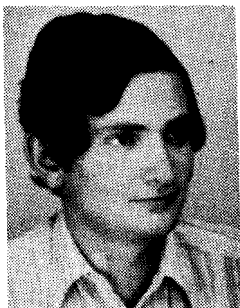


FORMATION AND OBSERVABLES OF THE QUARK-GLUON PLASMA

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The hadronic gas and the perturbative quark-gluon plasma phases of hadronic matter are described. Strangeness produced in the plasma may lead to observable anomalies in strange particle abundances.

1. Overview

What purpose could we follow when arguing for the study of high energy nuclear collisions¹⁾? It would appear that the complexity of such collisions involving several hundreds of valence quarks must cover up all the interesting features of fundamental interactions. I would like to argue in this lecture that the nature and properties of strong interactions can be studied by creating a new state of matter in the laboratory - the quark-gluon plasma²⁾. Unlike hadron-hadron collisions we anticipate that in an important fraction of nucleus-nucleus collisions each participating quark will scatter many times before joining into an asymptotic hadronic state. The associated simplification of the involved physics arises because we can use in such a case the well established methods of statistical physics in order to connect the microscopic world with effects and properties visible to experimentalists eyes. Alone the presumption of an approximate thermochemical equilibrium frees us from the dependence on details of quark wavefunctions in a small hadronic bag consisting of only few quarks.

There are several stages in this new and exciting field of high energy physics. The first one concerns the willingness to accept the fact that available energy is equipartitioned among accessible degrees of freedom. This means that there exists a domain in space, in which, in a proper Lorentz frame, the energy of the longitudinal motion has been largely transformed to transverse degrees of freedom. We call this region 'fireball'. The physical variables characterising a fireball are: energy density, baryon number density and its volume. The basic question concerns the internal structure of the fireball - it can consist either of individual hadrons or, instead, of quarks and gluons in a new physical phase: they are deconfined and can move freely over the volume of the fireball. It appears that the phase transition from the hadronic gas phase to the quark gluon plasma is mainly controlled by the energy density of the fireball. Several estimates²⁾, lead to $0.6-1 \text{ GeV/fm}^3$ for the critical energy density, to be compared with ca 0.16 GeV/fm^3 inside individual hadrons. Many theoretical questions about strong interactions will be settled if the parameters and nature of the phase transition are determined. We turn to these problems further below.

The second stage of the developments in this field concerns

the interaction of the experimentalists with the plasma. It is quite difficult to insert a thermometer and to measure baryon density at $T=150$ MeV and threefold or even higher nuclear compressions. We must either use only electromagnetically interacting particles³⁾ (photons, lepton pairs) in order to get them out of the plasma or study the heavy flavour abundance generated in the collision⁴⁾. To obtain a better impression of what is meant imagine that strange quarks are very abundant in the plasma (and indeed they are!). Then, since (sss)-state is bound and stable in the hot perturbative QCD-vacuum, it would be the most abundant baryon to emerge from the plasma. I doubt that such an Ω -sation of nuclear matter could leave any doubts about the occurrence of the phase transition. Other exotic hadrons⁵⁾ such as eg. $csq, c\bar{s}$ etc. would support this conclusion. But even the enhancement of the more accessible abundances of $\bar{\Lambda}$ may already be sufficient for our purposes.

But there is more to meet the eyes-restoration of the perturbative QCD vacuum may be followed at higher and higher energy densities by restoration of chiral symmetry, as shown qualitatively in Fig.1., than SU(2) symmetry and finally SU(5) symmetry. And if the fact that we can trace back the evolution of the universe⁶⁾ in the laboratory does not excite your fantasy, then remember that the

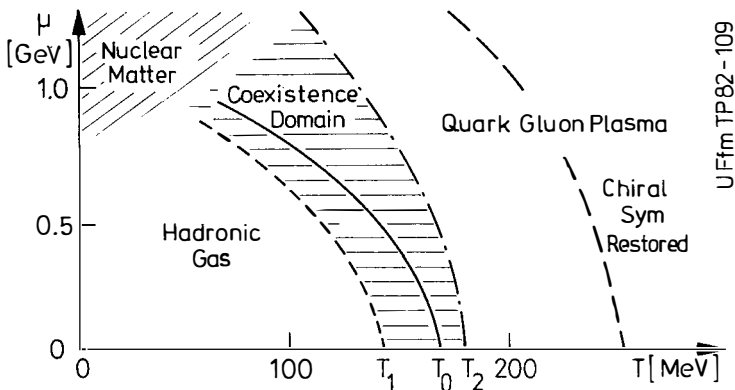


Fig. 1.: Phase diagramm of hadronic matter in the μ - T plane

plasma state is the only place I know of (after the universe was created) where we can burn baryon number, thus releasing the energy from the big bang stored in matter. Perhaps sufficiently extreme conditions that are here necessary are 'created' inside Quasars, leading to the enormous energies radiated by these stellar objects.

Coming back to earth we begin by recalling that in statistical description of matter the unhandy microscopical variables: energy, baryon number etc. are replaced by thermodynamical quantities; temperature T is a measure of energy per degree of freedom, baryon chemical potential μ controls the mean baryon density: Statistical quantities such as; entropy (=measure of the number of disposable states), pressure, heat capacity etc. will be also functions of T and μ to be determined. The theoretical techniques required for the description of both quite different phases: the hadronic gas and the quark gluon plasma, must allow for the formation of numerous hadronic resonances on the one side⁷⁾, which then dissolve at sufficiently high partial density in the state consisting of its constituents. At this point we must appreciate the importance and help provided by finite temperature. To obtain high particle density we may, instead of compressing the matter (which as it turns out is quite difficult), heat it up; many pions are generated easily, leading to the occurrence of the transition at moderate (even vanishing) baryon density⁸⁾.

2. Thermodynamics of Interacting Hadrons

The main hypothesis which allows to simplify the situation is to postulate the resonance dominance of hadron-hadron interactions⁷⁾ - in this case the hadronic gas phase is practically a superposition of an infinity of different hadronic gases and all information about the interaction is hidden in the mass spectrum $\tau(m^2, b)$ which describes the number of hadrons of baryon number b in a mass interval dm^2 ⁹⁾.

We pursue in the following the developments of Ref. (8,9): Assume that the mass spectrum $\tau(m^2, b)$ is already known, then grand microcanonical level density is given by an invariant phase space integral. The extreme richness of the spectrum $\tau(m^2, b) \sim \exp(m/T_0)$ enables us to neglect Fermi and Bose statistics above $T \approx 50$ MeV and to treat all particles as "Boltzmannions". We find:

$$\sigma(p, V_{\text{ex}}, b) = \delta^4(p) \delta_K(b) + \sum_{N=1}^{\infty} \frac{1}{N!} \delta^4(p - \sum_{i=1}^N p_i) \sum_{\{b_i\}} \delta_K(b - \sum_{i=1}^N b_i) \prod_{i=1}^N \frac{2\Delta_i p_i^\mu}{(2\pi)^3} \tau(p_i^2, b_i) d^4 p_i \quad (1)$$

Above, the first term corresponds to the vacuum state. The N^{th} term is the sum over all possible partitions of the total baryon number and the total momentum p among N Boltzmannions, each having an internal number of quantum states given by $\tau(p_i^2, b_i)$. These Boltzmannions are hadronic resonances of baryon number b_i ($-\infty < b_i < \infty$). Every resonance can move freely in the remaining volume Δ left over from the external volume V_{ex} after subtracting the proper volumina V_c of all hadrons:

$$\Delta^\mu = V_{\text{ex}}^\mu - \sum_{i=1}^N V_{c,i}^\mu \quad (2)$$

V^μ is a covariant generalisation of V_i ; in the restframe we have $V_\mu = (V, 0)$.

In the generalisation (1) of the popular phase space formula, three essential features of hadronic interactions are now explicitly included:

- a) The dense set of hadronic resonances dominating particle scattering via $\tau(m_i^2, b_i)$.
- b) The proper natural volumes of hadronic resonances via Δ^μ .
- b) Conservation of baryon number and clustering of hadrons into lumps of matter with $|b| > 1$.

Thermodynamic properties of hot hadronic gas follow from the study of the grand partition function $Z(\beta, V, \lambda)$, as obtained from the level density $\sigma(p, V, b)$:

$$Z(\beta, V, \lambda) = \sum_{b=-\infty}^{\infty} \lambda^b \int e^{-\beta \cdot p} \sigma(p, V, b) d^4 p \quad (3)$$

Covariant generalisation of thermodynamics with the inverse temperature four vector β_μ has been here used. In the rest frame of the relativistic baryon chemical potential μ :

$$\lambda = \exp(\mu/T) ; \quad (4)$$

it is introduced in order to conserve the baryon number in the

statistical ensemble. All quantities of physical interest can be derived as usual by differentiating $\ln Z$ with respect to its variables.

Eqs. 1-3 leave us with the task of finding the mass spectrum τ . Experimental knowledge of τ is limited to low excitations and/or low baryon number. Following Hagedorn I introduce here a theoretical model: "the statistical bootstrap", in order to obtain a mass spectrum consistent with direct and indirect experimental evidence. The qualitative arguments leading to an integral equation for $\tau(m^2, b)$ are the following: when V_{ex} in Eq.(1) is just the proper volume V_c of a hadronic cluster, then σ in Eq.(1), up to a normalization factor, is essentially the mass spectrum τ : indeed, how could we distinguish between a composite system [as described by Eq.(1)] compressed to the natural volume of a hadronic cluster and an "elementary" cluster having the same quantum numbers? Thus we demand

$$\sigma(p, V, b) \Big|_{V=V_c} \triangleq H\tau(p^2, b) \quad (5)$$

where the "bootstrap constant" H is to be determined below. It is not sufficient simply to insert Eq.(5) into Eq.(1) to obtain the bootstrap equation for τ ; more involved arguments are necessary^{8,9)} in order to obtain the following "bootstrap equation" for the mass spectrum:

$$H\tau(p^2, b) = H z_b \delta_o(p^2 - M_b^2) + \sum_{N=2}^{\infty} \frac{1}{N!} \int d^4(p - \sum_{i=1}^N p_i) \sum_{\{b_i\}} \delta_K(b - \sum_{i=1}^N b_i) \prod_{i=1}^N H\tau(p_i^2, b_i) d^4 p_i \quad (6)$$

The first term is the lowest one-particle contribution to the mass spectrum, z_b is its statistical weight $(2I+1)(2J+1)$. The index "o" restricts the δ function to the positive root only. Only terms with $b=0, \pm 1$, corresponding to lowest energy $q\bar{q}$ (pions) and qqq (nucleons) states contribute in the first term of Eq.(6). All excitations are contained in the second term as arbitrary quark constant can be achieved by combining $[(q\bar{q})^n (qqq)^m]$. Heavy flavours are ignored at this point and can easily be introduced - however they do not essentially influence the behaviour of τ . In the course of deriving the bootstrap equation (6) it turns out that the cluster volume V_c grows proportional to the invariant cluster mass⁹⁾

$$v_c(p^2) = \sqrt{p^2}/(4B) . \quad (7)$$

The proportionality constant has been called $4B$ in order to establish a close relationship with the quark bag model¹⁰⁾. The value of B can be derived from different considerations involving the true and perturbative QCD states. While the original MIT-bag fit has been $B^{1/4} = 145$ MeV the most generally accepted value today is perhaps

$$\begin{aligned} B^{1/4} &= 190 \text{ MeV} \\ B &= 170 \text{ MeV/fm}^3 \end{aligned} \quad (8)$$

The bootstrap constant H and the bag constant B are the only seemingly free parameters of this approach. As just pointed out, B is determined from other considerations, while H turns out to be inversely proportional to B . Hence if one wishes to believe the last detail of the statistical bootstrap approach, there remains no free parameter in this approach. What this means for the transition from gas to plasma will be now shown.

Instead of solving Eq.(6), which will lead us to the exponential mass spectrum⁷⁾,

$$\tau(m^2, b) \sim e^{m/T_0} \quad (9)$$

we wish to concentrate here on the double integral (Laplace) transform of Eq.(6) which will be all we need to establish the physical properties of the hadronic gas phase. Introducing the transforms of the one particle term, Eq.(6)

$$\varphi(\beta, \lambda) := \sum_{b=-\infty}^{\infty} \lambda^b \text{Hz}_b \delta_0(p^2 - M_b^2) e^{-\beta \cdot p} d^4 p \quad (10)$$

with pions and nucleons only

$$\varphi(\beta, \lambda) = 2\pi HT \left[3m K_1\left(\frac{m}{T}\right) + 4\left(\lambda + \frac{1}{\lambda}\right) m_N K_1\left(\frac{m_N}{T}\right) \right], \quad (11)$$

and of the mass spectrum:

$$\phi(\beta, \lambda) := \sum_{b=-\infty}^{\infty} \lambda^b \int H \tau(p^2, b) e^{-\beta \cdot p} d^4 p \quad (12)$$

We find for the entire Eq.(6) the simple relation

$$\phi(\beta, \lambda) = \varphi(\beta, \lambda) + \exp[\phi(\beta, \lambda)] - \phi(\beta, \lambda) - 1 \quad (13)$$

To study the behaviour of $\phi(\beta, \lambda)$ we make use of the apparent implicit dependence:

$$\phi(\beta, \lambda) = G(\varphi(\beta, \lambda)) \quad (14a)$$

with function G being defined by Eq. (13)

$$\varphi = 2G + 1 - \exp[G] \quad (14b)$$

This function $G(\varphi)$ is shown in Fig. 2

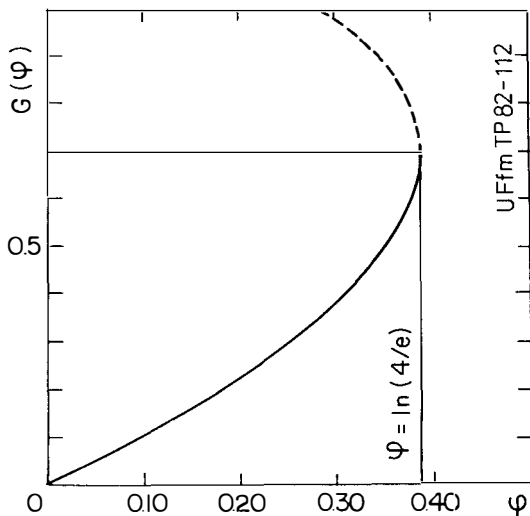


Fig. 2: Bootstrap function $G(\varphi)$ - the dashed line represents the unphysical branch.

As is apparent there is a maximal value φ_0

$$\varphi_0 = \ln(4/e) = 0.3863.. \quad (14c)$$

beyond which the function G has no real solutions. Recalling the

physical meaning of G , Eq. (14a,12) we conclude that Eq.(14c) establishes a boundary of λ (i.e. μ) and T values beyond which the hadronic gas phase cannot exist. This boundary is implicitly given by the relation (11):

$$\ln(4/e) = 2\pi HT_{cr} \left[3m_{\pi} K_1 \left(\frac{m_{\pi}}{T_{cr}} \right) + 8 \cosh \left(\frac{\mu_{cr}}{T_{cr}} \right) m_N K_1 \left(\frac{m_N}{T_{cr}} \right) \right] \quad (15)$$

shown in Fig. 3.

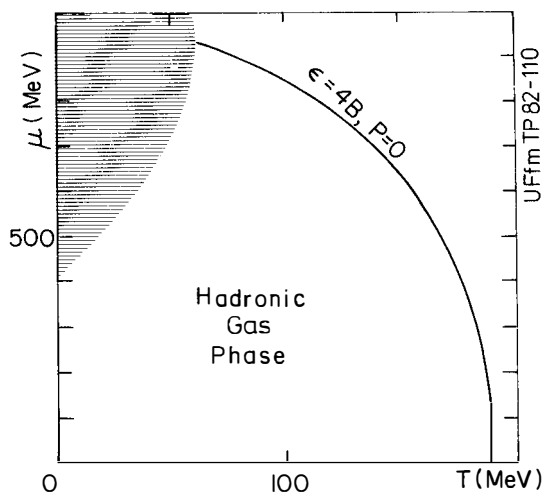


Fig. 3: Boundary to the 'hadronic gas phase' in the bootstrap model. In the shaded region quantum statistics can not be neglected.

The region denoted 'Hadronic Gas Phase' is described by our current approach. For the discussed choice of the parameters we find that

$$T_{cr}(\mu_{cr}=0) = T_0 \sim 160-170 \text{ MeV}. \quad (16)$$

Note that $\mu=0$ implies zero baryon number of the plasma state. For $\mu_0 = \mu_{cr}(T_{cr}=0)$ the solution of Eq.(15) is simply $\mu_{cr} \sim m_N$ since no quantum statistics effects have been included. Thus the dashed region in Fig. 2 'nuclear matter' is excluded from our considerations. As we shall shortly see, the boundary to the hadronic gas

phase is also characterized by a constant energy density $\epsilon=4B$.

Given the function $G(\varphi) = \phi(\beta, \lambda)$ we can in principle study the form of the hadronic mass spectrum. As it turns out we can obtain the partition function directly from ϕ : The formal similarity between Eq.(3) and Eq.(12) can be exploited to derive a relation between their integral transforms¹⁾; (from here on: $\beta = \sqrt{\beta_\mu \beta_\mu}$)

$$\ln Z(\beta, V_{\text{ex}}, \lambda) = - \frac{2\Delta(V_{\text{ex}})}{H \cdot (2\pi)^3} \frac{\partial}{\partial \beta} \phi(\beta, \lambda) \quad (17)$$

which can also be written in a form which makes the different physical inputs more explicit:

$$\ln Z(\beta, V_{\text{ex}}, \lambda) = \frac{\Delta(V_{\text{ex}})}{V_{\text{ex}}} \cdot \frac{\partial G(\varphi)}{\partial \varphi} \cdot Z_1(\beta, \lambda, V) \quad (18)$$

In absence of finite hadronic volume and interactions described by the first two terms in turn, we would simply have an ideal Boltzmann gas, described by the one particle partition function Z_1 :

$$Z_1 = Z_{\text{qq}}^- + 2 \cosh(\mu/T) Z_{\text{qqq}} \quad (19)$$

where

$$Z_{\text{qqq}}^- / Z_{\text{qq}} \approx (2I+1)(2S+1) \frac{V_T^3}{2\pi^2} \left(\frac{m_\pi/N}{T}\right)^2 K_2\left(\frac{m_\pi/N}{T}\right) \quad (20)$$

Let us now briefly discuss the rôle of the available volume: as we have explicitly assumed, all hadrons have an internal energy density $4B$ (actually at finite pressure there is a small correction, see Ref.4a for details). Hence the total energy of the fireball E_F can be written as

$$E_F \equiv \epsilon V_{\text{ex}} = 4B(V_{\text{ex}} - \Delta) \quad (21)$$

where $V_{\text{ex}} - \Delta$ is the volume occupied by hadrons. We thus find

$$\Delta = V_{\text{ex}} - E_F / 4B = V_{\text{ex}} (1 - \epsilon / (4B)) \quad (22)$$

when working out the relevant physical consequences we must always remember that the fireball is an isolated physical system, for which the statistical approach has been taken in view of the internal disorder (high number of available states) rather than because of a coupling to a heat bath.

The remainder of the discussion of the hadronic gas is a simple application of the rules of statistical thermodynamics. By investigating the meaning of the thermodynamic averages it turns out that the apparent (β, λ) dependence of the available volume Δ in Eq.(18) must be disregarded when differentiating $\ln Z$ with respect to β and λ . As Eq.(1) shows explicitly, the density of states of extended particles in V_{ex} is the same as that of point particles in Δ . Therefore also

$$\ln Z(\beta, V_{ex}, \lambda) \equiv \ln Z_{pt}(\beta, \Delta, \lambda) \quad (23)$$

We thus first calculate the point particle energy, and baryon number densities, pressure, and entropy density

$$\epsilon_{pt} = - \frac{1}{\Delta} \frac{\partial}{\partial \beta} \ln Z_{pt} = - \frac{2}{H (2\pi)^3} \frac{\partial^2}{\partial \beta^2} \phi(\beta, \lambda) \quad (24)$$

$$v_{pt} = \frac{1}{\Delta} \lambda \frac{\partial}{\partial \lambda} \ln Z_{pt} = - \frac{2}{H (2\pi)^3} \lambda \frac{\partial^2}{\partial \lambda \partial \beta} \phi(\beta, \lambda) \quad (25)$$

$$p_{pt} = \frac{T}{\Delta} \ln Z_{pt} = - \frac{2T}{H (2\pi)^3} \frac{\partial}{\partial \beta} \phi(\beta, \lambda) \quad (26)$$

$$s_{pt} = \frac{1}{\Delta} \frac{\partial}{\partial T} (T \ln Z_{pt}) = \frac{p_{pt}}{T} + \frac{\epsilon_{pt} - \mu v_{pt}}{T} \quad (27)$$

From this, we easily find the energy density, as

$$\epsilon = \frac{\langle E \rangle}{V_{ex}} = - \frac{1}{V_{ex}} \frac{\partial}{\partial \beta} \ln Z(\beta, V_{ex}, \lambda) = \frac{\Delta}{V_{ex}} \cdot \epsilon_{pt} \quad (28)$$

Inserting Eq.(22) into Eq.(28) and solving for ϵ we find:

$$\epsilon(\beta, \lambda) = \frac{\epsilon_{pt}(\beta, \lambda)}{1 + \epsilon_{pt}(\beta, \lambda)/4B} \quad (29)$$

And hence also another form of Eq.(22):

$$V_{ex} = \Delta \cdot (1 + \epsilon_{pt}(\beta, \lambda)/4B) \quad (30)$$

and similary for the baryon density, pressure and entropy density

$$v = \frac{v_{pt}}{1 + \epsilon_{pt}/4B} \quad (31)$$

$$p = \frac{p_{pt}}{1 + \epsilon_{pt}/4B} \quad (32)$$

$$s = \frac{s_{pt}}{1 + \epsilon_{pt}/4B} \quad (33)$$

We now have a complete set of equations of state for observable quantities as functions of the chemical potential μ , temperature T and external volume V_{ex} . While these equations are semi-analytic, one has to evaluate the different quantities numerically due to the implicit definition of $\phi(\beta, \lambda)$ that determines $\ln Z$. However, when β, λ approach the critical curve, Fig. 3, we easily find from the singularity of ϕ that ϵ_{pt} diverges and therefore

$$\begin{aligned} \epsilon &\longrightarrow 4B \\ p &\longrightarrow 0 \\ \Delta &\longrightarrow 0 \end{aligned} \quad (34)$$

These limites indicate that at the critical line, matter has lumped into one large cluster with the energy density $4B$. No free volume is left, and as only one cluster is present, the pressure has vanished. However, the baryon density varies along the critical curve; it falls with increasing temperature. This is easily understood: as temperature is increased, more mesons are produced that take up some of the available space. Therefore hadronic matter can saturate at lower baryon density. We further note here that in order to properly understand the approach to the phase boundary, one has to incorporate and understand the properties of the hadronic world beyond the critical curve. We turn now to the study of the perturbative quark-gluon plasma phase.

3. QCD and the Quark-Gluon Plasma

We begin with a summary of the relevant postulates and results that characterize the current understanding of strong interactions in quantum chromodynamics (QCD). The most important postulate is that the proper vacuum state in QCD is not the (trivial) perturbative state that we (naively) imagine to exist everywhere and which is little changed when the interactions are turned off/on. In QCD the true vacuum state is believed to have a complicated structure which originates in the glue (gauge) sector of the theory. The perturbative vacuum is an excited state with an energy density B above the true vacuum. It is to be found inside hadrons where perturbative quanta of the theory, in particular quarks, can therefore exist.

The occurrence of the true vacuum state is intimately connected to the glue-gluon interaction; gluons also carry the colour charge that is responsible for the quark-quark interaction. In the above discussion, the confinement of quarks is a natural feature of the hypothetical structure of the true vacuum.

Another feature of the true vacuum is that it exercises a pressure on the surface of the region of the perturbative vacuum to which quarks are confined. Indeed, this is just the idea of the original MIT bag model¹⁰⁾. The Fermi pressure of almost massless light quarks is in equilibrium with the vacuum pressure B . When many quarks are combined to form a giant quark bag, then their properties inside can be obtained using standard methods of many-body theory²⁾. In particular, this also allows the inclusion of the effect of internal excitation through a finite temperature and through a change in the chemical composition.

A further effect that must be taken into consideration is the quark-quark interaction. We shall use here the first order contribution in the QCD running coupling constant $\alpha_s(q^2) = g^2/4\pi$. However, as $\alpha_s(q^2)$ increases when the average momentum exchanged between quarks decreases, this approach will have only limited validity at relatively low densities and/or temperatures. The collective screening effects in the plasma are of comparable order of magnitude and should reduce the importance of perturbative contribution.

As u and d quarks are almost massless inside a bag, they can be produced in pairs and at moderate temperatures many $q\bar{q}$ pairs will be present. In particular also $s\bar{s}$ pairs will be produced and we will return to this point below. Furthermore, real gluons can be present when $T \neq 0$ and will be included here in our considerations.

As was outlined in the previous section, a complete description of the thermodynamical behaviour of a many-particle system can be derived from the grand partition function Z . For the case of the quark-gluon plasma in the perturbative vacuum, one finds an analytic expression through first order in α_s neglecting the quark masses. We obtain for the quark Fermi gas^{2b)}

$$\begin{aligned} \ln Z_q(\beta, \lambda) = & \frac{gV}{6\pi^2} \beta^{-3} \left[\left(1 - \frac{2\alpha_s}{\pi}\right) \left(\frac{1}{4} \ln^4 \lambda_q + \frac{\pi^2}{2} \ln^2 \lambda_q\right) + \right. \\ & \left. + \left(1 - \frac{50}{21} \frac{\alpha_s}{\pi}\right) \frac{7\pi^4}{60} \right] \end{aligned} \quad (35)$$

where $g = (2s+1)(2I+1)N=12$ counts the number of the components of the quark gas, and λ_q is the fugacity related to the quark number.

As each quark has baryon number $1/3$, we find

$$\lambda_q^3 = \lambda = e^{\mu/T} \quad (36)$$

where λ , as in last section, allows the conservation of the baryon number. Consequently

$$3\mu_q = \mu \quad (37)$$

The glue contribution is²⁾

$$\ln Z_g(\beta, \lambda) = V \frac{8\pi^2}{45} \beta^{-3} \left(1 - \frac{15\alpha_s}{4\pi}\right). \quad (38)$$

We notice the two relevant differences with the photon gas: (i) the occurrence of the factor eight associated with the number of gluons; (ii) the glue-gluon interaction since gluons carry the colour charge.

Finally, let us introduce the true vacuum term as

$$\ln Z_{\text{vac}} = -\beta B V \quad (39)$$

This leads to the required positive energy density B within the volume occupied by the coloured quarks and gluons and to a negative pressure on the surface of this region. At this stage, this term is entirely phenomenological as discussed above. The equations of state for the quark-gluon plasma are easily obtained by differentiating

$$\ln Z = \ln Z_q + \ln Z_g + \ln Z_{\text{vac}} \quad (40)$$

with respect to β , λ and V . The energy density, baryon number density, pressure and entropy density are respectively, written in terms of μ and T

$$\begin{aligned} \epsilon = \frac{6}{\pi^2} \left[\left(1 - \frac{2\alpha_s}{\pi}\right) \left(\frac{1}{4}\left(\frac{\mu}{3}\right)^4 + \frac{1}{2}\left(\frac{\mu}{3}\right)^2 (\pi T)^2\right) + \left(1 - \frac{50}{21} \frac{\alpha_s}{\pi}\right) \frac{7}{60} (\pi T)^4 \right] \\ + \frac{8}{15\pi^2} (\pi T)^4 \left(1 - \frac{15}{4} \frac{\alpha_s}{\pi}\right) + B \end{aligned} \quad (41)$$

$$v = \frac{2}{3\pi^2} \left[\left(1 - \frac{2\alpha_s}{\pi}\right) \left(\left(\frac{\mu}{3}\right)^3 + \frac{\mu}{3} (\pi T)^2\right) \right] \quad (42)$$

$$p = \frac{1}{3}(\epsilon - 4B) \quad (43)$$

$$s = \frac{2}{\pi} \left(1 - \frac{2\alpha_s}{\pi}\right) \left(\frac{\mu}{3}\right)^2 (\pi T)^2 + \frac{14}{15\pi} \left(1 - \frac{50}{21} \frac{\alpha_s}{\pi}\right) (\pi T)^3 + \frac{32}{45\pi} \left(1 - \frac{15}{4} \frac{\alpha_s}{\pi}\right) (\pi T)^3 \quad (44)$$

In Eqs. (41,44) the second T^4 respect. T^3 term originates in the gluonic degrees of freedom. In Eq.(43) we have right away used the relativistic relation between the quark and gluon energy density and pressure

$$p_q = \frac{1}{3} \epsilon_q, \quad p_g = \frac{1}{3} \epsilon_g \quad (45)$$

in order to derive this simple form of the equation of state.

This simple equation of state of the quark-gluon plasma is slightly modified when the finite quark masses are considered, or when the QCD coupling constant α_s is dependent on the dimensional parameter Λ . From Eq.(43) it follows that when the pressure vanishes, the energy density is $4B$, independent of the values of μ and T which fix the line $P = 0$. We recall that this has been precisely the kind of behaviour found for the hadronic gas. This coincidence of the physical observables strongly suggests that in the exact calculations both lines $P = 0$ coincide; we will return to this point again below. For $P > 0$ we have $\epsilon > 4B$ - we recall that in the hadronic gas we always had $\epsilon \leq 4B$. Thus, in this domain of $\mu - T$ we have the quark-gluon plasma exposed to an external force.

In order to obtain an idea of the form of the $P = 0$ critical curve in the μ - T plane for the quark-gluon plasma, we rewrite Eq. (43) for $P=0$:

$$B = \frac{(1 - \frac{2\alpha_s}{\pi})}{162\pi^2} [\mu^2 + (3\pi T)^2]^2 - \frac{T^4 \pi^2}{45} \left[\left(1 - \frac{5\alpha_s}{3\pi}\right)^2 - \left(1 - \frac{15\alpha_s}{4\pi}\right) \cdot 8 \right] \quad (46)$$

Here, the last term is the glue pressure contribution. We find that the greatest lower bound on temperature T_q at $\mu=0$ is about ($\alpha_s=1/2$)

$$T_q \approx .83B^{1/4} \sim 160 \text{ MeV} \approx T_0 \quad (47)$$

This result shows the expected order of magnitude. The most remarkable point is, that it leads for $B^{1/4}=190 \text{ MeV}$ to almost exactly the same value as found in the hadronic gas study of

the last section.

Let us further note here that for $T \ll \mu$ the baryon chemical potential tends to

$$\mu_B = 3\mu_q \Rightarrow 3B^{1/4} \left[\frac{2\pi^2}{2\alpha_s} \right]^{1/4} = 1320 \text{ MeV} [\alpha_s=1/2, B^{1/4}=190 \text{ MeV}] \quad (48)$$

$$(1 - \frac{\alpha_s}{\pi})$$

In concluding this discussion of the $P=0$ line of the quark-gluon plasma, let us note that the choice $\alpha_s \sim 1/2$ is motivated by the fits to the charmonium and ypsilononium spectra as well as to deep inelastic scattering. In both these cases spacelike domains of momentum transfer are explored. The much smaller value of $\alpha_s \sim 0.2$ is found in timelike regions of momentum transfer in $e^+e^- \rightarrow$ hadrons experiments. In the quark-gluon plasma, as described up to first order perturbation theory, positive and negative momenta transfers occur: the perturbative corrections to the radiative T^4 contribution is dominated by timelike momenta transfers, while the correction to the μ^4 term originates from spacelike quark-quark scattering. Finally we consider the energy density at $\mu=0$. We find the simple result, restating some factors again:

$$\epsilon = B + \frac{\pi^2}{30} T^4 \left[2_s \cdot 8_c \cdot \left(1 - \frac{15}{4} \frac{\alpha_s}{\pi}\right) + 2_l \cdot 2_s \cdot 2_c \frac{7}{4} \left(1 - \frac{50}{21} \frac{\alpha_s}{\pi}\right) \right] \quad (49)$$

We note that in both quarks and gluons the interaction conspires to reduce the effective number of degrees of freedom disponible. At $\alpha_s=0$ we find the handy relation

$$\epsilon_q + \epsilon_g = \left(\frac{T}{160 \text{ MeV}} \right)^4 \left[\frac{\text{GeV}}{\text{fm}^3} \right] \quad (50)$$

At $\alpha_s=1/2$ we are seemingly left with only ~50% of the degrees of freedom, and the temperature 'unit' in the above formula drops to 135 MeV.

I have so far neglected to include heavy flavours into the description. For charm, with the mass of about 1500 MeV the thermodynamic abundance is sufficiently low to ignore its influence on the properties of the plasma. Also, even the equilibrium abundance is quite small - I find evaluating the phase-space integrals that the ratio of charm to light antinflavour (either $\bar{u}$ or $\bar{d}$) is

$$c/\bar{q} = \bar{c}/\bar{q} = e^{-(m_c - \mu/3)T} \left(\frac{m_c}{T} \right)^{3/2} \frac{1}{2} \sqrt{\frac{\pi}{2}} \quad (51)$$

Taking as a numerical example $m_c = 1500$ MeV, $T = 200$ MeV, $\mu = 0$ one finds with $c/\bar{q} = 7 \cdot 10^{-3}$ a small, but still quite significant abundance. However, the approach to chemical equilibrium (see below) is to be studied to establish if chemical equilibrium assumption is justified. We note that the energy fraction carried by the intrinsic plasma charm would be $\sim 0,2\%$ in the above example.

Clearly, we must turn our attention to strangeness - with a current quark mass of about 180 MeV we are actually above the threshold and indeed one finds that there is a quite appreciable s-abundance (see over next section). An explicit calculation^{4b)} has shown that chemical equilibrium will be reached during the short time of a heavy ion reaction. The motion of the particles being already semirelativistic, an increase of about 15% of the number of available degrees of freedom, Eq. (49), is recorded due to $s\bar{s}$ production. The appearance of strangeness is a very important qualitative factor and we will return to its discussion in the section 5.

4. Phase Transition from Hadronic Gas to the Quark-Gluon Plasma

We have now shown that two inherently different descriptions lead to the prediction of qualitatively similar region for the transition between both phases of hadronic matter. From our results we can not deduce the order of the phase transition. However, the physics which went into these theoretical approaches requires that this is a first order phase transition.

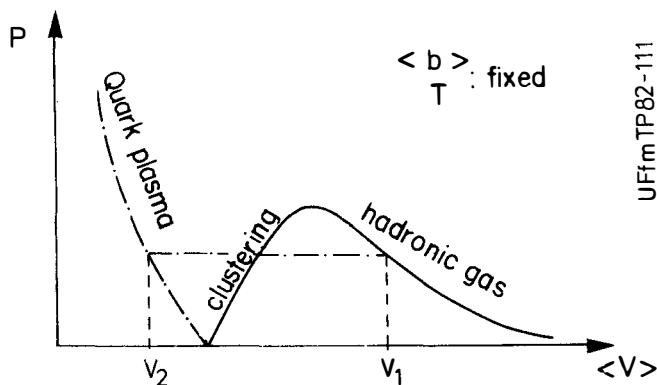


Fig. 4: p - V diagram for the gas-plasma first order transition.

Consider the p-V diagram shown in Fig.4. Here we distinguish three domains - the hadronic gas region is simply a Boltzmann gas where the pressure raises with reduction of the volume. However, when the internal excitation rises, the individual hadrons begin to cluster, reducing the increase in the Boltzmann pressure since smaller number of particles exerce smaller pressure. In the proper description we would have to describe this situation by allowing a coexistence of hadrons with the plasma state - this becomes necessary when the clustering overwhelms the compressive effects and the pressure falls to zero as V reaches the proper volume of hadronic matter. At this point the pressure rises again very quickly, since we now compress the hadronic constituents. By performing the Maxwell construction as indicated in the Fig. 4 between volumina V_1 and V_2 we can find the most likely way taken by the compressed hadronic gas in nuclear collision - which seems to be a firstorder transition in our approach. We should remember, that on the way out, during the expansion of the plasma state the entropy generated in the plasma (e.g. by s-production, shocks etc) may require that the isolated plasma state must expand to close to $P=0$ before it can desintegrate into individual hadrons. In an extreme situation this desintegration may be a quite slow process of successive fissions!

5. Strangness in the Plasma

In order to observe properties of quark-gluon plasma we must design a thermometer, an isolated degree of freedom weakly coupled to the hadronic matter. Nature has provided several such thermometers: leptons, direct photons and heavy flavours of quarks. We would like to point here to a particular phenomenon perhaps quite uniquely characteristic of quark matter; first we note that, at a given temperature, the quark-gluon plasma will contain an equal number of strange (s) quarks and antistrange ($\bar{s}$) quarks, naturally assuming that the hadronic collision time is much too short to allow for light flavour weak interaction conversion to strangeness. Assuming chemical equilibrium in the quark plasma, we find the density of the strange quarks to be (two spins and three colour):

$$\frac{s}{V} = \frac{\bar{s}}{V} = 6 \int \frac{d^3 p}{(2\pi)^3} e^{-\sqrt{p^2 + m_s^2}/T} = 3 \frac{T m_s^2}{\pi^2} K_2\left(\frac{m_s}{T}\right) \quad (52)$$

(neglecting, for the time being, the perturbative corrections). As

the mass of the strange quarks, m_s , in the perturbative vacuum is believed to be of the order of 150-280 MeV, the assumption of equilibrium for $m_s/T \sim 2$ may indeed be correct. In Eq. (52) Boltzmann distribution can be used, as the density of strangeness is relatively low. Similarly, there is a certain light antiquark density ($\bar{q}$ stands for either $\bar{u}$ or $\bar{d}$):

$$\frac{\bar{q}}{V} \approx 6 \int \frac{d^3 p}{(2\pi)^3} e^{-|p|/T - \mu_q/T} = e^{-\mu_q/T} \cdot T^3 \frac{6}{\pi} \quad (53)$$

where the quark chemical potential is $\mu_q = \mu/3$. This exponent suppresses the $q\bar{q}$ pair production as only for energies higher than μ_q is there a large number of empty states available for the q .

What I now intend to show is that there are many more $\bar{s}$ quarks than antiquarks of each light flavour. Indeed:

$$\frac{\bar{s}}{\bar{q}} = \frac{1}{2} \left(\frac{m_s}{T} \right)^2 K_2 \left(\frac{m_s}{T} \right) e^{\mu/3T} \quad (54)$$

The function $x^2 K_2(x)$ varies between 1.3 and for $x = m_s/T$ between 1.5 and 2. Thus, we almost always have more $\bar{s}$ than $\bar{q}$ quarks and, in many cases of interest, $\bar{s}/\bar{q} \sim 5$. As $\mu \rightarrow 0$ there are about as many $\bar{u}$ and $\bar{q}$ quarks as there are $\bar{s}$ quarks. We show this quantitatively in Fig. 5.

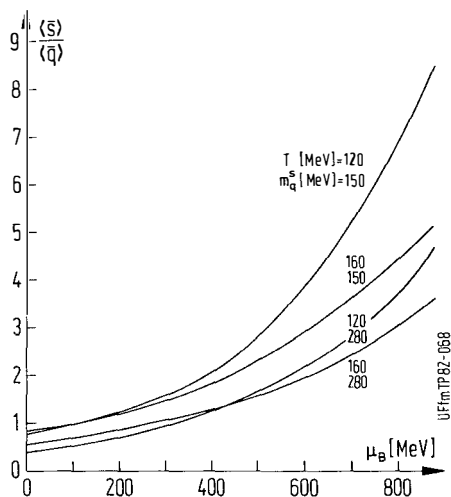


Fig. 5: Abundance of strange (=antistrange) quarks relative to light quark abundance as function of μ for several choices of T ($=120, 160$ MeV) and strange quark mass ($m_q^s = 150, 280$ MeV)

Another here important aspect is the total strangeness abundance - for $T=200$ MeV, $m_s=150$ MeV the chemical equilibrium predicts at about twice normal baryon density: $s/b = 0.4$; hence there are as many strange and antistrange quarks as there are baryons in the hadronic gas, or even much more, if we are in the 'radiation' i.e. baryonnumber depleted region.

The crucial question arises - is there enough time to create $s\bar{s}$ pairs in nuclear collisions? To answer this one I have to compute^{4b)} at least in lowest order in perturbative QCD the two contributing invariant reaction rates per unit time and unit volume:

$$\begin{aligned} A_{q\bar{q}} : q\bar{q} &\rightarrow s\bar{s} \\ A_{gg} : gg &\rightarrow s\bar{s} \end{aligned} \quad (55)$$

The contributing diagrams are shown in Fig. 6 a and b, respectively. The answer is dominated by the glue-gluon reaction and is at $T=200$ MeV, $m_s=150$ MeV, $\alpha_s=.6$: $A_{gg} \approx 16/\text{fm}^4$. This is quite a large rate indicating that the typical relaxation time

$$\tau = n(\infty) / A \quad (56)$$

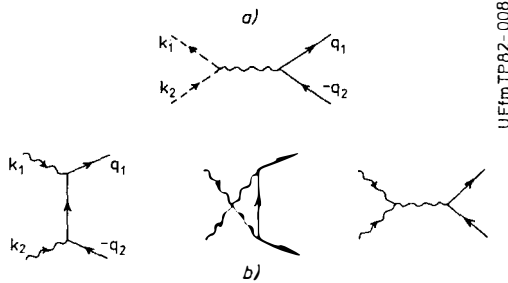


Fig. 6: First order diagrams for $s\bar{s}$ production reactions.

- a) $q\bar{q} \rightarrow s\bar{s}$
- b) $gg \rightarrow s\bar{s}$

($n(\infty)$ is the density at infinite time) will be about 10^{-23} sec.

In Fig. 7 the strangeness population evolution is shown as function of time at fixed $\mu=900$ MeV. during the minimal anticipated lifetime

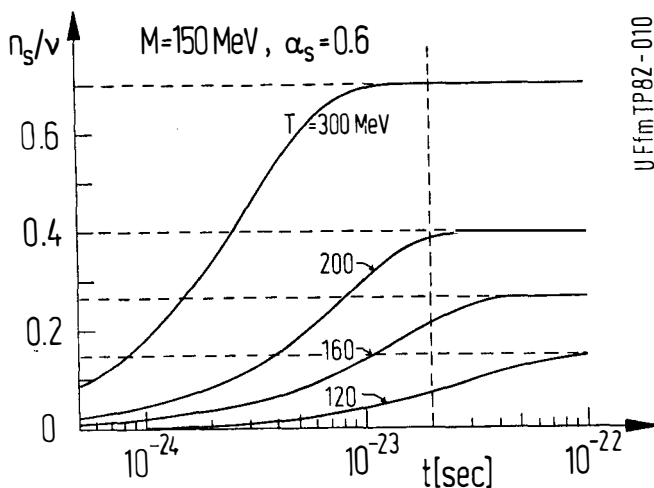


Fig. 7: Evolution of relative s population per baryon number as function of time in the plasma. For $T \geq 160$ MeV chemical saturation is noticeable in about $2.4 \cdot 10^{-23}$ sec, the anticipated minimal plasma lifetime.

of the plasma we find that the strange quark abundance saturates at its chemical equilibrium point.

One can study now how much more total strangeness is found in the quark gluon plasma as compared the hadronic gas phase. While the total yields are up to 5-7 times higher (again depending on some parameters) it is more appropriate to concentrate the attention on those reaction channels which will be particularly strongly populated when the quark plasma dissociates into hadrons. Here, in particular it appears that otherwise quite rare multistrange hadrons will be enhanced, on the one side by the relative high phase space density of strangeness in the plasma, on the other side recording the attractive ss -QCD interaction in the $\bar{3}_C$ and $\bar{s}s$ in 1_C channels. Hence we should search for the rise of the abundances of particles like $\Xi, \bar{\Xi}, \Omega, \bar{\Omega}, \phi$ and perhaps in highly strange pieces of baryonic matter, rather than in the K -channels. However, it appears that even such abundance ratios as $\bar{\Lambda}/\Lambda$ would be a carrier of the signal. Not to be forgotten are secondary effects, e.g. from the $s\bar{s}$

annihilation into γ (and infrared glue) in the plasma. Different experiments will be advantageous at different available energies.

6. Summary and outlook

Our aim has been to obtain a description of highly excited hadronic matter. By postulating the kinetic and chemical equilibrium we have been able to develop a thermodynamic description valid for high temperatures and different chemical compositions. In our work we have found two physically different domains; firstly the hadronic gas phase, in which individual hadrons can exist as separate entities, but are sometimes combined to larger hadronic clusters; and in the second domain, individual hadrons dissolve into one large cluster consisting of hadronic constituents - the quark-gluon plasma.

In order to obtain a theoretical description of both phases we have used some 'common' knowledge and plausible interpretations of the currently available experimental observation. In particular, in the case of hadronic gas we have completely abandoned a more conventional Lagrangian approach in favour of a semiphenomenological statistical bootstrap model of hadronic matter that incorporates those properties of hadronic interaction that are, in our opinion, most important.

In particular, the attractive interactions are included through the rich, exponentially growing hadronic mass spectrum $\tau(m^2, b)$ while the introduction of the finite volume of each hadron is responsible for an effective short-range repulsion. We neglect quantum statistics in the hadronic gas phase since a quantitative study has revealed that this is allowed above $T \approx 50$ MeV. But we allow particle production, which introduces a quantum physical aspect into the otherwise 'classical' theory of Boltzmann particles.

Our considerations lead us to the equations of state of hadronic matter which reflect what we have included in our considerations. It is the quantitative nature of our work that allows a detailed comparison with the experiment. It is important to observe that the predicted temperatures and mean transverse momenta of particles agree with the experimental results available at $E_{k,lab}/A = 2$ GeV [BEVELAC -] and at 100 GeV [ISR -] as much as a comparison is permitted.

The internal theoretical consistency of our description of the gas phase leads us, in a straightforward fashion, to the postulate

of a first order phase transition to the quark-gluon plasma. This second phase is treated by a quite different method; in addition to the standard Lagrangian quantum field theory of ('weakly') interacting particles at finite temperature and density, we also introduce the phenomenological vacuum pressure and energy density B . This term is required in a consistent theory of hadronic structure. It turns out that $B^{1/4} \sim 190$ MeV is just, to within 20%, the temperature of quark phase before dissociation into hadrons. This is similar to the maximal hadronic temperature $T_0 \approx 160$ MeV.

Perhaps the most interesting aspect of our work is the realization that the transition to quark matter will occur at very much lower baryon density for highly excited hadronic matter than for matter in the ground state ($T=0$). Using the currently accepted value B we find that at $v \sim 2-3v_0$, $T=150$ MeV, the quark phase may indeed already be formed. The detailed study of the different aspects of this phase transition must still be carried out. However, our initial results look very encouraging, as the required baryon density and temperatures are well within the range of the 10-300 GeV per nucleon fixed target heavy nucleon collisions. We look forward to the next generation of heavy ion facilities which may provide us with the required experimental information.

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