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Deterministic models of quantum fields

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Abstract. Deterministic dynamical models are discussed which can be described in quantum mechanical terms. – In particular, a local *quantum* field theory is presented which *is* a supersymmetric *classical* model [1]. The Hilbert space approach of Koopman and von Neumann is used to study the classical evolution of an ensemble of such systems. Its Liouville operator is decomposed into two contributions, with positive and negative spectrum, respectively. The unstable negative part is eliminated by a constraint on physical states, which is invariant under the Hamiltonian flow. Thus, choosing suitable variables, the classical Liouville equation becomes a functional Schrödinger equation of a genuine quantum field theory. – We briefly mention an U(1) gauge theory with "varying alpha" or dilaton coupling where a corresponding quantized theory emerges in the phase space approach [2]. It is energy-parity symmetric and, therefore, a prototype of a model in which the cosmological constant is protected by a symmetry.

1. Introduction

The (dis)similarity between the classical Liouville equation and the Schrödinger equation has recently been discussed, considering this an appropriate starting point for attempts to derive quantum from classical dynamics, i.e. for *emergent quantum theory* [1, 2].

In suitable coordinates both equations are remarkably similar, apart from the characteristic doubling of the classical phase space degrees of freedom as compared to the quantum mechanical case. The Liouville operator is Hermitian in the operator approach to classical statistical mechanics developed by Koopman and von Neumann [3]. However, unlike the case of the quantum mechanical Hamiltonian, its spectrum is generally not bounded from below. Therefore, attempts to find a deterministic foundation of quantum theory must pay special attention to the construction of a stable ground state, if they are based on a classical ensemble theory.

Research in this direction is suggested by work of 't Hooft, demonstrating several examples of systems which can be faithfully described as quantum mechanical and yet present deterministic dynamical models. It has been argued in favour of such model building that it may lead to a new approach in trying to understand and possibly resolve the persistent clash between general relativity and quantum theory, by questioning the fundamental character of the latter [4].

There have always been speculations about the (im)possibility of deriving quantum theory from more fundamental and deterministic dynamical structures. The discourse running from Einstein, Podolsky and Rosen to Bell, with numerous successors, is well known. Much of this has come under experimental scrutiny and no disagreement with quantum theory has been observed in the laboratory experiments on scales very large compared to the Planck scale. Nevertheless, it is conceivable that quantum mechanics emerges only on sufficiently large scales, where it describes effectively the fundamental deterministic degrees of freedom.

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A class of particularly simple emergent quantum models comprises systems which classically evolve in discrete time steps [4, 5]. Pointing towards a general nonlocality feature is the finding here that the coordinate eigenstates of the emergent quantum system are related to superpositions of underlying "primordial" states, which refer to the position of the classical degree of freedom. – Employing the path integral formulation of classical mechanics introduced by Gozzi and collaborators [6], it has been shown that classical models of Hamiltonian dynamics similarly turn into unitary quantum mechanical ones, if the corresponding Liouville operator governing the evolution of phase space densities is discretized [7]. Here, the arbitrariness in such discretizations helps to find a stable groundstate. Models of intrinsically discrete nature should be interesting to study in this context, such as causal sets.

Furthermore, it has been observed that classical systems with Hamiltonians which are linear in the momenta, can generally be represented in quantum mechanical terms. However, a new kind of gauge fixing or constraints implementing "information loss" at a fundamental level have to be invoked, in order to provide a groundstate for such systems [4, 8, 9]. – Various other arguments for deterministically induced quantum features have been proposed recently; see works collected in Part III of Ref. [10], for example, or Refs. [11, 12], concerning statistical and/or dissipative systems, quantum gravity, and matrix models. In all cases, the unifying dynamical principle leading to the necessary truncation of the Hilbert space is still missing.

Presently, I present a deterministic field theory from which a corresponding *quantum theory emerges by constraining the classical dynamics*. A functional Schrödinger equation is obtained with a positive Hamilton operator, which involves a standard scalar boson part. Main ingredient is a splitting of the Liouville operator into positive and negative energy contributions. The latter would render the to-be-quantum field theory unstable and are eliminated by a constraint on the physical states, based on the supersymmetry of the classical system. This is analogous to the "loss of information" condition in 't Hooft's and subsequent work [4, 8, 9]. We hope that the study of interacting fields will lead to better understand the dynamical origin of such a constraint. A dissipative information loss mechanism is plausible, yet a dynamical symmetry breaking may be an alternative [2].

It seems worth while to point out still another perspective on the emergence of quantum mechanics from possibly more fundamental physics. Textbooks explain *how to quantize* a given classical system, following the rules of imposing commutators or of setting up a Feynman path integral, etc. These reflect the status of the experimentally acquired knowledge. However, where do the quantization rules come from? – This question surfaces now and then since the early days of quantum theory. It must not be forgotten that quantum theory is beset with serious problems, other than the incompatibility with general relativity. – The infinities of quantum field theory are dealt with by renormalization. This procedure has become familiar to the extent of not perceiving it as problematic anymore, even if it may rule out a theoretical determination of its basic parameters (masses, couplings, etc.). – The emergence of the classical world of our experience from the quantum mechanical picture was a problem that has been solved. It is understood through *environment induced decoherence*, i.e., as being due to the interaction of quantum mechanical systems with the "rest of the universe" [13, 14, 15]. However, related is the measurement problem which states that a measurement on a quantum system which leads to a classical apparatus reading is a process which cannot be described entirely and consistently within quantum theory itself [12, 16, 17]. This problem has not been solved. It has given rise to a number of dynamical wave function collapse or reduction models, however, with no generally accepted completion of quantum physics in this respect [18, 19]. – Clearly, there is a need to better understand or change the foundations of quantum theory.

2. A quantum field as a supersymmetric classical one

We will make use of "pseudoclassical mechanics" or, rather, pseudoclassical field theory [1]. These notions have been introduced through the work of Refs. [20], considering a *Grassmann variant of classical mechanics* and studying the dynamics of spin degrees of freedom classically (and after quantization in the usual way). – Consider a "fermionic" field ψ, together with a real scalar field ϕ. The former is represented by the nilpotent generators of an infinite dimensional Grassmann algebra. They obey:

$$\{\psi(x),\psi(x')\}_+\equiv\psi(x)\psi(x')+\psi(x')\psi(x)=0\ \ , \tag{1}$$

where x, x' are coordinate labels in Minkowski space. All elements are real. Then, the classical model to be studied is defined by the action:

$$S\equiv\int \mathrm{d}^4x\ \Big(\dot{\phi}\dot{\psi}-\phi(\,-\,\Delta+m^2+v(\phi))\psi\Big)\equiv\int \mathrm{d}t\ L\ \ , \tag{2}$$

where dots denote time derivatives, and $v(\phi)$ may be a polynomial in ϕ, for example. Note that the action is Grassmann odd; such kind of models have been studied in different context by Volkov et al.

In terms of canonical momenta, $P_\phi\equiv\delta L/\delta\dot{\phi}=\dot{\psi}$, $P_\psi\equiv\delta L/\delta\dot{\psi}=\dot{\phi}$, the Hamiltonian is:

$$H=\int \mathrm{d}^3x\ \Big(P_\phi\dot{\phi}+P_\psi\dot{\psi}\Big)-L=\int \mathrm{d}^3x\ \Big(P_\phi P_\psi+\phi K\psi\Big)\ \ , \tag{3}$$

where $K\equiv-\Delta+m^2+v(\phi)$ and, for later use, $K'\equiv K+\phi\mathrm{d}v(\phi)/\mathrm{d}\phi$. Then, Hamilton's equations can be shown to be invariant under two global symmetry transformations,

$$\phi\longrightarrow\phi+\epsilon\psi\ \ ;\quad \psi\longrightarrow\psi+\epsilon\dot{\phi}\ \ , \tag{4}$$

where ϵ is an infinitesimal real parameter. Associated are the conserved Noether charges:

$$C_1\equiv\int \mathrm{d}^3x\ P_\phi\psi\ \ ;\quad C_2\equiv\int \mathrm{d}^3x\ \Big(\frac{1}{2}P_\psi^2+V(\phi)\Big)\ \ . \tag{5}$$

The second one is the total energy of the scalar field, with $\mathrm{d}V(\phi)/\mathrm{d}\phi\equiv K\phi$.

We introduce the Poisson bracket of observables A,B defined over phase space:

$$\{A,B\}\equiv A\int \mathrm{d}^3x\ \Big(\frac{\overleftarrow{\delta}}{\delta P_\phi}\frac{\overrightarrow{\delta}}{\delta\phi}+\frac{\overleftarrow{\delta}}{\delta P_\psi}\frac{\overrightarrow{\delta}}{\delta\psi}-\frac{\overleftarrow{\delta}}{\delta\phi}\frac{\overrightarrow{\delta}}{\delta P_\phi}-\frac{\overleftarrow{\delta}}{\delta\psi}\frac{\overrightarrow{\delta}}{\delta P_\psi}\Big)B\ \ . \tag{6}$$

Functional derivatives refer to the same space-time point and act in the indicated direction; it coincides with their fermionic left/right-derivative character. Embodying Hamilton's equations of motion, this yields the familiar relation:

$$\frac{\mathrm{d}}{\mathrm{d}t}A=\{H,A\}+\partial_tA\ \ . \tag{7}$$

For the Hamiltonian and Noether charge densities, identified by $H\equiv\int\mathrm{d}^3xH(x)$ and $C_j\equiv\int\mathrm{d}^3xC_j(x)|_{j=1,2}$, respectively, one finds a local (equal-time) supersymmetry algebra, see the second of Refs. [1]. A Hilbert space version of the symmetry algebra will be obtained shortly. – An important example of Eq. (7) is the Liouville equation. Considering an ensemble of systems, this equation governs the evolution of its phase space density ρ:

$$0=i\frac{\mathrm{d}}{\mathrm{d}t}\rho=i\partial_t\rho-\hat{\mathcal{L}}\rho\ \ , \tag{8}$$

where a convenient factor i has been introduced, and the Liouville operator $\hat{\mathcal{L}}$ is defined by:

$$-\hat{\mathcal{L}}\rho \equiv i\{H,\rho\} \quad . \tag{9}$$

These equations summarize the classical statistical mechanics of a conservative system.

An equivalent Hilbert space formulation is due to Koopman and von Neumann [3]. It will be modified appropriately for our supersymmetric classical field theory. Two *postulates* are put forth: (**A**) the phase space density functional can be factorized in the form $\rho \equiv \Psi^*\Psi$; (**B**) the Grassmann valued and, in general, complex state functional Ψ itself obeys the Liouville Eq. (8). – Furthermore, the complex valued inner product of such state functionals is defined by:

$$\langle\Psi|\Phi\rangle \equiv \int \mathcal{D}\phi\mathcal{D}P_\psi\mathcal{D}\psi\mathcal{D}P_\phi\ \Psi^*\Phi = \langle\Phi|\Psi\rangle^* \quad , \tag{10}$$

i.e., by functional integration over all phase space variables. Since there are Grassmann valued variables, the $*$-operation defining the dual Ψ^* needs to be treated carefully. – Given the Hilbert space structure, the Liouville operator of a conservative system is Hermitian and the overlap $\langle\Psi|\Psi\rangle$ is a conserved quantity. Then, the Liouville equation also applies to $\rho = |\Psi|^2$, due to its linearity, and ρ may be interpreted as a phase space density, as before [3].

Certainly, one is reminded here of quantum mechanics. – In order to expose the striking similarity as well as the remaining crucial difference, further transformations of the functional Liouville equation are useful [1, 2]. Fourier transformation replaces the momentum P_ψ by a second scalar field $\bar{\phi}$. Furthermore, define $\bar{\psi} \equiv P_\phi$. Thus, the Eqs. (8)–(9) yield:

$$i\partial_t\Psi = \hat{\mathcal{H}}\Psi \quad , \tag{11}$$

where Ψ is considered as a functional of $\phi, \bar{\phi}, \psi, \bar{\psi}$, and with the *emergent* "Hamilton operator":

$$\hat{\mathcal{H}}\Psi \equiv -i\int \mathcal{D}P_\psi\ \exp(iP_\psi\cdot\bar{\phi})\{H,\Psi\} = \int \mathrm{d}^3x\ \Big(-\delta_{\bar{\phi}}\delta_\phi + \bar{\phi}K\phi - i(\bar{\psi}\delta_\psi - \psi K'\delta_{\bar{\psi}})\Big)\Psi \quad , \tag{12}$$

using $f\cdot g \equiv \int \mathrm{d}^3x\ f(x)g(x)$. The Hamiltonian (density) is Grassmann even. – While Eq. (11) appears as a *functional Schrödinger equation*, several remarks are in order here. First, following the transformation, $\phi \equiv (\sigma+\kappa)/\sqrt{2}$ and $\bar{\phi} \equiv (\sigma-\kappa)/\sqrt{2}$, one finds a "bosonic" kinetic energy:

$$-\frac{1}{2}\int \mathrm{d}^3x\ \left(\delta_\sigma^2 - \delta_\kappa^2\right) \quad , \tag{13}$$

which is *not bounded from below*. This means that the Hamiltonian lacks a lowest energy state. Secondly, the $*$-operation mentioned before amounts to complex conjugation for a bosonic state functional, in analogy with a quantum mechanical wave function. However, based on complex conjugation alone, the fermionic part of the Hamiltonian (12) would not be Hermitian. Instead, a detailed construction of the inner product for functionals of Grassmann valued fields has been given and applied, respectively, in Refs. [21] and [1, 2]. Considering the *noninteracting case* with $K' = K$, i.e., with $v(\phi) = 0$ in Eq. (2), the construction of Floreanini and Jackiw suffices here: the Hermitian conjugate of ψ is $\psi^\dagger = \delta_\psi$ and of $\bar{\psi}$ it is $\bar{\psi}^\dagger = \delta_{\bar{\psi}}$. Furthermore, rescaling $\bar{\psi} \longrightarrow \bar{\psi}\sqrt{K}$, the Hermitian fermionic part of the Hamiltonian (12) becomes:

$$\hat{\mathcal{H}}_{\bar{\psi}\psi} \equiv i(\psi\sqrt{K}\delta_{\bar{\psi}} - \bar{\psi}\sqrt{K}\delta_\psi) \quad . \tag{14}$$

In the presence of interactions, with $K' \neq K$, additional modifications are necessary, which will not be considered here. In any case, the eigenvalues of $\hat{\mathcal{H}}_{\bar{\psi}\psi}$ will not have a lower bound either.

To summarize, the emergent Hamiltonian $\hat{\mathcal{H}}$ is unbounded from below, lacking a groundstate. This difficulty has been encountered in various other attempts to build deterministic quantum models [2, 4, 5, 7, 8, 9]. For our case, it will be solved next.

3. Groundstate construction for the emergent quantum model

We proceed with equal-time operator relations for the interacting case, which are related to the supersymmetry algebra mentioned before [1]. Using Eqs. (5) and $\bar{\psi} \equiv P_\phi$, as before, one obtains:

$$\hat{\mathcal{C}}_1(x)\Psi \equiv \int \mathcal{D}P_\psi \ \exp(iP_\psi \cdot \bar{\phi})\{C_1(x), \Psi\} = (\,-\psi\delta_\phi + i\bar{\psi}\bar{\phi})_{(x)}\Psi \ \ . \tag{15}$$

Analogously, we calculate: $\hat{\mathcal{C}}_2(x)\Psi \ \equiv \ (-i\delta_{\bar{\phi}}\delta_\psi - \phi K \delta_{\bar{\psi}})_{(x)}\Psi$. Both operators are Grassmann odd and obey: $\{\hat{\mathcal{C}}_j(x), \hat{\mathcal{C}}_j(x')\}_+ = 0$, for $j = 1, 2$. Furthermore, one finds the vanishing commutator:

$$[\hat{\mathcal{H}}(x), \hat{\mathcal{H}}(x')] = 0 \ \ , \tag{16}$$

i.e., the emergent theory is *local*. However, the Hamilton operator, Eq. (12), involves a functional Fourier transform. Therefore, the emergent quantum field theory, which is local in the usual sense, is *nonlocal* with respect to the space of fields of the underlying classical system. This is analogous to what has been found in several models mentioned in the Introduction [4, 5, 11]; see also the third of Refs. [1] and [2] for more detailed discussions. – Finally, we find:

$$[\hat{\mathcal{H}}(x), \hat{\mathcal{C}}_j(x')] = 0 \ \ , \ \ \text{for } j = 1, 2 \ \ ; \ \ \ \{i\hat{\mathcal{C}}_1(x), \hat{\mathcal{C}}_2(x')\}_+ = \hat{\mathcal{H}}(x)\delta^3(x - x') \ \ . \tag{17}$$

One may complete these with relations for the full set of operators generating the space-time symmetries of our model. However, they do not play a special role in the following.

Since the emergent Hamiltonian lacks a proper groundstate, we cannot yet interpret the model as a quantum mechanical one, despite close formal similarities. – This problem is solved, if we find a positive definite local operator $\hat{P}$ that obeys $[\hat{\mathcal{H}}(x), \hat{P}(x')] = 0$. Then, the Hamiltonian can be split into contributions with positive and negative spectrum:

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_+ - \hat{\mathcal{H}}_- \ \ , \tag{18}$$

where, in the simplest way, $\hat{\mathcal{H}}_\pm(x) \equiv (\hat{\mathcal{H}}(x) \pm \hat{P}(x))^2/4\hat{P}(x)$. With this, the spectrum of the Hamiltonian $\hat{\mathcal{H}}$ is made bounded from below by imposing the "positivity constraint":

$$\hat{\mathcal{H}}_-\Psi = 0 \ \ , \tag{19}$$

which is conserved, since $[\hat{\mathcal{H}}_+(x), \hat{\mathcal{H}}_-(x)] = 0$, by construction. In this way, the *physical states* are selected which are based on the existence of a *quantum mechanical groundstate.*

For our field theory, the noninteracting and interacting cases have been studied separately in the second of Refs. [1]. We specialize here to the noninteracting case. As mentioned before, with $v(\phi) = 0$ in Eq. (2), and therefore $K' = K = -\Delta + m^2$, the rescaling $\bar{\psi} \ \longrightarrow \ \bar{\psi}\sqrt{K}$ is useful, and we consider the operators: $\hat{\mathcal{H}}(x) = (\,-\delta_{\bar{\phi}}\delta_\phi + \bar{\phi}K\phi)_{(x)} + \hat{\mathcal{H}}_{\bar{\psi}\psi}(x)$, $i\hat{\mathcal{C}}_1(x) = (\,-i\psi\delta_\phi - \bar{\psi}\sqrt{K}\bar{\phi})_{(x)}$, $\hat{\mathcal{C}}_2(x) \ = \ (\,-\,i\delta_{\bar{\phi}}\delta_\psi \,-\, \phi\sqrt{K}\delta_{\bar{\psi}})_{(x)}$, with $\hat{\mathcal{H}}_{\bar{\psi}\psi}$ from Eq. (14). Completing the $\hat{\mathcal{C}}_j$ to Hermitian operators, with $\psi^\dagger \ = \ \delta_\psi$ and $\bar{\psi}^\dagger \ = \ \delta_{\bar{\psi}}$, a suitable combination yields an operator $\hat{P}$ with the required properties; in terms of *"square-root of harmonic oscillator"* operators, $\hat{\mathcal{C}}_{1+}(x) \ \equiv \ (i\hat{\mathcal{C}}_1(x) + (i\hat{\mathcal{C}}_1(x))^\dagger)$ and $\hat{\mathcal{C}}_{2+}(x) \ \equiv \ (\hat{\mathcal{C}}_2(x) + (\hat{\mathcal{C}}_2(x))^\dagger)$: $\hat{P}(x) \equiv (\hat{\mathcal{C}}^2_{1+}(x) + \hat{\mathcal{C}}^2_{2+}(x))/\delta^3(0)$. This results in: $\hat{\mathcal{H}}_\pm(x) = (\,-\delta^2_\phi + \phi K\phi - \delta^2_{\bar{\phi}} + \bar{\phi}K\bar{\phi})/4 \pm \hat{\mathcal{H}}(x)/2 + \hat{\mathcal{H}}^2(x)/4\hat{P}(x)$, cf. Eq. (18). Finally, with $\phi \equiv (\sigma + \kappa)/\sqrt{2}$ and $\bar{\phi} \equiv (\sigma - \kappa)/\sqrt{2}$, the Hamiltonian density becomes:

$$\hat{\mathcal{H}}_+(x) = \frac{1}{2}\Big(\,-\,\delta^2_\sigma + \sigma K\sigma + \hat{\mathcal{H}}_{\bar{\psi}\psi} + \frac{1}{2}\hat{\mathcal{H}}^2/\hat{P}\Big)_{(x)} \ \ . \tag{20}$$

The only trace of the previous instability is now relegated to the last term. Similarly, the constraint operator becomes: $\hat{\mathcal{H}}_-(x) = (-\delta_\kappa^2 + \kappa K\kappa - \hat{\mathcal{H}}_{\bar{\psi}\psi} + \hat{\mathcal{H}}^2/2\hat{P})_{(x)}/2$. Note the symmetry with $\hat{\mathcal{H}}_+$ $(-\hat{\mathcal{H}}_{\bar{\psi}\psi} = \hat{\mathcal{H}}_{\psi\bar{\psi}})$. Thus, elimination of part of the Hilbert space, Eq. (19), may be related to a symmetry breaking [1]. – The resulting Hamilton operator $\hat{\mathcal{H}}_+$ now has a positive spectrum, by construction, and the leading terms are those of a *free bosonic quantum field* together with a *fermion doublet* in the Schrödinger representation. They dominate at low energy.

A different solution of the lacking groundstate problem has been proposed in Ref. [2]: It is shown that energy-parity arises in the Hilbert space representation of *classical phase space dynamics* of matter. (This symmetry was introduced to protect the cosmological constant against large matter contributions [22].) Generalizing an Abelian gauge theory by a varying coupling, as in "varying alpha" or dilaton models, classical matter fields can turn into *quantum fields* (Schrödinger picture), accompanied by a gauge symmetry change – U(1) → U(1) x U(1). The transition between classical ensemble theory and quantum field theory is governed here by the varying coupling through correction terms that introduce diffusion and dissipation.

4. Conclusions

We discussed *deterministic (ensemble) theories* trying to reconstruct quantum mechanics as an emergent phenomenon, which may challenge common wisdom about its foundations and limitations . – More immediate consequences for the measurement process, reduction, and the "collapse of the wave function" [13, 15, 16, 17] need to be explored, as well as extensions to more realistic theories, such as the Standard Model including gravity.

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