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Research Article

Generalized Nuclear Woods-Saxon Potential under Relativistic Spin Symmetry Limit

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By using the Pekeris approximation, we present solutions of the Dirac equation with the generalized Woods-Saxon potential with arbitrary spin-orbit coupling number κ under spin symmetry limit. We obtain energy eigenvalues and corresponding eigenfunctions in closed forms. Some numerical results are given too.

1. Introduction

The Dirac equation, which describes the motion of a spin 1/2 particle, has been used in solving many problems of nuclear and high-energy physics. One of the most important concepts for understanding the traditional magic numbers in stable nuclei is the spin symmetry breaking [1–4]. On the other hand, the p-spin symmetry observed originally almost 40 years ago as a mechanism to explain different aspects of the nuclear structure is one of the most interesting phenomena in the relativistic quantum mechanics. Within the framework of Dirac equation, p-spin symmetry used to feature deformed nuclei, superdeformation, and to establish an effective shell model [5–8], whereas spin symmetry is relevant for mesons [9, 10]. Spin symmetry occurs when $V_S \approx V_V$ and pseudospin symmetry occurs when $V_S \approx -V_V$, [11, 12], where S is scalar potential and V_V is vector potential. Pseudospin symmetry is exact under the condition of $d(V(r) + S(r))/dr = 0$, and spin symmetry is exact under the condition of $d(V(r) - S(r))/dr = 0$ [13]. For the first time, the spin symmetry tested in the realistic nuclei [14], and then this symmetry is investigated by examining the radial wave functions [15–17]. The pseudospin symmetry refers to a quasi-degeneracy of single nucleon doublets with nonrelativistic quantum numbers $(n, l, j = l + 1/2)$ and $(n - 1, l + 2, j = l + 3/2)$, where n, l, and j are single nucleon radial, orbital, and total angular quantum numbers, respectively. The total angular momentum is $j = \tilde{l} + \tilde{s}$, where $\tilde{l} = l + 1$ is pseudo-angular momentum and $\tilde{s}$ is pseudospin angular momentum [12, 18–20].

On the other hand, some typical physical models have been studied like harmonic oscillator [19, 20], Woods-Saxon potential [21, 22], Morse potential [23–25], Eckart potential [26–28], Pöschl-Teller potential [29], Manning-Rosen potential [30], and so forth [31–36].

The interactions between nuclei are commonly described by using a potential that consists of the Coulomb and the nuclear potentials. These potentials are usually taken to be of the Woods-Saxon form. The Coulomb plus Woods-Saxon potentials are well known as modified Woods-Saxon potential that plays a great role in nuclear physics. Fusion barriers for a large number of fusion reactions from light to heavy systems can be described well with this potential [37–40]. But the modified Woods-Saxon potential has no exact or approximate solutions. The generalized Woods-Saxon is near the modified Woods-Saxon potential, and therefore we can solve the generalized Woods-Saxon instead of the modified Woods-Saxon potential. The form of the generalized Woods-Saxon potential is as follows [41, 42]:

$$V(r) = -\frac{V_0}{1 + e^{(r-R_0)/a}} - \frac{Ce^{(r-R_0)/a}}{\left(1 + e^{(r-R_0)/a}\right)^2}, \qquad (1)$$

where V_0 and C determine the potential depth, R_0 is the width of the potential, a is the surface thickness, and $0 < C < 150\,\text{MeV}$ [42]. In Figure 1, we illustrated the shape of the Woods-Saxon potential ($C = 0$) and its generalized form in (1) ($C \neq 0$).

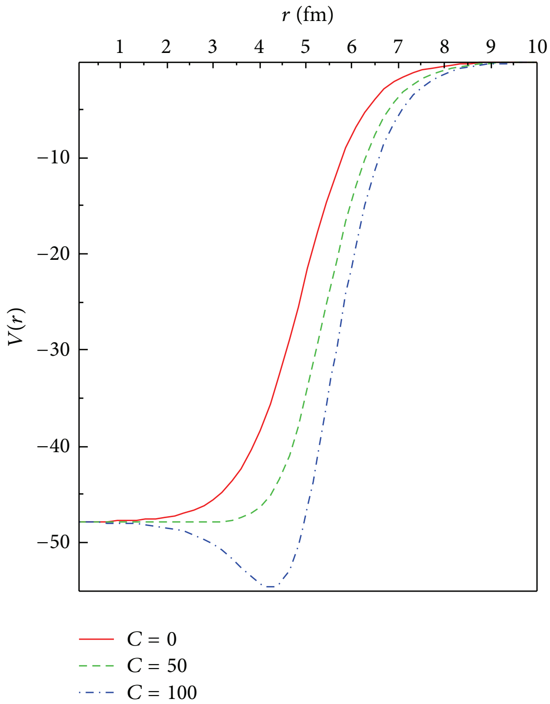

FIGURE 1: Shape of the generalized Woods-Saxon potential.

The purpose of this work is devoted to studying the spin symmetry solutions of the Dirac equation for arbitrary spin-orbit coupling quantum number κ with the generalized Woods-Saxon potential, which was not considered before.

This paper is organized as follows. In Section 2, we briefly introduced the Dirac equation with scalar and vector potential with arbitrary spin-orbit coupling number κ under spin symmetry limit. The generalized parametric Nikiforov-Uvarov method is presented in Section 3. The energy eigenvalue equations and corresponding eigenfunctions are obtained in Section 4. In this section, some remarks and numerical results are given too. Finally, conclusion is given in Section 5.

2. Dirac Equation under Spin and Pseudospin Symmetry Limit

The Dirac equation with scalar potential $S(r)$ and vector potential $V(r)$ is

$$\left[c\vec{\alpha}\cdot\vec{p}+\beta\left(Mc^2+S(r)\right)\right]\psi(r)=\left[E-V(r)\right]\psi(r), \tag{2}$$

where E is the relativistic energy of the system and $\vec{p}=-i\vec{\nabla}$ is the three-dimensional momentum operator. $\vec{\alpha}$ and β are the 4×4 usual Dirac matrices given as

$$\vec{\alpha}=\begin{pmatrix}0 & \vec{\sigma}\\ \vec{\sigma} & 0\end{pmatrix}, \qquad \beta=\begin{pmatrix}I & 0\\ 0 & -I\end{pmatrix}, \tag{3}$$

where $\vec{\sigma}$ is Pauli matrix and I is 2×2 unitary matrix. The total angular momentum operator $\vec{J}$ and spin-orbit operator $K=(\vec{\sigma}\cdot\vec{L}+1)$, where $\vec{L}$ is orbital angular momentum operator and commutes with Dirac Hamiltonian. The eigenvalues of spin-orbit coupling operator are $\kappa=(j+1/2)>0$ and $\kappa=-(j+1/2)<0$ for unaligned spin $j=l-1/2$ and the aligned spin $j=l+1/2$, respectively. (H^2,K,J^2,J_z) can be taken as a complete set of the conservative quantities. Thus, the Dirac spinors can be written according to radial quantum number n and spin-orbit coupling number κ as follows:

$$\psi_{n\kappa}(\vec{r})=\begin{pmatrix}f_{n\kappa}(\vec{r})\\ g_{n\kappa}(\vec{r})\end{pmatrix}=\begin{pmatrix}\dfrac{F_{n\kappa}(r)}{r}Y^{l}_{jm}(\theta,\varphi)\\ i\dfrac{G_{n\kappa}(r)}{r}Y^{\tilde{l}}_{jm}(\theta,\varphi)\end{pmatrix}, \tag{4}$$

where $f_{n\kappa}(\vec{r})$ is the upper (large) component and $g_{n\kappa}(\vec{r})$ is the lower (small) component of the Dirac spinors. $Y^{l}_{jm}(\theta,\varphi)$ and $Y^{\tilde{l}}_{jm}(\theta,\varphi)$ are the spherical harmonic functions coupled to the total angular momentum j and its projection m on the z axis. Substituting (4) into (2) with the usual Dirac matrices, one obtains two coupled differential equations for the upper and the lower radial wave functions $F_{n\kappa}(r)$ and $G_{n\kappa}(r)$ as

$$\left(\frac{d}{dr}+\frac{\kappa}{r}\right)F_{n\kappa}(r)=\frac{1}{\hbar c}\left[Mc^2+E_{n\kappa}-\Delta(r)\right]G_{n\kappa}(r), \tag{5a}$$

$$\left(\frac{d}{dr}-\frac{\kappa}{r}\right)G_{n\kappa}(r)=\frac{1}{\hbar c}\left[Mc^2-E_{n\kappa}+\Sigma(r)\right]F_{n\kappa}(r), \tag{5b}$$

where

$$\Delta(r)=V(r)-S(r), \tag{6a}$$

$$\Sigma(r)=V(r)+S(r). \tag{6b}$$

Eliminating $F_{n\kappa}(r)$ and $G_{n\kappa}(r)$ from (5a) and (5b), we obtain the following second-order Schrödinger-like differential equations for the upper and lower components of the Dirac wave functions, respectively:

$$\begin{aligned}&\left[\frac{d^2}{dr^2}-\frac{\kappa(\kappa-1)}{r^2}\right]G_{n\kappa}(r)\\ &\quad+\left[-\frac{1}{\hbar^2c^2}\left(Mc^2+E_{n\kappa}-\Delta(r)\right)\left(M-E_{n\kappa}+\Sigma(r)\right)\right.\\ &\qquad\left.-\frac{1}{\hbar^2c^2}\frac{(d\Sigma(r)/dr)(d/dr-\kappa/r)}{Mc^2-E_{n\kappa}+\Sigma(r)}\right]G_{n\kappa}(r)=0,\end{aligned} \tag{7a}$$

$$\begin{aligned}&\left[\frac{d^2}{dr^2}-\frac{\kappa(\kappa+1)}{r^2}\right]F_{n\kappa}(r)\\ &\quad+\left[-\frac{1}{\hbar^2c^2}\left(Mc^2+E_{n\kappa}-\Delta(r)\right)\left(Mc^2-E_{n\kappa}+\Sigma(r)\right)\right.\\ &\qquad\left.+\frac{1}{\hbar^2c^2}\frac{(d\Delta(r)/dr)(d/dr+\kappa/r)}{Mc^2+E_{n\kappa}-\Delta(r)}\right]F_{n\kappa}(r)=0,\end{aligned} \tag{7b}$$

where $\kappa(\kappa-1)=\tilde{l}(\tilde{l}+1)$ and $\kappa(\kappa+1)=l(l+1)$.

2.1. Spin Symmetry Limit. In the spin symmetry limit $d\Delta(r)/dr = 0$ or $\Delta(r) = C_s =$ constant [14], then (7b) becomes

$$\left[\frac{d^2}{dr^2} - \frac{\kappa(\kappa+1)}{r^2} - \frac{1}{\hbar^2 c^2}\left(Mc^2 + E_{n\kappa} - C_s\right) \times \left(Mc^2 - E_{n\kappa} + \Sigma(r)\right)\right] F_{n\kappa}(r) = 0, \tag{8}$$

where $\kappa = l$ and $\kappa = -l-1$ for $\kappa < 0$ and $\kappa > 0$, respectively. In (8), $\Sigma(r)$ can be taken as generalized Woods-Saxon potential, and it is reduced to

$$\left[\frac{d^2}{dr^2} - \frac{\kappa(\kappa+1)}{r^2} + \eta\left(\frac{V_0}{1+e^{(r-R_0)/a}} + \frac{Ce^{(r-R_0)/a}}{\left(1+e^{(r-R_0)/a}\right)^2}\right) - \xi^2\right] F_{n\kappa}(r) = 0, \tag{9}$$

where

$$\eta = \frac{1}{\hbar^2 c^2}\left(E_{n\kappa} + Mc^2 - C_s\right), \tag{10a}$$

$$\xi^2 = \frac{1}{\hbar^2 c^2}\left(Mc^2 - E_{n\kappa}\right)\left(Mc^2 + E_{n\kappa} - C_s\right). \tag{10b}$$

For the solution of (9), we will use the Nikiforov-Uvarov method which is briefly introduced in the following section.

2.2. Pekeris-Type Approximation to the Spin-Orbit Coupling Term. Because of the spin-orbit coupling term, that is, $\kappa(\kappa+1)/r^2$, (9) cannot be solved analytically, except for $\kappa = 0, -1$. Therefore, we shall use the Pekeris approximation [43] in order to deal with the spin-orbit coupling terms, and we may express the spin-orbit term as follows:

$$V_{so}(r) = \frac{\kappa(\kappa+1)}{r^2} = \frac{\kappa(\kappa+1)}{(1+x/R_0)^2} \cong \frac{\kappa(\kappa+1)}{R_0^2}\left(1 - 2\frac{x}{R_0} + 3\left(\frac{x}{R_0}\right)^2 + \cdots\right). \tag{11}$$

In addition, we may also approximately express it in the following way:

$$\widetilde{V}_{so}(r) = \frac{\kappa(\kappa+1)}{r^2} \cong \frac{\kappa(\kappa+1)}{R_0^2}\left(D_0 + \frac{D_1}{1+e^{\nu x}} + \frac{D_2}{(1+e^{\nu x})^2}\right), \tag{12}$$

where $x = r - R_0$, $\nu = 1/a$, and D_i is constant ($i = 0, 1, 2$) [44–48]. If we expand the expression of (12) around $x = 0$ up to the second-order term and next compare it with (11), we can obtain expansion coefficients D_0, D_1, and D_2 as follows:

$$D_0 = 1 - \frac{4}{\nu R_0} - \frac{12}{\nu^2 R_0^2}, \quad D_1 = \frac{8}{\nu R_0} - \frac{48}{\nu^2 R_0^2}, \quad D_2 = \frac{48}{\nu^2 R_0^2}. \tag{13}$$

Now, we can take the potential $\widetilde{V}_{so}$ (12) instead of the spin-orbit coupling potential (11).

3. The Parametric Generalization Nikiforov-Uvarov Method

This powerful mathematical tool solves second-order differential equations. Let us consider the following differential equation [49–51]:

$$\left(\frac{d^2}{dz^2} + \frac{\alpha_1 - \alpha_2 z}{z(1-\alpha_3 z)}\frac{d}{dz} + \frac{1}{z^2(1-\alpha_3 z)^2} \times \left[-p_2 z^2 + p_1 z - p_0\right]\right)\psi(z) = 0. \tag{14}$$

According to the Nikiforov-Uvarov method, the eigenfunctions and eigenenergies, respectively, are

$$\psi(z) = z^{\alpha_{12}}(1-\alpha_3 z)^{-\alpha_{12}-\alpha_{13}/\alpha_3} P_n^{(\alpha_{10}-1,\,\alpha_{11}/\alpha_3-\alpha_{10}-1)}(1-2\alpha_3 z), \tag{15}$$

$$\alpha_2 n - (2n+1)\alpha_5 + (2n+1)\left(\sqrt{\alpha_9} - \alpha_3\sqrt{\alpha_8}\right) + n(n-1)\alpha_3 + \alpha_7 + 2\alpha_3\alpha_8 - 2\sqrt{\alpha_8\alpha_9} = 0, \tag{16}$$

where

$$\begin{gathered}\alpha_4 = \frac{1}{2}(1-\alpha_1), \qquad \alpha_5 = \frac{1}{2}(\alpha_2 - 2\alpha_3), \\ \alpha_6 = \alpha_5^2 + p_2, \qquad \alpha_7 = 2\alpha_4\alpha_5 - p_1, \\ \alpha_9 = \alpha_3\alpha_7 + \alpha_3^2\alpha_8 + \alpha_6, \\ \alpha_8 = \alpha_4^2 + p_0, \\ \alpha_{10} = \alpha_1 + 2\alpha_4 + 2\sqrt{\alpha_8}, \\ \alpha_{11} = \alpha_2 - 2\alpha_5 + 2\left(\sqrt{\alpha_9} + \alpha_3\sqrt{\alpha_8}\right), \\ \alpha_{12} = \alpha_4 + \sqrt{\alpha_8}, \\ \alpha_{13} = \alpha_5 - \left(\sqrt{\alpha_9} + \alpha_3\sqrt{\alpha_8}\right).\end{gathered} \tag{17}$$

In the rather more special case of $\alpha_3 = 0$ [50, 51],

$$\lim_{\alpha_3 \to 0} P_n^{(\alpha_{10}-1,\,\alpha_{11}/\alpha_3-\alpha_{10}-1)}(1-2\alpha_3 z) = L_n^{\alpha_{10}-1}(\alpha_{11} z), \tag{18a}$$

$$\lim_{\alpha_3 \to 0}(1-\alpha_3 z)^{-\alpha_{12}-\alpha_{13}/\alpha_3} = e^{\alpha_{13} z}, \tag{18b}$$

Table 1: The approximate bound state energy eigenvalues (in unit of MeV) of the generalized Woods-Saxon potential within the spin symmetry limit for several values of n and κ.

l	$n, \kappa < 0, \kappa > 0$	State	$E_{n\kappa}$ $C = 0$	$E_{n\kappa}$ $C = 50$ MeV	$E_{n\kappa}$ $C = 100$ MeV	$E_{n\kappa}$ $C = 150$ MeV
1	0, −2, 1	$0p_{3/2}, 0p_{1/2}$	−939.0045195	−939.0074037	−939.0102535	−939.0130695
2	0, −3, 2	$0d_{5/2}, 0d_{3/2}$	−938.5761054	−938.5822985	−938.5884052	−938.5944276
3	0, −4, 3	$0f_{7/2}, 0f_{5/2}$	−938.4033046	−938.4111487	−938.4188786	−938.4264972
4	0, −5, 4	$0g_{9/2}, 0g_{7/2}$	−938.532527	−938.5397300	−938.5468274	−938.5538221
1	1, −2, 1	$1p_{3/2}, 1p_{1/2}$	−906.9952581	−907.4067290	−907.8063989	−908.1948041
2	1, −3, 2	$1d_{5/2}, 1d_{3/2}$	−905.7621398	−906.1743738	−906.5749872	−906.9645019
3	1, −4, 3	$1f_{7/2}, 1f_{5/2}$	−904.2904841	−904.7048150	−905.1077390	−905.4997600
4	1, −5, 4	$1g_{9/2}, 1g_{7/2}$	−902.7908284	−903.2078994	−903.6137936	−904.0089945
1	2, −2, 1	$2p_{3/2}, 2p_{1/2}$	−835.7682302	−837.2498452	−838.6826152	−840.0691370
2	2, −3, 2	$2d_{5/2}, 2d_{3/2}$	−834.4172283	−835.8982294	−837.3306378	−838.7170294
3	2, −4, 3	$2f_{7/2}, 2f_{5/2}$	−832.5628129	−834.0439814	−835.4769039	−836.8641261
4	2, −5, 4	$2g_{9/2}, 2g_{7/2}$	−830.3570797	−831.8396528	−833.2743934	−834.6638100

and, from (15), we find for the wave function

$$\psi = z^{\alpha_{12}} e^{\alpha_{13} z} L_n^{\alpha_{10}-1}(\alpha_{11} z). \quad (19)$$

4. Bound States of the Generalized Woods-Saxon Potential with Arbitrary κ

Substituting (12) into (9) and using transformation $z = e^{\nu x}$, we find the following second order deferential equation for the upper component of the Dirac spinor as

$$\left\{\frac{d^2}{dz^2} + \frac{1+z}{z(1+z)}\frac{d}{dz} - \frac{\kappa(\kappa+1)}{\nu^2 z^2 R_0^2}\left(D_0 + \frac{D_1}{(1+z)} + \frac{D_2}{(1+z)^2}\right) + \frac{1}{\nu^2 z^2}\left(\frac{\eta V_0}{(1+z)} + \frac{\eta C z}{(1+z)^2} - \xi^2\right)\right\} F(z) = 0. \quad (20)$$

Comparing the previous equation with (14), one can find the following parameters:

$$\alpha_1 = 1, \qquad \alpha_2 = -1, \qquad \alpha_3 = -1,$$
$$p_2 = \left(\frac{\kappa(\kappa+1)}{\nu^2 R_0^2} D_0 + \frac{\xi^2}{\nu^2}\right),$$
$$p_1 = -\frac{\kappa(\kappa+1)}{\nu^2 R_0^2}(2D_0 + D_1) + \frac{1}{\nu^2}\left(\eta V_0 + \eta C - 2\beta^2\right), \quad (21)$$
$$p_0 = \frac{\kappa(\kappa+1)}{\nu^2 R_0^2}(D_0 + D_1 + D_2) - \frac{1}{\nu^2}\left(\eta V_0 - \xi^2\right),$$

and also

$$\alpha_4 = 0, \qquad \alpha_5 = \frac{1}{2},$$
$$\alpha_6 = \frac{1}{4} + p_2, \qquad \alpha_7 = -p_1,$$
$$\alpha_8 = p_0, \qquad \alpha_9 = p_2 - p_1 + p_0 + \frac{1}{4},$$
$$\alpha_{10} = 1 + 2\sqrt{p_0}, \quad (22)$$
$$\alpha_{11} = -2 + 2\left(\sqrt{p_2 - p_1 + p_0 + 1/4} - \sqrt{p_0}\right),$$
$$\alpha_{12} = \sqrt{p_0},$$
$$\alpha_{13} = \frac{1}{2} - \left(\sqrt{p_2 - p_1 + p_0 + 1/4} - \sqrt{p_0}\right).$$

From (16), (21), and (22), we obtained the closed form of the energy eigenvalues for the generalized Woods-Saxon potential in spin symmetry as

$$\left(n + \frac{1}{2} + \sqrt{\left(\kappa(\kappa+1)/\nu^2 R_0^2\right) D_2 - \eta C/\nu^2 + 1/4} + \sqrt{\left(\kappa(\kappa+1)/\nu^2 R_0^2\right)\left(D_0 + D_1 + D_2\right) - \left(1/\nu^2\right)\left(\eta V_0 - \xi^2\right)}\right)^2 = \left(\kappa(\kappa+1)/\nu^2 R_0^2\right) D_0 + \xi^2/\nu^2. \quad (23)$$

Recalling η and ξ from (10a) and (10b) the previous equation becomes a quadratic algebraic equation in $E_{n\kappa}$. Thus, the solution of this algebraic equation with respect to $E_{n\kappa}$ can be obtained in terms of particular values of n and κ. In Table 1, we have calculated some energy levels of the generalized Woods-Saxon potential under the spin symmetry limit for different

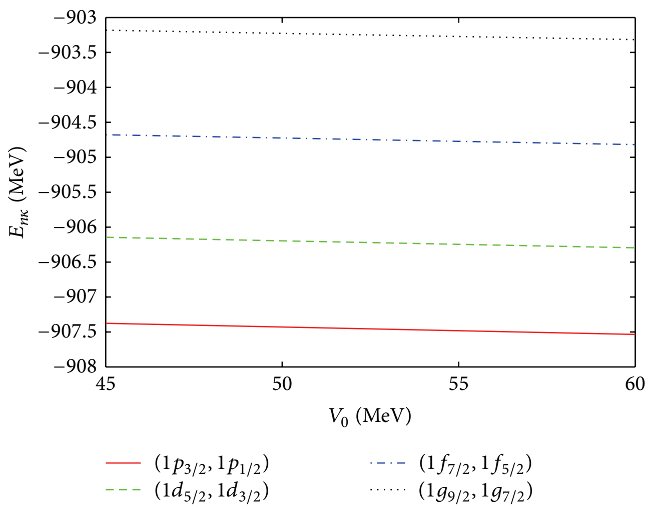

FIGURE 2: Computed energies as a function of V_0.

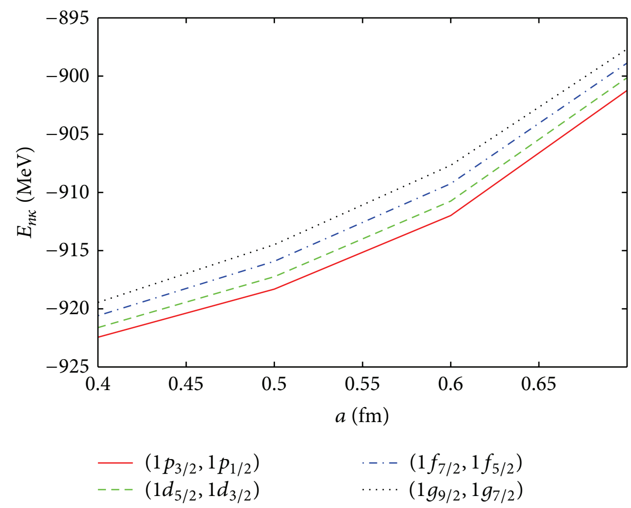

FIGURE 4: Computed energies as a function of a.

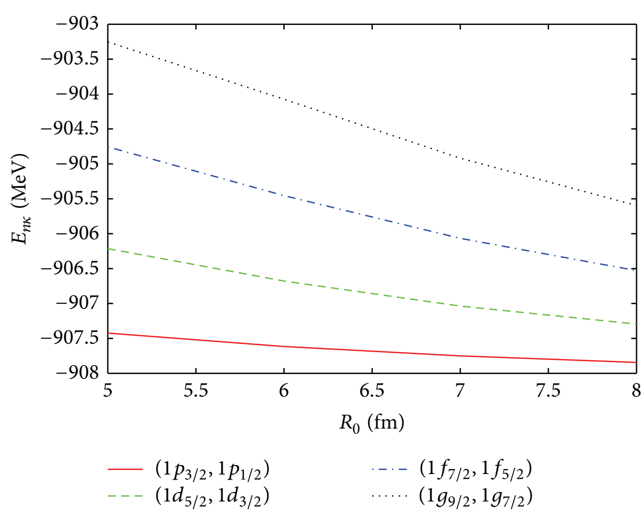

FIGURE 3: Computed energies as a function of R_0.

values of C. The empirical values that can be found in [52], as $r_0 = 1.285$ fm, $a = 0.65$ fm, $M_n = 1.00866$ u, and $V_0 = 4.5 + 0.13A$ (MeV), are used. Here, A is the mass number of target nucleus, and $R_0 = r_0 A^{1/3}$. From Table 1, we can see that pairs $(np_{1/2}, np_{3/2})$,$(nd_{3/2}, nd_{5/2})$, $(nf_{5/2}, nf_{7/2})$, $(ng_{7/2}, ng_{9/2})$, and so forth are degenerate. Thus, each pair is considered as spin doublet and has negative energy, and also when C increases, the energy decreases. The reason is that when C increases, the depth of the potential well becomes deeper and then the energy levels decrease. In Figure 2, the results are shown as a function of V_0. As we saw in Table 1, when the depth of the potential well increases, then the energy levels decrease. In Figure 3, the results are presented as a function of width of the potential R_0. Here, when the width of the potential increases, the energy levels decrease. Finally we showed the numerical results as a function of surface thickness a in Figure 4, and one can observe that when the surface thickness increases, the energy levels decrease too.

To find corresponding wave functions, referring to (15), (21), and (22), we find the upper component of the Dirac spinor as

$$F_{n\kappa}(z) = z^{\alpha_{12}} (1 - \alpha_3 z)^{-\alpha_{12} - \alpha_{13}/\alpha_3} P_n^{(\alpha_{10}-1, \alpha_{11}/\alpha_3 - \alpha_{10} - 1)} \times (1 - 2\alpha_3 z)$$
$$= z^{\sqrt{\xi_3}} (1 + z)^{1/2 + \sqrt{(\kappa(\kappa+1)/\nu^2 R_0^2) D_2 - (\eta C/\nu^2) + 1/4}} \times P_n^{(2\sqrt{p_0}, -2\sqrt{(\kappa(\kappa+1)/\nu^2 R_0^2) D_2 - \eta C/\nu^2 + 1/4})} (1 + 2z) \quad (24)$$

or equivalently

$$F_{n\kappa}(r) = e^{\sqrt{p_0}((r-R_0)/a)} \left(1 + e^{(r-R_0)/a}\right)^{1/2 + \sqrt{(\kappa(\kappa+1)/\nu^2 R_0^2) D_2 - \eta C/\nu^2 + 1/4}} \times P_n^{(2\sqrt{p_0}, -2\sqrt{(\kappa(\kappa+1)/\nu^2 R_0^2) D_2 - \eta C/\nu^2 + 1/4})} \left(1 + 2e^{(r-R_0)/a}\right). \quad (25)$$

Finally, the lower component of the Dirac spinor can be calculated as

$$G_{n\kappa}(r) = \frac{\hbar c}{Mc^2 + E_{n\kappa} - C_s} \left(\frac{d}{dr} + \frac{\kappa}{r}\right) F_{n\kappa}(r), \quad (26)$$

where $E_{n\kappa} \neq -Mc^2 + C_s$ [20].

5. Conclusion

In this paper we have studied the spin symmetry of a Dirac nucleon subjected to scalar and vector generalized Woods-Saxon potentials. The quadratic energy equation and spinor wave functions for bound states have been obtained by parametric form of the Nikiforov-Uvarov method. It is shown that there exist negative-energy bound states in the case of exact spin symmetry ($C_s = 0$). We gave some numerical results of the energy eigenvalues too.

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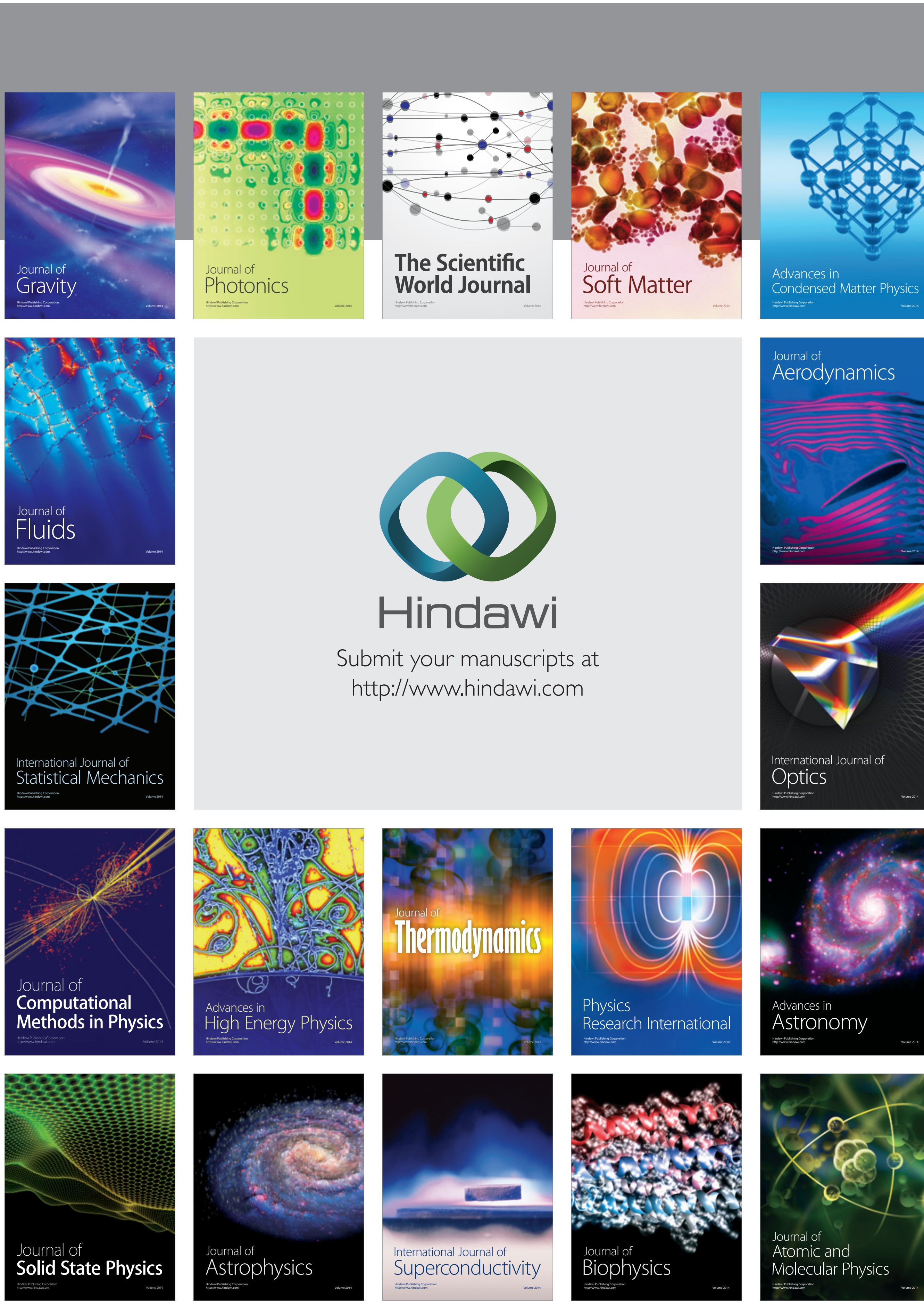
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