

Gravitational Radiation from Minihole Coalescence and Quadrupole Transitions of Hydrogen-Like Atoms

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Abstract. The minihole coalescences and quadrupole transitions of hydrogen atoms and hydrogen-like graviatoms are considered to be sources of the gravitational background radiation. The total energies of the gravitational radiation of these microsystems have been calculated. This gravitational background proves to be comparable with the observed gravitational radiation from the coalescence of stellar mass black holes.

1. Introduction

The gravitational radiation from microsystems is considered. A classical gravitational radiation arises from coalescence of the primordial black holes of minor mass, i.e. miniholes. The latter may capture particles forming graviatoms [1]. A quantum gravitational radiation arises from the hydrogen atoms and hydrogen-like graviatoms performing quadrupole transitions. The total energy of the gravitational radiation background in the gamma, X-ray and optical ranges from the minihole coalescence and quadrupole transitions will be compared with the observed gravitational radiation from the coalescence of stellar mass black holes [2,3].

2. Minihole coalescence

The masses of black holes vary over a wide range: from 10^9 stellar masses to the Planckian mass. The coalescence of stellar mass black holes accompanied by gravitational radiation has been observed recently. Of interest is to consider coalescences of primordial black holes including those of minor mass, i.e. miniholes.

The number of coalescences of identical miniholes in a unit volume and a unit time takes the form:

$$N_{coa} = \frac{1}{2} n^2 v \sigma_{coa}(v), \quad (1)$$

where

$$n = \frac{3\Omega_{dm} f c^3}{8\pi G M H a^3} \quad (2)$$

is the number of miniholes in a unit volume, Ω_{dm} the average density of dark matter in units of the critical density

$$\rho_{cr} = \frac{3H^2}{8\pi G}, \quad (3)$$

f the contribution of miniholes to dark matter [4], M - the minihole mass, H - Hubble's constant, a - the scale factor, v - the relative velocity of miniholes,

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$$\sigma_{coa}(v) = \pi r_g^2 \left(\frac{c}{v}\right)^{18/7} \quad (4)$$

is the minihole coalescence cross-section [5],

$$r_g = \frac{2GM}{c^2} \quad (5)$$

is the gravitational radius of a minihole. The total number of minihole coalescences in the Universe

$$N_{tot} = \frac{2}{3}\pi \int_{t_0}^{1/H} n^2 v \sigma_{coa}(v) a^3 dt, \quad (6)$$

where

$$t_0 = \frac{r_g}{c} \quad (7)$$

is the minihole formation time. Since the coalescences of miniholes at the radiation-dominant stage are prevailing, formula (2) takes the form:

$$N_{tot} = \frac{2}{3}\pi^2 c^7 r_g^2 \left(\frac{3\Omega_{dm} f}{8\pi GMH}\right)^2 \int_{t_0}^{t_{rec}} \frac{dt}{a^3}, \quad (8)$$

where

$$a = a_{t_{rec}} \left(\frac{t}{t_{rec}}\right)^{1/2}, \quad (9)$$

t_{rec} is the recombination time.

$$t_0 \ll t_{rec} \ll \frac{1}{H}. \quad (10)$$

2.1. Coalescences of miniholes capable to capture electrons

The hydrogen-like graviatoms satisfies the relation

$$a_g = \frac{GMm}{\hbar c} \quad (11)$$

between the minihole mass M and the mass m of the particle it captures, where

$$0.5 < \alpha_g < 0.7, \quad (12)$$

which provides the energy levels being outside the event horizon and unperturbed by Hawking's radiation [1]. Consider miniholes with the mass

$$M = \frac{\alpha_g \hbar c}{G m_e}, \quad (13)$$

which can capture electrons. For such miniholes we have:

$$N_{tot} = \frac{3\Omega_{dm}^2}{4\sqrt{2}} \frac{(ct_{rec})^{3/2}}{a_{rec}^3 (\alpha_g \lambda_c^e)^{1/2}} \frac{c^2 f^2}{H^2}, \quad (14)$$

where $\lambda_c^e = \frac{\hbar}{m_e c}$ is Compton's wavelength of the electron.

2.2. Gravitational radiation of miniholes

The energy of gravitons

$$\hbar\omega_{gm} = \frac{\pi}{\alpha_g} m_e c^2. \quad (15)$$

The energy of gravitational radiation emitted in the coalescence of two identical black holes of mass M is estimated [2,3] as

$$\Delta E = 0.1 M c^2. \quad (16)$$

The total energy of gravitational radiation in minihole coalescences reads

$$E_{tot} = N_{tot} \Delta E. \quad (17)$$

Substituting the constants

$$H = 67.8 \frac{km}{s \cdot Mpc}; \quad t_{rec} = 3.78 \cdot 10^5 y; \quad a_{t_{rec}} = 8 \cdot 10^{24} cm; \quad \Omega_{dm} = 0.21, \quad \alpha_g = 0.6,$$

we obtain for $f < 0.1$

$$N_{tot} < 2.52 \cdot 10^{18}; \quad M = 3.13 \cdot 10^{17} g; \quad \hbar\omega_{gm} = 2.68 MeV; \quad E_{tot} < 7.08 \cdot 10^{55} erg.$$

3. Gravitatom formation

Consider formation of graviatoms, while capturing electrons by miniholes. At the radiation-dominant stage, the number of electrons (equal to that of protons for an electroneutral plasma) has the form

$$n_p = \frac{2\Omega_b c^3}{8\pi G m_p H a^3}, \quad (18)$$

where Ω_b is the average density of baryons in critical density units. The number of electrons much exceeds the number of miniholes capturing them, since

$$\frac{n_p}{n} = \frac{a_g \Omega_b m_{pl}^2}{\Omega_{dm} m_p m_e} \gg 1, \quad (19)$$

whence the total number of hydrogen-like equals that of miniholes capturing electrons

$$N_{gr} = \frac{f \Omega_{dm} m_e c^2}{2\alpha_g H \hbar}. \quad (20)$$

Finally, we obtain: $N_{gr} < 6.25 \cdot 10^{36}$.

4. Quadrupole transitions of hydrogen-like atoms

Consider the gravitational radiation in quadrupole transitions $3d \rightarrow 1s$.

4.1. Gravitational radiation of hydrogen atoms

The intensity of the gravitational radiation of the hydrogen atom is written as

$$I_{gH} = \frac{3\alpha_e^8 \alpha_{gH} m_e^2 c^4}{27\hbar}, \quad (21)$$

where

$$\alpha_e = \frac{e^2}{\hbar c}, \quad (22)$$

is the fine structure constant,

$$\alpha_{gH} = \frac{Gm_p m_e}{\hbar c} \quad (23)$$

the gravitational equivalent of the fine structure constant of the hydrogen atom. The energy of gravitons

$$\hbar\omega_{13}^g = \frac{4\alpha_e^2}{9} m_e c^2. \quad (24)$$

The total energy of the gravitational radiation of hydrogen atoms for Metagalaxy's lifetime is estimated as

$$E_{gH} = \frac{I_{gH} E_{em}}{I_q}, \quad (25)$$

where

$$\frac{I_{gH}}{I_q} = 1.76 \cdot 10^{-39}, \quad (26)$$

I_q is the intensity of electric quadrupole radiation for the transition $3d \rightarrow 1s$,

$$E_{em} = \frac{c^3 \Omega_b L_{bg} \delta}{2GH^2 M_\odot} \quad (27)$$

the total energy of electric quadrupole radiation emitted in the transition $3d \rightarrow 1s$ by blue giants, whose fraction of all stars $\delta = 3 \cdot 10^{-7}$, luminosities

$$L_{bg} = 4\pi R_{bg}^2 \sigma T_{bg}^4, \quad (28)$$

where σ is Stefan-Bolzmann's constant, radii $R_{bg} = 15R_\odot$ and temperatures

$$T_{bg} = \frac{\hbar\omega_{13}}{2.822k}. \quad (29)$$

Finally we obtain:

$$E_{em} = 1.54 \cdot 10^{72} \text{ erg}, \quad E_{gH} = 2.71 \cdot 10^{33} \text{ erg}.$$

4.2. Gravitational radiation of hydrogen-like graviatoms

The intensity of the gravitational radiation of graviatoms exceeds its electromagnetic radiation, since $\alpha_g \gg \alpha_e$, thus its gravitational background may be considerable.

The intensity of the gravitational radiation of the graviatom containing an electron:

$$I_{ga} = \frac{3\alpha_g^9 m_e^2 c^4}{27\hbar}. \quad (30)$$

The total energy of the gravitational radiation of the graviatoms is estimated as

$$E_{ga} = N_{gr} I_{ga} t_T, \quad (31)$$

where

$$t_T = \frac{\hbar m_{pl} c^2}{(kT)^2} \left(\frac{45}{32\pi^3 g_{eff}} \right)^{1/2} \quad (32)$$

is the time corresponding to the temperature T of the Planckian radiation, whose intensity maximum falls on the frequency of the above-mentioned transition, g_{eff} the number of the degrees of freedom of ultrarelativistic particles.

Assuming $g_{eff} = 10$, we obtain:

$$I_{ga} = 1.49 \cdot 10^{11} \text{ erg} \cdot \text{s}^{-1}; \quad \hbar\omega_{13} = 0.0817 \text{ MeV}; \quad t_T = 645 \text{ s}; \quad E_{ga} < 5 \cdot 10^{50} \text{ erg}.$$

5. Conclusion

The total energies of the background gravitational radiation of microsystems emitted in the gamma and X-ray ranges prove to be comparable with the energy of the gravitational radiation in the lowfrequency range from the coalescences of two stellar mass black holes.

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