

Unification of gauge and Yukawa couplings

Aldo Deandrea

Université de Lyon, France; Université Lyon 1, CNRS/IN2P3, UMR5822 IPNL,
F-69622 Villeurbanne Cedex, France.

E-mail: deandrea@ipnl.in2p3.fr

Abstract. In this talk we describe how unification of gauge and Yukawa couplings can be obtained in gauge-Higgs unification models in extra-dimensions using a toy model. It is usually considered that such unification is difficult to obtain using simple group theory arguments. We consider a toy model based on the $SU(3)$ symmetry and show that including the renormalisation group running at one loop for the couplings improve the result. The gauge couplings unify asymptotically at high energies, and this may result from the presence of an UV fixed point. The Yukawa coupling is enhanced at low energies, showing that a genuine unification of gauge and Yukawa couplings may be achieved. Some other arguments and directions of study are also mentioned to tackle the problem of producing a scalar potential allowing to accommodate the measured Higgs mass at 125 GeV.

1. Introduction

The Higgs boson discovery at the LHC and the measurements of its mass and couplings allows probing of the electroweak (EW) symmetry breaking sector of the Standard Model (SM). This vast experimental program is still ongoing, but has already allowed to test at least partially the origin of symmetry breaking in the SM and its extensions. Fundamental scalar field mass terms are not protected by quantum symmetry, therefore many extensions of the SM focus on finding a more satisfactory origin of the scalar sector. Another intriguing property implemented in Beyond the Standard Model (BSM) models is the unification of gauge couplings at high energies, due to the renormalisation group evolution of the couplings. This brought to the development of Grand Unified Theories (GUT). Moreover the fact that the mass of the top quark is close to the electroweak scale may suggest that the Yukawa coupling of the top quark can have a similar origin.

In the following we shall follow the attractive ideas developed in Gauge-Higgs Unification (GHU) models [1, 2, 3]. Extra-dimensional models contain a peculiar class of scalar fields, arising as additional polarisations of vector gauge fields from the extra compact space. If the Higgs is identified as such a scalar, its couplings are related to the gauge couplings. A mass terms for the Higgs boson is forbidden by gauge invariance in the extra-dimensions solving the problem of the presence of a massive and fundamental scalar field mentioned above. If the gauge symmetry is broken by boundary conditions in a suitable way, a massless scalar is present in the physical spectrum of the theory when reduced to four space-time dimensions, and its potential is radiatively generated and finite [4, 5]. This is an attractive property as it addresses at the same time gauge-Yukawa unification and naturalness of the scalar sector.

However attempts in this sense are challenging as one has to find a gauge group, \mathcal{G}_{GHU} , that predicts the measured values of the SM couplings and the Higgs mass value. Indeed most



Table 1. Gauge and bulk fermion Yukawa couplings in the SU(3) GHU model compared to the SM values at M_Z .

	SU(2) _L	U(1) _Y	Yuk.	SU(3) _c
	g	g'	y	g_s
SU(3) GHU	g_{GHU}	$\sqrt{3} g_{\text{GHU}}$	$g_{\text{GHU}}/\sqrt{2}$	-
SM	0.66	0.35	1.0	1.2

of the models seem to give incorrect predictions [6]. This conclusion can be at least partially modified if the energy evolution of the couplings is taken into account. The extra-dimensional model is actually an effective theory, and its unified predictions are only valid close to the cut-off scale of the theory while the experimental values are measured at the EW scale, and the couplings change due to the running between the two scales. This fact is studied in extra-dimensional GUTs [7, 8]. The running in extra-dimensions is not logarithmic but follows a power law [7, 9, 10]. It is therefore much faster than in four dimensions. Taking into account the running, the tree-level predictions can be strongly modified and the low energy values of the SM couplings may match the experimental values for some choices of the gauge group and matter representations. In the case of the top-quark Yukawa coupling, the running tends to reduce the tension between the value of the large Yukawa coupling and the small gauge coupling at low energy.

2. The minimal SU(3) toy model

The simplest group allowing to embed the EW symmetry and the Higgs boson is $\mathcal{G}_{\text{GHU}} = \text{SU}(3)_W$ [11]. This group is of rank two and contains an $SU(2) \times U(1)$ subgroup that can be identified with the gauged EW group of the SM. We insert in the model a quark as a bulk field, while the other SM fermions can be added as localised fields [6, 11, 12]. Their couplings to the bulk Higgs will be suppressed, thus implying for those fermion fields masses below the EW scale. Gauge and Yukawa couplings, in terms of the unified SU(3) coupling g_{GHU} , are given in table 1. For the Yukawa coupling we consider the top quark as our bulk field because it has the largest one. It is clear that the tree-level GHU predictions are different from the SM values, however, they only apply at the cut-off of the effective theory, which may be different from the EW scale. We consider the running effects with a single extra-dimension compactified on an interval S^1/\mathbb{Z}_2 . The boundary conditions are such that the initial group is broken to the EW one. The spectrum of the theory contains massless gauge bosons and a massless scalar associated to the broken generators. The bulk fermion transforming as the fundamental of $\text{SU}(3)_W$ is assigned boundary conditions such that only two massless fermions appear and we identify them with the third generation quark doublet and down-type singlet (the other fermions are assumed to be localised). At low energy the spectrum of the theory matches the one of the SM. The running of the couplings is modified by the presence of the Kaluza-Klein (KK) fields once the mass thresholds are met, starting at $m_{\text{KK}} = 1/R$ (where R is the compactification radius). The results presented in this talk are those obtained in [12]. The running of the gauge and Yukawa couplings is shown in figure 1 as a function of the energy scale μ , normalised as in table 1:

$$\{g_1, g_2, g_3, g_y\} = \left\{ \frac{g'}{\sqrt{3}}, g, g_s, \sqrt{2} y \right\}. \quad (1)$$

The normalisations follow from the structure of the $\text{SU}(3)_W$ matrices, while the QCD coupling is unrelated in this simple SU(3) model, even if it possible to include it using a larger symmetry

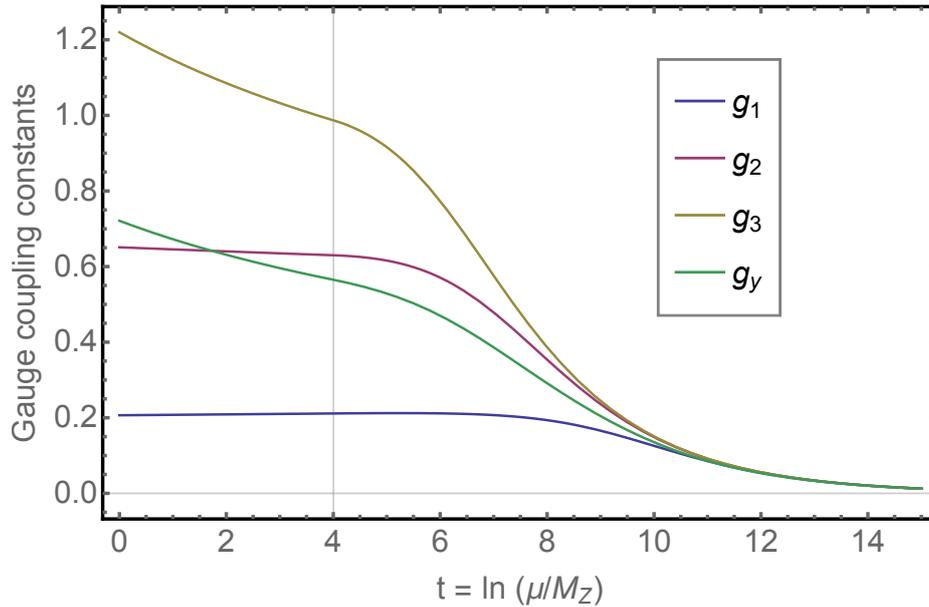


Figure 1. Running of the normalised gauge and Yukawa couplings for the SU(3) model, with $1/R = 5$ TeV. The first KK mode is at the scale $t_{\text{KK}} \sim 4.0$.

group. The couplings follow the SM evolution up to the scale of the first KK resonances:

$$t_{\text{KK}} = \ln \frac{1}{M_Z R}. \quad (2)$$

and from there the running is modified by the extra-dimensions and the gauge couplings asymptotically tend to the same value.

In figure 2 we show an estimate of the 5-dimensional loop factor based on naive dimensional analysis (NDA) [13, 14]:

$$\alpha_i^{\text{NDA}}(\mu) \sim \frac{g_i^2(\mu)}{8\pi} \mu R. \quad (3)$$

The loop factor, which can be thought of as a 5D 't Hooft coupling, can be used as an indication of the energy where the extra-dimensional theory becomes unreliable. The value stays actually small suggesting that the theory may have a more extended validity than usually estimated.

The strong coupling also falls very close as actually the GHU model contains two SU(3) gauge structures, one associated to QCD and the other one to the EW gauge sector, and the bulk fermion is a bi-fundamental. This allows a \mathbb{Z}_2 exchange symmetry between the two sectors at high energy that implies equal couplings.

The initial value of the Yukawa coupling ($y(m_Z) = 0.51$), is selected to achieve unification in the UV. This value depends mildly on the scale of the extra-dimension $1/R$. The running of the Yukawa coupling does not follow the gauge ones at high energy, as the compactification of the extra-dimension singles out the scalar component of the gauge field in the bulk. The effect of the running shows that the value of the Yukawa coupling at low energy is larger than the values at unification, $y = g_2/\sqrt{2}$, even if the enhancement is not enough to explain the Yukawa coupling of the top, $y = 1$ in this toy model. Two loop corrections, the embedding of the top in a more realistic model, or even a different choice of the compactified geometry, may further improve the agreement.

We also performed the running in a different set-up, following the same technique. Using a larger representation that contains a singlet with the correct hypercharge to match the right-

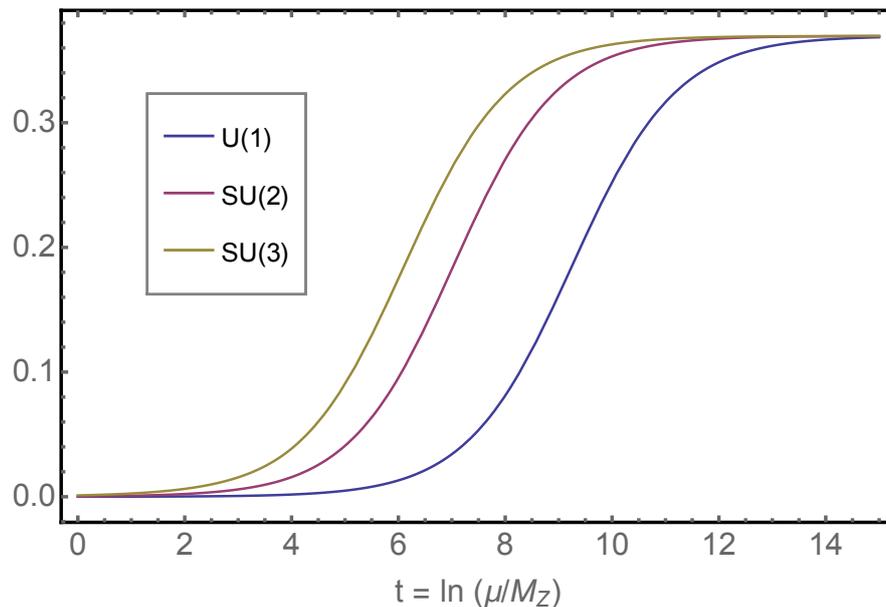


Figure 2. 5D NDA loop factor as a function of the energy, for $1/R = 5$ TeV.

handed top the minimal possibility is to use a 2-index symmetric representation. However, the NDA loop factor estimate in that case goes to non-perturbative values well before unification. This seems to indicate that only models with small representations of the bulk gauge symmetries can be realistic candidates to implement GHU.

3. Conclusions

Simple models of GHU where both the EW gauge couplings and the top Yukawa unify can be built and the running of couplings from the EW scale to the extra-dimension scale is an important ingredient in order to obtain reliable results. We have described a model in five-dimensions, compactified on S^1/\mathbb{Z}_2 , with gauge groups $SU(3)_c \times SU(3)_W$ and a bulk fermion transforming as a bi-fundamental which constitutes a toy example for including a fermion with a large Yukawa coupling at low energy. The fermions contained in the bulk fermion have the quantum numbers of a down-type quark, and the effective Yukawa coupling is enhanced at low energies due to the running. This structure is good enough to describe the EW gauge sector unified in $SU(3)_W$. The running allows to match the value of the Weinberg angle at the EW scale, as well as large Yukawa couplings. The values of the couplings appear as an attractor in the UV, providing asymptotic unification. The QCD gauge coupling also unifies, suggesting that the double- $SU(3)$ structure may be embedded in a larger group. Another important test in more realistic models is the calculation of the Higgs mass, as indeed typical GHU models fail to obtain a large enough Higgs boson mass to reach the experimentally observed value, unless extra ingredients are added.

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