Effective potential and universality in GUT-inspired gauge-Higgs unification

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(Received 28 April 2020; accepted 17 June 2020; published 14 July 2020)

The effective potential for the Aharonov-Bohm phase θ_H in the fifth dimension in GUT-inspired $SO(5) \times U(1) \times SU(3)$ gauge-Higgs unification is evaluated to show that dynamical electroweak symmetry breaking takes place with $\theta_H \neq 0$, where the four-dimensional Higgs boson mass of 125 GeV is generated at the quantum level. The cubic and quartic self-couplings (λ_3, λ_4) of the Higgs boson are found to satisfy universal relations, i.e., they are determined, to high accuracy, solely by θ_H, irrespective of the values of other parameters in the model. For $\theta_H = 0.1$ (0.15), λ_3 and λ_4 are smaller than those in the standard model by 7.7% (8.1%) and 30% (32%), respectively.

DOI: 10.1103/PhysRevD.102.015005

I. INTRODUCTION

The Higgs boson is responsible for the electroweak symmetry breaking in the standard model (SM). The Higgs potential is arranged such that the Higgs field spontaneously develops a nonvanishing vacuum expectation value. Its couplings to quarks and leptons (Yukawa couplings) are determined such that the observed quark-lepton mass spectrum is reproduced. Although the SM seems consistent with almost all experimental data so far obtained, it is yet to be seen whether or not the Higgs boson is exactly what is postulated in the SM. While the gauge sector in the SM is regulated by the gauge principle, the Higgs sector lacks such a principle, which leaves arbitrariness in the theory. The Higgs boson mass acquires large quantum corrections which must be canceled by fine-tuning of parameters in the model.

One approach to overcome these difficulties is gauge-Higgs unification (GHU) in which the four-dimensional (4D) Higgs boson is identified with the 4D fluctuation mode of an Aharonov-Bohm (AB) phase in the fifth dimension (5D). The 4D Higgs field is contained in the extra-dimensional component of gauge potentials. As an AB phase the Higgs boson is massless at the tree level, but acquires a finite mass at the quantum level, independent of the cutoff scale and regularization method. The gauge hierarchy problem is naturally solved [1–6].

Recently, substantial advances have been made in gauge-Higgs unification. Realistic models have been constructed which yield nearly the same phenomenology as the SM at low energies and give many predictions to be explored at the LHC and ILC. Most gauge-Higgs unification models are constructed on orbifolds such as $M^4 \times (S^1/Z_2)$ and the Randall-Sundrum (RS) warped space. Chiral fermions naturally emerge on orbifolds [7]. The $SU(2)_L$ doublet Higgs field must appear as a zero mode of the fifth-dimensional component of gauge fields. This condition leads to gauge groups such as $SU(3) \times U(1)_X \times SU(3)_C$ or $SO(5) \times U(1)_X \times SU(3)_C$, among which the latter accommodates the custodial symmetry in the Higgs sector. Quark-lepton multiplets are introduced such that with orbifold conditions specified zero modes appear precisely for quarks and leptons, but not for exotic light fermions. They must have observed couplings to W and Z bosons, and their masses must be reproduced. Further, the effective potential for the AB phase θ_H must have a global minimum at $\theta_H \neq 0$ so that the electroweak gauge symmetry is dynamically broken to $U(1)_{\rm EM}$. As a model satisfying these conditions, $SO(5) \times U(1)_X \times SU(3)_C$ gauge-Higgs unification is formulated in the RS space [8–20].

In the RS space, which is an anti–de Sitter (AdS) spacetime sandwiched by UV and IR branes, wave functions of dominant components of W and Z bosons are almost constant in the bulk region so that gauge couplings of quarks and leptons are nearly the same as those in the SM. The hierarchy between the Kaluza-Klein (KK) mass scale ($\sim$10 TeV) and the weak scale ($\sim$100 GeV) naturally emerges. Two typical ways of introducing fermions have been investigated. In one type of model (the A model) quarks and leptons are introduced in the vector representation of $SO(5)$. The model predicts large parity violation in the Z' couplings of quarks and leptons, which can be checked in the early stage of the ILC experiments with polarized electron and positron beams [14,17,21,22].

It has been noticed, however, that there arises a difficulty in promoting the A model to grand unification [23–27]. The natural extension of the $SO(5) \times U(1)_X \times SU(3)_C$ model is $SO(11)$ gauge-Higgs grand unification [24]. Up-type quarks are contained in the spinor representation of $SO(11)$, but not in the vector representation so that up-type quarks in the A model do not appear from the $SO(11)$ gauge-Higgs unification. A new way of introducing fermion multiplets has been found which can be embedded into the $SO(11)$ gauge-Higgs grand unification [16]. In this grand unified theory (GUT)-inspired model, or the B model, quarks and leptons are introduced in the spinor and singlet representations of $SO(5)$. It has been shown that quarks and leptons have almost the same gauge couplings as in the SM. Furthermore, the flavor mixing is nicely incorporated with gauge-invariant brane interactions in the B model. The Cabibbo-Kobayashi-Maskawa (CKM) matrix is obtained, and remarkably flavor-changing neutral-current (FCNC) interactions are naturally suppressed [20].

In this paper we evaluate the effective potential $V_{\rm eff}(\theta_H)$ in GUT-inspired $SO(5) \times U(1)_X \times SU(3)_C$ gauge-Higgs unification. It will be shown that with appropriate choices of parameters $V_{\rm eff}(\theta_H)$ has a global minimum at $\theta_H \neq 0$ and the Higgs boson mass $m_H = 125$ GeV is obtained. The cubic and quartic self-couplings of the Higgs boson are determined from $V_{\rm eff}(\theta_H)$. We shall show that those cubic and quartic self-couplings are, to high accuracy, determined as functions of θ_H only. They do not depend on other parameters of the theory. It will be explained how this property appears in the model.

The effective potential $V_{\rm eff}(\theta_H)$ is important in discussing phase transitions at finite temperature as well. Recently, a possibility of having first-order phase transitions in gauge-Higgs unification has been argued [28]. At the moment the nature of phase transitions at finite temperature in $SO(5) \times U(1) \times SU(3)$ gauge-Higgs unification remains unclear.

The Higgs boson as an AB phase in gauge-Higgs unification is similar to that in composite Higgs models in which the Higgs boson appears as a pseudo-Nambu-Goldstone boson [9,29,30]. In both scenarios the Higgs boson field has the character of a phase, but has a quite different mechanism for acquiring its mass. In gauge-Higgs unification the Higgs boson mass is generated by the gauge-invariant dynamics of the AB phase, whereas it results from the ungauged part of global symmetry in composite Higgs models. Further, in gauge-Higgs unification left-handed and right-handed components of quarks and leptons are normally localized in opposite branes: if left-handed components are localized near the UV (IR) brane, then right-handed components are localized near the IR (UV) brane. In typical composite Higgs models all light quarks and leptons are assumed to be localized near the UV brane. This leads to big difference in phenomenology associated with Z' or technirho bosons. In gauge-Higgs unification in RS space there is large parity violation in Z' couplings of quarks and leptons [14,31], whereas such asymmetry is absent in composite Higgs models. Gauge-Higgs unification is strictly regulated by the gauge principle.

The paper is organized as follows. In Sec. II the model is introduced. In Sec. III the effective potential $V_{\rm eff}(\theta_H)$ is evaluated. We show that dynamical electroweak symmetry breaking takes place. The cubic and quartic self-couplings, λ_3 and λ_4, of the Higgs boson are evaluated from $V_{\rm eff}(\theta_H)$. It is observed there that λ_3 and λ_4 are determined to high accuracy as functions of θ_H, irrespective of other parameters in the model. The origin of the θ_H universality in the RS space is clarified in Sec. IV. In Sec. V the spectrum of dark fermions is evaluated. Section VI is devoted to a summary. The mass spectra of all fields in the model are summarized in Appendix A. The functions used for the evaluation of $V_{\rm eff}(\theta_H)$ are summarized in Appendix B.

II. MODEL

The GUT-inspired $SO(5) \times U(1)_X \times SU(3)_C$ gauge-Higgs unification was introduced in Refs. [16,20]. It is defined in the RS warped space with metric given by [32]

$$ds^2 = g_{MN} dx^M dx^N = e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \qquad (2.1)$$

where $M, N = 0, 1, 2, 3, 5$, $\mu, \nu = 0, 1, 2, 3$, $y = x^5$, $\eta_{\mu\nu} = {\rm diag}(-1, +1, +1, +1)$, $\sigma(y) = \sigma(y + 2L) = \sigma(-y)$, and $\sigma(y) = ky$ for $0 \leq y \leq L$. In terms of the conformal coordinate $z = e^{ky}$ $(1 \leq z \leq z_L = e^{kL})$ in the region $0 \leq y \leq L$,

$$ds^2 = \frac{1}{z^2} \left(\eta_{\mu\nu} dx^\mu dx^\nu + \frac{dz^2}{k^2} \right). \qquad (2.2)$$

The bulk region $0 < y < L$ $(1 < z < z_L)$ is AdS spacetime with a cosmological constant $\Lambda = -6k^2$, which is sandwiched by the UV brane at $y = 0$ $(z = 1)$ and the IR brane at $y = L$ $(z = z_L)$. The KK mass scale is $m_{\rm KK} = \pi k/(z_L - 1) \simeq \pi k z_L^{-1}$ for $z_L \gg 1$.

In addition to the gauge fields $A_M^{SU(3)_C}$, $A_M^{SO(5)}$, and $A_M^{U(1)_X}$ of $SU(3)_C$, $SO(5)$, and $U(1)_X$, we introduce the matter fields listed in Table I. Fields defined in the bulk satisfy orbifold boundary conditions. Each gauge field satisfies

TABLE I. The $SU(3)_C \times SO(5) \times U(1)_X$ content of matter fields is shown in the GUT-inspired B model and previous A model. In the A model only $SU(3)_C \times SO(4) \times U(1)_X$ symmetry is preserved on the UV brane so that the $SU(2)_L \times SU(2)_R$ content is shown for brane fields. The B model is analyzed in the present paper.

	B model	A model
Quark	$\Psi^{\alpha}_{(\mathbf{3},\mathbf{4})}:(\mathbf{3},\mathbf{4})_{\frac{1}{6}},\ \Psi^{\pm\alpha}_{(\mathbf{3},\mathbf{1})}:(\mathbf{3},\mathbf{1})^{\pm}_{-\frac{1}{3}}$	$\Psi^{\alpha}_1:(\mathbf{3},\mathbf{5})_{\frac{2}{3}},\ \Psi^{\alpha}_2:(\mathbf{3},\mathbf{5})_{-\frac{1}{3}}$
Lepton	$\Psi^{\alpha}_{(\mathbf{1},\mathbf{4})}:(\mathbf{1},\mathbf{4})_{-\frac{1}{2}}$	$\Psi^{\alpha}_3:(\mathbf{1},\mathbf{5})_{-1}\ \Psi^{\alpha}_4:(\mathbf{1},\mathbf{5})_0$
Dark fermion	$\Psi^{\beta}_F:(\mathbf{3},\mathbf{4})_{\frac{1}{6}},\ \Psi^{\pm\gamma}_{(\mathbf{1},\mathbf{5})}:(\mathbf{1},\mathbf{5})^{\pm}_0$	$\Psi^{\delta}_F:(\mathbf{1},\mathbf{4})_{\frac{1}{2}}$
Brane fermion	$\chi^{\alpha}:(\mathbf{1},\mathbf{1})_0$	$\hat{\chi}^{q}_{1,2,3R}:(\mathbf{3},[\mathbf{2},\mathbf{1}])_{\frac{7}{6},\frac{1}{6},-\frac{5}{6}}$
		$\hat{\chi}^{l}_{1,2,3R}:(\mathbf{1},[\mathbf{2},\mathbf{1}])_{-\frac{3}{2},\frac{1}{2},-\frac{1}{2}}$
Brane scalar	$\Phi_{(\mathbf{1},\mathbf{4})}:(\mathbf{1},\mathbf{4})_{\frac{1}{2}}$	$\hat{\Phi}:(\mathbf{1},[\mathbf{1},\mathbf{2}])_{\frac{1}{2}}$
Symmetry of brane interactions	$SU(3)_C \times SO(5) \times U(1)_X$	$SU(3)_C \times SO(4) \times U(1)_X$

$$\begin{pmatrix} A_\mu \\ A_y \end{pmatrix}(x, y_j - y) = P_j \begin{pmatrix} A_\mu \\ -A_y \end{pmatrix}(x, y_j + y) P_j^{-1}, \tag{2.3}$$

where $(y_0, y_1) = (0, L)$, $P_0 = P_1 = I_3$ for $A_M^{SU(3)_C}$, and $P_0 = P_1 = 1$ for $A_M^{U(1)_X}$. $P_0 = P_1 = P_{\mathbf{5}}^{SO(5)} = \mathrm{diag}(I_4, -1)$ for $A_M^{SO(5)}$ in the vector representation and $P_0 = P_1 = P_{\mathbf{4}}^{SO(5)} = \mathrm{diag}(I_2, -I_2)$ in the spinor representation. Quark and lepton multiplets satisfy

$$\begin{aligned}
\Psi^{\alpha}_{(\mathbf{3},\mathbf{4})}(x, y_j - y) &= -P_{\mathbf{4}}^{SO(5)} \gamma^5 \Psi^{\alpha}_{(\mathbf{3},\mathbf{4})}(x, y_j + y), \\
\Psi^{\pm\alpha}_{(\mathbf{3},\mathbf{1})}(x, y_j - y) &= \mp \gamma^5 \Psi^{\pm\alpha}_{(\mathbf{3},\mathbf{1})}(x, y_j + y), \\
\Psi^{\alpha}_{(\mathbf{1},\mathbf{4})}(x, y_j - y) &= -P_{\mathbf{4}}^{SO(5)} \gamma^5 \Psi^{\alpha}_{(\mathbf{1},\mathbf{4})}(x, y_j + y),
\end{aligned} \tag{2.4}$$

where $\alpha = 1 \sim 3$. Dark fermion multiplets satisfy

$$\begin{aligned}
\Psi^{\beta}_F(x, y_j - y) &= (-1)^j P_{\mathbf{4}}^{SO(5)} \gamma^5 \Psi^{\beta}_F(x, y_j + y), \\
\Psi^{\pm\gamma}_{(\mathbf{1},\mathbf{5})}(x, y_j - y) &= \pm P_{\mathbf{5}}^{SO(5)} \gamma^5 \Psi^{\pm\gamma}_{(\mathbf{1},\mathbf{5})}(x, y_j + y),
\end{aligned} \tag{2.5}$$

where $\beta = 1 \sim N_F$ and $\gamma = 1 \sim N_V$.

The bulk action of each gauge field ($A_M^{SU(3)_C}$, $A_M^{SO(5)}$, or $A_M^{U(1)_X}$) is given by

$$S_{\text{bulk}}^{\text{gauge}} = \int d^5x \sqrt{-\det G} \left[-\mathrm{tr}\left(\frac{1}{4} F^{MN} F_{MN} + \frac{1}{2\xi} (f_{\text{gf}})^2 + \mathcal{L}_{\text{gh}} \right) \right], \tag{2.6}$$

where $\sqrt{-\det G} = 1/kz^5$ and $F_{MN} = \partial_M A_N - \partial_N A_M - ig[A_M, A_N]$ with each 5D gauge coupling constant g. The gauge fixing f_{gf} and ghost terms $\mathcal{L}_{\text{gh}}$ have been specified in Ref. [16]. Each fermion multiplet $\Psi(x, y)$ in the bulk has its own bulk-mass parameter c [33]. The covariant derivative is given by

$$\begin{aligned}
\mathcal{D}(c) &= \gamma^A e_A{}^M \left(D_M + \frac{1}{8} \omega_{MBC} [\gamma^B, \gamma^C] \right) - c\sigma'(y), \\
D_M &= \partial_M - ig_S A_M^{SU(3)} - ig_A A_M^{SO(5)} - ig_B Q_X A_M^{U(1)}.
\end{aligned} \tag{2.7}$$

Here $\sigma' = d\sigma(y)/dy$ and $\sigma'(y) = k$ for $0 < y < L$. g_S, g_A, and g_B are $SU(3)_C$, $SO(5)$, and $U(1)_X$ gauge coupling constants. The bulk part of the action for the fermion multiplets, with $\bar{\Psi} = i\Psi^\dagger \gamma^0$, is given by

$$\begin{aligned}
S_{\text{bulk}}^{\text{fermion}} = \int d^5x \sqrt{-\det G} \Bigg\{ & \sum_J \bar{\Psi}^J \mathcal{D}(c_J) \Psi^J \\
& - \sum_\alpha \left(m_{D_\alpha} \bar{\Psi}^{+\alpha}_{(\mathbf{3},\mathbf{1})} \Psi^{-\alpha}_{(\mathbf{3},\mathbf{1})} + \text{H.c.} \right) \\
& - \sum_\gamma \left(m_{V_\gamma} \bar{\Psi}^{+\gamma}_{(\mathbf{1},\mathbf{5})} \Psi^{-\gamma}_{(\mathbf{1},\mathbf{5})} + \text{H.c.} \right) \Bigg\},
\end{aligned} \tag{2.8}$$

where the sum $\sum_J$ extends over $\Psi^J = \Psi^{\alpha}_{(\mathbf{3},\mathbf{4})}, \Psi^{\alpha}_{(\mathbf{1},\mathbf{4})}, \Psi^{\pm\alpha}_{(\mathbf{3},\mathbf{1})}, \Psi^{\beta}_F$, and $\Psi^{\pm\gamma}_{(\mathbf{1},\mathbf{5})}$.

The action for the brane scalar field $\Phi_{(\mathbf{1},\mathbf{4})}(x)$ is given by

$$\begin{aligned}
S_{\text{brane}}^{\Phi} = \int d^5x \sqrt{-\det G}\, \delta(y) & \\
\times \Big\{ & -(D_\mu \Phi_{(\mathbf{1},\mathbf{4})})^\dagger D^\mu \Phi_{(\mathbf{1},\mathbf{4})} \\
& - \lambda_{\Phi_{(\mathbf{1},\mathbf{4})}} \left(\Phi^\dagger_{(\mathbf{1},\mathbf{4})} \Phi_{(\mathbf{1},\mathbf{4})} - |w|^2 \right)^2 \Big\},
\end{aligned} \tag{2.9}$$

where $D_\mu = \partial_\mu - ig_A A_\mu^{SO(5)} - i\frac{1}{2} g_B A_\mu^{U(1)}$. The action for the gauge-singlet brane fermion $\chi^\alpha(x)$ is

$$S_{\text{brane}}^{\chi} = \int d^5x \sqrt{-\det G}\, \delta(y) \left\{ \frac{1}{2} \bar{\chi}^\alpha \gamma^\mu \partial_\mu \chi^\alpha - \frac{1}{2} M^{\alpha\beta} \bar{\chi}^\alpha \chi^\beta \right\}. \tag{2.10}$$

$\chi^\alpha(x)$ satisfies the Majorana condition $\chi^c = \chi$:

$$\chi = \begin{pmatrix} \xi \\ \eta \end{pmatrix}, \qquad \chi^c = \begin{pmatrix} +\eta^c \\ -\xi^c \end{pmatrix} = e^{i\delta_C} \begin{pmatrix} +\sigma^2 \eta^* \\ -\sigma^2 \xi^* \end{pmatrix}. \qquad (2.11)$$

On the UV brane there are $SU(3)_C \times SO(5) \times U(1)_X$-invariant brane interactions among the bulk fermion, brane fermion, and brane scalar fields. The relevant parts of the brane interactions are given by

$$S^{\rm int}_{\rm brane} = -\int d^5x \sqrt{-\det G}\,\delta(y) \times \Big\{ \kappa^{\alpha\beta} \bar{\Psi}^{\alpha}_{(\mathbf{3},\mathbf{4})} \Phi_{(\mathbf{1},\mathbf{4})} \cdot \Psi^{+\beta}_{(\mathbf{3},\mathbf{1})} + \tilde{\kappa}^{\alpha\beta} \bar{\chi}^{\beta} \tilde{\Phi}^{\dagger}_{(\mathbf{1},\mathbf{4})} \Psi^{\alpha}_{(\mathbf{1},\mathbf{4})} + {\rm H.c.} \Big\}, \qquad (2.12)$$

where the κ's and $\tilde{\kappa}$'s are coupling constants and

$$\Phi_{(\mathbf{1},\mathbf{4})} = \begin{pmatrix} \Phi_{[\mathbf{2},\mathbf{1}]} \\ \Phi_{[\mathbf{1},\mathbf{2}]} \end{pmatrix}, \qquad \tilde{\Phi}_{(\mathbf{1},\mathbf{4})} = \begin{pmatrix} i\sigma^2 \Phi^*_{[\mathbf{2},\mathbf{1}]} \\ -i\sigma^2 \Phi^*_{[\mathbf{1},\mathbf{2}]} \end{pmatrix}. \qquad (2.13)$$

When $\langle \Phi_{(\mathbf{1},\mathbf{4})} \rangle = (0,0,0,w)^t$, Eq. (2.12) generates additional mass terms:

$$\int d^5x \sqrt{-\det G}\,\delta(y) \Big\{ 2\mu^{\alpha\beta} \bar{d}'^{\alpha}_R D^{+\beta}_L + {\rm H.c.} \Big\}, \quad \mu^{\alpha\beta} = \frac{\kappa^{\alpha\beta} w}{\sqrt{2}} \qquad (2.14)$$

in the down-type quark sector, and

$$-\int d^5x \sqrt{-\det G}\,\delta(y) \frac{m_B^{\alpha\beta}}{\sqrt{k}} (\bar{\chi}^{\beta} \nu'^{\alpha}_R + \bar{\nu}'^{\alpha}_R \chi^{\beta}), \quad m_B^{\alpha\beta} = \tilde{\kappa}^{\alpha\beta} w \sqrt{k} \qquad (2.15)$$

in the neutrino sector. With the Majorana masses in Eq. (2.10), the mass term (2.15) induces the inverse-seesaw mechanism in the neutrino sector [26]. Further, $\langle \Phi_{(\mathbf{1},\mathbf{4})} \rangle \neq 0$ breaks $SO(4) \times U(1)_X$ down to $SU(2)_L \times U(1)_Y$. We assume that $w \gg m_{\rm KK}$. The 4D $SU(2)_L$ gauge coupling is given by $g_w = g_A/\sqrt{L}$. The 5D gauge coupling $g_Y^{\rm 5D}$ of $U(1)_Y$ and the 4D bare Weinberg angle at the tree level, θ_W^0, are given by

$$g_Y^{\rm 5D} = \frac{g_A g_B}{\sqrt{g_A^2 + g_B^2}}, \qquad \sin\theta_W^0 = \frac{s_\phi}{\sqrt{1 + s_\phi^2}}, \qquad s_\phi = \frac{g_B}{\sqrt{g_A^2 + g_B^2}}. \qquad (2.16)$$

The bare Weinberg angle θ_W^0 with a given θ_H is determined to fit the LEP1 data for $e^+e^- \to \mu^+\mu^-$ at $\sqrt{s} = m_Z$ [34]. Approximately, $\sin^2\theta_W^0 \simeq 0.1140 + 0.1186 \cos\theta_H - 0.0014 \cos 2\theta_H$. The values of the gauge couplings turn out to be very close to those in the SM, with $\sin^2\theta_W = 0.2312$ [20].

The 4D Higgs boson $\Phi_H(x)$ is contained in the $SO(5)/SO(4)$ part of $A_y^{SO(5)}$. In the z coordinate $A_z = (kz)^{-1} A_y$ $(1 \le z \le z_L)$, and

$$A_z^{(j5)}(x,z) = \frac{1}{\sqrt{k}} \phi_j(x) u_H(z) + \cdots, \qquad u_H(z) = \sqrt{\frac{2}{z_L^2 - 1}}\, z, \qquad \Phi_H(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_2 + i\phi_1 \\ \phi_4 - i\phi_3 \end{pmatrix}. \qquad (2.17)$$

At the quantum level Φ_H develops a nonvanishing expectation value. Without loss of generality, we assume $\langle\phi_1\rangle, \langle\phi_2\rangle, \langle\phi_3\rangle = 0$ and $\langle\phi_4\rangle \neq 0$, which is related to the AB phase θ_H in the fifth dimension. The eigenvalues of

$$\hat{W} = P \exp\Big\{ i g_A \int_{-L}^{L} dy A_y \Big\} \cdot P_1 P_0 \qquad (2.18)$$

are gauge invariant. For $A_y = (2k)^{-1/2} \phi_4(x) v_H(y) T^{(45)}$, where $v_H(y) = k e^{ky} u_H(z)$ for $0 \le y \le L$ and $v_H(-y) = v_H(y) = v_H(y + 2L)$, one finds

$$\hat{W} = \exp\{ i f_H^{-1} \phi_4(x) \cdot 2T^{(45)} \}, \qquad f_H = \frac{2}{g_A} \sqrt{\frac{k}{z_L^2 - 1}} = \frac{2}{g_w} \sqrt{\frac{k}{L(z_L^2 - 1)}}, \qquad \theta_H = \frac{\langle\phi_4\rangle}{f_H}. \qquad (2.19)$$

Note that

$$A_z^{(45)}(x,z) = \frac{1}{\sqrt{k}} \{ \theta_H f_H + H(x) \} u_H(z) + \cdots, \qquad (2.20)$$

where $H(x)$ is the neutral Higgs boson field. There is a large gauge transformation which shifts θ_H by 2π, preserving the boundary conditions. The physics is invariant under $\theta_H \to \theta_H + 2\pi$. We shall evaluate the effective potential $V_{\rm eff}(\theta_H)$ in the next section.

III. EFFECTIVE POTENTIAL

The effective potential $V_{\rm eff}(\theta_H)$ at the one-loop level is evaluated from the mass spectra of all fields which depend on θ_H. After the Wick rotation into the Euclidean signature it is expressed as

$$V_{\rm eff}(\theta_H) = \sum \pm \frac{1}{2}\int \frac{d^4p_E}{(2\pi)^4}\sum_n \ln\{p_E^2 + m_n(\theta_H)^2\}, \tag{3.1}$$

where the sign + (−) corresponds to bosons (fermions). When the KK spectrum $\{m_n(\theta_H)\}$ is determined by zeros of a function $\rho(z;\theta_H)$, namely, by

$$\rho(m_n;\theta_H) = 0 \ (n = 1, 2, 3, \ldots), \tag{3.2}$$

$V_{\rm eff}(\theta_H)$ is given [35] by

$$V_{\rm eff}(\theta_H) = \sum \pm \frac{1}{(4\pi)^2}\int_0^\infty dy\, y^3 \ln \rho(iy;\theta_H). \tag{3.3}$$

The θ_H-dependent part of $V_{\rm eff}^{\rm 1loop}(\theta_H)$ is finite, independent of the cutoff and regularization method employed.

The spectrum-determining functions $\rho(z;\theta_H)$ for all fields in the model were given in Ref. [16]. They are summarized in Appendix A for convenience. Relevant contributions come from W and Z gauge fields, top-bottom quark multiplets, and dark fermions in the spinor and vector representations. Contributions from light quarks and leptons are negligible. To avoid unnecessary confusion in the following argument, we denote the effective potential as $V_{\rm eff}(\theta)$. The physical value θ_H corresponds to the global minimum of $V_{\rm eff}(\theta)$, namely, $dV_{\rm eff}/d\theta|_{\theta=\theta_H} = 0$. One finds

$$\begin{aligned} V_{\rm eff}(\theta) &= 2(3-\xi^2)A_W(\theta) + (3-\xi^2)A_Z(\theta) + 3\xi^2 A_S(\theta) \\ &\quad - 12A_{\rm top}(\theta) - 12A_{\rm bottom}(\theta) - 12n_F A_F(\theta) \\ &\quad - 8n_V A_V(\theta), \\ A_p(\theta) &\equiv \frac{(kz_L^{-1})^4}{(4\pi)^2}\int_0^\infty dq\, q^3 \ln\left\{1 + \sum_{n=1}^2 Q_p^{(n)}(q)\cos(n\theta)\right\}, \end{aligned} \tag{3.4}$$

where n_F and n_V are the number of Ψ_F and $\Psi^\pm_{(1,5)}$, and ξ is a gauge parameter in the generalized R_ξ gauge. The integration variable has been changed from y in Eq. (3.3) to $q = k^{-1}z_L y$. In the following we take $z_L = e^{kL} = 10^{10}$ and $\xi = 0$. The contributions from W and Z towers and the Goldstone boson tower are given by

$$\begin{aligned} Q_W^{(1)}(q) &= Q_Z^{(1)}(q) = Q_S^{(1)}(q) = 0, \\ Q_W^{(2)}(q) &= -\frac{1}{-4iz_L q^{-1}\hat{C}'(q)\hat{S}(q) + 1}, \\ Q_Z^{(2)}(q) &= -\frac{1 + s_\phi^2}{-4iz_L q^{-1}\hat{C}'(q)\hat{S}(q) + 1 + s_\phi^2}, \\ Q_S^{(2)}(q) &= -\frac{1}{-2iz_L q^{-1}\hat{C}(q)\hat{S}(q) + 1}. \end{aligned} \tag{3.5}$$

$\hat{C}(q)$, $\hat{S}(q)$, etc. in the expressions above and $\hat{C}_L(q,c)$, $\hat{S}_L(q,c)$, etc. in the expressions below are given in Appendix B. They are expressed in terms of modified Bessel functions.

The top- and bottom-quark contributions are given by

$$\begin{aligned} Q_{\rm top}^{(2)}(q) &= Q_{\rm bottom}^{(2)}(q) = 0, \\ Q_{\rm top}^{(1)}(q) &= -\frac{1}{2\hat{S}_L(q;c_t)\hat{S}_R(q;c_t) + 1}, \\ Q_{\rm bottom}^{(1)}(q) &= -\frac{\widetilde{S_L S_R}}{2\hat{S}_L(q;c_t)\hat{S}_R(q;c_t)\widetilde{S_L S_R} + 2|\mu|^2\hat{C}_R(q;c_t)\hat{S}_R(q;c_t)\widetilde{C_L S_L} + 1}, \\ \widetilde{S_L S_R} &= \hat{S}_L(q;c_{D_b} + \tilde{m}_{D_b})\hat{S}_R(q;c_{D_b} - \tilde{m}_{D_b}) + \hat{S}_L(q;c_{D_b} - \tilde{m}_{D_b})\hat{S}_R(q;c_{D_b} + \tilde{m}_{D_b}) \\ &\quad + \hat{C}_L(q;c_{D_b} + \tilde{m}_{D_b})\hat{C}_R(q;c_{D_b} - \tilde{m}_{D_b}) + \hat{C}_L(q;c_{D_b} - \tilde{m}_{D_b})\hat{C}_R(q;c_{D_b} + \tilde{m}_{D_b}) - 2, \\ \widetilde{C_L S_L} &= \hat{C}_L(q;c_{D_b} + \tilde{m}_{D_b})\hat{S}_L(q;c_{D_b} - \tilde{m}_{D_b}) + \hat{C}_L(q;c_{D_b} - \tilde{m}_{D_b})\hat{S}_L(q;c_{D_b} + \tilde{m}_{D_b}). \end{aligned} \tag{3.6}$$

In the above expressions we have assumed that the brane interaction term (2.12) is diagonal in generation space. c_{D_α} is the bulk mass parameter of $\Psi^{\pm\alpha}_{(3,1)}$ and $\tilde{m}_{D_\alpha} = m_{D_\alpha}/k$. Numerically, the bottom-quark contribution is very small and may be ignored. There are two kinds of dark fermions: Ψ_F^β and $\Psi^{\pm\gamma}_{(1,5)}$. Their contributions are given by

$$
\begin{aligned}
Q_F^{(1)}(q) &= \frac{1}{2\hat{S}_L(q;c_F)\hat{S}_R(q;c_F)+1}, \\
Q_F^{(2)}(q) &= 0, \\
Q_V^{(1)}(q) &= 0, \\
Q_V^{(2)}(q) &= -\frac{2}{\hat{B}_0(q;c_V,\tilde{m}_V)}, \\
\hat{B}_0(q;c_V,\tilde{m}_V) &= \hat{C}_L(q;c_V+\tilde{m}_V)\hat{C}_R(q;c_V-\tilde{m}_V) \\
&\quad + \hat{C}_L(q;c_V-\tilde{m}_V)\hat{C}_R(q;c_V+\tilde{m}_V) \\
&\quad + \hat{S}_L(q;c_V+\tilde{m}_V)\hat{S}_R(q;c_V-\tilde{m}_V) \\
&\quad + \hat{S}_L(q;c_V-\tilde{m}_V)\hat{S}_R(q;c_V+\tilde{m}_V). \qquad (3.7)
\end{aligned}
$$

For the sake of simplicity we set degenerate bulk mass parameters c_F for Ψ_F^β, and degenerate masses $m_V = k\tilde{m}_V$ and bulk mass parameters c_V for $\Psi_{(1,5)}^{\pm\gamma}$. We note that contributions from gauge bosons and $\Psi_{(1,5)}^{\pm\gamma}$ fields to $V_{\rm eff}(\theta)$ are periodic in θ with a period π, whereas those from top- and bottom-quark fields and Ψ_F^β fields are periodic with a period 2π.

The parameters of the model are determined in the following steps. (i) We pick the value of θ_H. In other words, we adjust the parameters of the model such that $V_{\rm eff}(\theta)$ has a global minimum at $\theta = \theta_H$. (ii) We take $z_L = 10^{10}$. Then, k is determined so as to reproduce m_Z, and the KK mass scale $m_{\rm KK} = \pi k(z_L - 1)^{-1}$ is fixed. (iii) The bulk mass parameters of $\Psi_{(3,4)}^\alpha$ and $\Psi_{(1,4)}^\alpha$ are fixed from the masses of up-type quarks and charged leptons. In particular, c_t is determined by m_t. (iv) The bulk mass parameters c_{D_α} of $\Psi_{(3,1)}^{\pm\alpha}$ and brane interaction coefficients $\mu^{\alpha\beta}$ are determined so as to reproduce the masses of down-type quarks and the CKM matrix. Similarly, the Majorana mass terms $M^{\alpha\beta}$ and brane interactions $\tilde{\kappa}^{\alpha\beta}$ are determined so as to reproduce neutrino masses and the Pontecorvo-Maki-Nakagawa-Sakata matrix. As remarked above, these parameters are numerically irrelevant for $V_{\rm eff}(\theta)$. (v) At this stage there remain five parameters to be determined: (n_F, c_F) of Ψ_F^β and $(n_V, c_V, \tilde{m}_V)$ of $\Psi_{(1,5)}^{\pm\gamma}$. There are two conditions to be satisfied:

$$
\begin{aligned}
(a):\ & \left.\frac{dV_{\rm eff}}{d\theta}\right|_{\theta=\theta_H} = 0, \\
(b):\ & m_H^2 = \frac{1}{f_H^2}\left.\frac{d^2V_{\rm eff}}{d\theta^2}\right|_{\theta=\theta_H}, \qquad (3.8)
\end{aligned}
$$

where $m_H = 125.1$ GeV. The second condition for the Higgs boson mass m_H follows from the fact that the effective potential for the 4D Higgs field $H(x)$ is given by $V_{\rm eff}(\theta_H + f_H^{-1}H)$, as inferred from Eq. (2.20). The conditions (3.8) give two constraints to be satisfied among the five parameters $(n_F, c_F, n_V, c_V, \tilde{m}_V)$. We first fix, for instance, (n_F, n_V, c_V) and determine $(c_F, \tilde{m}_V)$ by Eq. (3.8).

One may wonder whether the arbitrary choice of the parameters in the last step diminishes the predictive power of the model. Quite surprisingly, many of the physical quantities do not depend on such details of the parameter choice, being determined solely by θ_H. There appears the θ_H universality, which will be explained in the next section.

Here we give some examples. The parameters fixed in steps (i)–(iv) above are tabulated in Table II. In Fig. 1 the effective potential for $\theta_H = 0.1$, $n_F = n_V = 2$, and $c_V = 0$ is displayed. $c_F = 0.319$ and $\tilde{m}_V = 0.0806$ are chosen to satisfy Eq. (3.8). One observes that the electroweak symmetry is dynamically broken. In Fig. 2 the contributions of relevant fields to the effective potential $V_{\rm eff}(\theta)$ are displayed. There is a lower bound for θ_H in order to reproduce the top-quark mass. $\theta_H \geq \theta_{c1}$, where $\theta_{c1} \sim 0.015$ for $z_L = 10^{10}$. Similarly, there is a constraint for the warp factor. For $\theta_H = 0.1$, $n_F = n_V = 2$, the top-quark mass is reproduced only if $z_L \geq z_{L1} \sim 10^{8.1}$ and dynamical electroweak symmetry breaking is achieved only if $z_L \leq z_{L2} \sim 10^{15.5}$.

The effective potential $V_{\rm eff}(\theta)$ has more information on the Higgs self-couplings. By expanding $V_{\rm eff}(\theta_H + H/f_H)$, one finds the Higgs self-couplings $\lambda_n H^n$. The nth self-coupling λ_n is given by

$$
\lambda_n \equiv \frac{1}{n!f_H^n}\left.\frac{d^nV_{\rm eff}}{d\theta^n}\right|_{\theta=\theta_H}. \qquad (3.9)
$$

The couplings λ_3 and λ_4 are plotted in Figs. 3 and 4 as functions of θ_H for $c_V = 0.2, n_F = n_V = 2$. The fitting curves are given by

$$
\begin{aligned}
\text{B model}:\ \lambda_3/\text{GeV} &= 39.6\cos\theta_H - 5.21(1+\cos 2\theta_H) \\
&\quad - 0.00911\cos 3\theta_H, \\
\lambda_4 &= -0.0695 + 0.0852\cos\theta_H \\
&\quad + 0.00725\cos 2\theta_H. \qquad (3.10)
\end{aligned}
$$

λ_3 and λ_4 in the A model are also plotted in Figs. 3 and 4, for which the fitting curves are given by

TABLE II. Parameters determined for $z_L = 10^{10}$ and $\theta_H = 0.05$, 0.1, 0.15. We set $\mu^{\alpha\beta} = \mu_\alpha\delta_{\alpha\beta}$, and take $c_{D_b} = 1.04$. μ_b is determined so as to reproduce m_b.

θ_H	k [10^{13} GeV]	$m_{\rm KK}$ [TeV]	c_t	c_{D_b}	μ_b
0.05	7.68	24.1	−0.226	1.04	0.106
0.10	3.84	12.1	−0.227	1.04	0.104
0.15	2.57	8.07	−0.230	1.04	0.0990

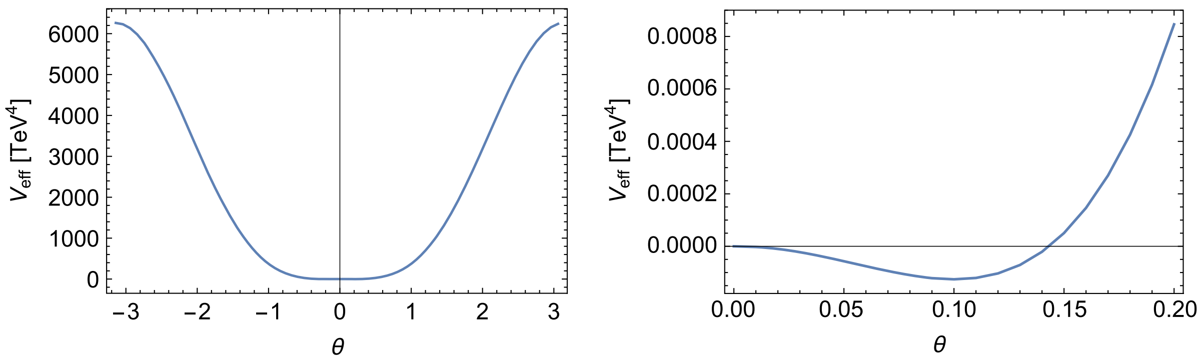

FIG. 1. The effective potential for $\theta_H = 0.1$, $n_F = 2$, $n_V = 2$, and $c_V = 0$. The global minimum is located at $\theta = \theta_H$.

$$
\begin{aligned}
\text{A model}:\ \lambda_3/\mathrm{GeV} &= 32.4\cos\theta_H - 2.26(1+\cos 2\theta_H) \\
&\quad - 1.1\cos 3\theta_H, \\
\lambda_4 &= -0.00264 - 0.0129\cos\theta_H \\
&\quad + 0.0363\cos 2\theta_H. \qquad (3.11)
\end{aligned}
$$

Note that λ_3 vanishes at $\theta_H = \frac{1}{2}\pi$ as a consequence of the H parity in GHU models [36]. For $\theta_H \gtrsim 0.6$, λ_4 becomes negative which, however, does not imply instability. The θ-dependent part of $V_{\mathrm{eff}}(\theta)$ is finite and bounded from below. Gauge-Higgs unification does not experience the vacuum instability problem that afflicts most 4D field theories. From the experimental constraints from LEP1, LEP2, and LHC data for the nonobservation of Z' events, it is inferred that $\theta_H \lesssim 0.11$. For $\theta_H \sim 0.1$ (0.15), λ_3 and λ_4 are smaller than those in the SM by 7.7% (8.1%) and 30% (32%), respectively. As explained above, there is a lower bound for θ_H in GHU, namely, $\theta_H > \theta_{c1}$, and there does not exist a $\theta_H \to 0$ limit. It is not surprising that λ_3 and λ_4 deviate from the values in the SM even for small θ_H. The couplings λ_n ($n \geq 5$) are generated at the one-loop level in both GHU and the SM, which turn out to be finite. The Higgs couplings to quarks, leptons, and W and Z bosons in GHU for small θ_H are very close to those in the SM. There are additional contributions coming from KK modes in GHU. It may be interesting as well to measure these couplings λ_n ($n \geq 5$) in future experiments to test predictions from GHU and the SM.

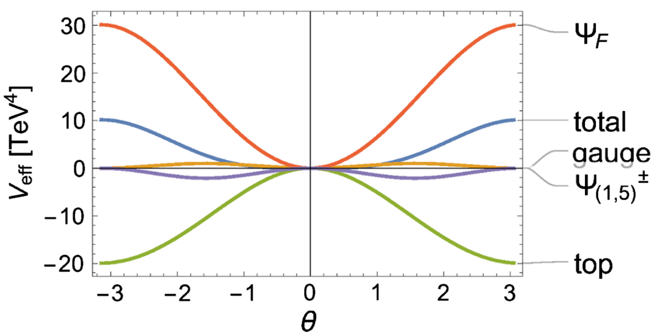

FIG. 2. Contributions of relevant fields to the effective potential for $\theta_H = 0.1$, $n_F = 2$, $n_V = 2$, and $c_V = 0$.

IV. θ_H UNIVERSALITY

As remarked in the previous section, there remains the arbitrariness in the choice of the parameters in the model. Among the five parameters $(n_F, c_F, n_V, c_V, \tilde{m}_V)$ there are only two conditions in Eq. (3.8) that must be obeyed. In the examples given in the previous section, we first fixed (n_F, n_V, c_V) and determined $(c_F, \tilde{m}_V)$ by Eq. (3.8). The Higgs cubic and quartic couplings, λ_3 and λ_4, are

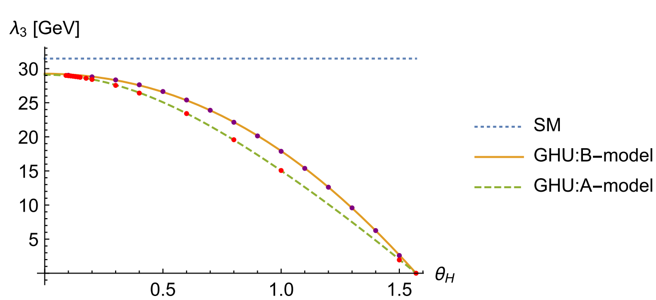

FIG. 3. The cubic coupling λ_3 of the Higgs boson. The fitting curves are given by $\lambda_3/\mathrm{GeV} = 39.6\cos\theta_H - 5.21(1+\cos 2\theta_H) - 0.00911\cos 3\theta_H$ for the B model and $\lambda_3/\mathrm{GeV} = 32.4\cos\theta_H - 2.26(1+\cos 2\theta_H) - 1.1\cos 3\theta_H$ for the A model. The SM value is $\lambda_{3,\mathrm{SM}} = 31.5$ GeV.

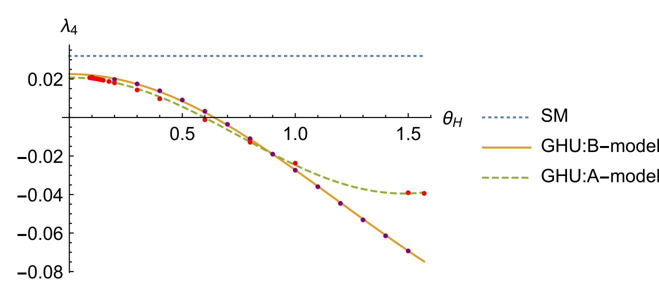

FIG. 4. The quartic coupling λ_4 of the Higgs boson. The fitting curves are given by $\lambda_4 = -0.0695 + 0.0852\cos\theta_H + 0.00725\cos 2\theta_H$ for the B model and $\lambda_4 = -0.00264 - 0.0129\cos\theta_H + 0.0363\cos 2\theta_H$ for the A model. The SM value is $\lambda_{4,\mathrm{SM}} = 0.0320$.

evaluated with this choice. One might wonder how λ_3 and λ_4 depend on the choice of the parameters (n_F, n_V, c_V).

In this section we show that λ_3 and λ_4 are determined, to high accuracy, as functions of θ_H only, and do not depend on the details of the parameter choice. It has been known in GHU that the three-point Higgs couplings to W, Z, quarks, and leptons also have the same property [37]. These physical quantities are determined by θ_H to high accuracy. This may be called the θ_H universality. The θ_H universality leads to profound predictive power. Once the value of θ_H is determined by one of the physical quantities, the values of other physical quantities can be predicted.

In Table III we list the values of (λ_3, λ_4) for $\theta_H = 0.1$ with various choices of (n_F, n_V, c_V). Although the values of c_F and $\tilde{m}_V$ depend on the choice of (n_F, n_V, c_V), the values of λ_3 and λ_4 are universal to high accuracy. λ_3 and λ_4 are determined as functions of θ_H only.

There is a reason for the θ_H universality. We first examine the global behavior of $V_{\rm eff}(\theta)$ with a given θ_H. Notice that the function $A_p(\theta)$ in Eq. (3.4) is expanded as

$$A_p(\theta) = \frac{(kz_L^{-1})^4}{(4\pi)^2} \sum_{\ell=1}^{\infty} \sum_{n=1}^{2} \alpha_p^{(n,\ell)} \cos^\ell n\theta,$$
$$\alpha_p^{(n,\ell)} \equiv \frac{(-1)^{\ell+1}}{\ell} \int_0^\infty dq q^3 (Q_p^{(n)}(q))^\ell. \tag{4.1}$$

As either $Q_p^{(1)}(q)$ or $Q_p^{(2)}(q)$ for a given p vanishes in Eq. (3.4), $\alpha_p^{(1,\ell)}$ or $\alpha_p^{(2,\ell)} = 0$ for each p in Eq. (4.1).

To understand the qualitative behavior of $V_{\rm eff}(\theta)$, let us approximate $A_p(\theta)$ in Eq. (3.4) by

$$\frac{(4\pi)^2}{(kz_L^{-1})^4} A_p(\theta) = \begin{cases} \alpha_p^{(2,1)} \cos 2\theta & \text{for } p = W, Z, S, V, \\ \alpha_p^{(1,1)} \cos\theta + \alpha_p^{(1,2)} \cos^2\theta & \text{for } p = \text{top}, F. \end{cases} \tag{4.2}$$

TABLE III. θ_H universality in λ_3 and λ_4 for $\theta_H = 0.1$ and $z_L = 10^{10}$. With given (n_F, n_V, c_V), c_F and $\tilde{m}_V$ are determined to satisfy the condition (3.8), and λ_3 and λ_4 are evaluated using Eq. (3.9).

n_F	n_V	c_V	c_F	$\tilde{m}_V$	λ_3(GeV)	λ_4
2	2	0.	0.319	0.0806	29.03	0.02083
2	2	0.2	0.319	0.0777	29.03	0.02083
2	2	0.5	0.322	−0.0371	29.02	0.02078
4	2	0.	0.425	0.0794	29.02	0.02082
4	2	0.2	0.425	0.0765	29.02	0.02082
4	2	0.5	0.426	−0.0350	29.01	0.02076
2	4	0.	0.318	0.0964	29.03	0.02084
2	4	0.2	0.318	0.0937	29.03	0.02084
2	4	0.5	0.319	0.0615	29.03	0.02083

Note that $|\alpha_p^{(n,2)}/\alpha_p^{(n,1)}| < 0.05$. As the contributions from $p = \text{top}, F$ are 1 order of magnitude larger than those from $p = W, Z, S, V$, the $\cos^2\theta$ terms have been retained for the top and F. $V_{\rm eff}(\theta)$ in this approximation, denoted as $V_{\rm app}(\theta)$, is given by

$$V_{\rm app}(\theta) = \frac{(kz_L^{-1})^4}{(4\pi)^2}(-B_1\cos\theta + B_2\cos 2\theta),$$
$$B_1 = 12\alpha_{\rm top}^{(1,1)} + 12 n_F \alpha_F^{(1,1)},$$
$$B_2 = \alpha_{\rm gauge} - 8 n_V \alpha_V^{(2,1)} - 6\alpha_{\rm top}^{(1,2)} - 6 n_F \alpha_F^{(1,2)},$$
$$\alpha_{\rm gauge} = 2(3-\xi^2)\alpha_W^{(2,1)} + (3-\xi^2)\alpha_Z^{(2,1)} + 3\xi^2\alpha_S^{(2,1)}. \tag{4.3}$$

The condition (a) in Eq. (3.8) leads to $B_1 = 4B_2\cos\theta_H$. Then, the condition (b) in Eq. (3.8) implies that

$$B_2 \sim \frac{16\pi^2}{g_w^2(kL)^2}\left(\frac{m_H}{m_W}\right)^2, \tag{4.4}$$

where the relations $m_{\rm KK} \sim \pi k z_L^{-1}$, $m_W \sim (k/L)^{1/2} z_L^{-1} \sin\theta_H$, and $f_H \sim 2m_W/g_w \sin\theta_H$ have been used. It follows that

$$V_{\rm app}(\theta) = V_0 u(\theta),$$
$$u(\theta) = -4\cos\theta_H\cos\theta + \cos 2\theta,$$
$$V_0 = \frac{m_W^2 m_H^2}{g_w^2 \sin^4\theta_H}. \tag{4.5}$$

The cubic and quartic Higgs self-couplings are given by

$$\lambda_3^{\rm app} \sim \frac{g_w m_H^2}{4m_W}\cos\theta_H,$$
$$\lambda_4^{\rm app} \sim \frac{g_w^2 m_H^2}{96 m_W^2}(7\cos^2\theta_H - 4). \tag{4.6}$$

The approximate formulas (4.5) and (4.6) represent the qualitative behavior of the effective potential $V_{\rm eff}(\theta)$, but exhibit a slight deviation from the values in Table III and the fitting curves (3.10) and (3.11). We first note that the form of $V_{\rm app}(\theta)$ is fixed, once one makes the ansatz that $V_{\rm eff}(\theta)$ is expressed in terms of two functions: $\cos\theta$ and $\cos 2\theta$. The quantities relevant for determining λ_3 and λ_4 are B_1 and B_2, but not detailed values of the parameters in the models considered. In the A or B model the same universality relations (4.6) result in this approximation. It is easy to confirm that the formulas (4.6) reproduce the SM values at $\theta_H = 0$,

$$\lambda_3^{\rm app}|_{\theta_H=0} = \lambda_{3,\rm SM}, \qquad \lambda_4^{\rm app}|_{\theta_H=0} = \lambda_{4,\rm SM}. \tag{4.7}$$

We also note that $u(\pi) - u(0) = 8\cos\theta_H$ and $u(\theta_H) - u(0) = -2(1-\cos\theta_H)^2$. For small θ_H, $u(\pi) - u(0) \sim 8$ and

$u(\theta_H) - u(0) \sim -\frac{1}{2}\theta_H^4$, which explains the behavior of $V_{\rm eff}(\theta)$ for $\theta_H = 0.1$ seen in Fig. 1.

To understand the θ_H universality demonstrated in the previous section we must refine our arguments. The universality was first found in the A model of $SO(5) \times U(1)$ gauge-Higgs unification [13]. The mechanism that yields the θ_H universality was explained in Ref. [38]. We generalize the argument for the current B model. The important observation is that λ_3 and λ_4 are determined by the local behavior of the effective potential $V_{\rm eff}(\theta_H)$ in the vicinity of the global minimum at $\theta = \theta_H$, and the universality reflects the local—but not global—behavior of $V_{\rm eff}(\theta_H)$.

The effective potential $V_{\rm eff}(\theta)$ [Eq. (3.4)] is decomposed into three parts:

$$
\begin{aligned}
V_{\rm eff}(\theta) = \frac{(kz_L^{-1})^4}{(4\pi)^2}\{h_0(\theta) + n_F h_F(\theta; c_F, z_L) \\
+ n_V h_V(\theta; c_V, \tilde{m}_V, z_L)\},
\end{aligned}
\tag{4.8}
$$

where $h_0(\theta)$ represents the contributions from gauge and top-quark fields. With θ_H, z_L, and ξ specified, k is determined from m_Z and c_t is subsequently determined from m_t, so that $h_0(\theta)$ is fixed. All other parameters associated with quarks and leptons are irrelevant for $V_{\rm eff}(\theta)$. There remain five parameters $(n_F, c_F, n_V, c_V, \tilde{m}_V)$ to be specified in Eq. (4.8). They must be adjusted such that the two conditions in Eq. (3.8) are satisfied. The important feature in the RS space with $z_L \gg 1$ is that the θ dependence of $h_F(\theta; c_F, z_L)$ and $h_V(\theta; c_V, \tilde{m}_V, z_L)$ factorizes near $\theta = \theta_H$,

$$
\begin{aligned}
h_F(\theta; c_F, z_L) \simeq \alpha_F(c_F, z_L)\tilde{h}_F(\theta), \\
h_V(\theta; c_V, \tilde{m}_V, z_L) \simeq \alpha_V(c_V, \tilde{m}_V, z_L)\tilde{h}_V(\theta),
\end{aligned}
\tag{4.9}
$$

to very high accuracy. This can be confirmed numerically from the formula for $A_p(\theta)$ in Eq. (3.4). The relation (4.9) implies, for instance, that the ratio $h_F(\theta; c_F^{(1)}, z_L)/h_F(\theta; c_F^{(2)}, z_L)$ is θ independent near θ_H. For $\theta_H = 0.1$, $z_L = 10^{10}$, and $(c_F^{(1)}, c_F^{(2)}) = (0.3, 0.4)$, the ratio varies from 1.7916 to 1.7896 in the range $0.09 \le \theta \le 0.11$. The variation is only 0.1%. We stress that these factorization formulas are valid only locally, namely, near $\theta = \theta_H$, and $\tilde{h}_F(\theta)$ and $\tilde{h}_V(\theta)$ depend on θ_H. The z_L dependence of $h_F(\theta; c_F, z_L)$ and $h_V(\theta; c_V, \tilde{m}_V, z_L)$ is also tiny in the range $10^8 \lesssim z_L \lesssim 10^{15}$.

Let us pick a set of values (n_F, n_V, c_V) and determine $(c_F, \tilde{m}_V)$ by Eq. (3.8). Making use of Eq. (4.9), one finds

$$
\begin{aligned}
\left[\frac{dh_0}{d\theta} + n_F\alpha_F(c_F, z_L)\frac{d\tilde{h}_F}{d\theta} + n_V\alpha_V(c_V, \tilde{m}_V, z_L)\frac{d\tilde{h}_V}{d\theta}\right]_{\theta=\theta_H} &= 0, \\
\left[\frac{d^2h_0}{d\theta^2} + n_F\alpha_F(c_F, z_L)\frac{d^2\tilde{h}_F}{d\theta^2} + n_V\alpha_V(c_V, \tilde{m}_V, z_L)\frac{d^2\tilde{h}_V}{d\theta^2}\right]_{\theta=\theta_H} &= \frac{(4\pi)^2 m_H^2 f_H^2}{(kz_L^{-1})^4}.
\end{aligned}
\tag{4.10}
$$

We perform this procedure for two sets: $(n_F, n_V, c_V) = (n_F^{(1)}, n_V^{(1)}, c_V^{(1)})$ and $(n_F^{(2)}, n_V^{(2)}, c_V^{(2)})$. Then, Eq. (4.10) implies that

$$
\begin{aligned}
n_F^{(1)}\alpha_F\left(c_F^{(1)}, z_L\right) = n_F^{(2)}\alpha_F\left(c_F^{(2)}, z_L\right) \equiv \beta_F, \\
n_V^{(1)}\alpha_V\left(c_V^{(1)}, \tilde{m}_V^{(1)}, z_L\right) = n_V^{(2)}\alpha_V\left(c_V^{(2)}, \tilde{m}_V^{(2)}, z_L\right) \equiv \beta_V.
\end{aligned}
\tag{4.11}
$$

Although the values of $(c_F, \tilde{m}_V)$ depend on the choice of (n_F, n_V, c_V), $\beta_F = n_F\alpha_F(c_F, z_L)$ and $\beta_V = n_V\alpha_V(c_V, \tilde{m}_V, z_L)$ are universal, provided solutions exist. Consequently, one obtains

$$
\begin{aligned}
V_{\rm eff}(\theta) \simeq \frac{(kz_L^{-1})^4}{(4\pi)^2}\bar{h}(\theta), \\
\bar{h}(\theta) = h_0(\theta) + \beta_F\tilde{h}_F(\theta) + \beta_V\tilde{h}_V(\theta).
\end{aligned}
\tag{4.12}
$$

It immediately follows that

$$
\begin{aligned}
\lambda_3(\theta_H) = \frac{g_w m_H^2 \sin\theta_H}{12 m_W}\frac{\bar{h}^{(3)}(\theta_H)}{\bar{h}^{(2)}(\theta_H)}, \\
\lambda_4(\theta_H) = \frac{g_w^2 m_H^2 \sin^2\theta_H}{96 m_W^2}\frac{\bar{h}^{(4)}(\theta_H)}{\bar{h}^{(2)}(\theta_H)},
\end{aligned}
\tag{4.13}
$$

which explains the θ_H universality observed in the previous section. The relevant quantities for λ_3 and λ_4 are $\beta_F(\theta_H)$ and $\beta_V(\theta_H)$, but not (n_F, n_V, c_V). As mentioned above, the z_L dependence of $h_F(\theta; c_F, z_L)$ and $h_V(\theta; c_V, \tilde{m}_V, z_L)$ is weak. The θ_H universality stays valid to good approximation even for varying z_L. For instance, for $\theta_H = 0.1$ and $(n_F, n_V, c_V) = (2, 2, 0)$, the resultant (λ_3, λ_4) are (28.93 GeV, 0.02042) for $z_L = 1.237 \times 10^8$, which should be compared with (29.03 GeV, 0.02083) for $z_L = 10^{10}$.

The θ_H universality is observed in other physical quantities. The Higgs boson couplings g_{WWH} and g_{ZZH}

TABLE IV. KK mass and dark fermion masses are shown in units of TeV for various θ_H with $z_L = 10^{10}$, $n_F = n_V = 2$, and $c_V = 0.2$. Charged and neutral components of $\Psi^{\pm}_{(1,5)}$ have nearly the same masses.

θ_H	m_{KK}	Ψ_F	$\Psi^{\pm}_{(1,5)}$
0.05	24.1	6.30	5.60
0.10	12.1	3.42	2.84
0.15	8.07	2.38	1.91
0.20	6.08	1.82	1.45

to W and Z, and Yukawa couplings y_f to quarks and leptons are given, to good approximation, by [20,37]

$$g_{WWH} = g_w m_W \cos\theta_H,$$
$$g_{ZZH} = \frac{g_w m_Z}{\cos\theta_W^0}\cos\theta_H,$$
$$y_f = \begin{cases} \frac{m_f}{v_{\rm SM}}\cos\theta_H & \text{in the A model,} \\ \frac{m_f}{v_{\rm SM}}\cos^2\frac{1}{2}\theta_H & \text{in the B model,} \end{cases} \tag{4.14}$$

where $v_{\rm SM} = f_H \sin\theta_H = 2m_W/g_w$. For small θ_H, the deviations of the Higgs couplings in Eq. (4.14) are small, whereas the deviations of λ_3 and λ_4 become substantial. We remark that the relations in Eq. (4.14) have been derived in the composite Higgs model where the parameter $\sqrt{\xi} = v/f$ corresponds to θ_H in GHU [29,30].

V. DARK FERMIONS

Although the θ_H universality holds for various couplings associated with the Higgs boson, the masses of dark fermions Ψ_F and $\Psi^{\pm}_{(1,5)}$, for instance, sensitively depend on the choice of the parameters (n_F, n_V, c_V). They are determined by Eq. (A12) for Ψ_F and Eqs. (A13) and (A14) for charged and neutral components of $\Psi^{\pm}_{(1,5)}$. In Table IV we list their masses for various θ_H with $n_F = n_V = 2$. Dark fermions have relatively small masses compared with the KK mass scale $m_{\rm KK}$. The lightest neutral component of the dark fermions can be a candidate for dark matter.

In Fig. 5 the mass of Ψ_F is plotted as a function of θ_H for several n_F. The mass decreases as n_F increases. Similar behavior is obtained for $\Psi^{\pm}_{(1,5)}$ as n_V is varied with c_V fixed.

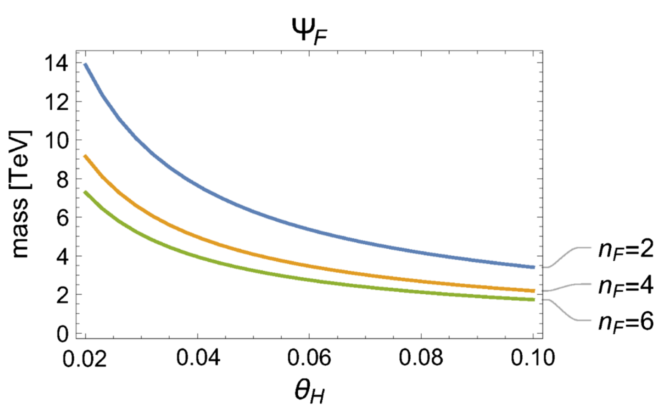

FIG. 5. θ_H and n_F dependence of the mass of the dark fermion Ψ_F. Here $z_L = 10^{10}$, $n_V = 2$, and $c_V = 0.2$.

VI. SUMMARY

In this paper we have examined the effective potential $V_{\rm eff}(\theta_H)$ in GUT-inspired $SO(5) \times U(1) \times SU(3)$ gauge-Higgs unification to confirm that electroweak symmetry breaking is dynamically induced by the Hosotani mechanism. From $V_{\rm eff}(\theta_H)$ the cubic and quartic self-couplings, λ_3 and λ_4, of the Higgs boson have been determined. We have shown the θ_H universality of these couplings, i.e., they are determined as functions of θ_H to high accuracy, irrespective of the details of other parameters in the theory. For $\theta_H = 0.1$ (0.15), λ_3 and λ_4 are smaller than those in the standard model by 7.7% (8.1%) and 30% (32%), respectively. The θ_H universality in λ_3 and λ_4 is understood as a result of the factorization property of each component in the contributions to the effective potential, which is valid to high accuracy in the Randall-Sundrum warped space with $z_L \gg 1$.

The θ_H universality gives the model great predictive power. Once the value of θ_H is determined by experimental data, many other physical quantities such as the masses and couplings of various particles can be predicted. It has been known that gauge-Higgs unification models in the RS space predict large parity violation in the couplings of quarks and leptons to Z' particles (KK modes of γ, Z, and Z_R). This effect can be clearly seen in electron-positron collision experiments with polarized electron/positron beams in which θ_H is the most important parameter. Z' particles can be directly produced at the LHC, and parity-violating couplings would manifest in the rapidity distribution in $t\bar{t}$ production. CKM mixing with natural FCNC suppression is also incorporated in the GUT-inspired gauge-Higgs unification. It would be interesting to pin down the behavior of the model at finite temperature and its implications for cosmology. $SO(5) \times U(1) \times SU(3)$ gauge-Higgs unification is one of the most promising scenarios beyond the standard model. We shall come back to these issues in the future.

ACKNOWLEDGMENTS

This work was supported in part by the European Regional Development Fund-Project Engineering Applications of Microworld Physics (No. CZ.02.1.01/0.0/0.0/16_019/0000766) (Y. O.), by the National Natural Science Foundation of China (Grant Nos. 11775092, 11675061, 11521064, 11435003 and 11947213) (S. F.), by the International Postdoctoral Exchange Fellowship Program (IPEFP) (S. F.), and by the Japan Society for the Promotion of Science, Grants-in-Aid for Scientific Research, No. 19K03873 (Y. H.) and Nos. 18H05543 and 19K23440 (N. Y.).

APPENDIX A: MASS SPECTRUM

In evaluating the effective potential $V_{\rm eff}(\theta_H)$ in Sec. III, one needs to know the mass spectrum of each KK tower of the fields in the model. It is sufficient to know the form of functions whose zeros determine the mass spectrum. These functions have been given in Ref. [16]. We summarize them in this appendix for convenience.

We first introduce

$$F_{\alpha,\beta}(u,v) \equiv J_\alpha(u)Y_\beta(v) - Y_\alpha(u)J_\beta(v), \tag{A1}$$

where $J_\alpha(u)$ and $Y_\alpha(u)$ are Bessel functions of the first and second kind. For gauge fields we define

$$\begin{aligned} C(z;\lambda) &= \frac{\pi}{2}\lambda z z_L F_{1,0}(\lambda z, \lambda z_L),\\ S(z;\lambda) &= -\frac{\pi}{2}\lambda z F_{1,1}(\lambda z, \lambda z_L),\\ C'(z;\lambda) &= \frac{\pi}{2}\lambda^2 z z_L F_{0,0}(\lambda z, \lambda z_L),\\ S'(z;\lambda) &= -\frac{\pi}{2}\lambda^2 z F_{1,1}(\lambda z, \lambda z_L). \end{aligned} \tag{A2}$$

For fermion fields with a bulk mass parameter c, we define

$$\begin{aligned} \begin{pmatrix} C_L \\ S_L \end{pmatrix}(z;\lambda,c) &= \pm\frac{\pi}{2}\lambda\sqrt{z z_L} F_{c+\frac12, c\mp\frac12}(\lambda z, \lambda z_L),\\ \begin{pmatrix} C_R \\ S_R \end{pmatrix}(z;\lambda,c) &= \mp\frac{\pi}{2}\lambda\sqrt{z z_L} F_{c-\frac12, c\pm\frac12}(\lambda z, \lambda z_L), \end{aligned} \tag{A3}$$

and

$$\begin{aligned} \mathcal{C}_{R1}(z;\lambda,c,\tilde m) &= C_R(z;\lambda,c+\tilde m) + C_R(z;\lambda,c-\tilde m),\\ \mathcal{C}_{R2}(z;\lambda,c,\tilde m) &= S_R(z;\lambda,c+\tilde m) - S_R(z;\lambda,c-\tilde m),\\ \mathcal{S}_{L1}(z;\lambda,c,\tilde m) &= S_L(z;\lambda,c+\tilde m) + S_L(z;\lambda,c-\tilde m),\\ \mathcal{S}_{L2}(z;\lambda,c,\tilde m) &= C_L(z;\lambda,c+\tilde m) - C_L(z;\lambda,c-\tilde m),\\ \mathcal{C}_{L1}(z;\lambda,c,\tilde m) &= C_L(z;\lambda,c+\tilde m) + C_L(z;\lambda,c-\tilde m),\\ \mathcal{C}_{L2}(z;\lambda,c,\tilde m) &= S_L(z;\lambda,c+\tilde m) - S_L(z;\lambda,c-\tilde m),\\ \mathcal{S}_{R1}(z;\lambda,c,\tilde m) &= S_R(z;\lambda,c+\tilde m) + S_R(z;\lambda,c-\tilde m),\\ \mathcal{S}_{R2}(z;\lambda,c,\tilde m) &= C_R(z;\lambda,c+\tilde m) - C_R(z;\lambda,c-\tilde m). \end{aligned} \tag{A4}$$

1. Gauge bosons

The mass spectrum $\{m_n = k\lambda_n\}$ of W and W_R towers is determined by

$$\begin{aligned} &W \text{ tower: } 2S(1;\lambda)C'(1;\lambda) + \lambda\sin^2\theta_H = 0,\\ &W_R \text{ tower: } C(1;\lambda) = 0. \end{aligned} \tag{A5}$$

The spectrum of γ, Z, Z_R, and A_z towers is determined by

$$\begin{aligned} &\gamma \text{ tower: } C'(1;\lambda) = 0,\\ &Z \text{ tower: } 2S(1;\lambda)C'(1;\lambda) + (1+s_\phi^2)\lambda\sin^2\theta_H = 0,\\ &Z_R \text{ tower: } C(1;\lambda) = 0,\\ &A_z \text{ tower: } S(1;\lambda)C'(1;\lambda) + \lambda\sin^2\theta_H = 0. \end{aligned} \tag{A6}$$

2. Fermions

With given up-type quark masses $m_Q = (m_u, m_c, m_t)$ the bulk mass parameter $c_Q = (c_u, c_c, c_t)$ of up-type quark multiplets is fixed by

$$S_L(1;\lambda,c_Q)S_R(1;\lambda,c_Q) + \sin^2\frac{\theta_H}{2} = 0, \tag{A7}$$

where $\lambda = \lambda_Q = m_Q/k$. Then the spectrum of up-type quark towers is determined by Eq. (A7). In the down-type quark sector there are brane interactions which mix d'^α and $D^{+\beta}$ through Eq. (2.14). When brane interactions are diagonal in the generation space, $\mu^{\alpha\beta} = \delta^{\alpha\beta}\mu_\alpha$, the spectrum of down-type quark towers is determined by

$$\begin{aligned} &\left(S_L^Q S_R^Q + \sin^2\frac{\theta_H}{2}\right)(\mathcal{S}_{L1}^D\mathcal{S}_{R1}^D - \mathcal{S}_{L2}^D\mathcal{S}_{R2}^D)\\ &\quad + |\mu_1|^2 C_R^Q S_R^Q(\mathcal{S}_{L1}^D\mathcal{C}_{L1}^D - \mathcal{S}_{L2}^D\mathcal{C}_{L2}^D) = 0,\\ &S_L^Q = S_L(1;\lambda,c_Q), \mathcal{S}_{L1}^D = \mathcal{S}_{L1}(1;\lambda,c_D,\tilde m_D), \quad \text{etc.}, \end{aligned} \tag{A8}$$

where $\mu_1 = (\mu_d, \mu_s, \mu_b)$, $c_D = (c_{D_d}, c_{D_s}, c_{D_b})$, and $\tilde m_D = (\tilde m_{D_d}, \tilde m_{D_s}, \tilde m_{D_b})$. The parameters $\mu_1, c_D, \tilde m_D$ are determined such that $\lambda = (\lambda_d, \lambda_s, \lambda_b) = k^{-1}(m_d, m_s, m_b)$ solves Eq. (A8) in each generation. For the third generation, for instance, we take $(\mu_b, c_{D_b}, \tilde m_{D_d}) = (0.1, 1.044, 1.0)$. Only the top-quark multiplet among quark multiplets gives a relevant contribution to $V_{\rm eff}(\theta_H)$. By considering general $\mu^{\alpha\beta}$ the CKM mixing is incorporated with natural FCNC suppression [20].

With given charged lepton masses $m_L = (m_e, m_\mu, m_\tau)$ the bulk mass parameter $c_L = (c_e, c_\mu, c_\tau)$ of charged lepton multiplets is fixed by

$$S_L(1;\lambda,c_L)S_R(1;\lambda,c_L) + \sin^2\frac{\theta_H}{2} = 0, \tag{A9}$$

where $\lambda = \lambda_L = m_L/k$. Then the spectrum of charged lepton towers is determined by Eq. (A9). In the neutrino sector brane interactions mix $\nu^\alpha, \nu'^\alpha, \chi^\alpha$. When both

$M^{\alpha\beta} = \delta^{\alpha\beta} M_\alpha$ in Eq. (2.10) and $m_B^{\alpha\beta} = \delta^{\alpha\beta} m_B^\alpha$ in Eq. (2.15) are diagonal, the spectrum of neutrino towers is determined by

$$(k\lambda - M)\left(S_L^L S_R^L + \sin^2\frac{\theta_H}{2}\right) + \frac{m_B^2}{k} S_R^L C_R^L = 0,$$
$$S_R^L = S_R(1;\lambda, c_L), \quad \text{etc.}, \tag{A10}$$

where $M = (M_1, M_2, M_3)$ and $m_B = (m_B^1, m_B^2, m_B^3)$. With $c_L < -\frac{1}{2}$ the light neutrino mass is given by

$$m_\nu \sim \frac{m_L^2 M}{(2|c_L| - 1) m_B^2} \tag{A11}$$

in each generation. Contributions from lepton multiplets to $V_{\rm eff}(\theta_H)$ are negligible.

The spectrum of dark fermion Ψ_F towers is determined by

$$S_L(1;\lambda, c_F) S_R(1;\lambda, c_F) + \cos^2\frac{\theta_H}{2} = 0. \tag{A12}$$

The spectrum of charged components of dark fermion $\Psi^{\pm}_{(1,5)}$ towers is determined by

$$\mathcal{S}_{L1}(1;\lambda,c_V)\mathcal{S}_{R1}(1;\lambda,c_V) - \mathcal{S}_{L2}(1;\lambda,c_V)\mathcal{S}_{R2}(1;\lambda,c_V) = 0, \tag{A13}$$

whereas the spectrum of neutral component towers is determined by

$$\{B_0(\lambda, c_V, \tilde{m}_V) - 2\cos 2\theta_H\}^2 = 0,$$
$$B_0(\lambda, c, \tilde{m}) = C_L(1;\lambda, c+\tilde{m}) C_R(1;\lambda, c-\tilde{m}) + C_L(1;\lambda, c-\tilde{m}) C_R(1;\lambda, c+\tilde{m})$$
$$+ S_L(1;\lambda, c+\tilde{m}) S_R(1;\lambda, c-\tilde{m}) + S_L(1;\lambda, c-\tilde{m}) S_R(1;\lambda, c+\tilde{m}). \tag{A14}$$

There are two degenerate towers.[1]

APPENDIX B: USEFUL FUNCTIONS

As shown in the formula (3.3), a mass-determining function $\rho(m;\theta_H)$ is analytically continued to $\rho(iy;\theta_H)$. We summarize the functions used in the evaluation of $V_{\rm eff}(\theta_H)$ in Sec. III. We introduce

$$\hat{F}_{\alpha,\beta}(u, v) \equiv I_\alpha(u) K_\beta(v) - e^{-i(\alpha-\beta)\pi} K_\alpha(u) I_\beta(v), \tag{B1}$$

where $I_\alpha(u)$ and $K_\alpha(u)$ are modified Bessel functions of the first and second kind. In terms of $\hat{F}_{\alpha,\beta}(u, v)$ we define

$$\hat{C}(q) = q\hat{F}_{1,0}(qz_L^{-1}, q),$$
$$\hat{S}(q) = iqz_L^{-1}\hat{F}_{1,1}(qz_L^{-1}, q),$$
$$\hat{C}'(q) = q^2 z_L^{-1}\hat{F}_{0,0}(qz_L^{-1}, q),$$
$$\hat{S}'(q) = -iq^2 z_L^{-2}\hat{F}_{1,1}(qz_L^{-1}, q) \tag{B2}$$

for gauge fields. For fermion fields with $c > 0$ we define

$$\hat{C}_L(q;c) = qz_L^{-1/2}\hat{F}_{c+\frac{1}{2},c-\frac{1}{2}}(qz_L^{-1}, q),$$
$$\hat{S}_L(q;c) = iqz_L^{-1/2}\hat{F}_{c+\frac{1}{2},c+\frac{1}{2}}(qz_L^{-1}, q),$$
$$\hat{C}_R(q;c) = qz_L^{-1/2}\hat{F}_{c-\frac{1}{2},c+\frac{1}{2}}(qz_L^{-1}, q),$$
$$\hat{S}_R(q;c) = -iqz_L^{-1/2}\hat{F}_{c-\frac{1}{2},c-\frac{1}{2}}(qz_L^{-1}, q). \tag{B3}$$

For $c < 0$, we use the relations

$$\hat{C}_L(q;-c) = \hat{C}_R(q;c), \hat{S}_L(q;-c) = -\hat{S}_R(q;c). \tag{B4}$$

[1]There was a typo in Eq. (D.16) of Ref. [16]. The last term in the second line, $s_H^2 c_H^2 (\mathcal{C}_{R1}^V \mathcal{C}_{L2}^V - \mathcal{C}_{R2}^V \mathcal{S}_{L1}^V)^2$, should be $s_H^2 c_H^2 (\mathcal{C}_{R1}^V \mathcal{S}_{L2}^V - \mathcal{C}_{R2}^V \mathcal{S}_{L1}^V)^2$. With this correction, Eq. (D.16) of Ref. [16] coincides with Eq. (A14) in the current paper.

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