high energy manifestations of the anomaly, Eq. (5). Because $\theta_\mu^{\ \mu}$ is a soft operator (or, in perturbation theory, by Weinberg's theorem[19]), the left hand side of Eq. (5) becomes negligible compared to the right hand side as p^2 becomes large. If there were no anomaly, then we would have asymptotically that

$$\left(2 - p \cdot \frac{\partial}{\partial p}\right) \Pi_{\mu\nu}(p) = 0 \tag{6}$$

which implies that the absorptive part of $\Pi_{\mu\nu}$ grows more slowly than p^2 and that $\sigma(e^+e^- \rightarrow \text{hadrons})$ falls faster than $\frac{1}{s}$, contradicting what we expect on the basis of canonical short distance behavior. This serves to emphasize the canonical nature of the anomaly. To regain the anticipated canonical behavior for $\sigma(e^+e^- \rightarrow \text{hadrons})$, we must include the anomaly in (6), so that the asymptotic equation is

$$\left(2 - p \cdot \frac{\partial}{\partial p}\right) \Pi_{\mu\nu}(p) = + \frac{R}{6\pi^2}\left(p_\mu p_\nu - g_{\mu\nu} p^2\right), \tag{7}$$

which as $p^2 \rightarrow \infty$ has the solution

$$\Pi_{\mu\nu}(p) = -\left(p_\mu p_\nu - g_{\mu\nu} p^2\right) \frac{R}{12\pi^2} \log p^2. \tag{8}$$

$\Pi_{\mu\nu}$ as given by (8) has a nonvanishing absorptive part, from which we regain the result expected from canonical short distance behavior of $\left[J^\mu(x), J^\nu(0)\right]$, that is,

$$\frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \xrightarrow[p^2 \to \infty]{} R. \qquad (9)$$

Indeed, anomalies of the form of (5) must be a general feature of trace identities involving fields with integral dimensions. Consider a field $\phi(x)$ with scale dimension d: the naive expectation is that the absorptive part of the propagator will scale as $(p^2)^{d-2}$. If d is not an integer then by analyticity the dispersive part will also scale as $(p^2)^{d-2}$. However, if d is integral and the absorptive part is nontrivial, then the dispersive part must scale as $(p^2)^{d-2}\log p^2$, †††† and we find for large p^2 that

$$\left(2d - 4 - p\,\frac{\partial}{\partial p}\right)G(p) = A \neq 0,$$

i.e., that the trace identity has an anomaly. In general there need not be such anomalies for fields with nonintegral dimensions: unless scale invariance is badly broken,[9] currents are required to have dimension 3, hence the anomaly (5).

We have shown that a canonical short-distance anomaly is connected with both the e^+e^- annihilation cross section at high energies and the low energy theorem for $\epsilon \to \gamma\gamma$. A presentation of the details of this work, together with a discussion of other trace identities involving currents and other questions is in preparation.

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FOOTNOTES

† As in the case of the $\pi^o \to 2\gamma$ anomaly, it is not possible to restore the trace identity by adding subtraction polynomials to either $\Delta^{(2)}_{\mu\nu}$ or $\Pi^{(2)}_{\mu\nu}$.

†† After this work was completed, we received a preprint by Crewther[14] which contains an elegant configuration space analysis of the $\pi^o \to \gamma\gamma$ anomaly and sketches a derivation of the trace anomaly (5).

††† Using $m_\epsilon \approx 700$ MeV and $F_\epsilon \approx 150$ MeV, we obtain $\Gamma(\epsilon \to 2\gamma) \approx 0.2\ R^2$ keV.

†††† In field algebra models, where the spatial components of the currents have dimension 1, the short-distance argument yields no anomaly. This reflects the fact that the absorptive part of the propagator is trivial at the canonical level, being $\propto \delta(p^2 - m^2)$, so that the dispersive part is not $\propto (p^2)^{-1} \log p^2$.

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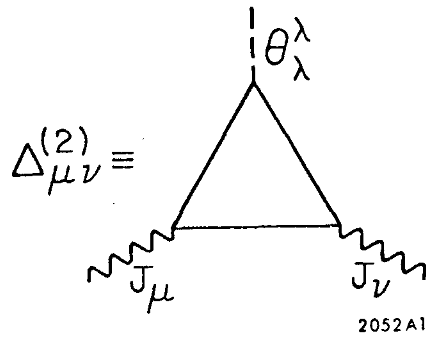

Fig. 1

Lowest order contribution to $\theta_\lambda{}^\lambda J_\mu J_\nu$ vertex.

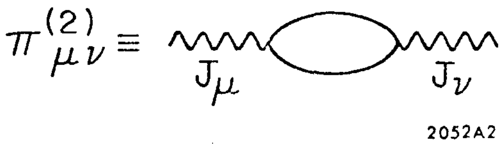

Fig. 2

Lowest order contribution to current propagator.