

THE COSMOLOGICAL CONSTANT PUZZLE: VACUUM ENERGIES FROM QCD TO DARK ENERGY*

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The accelerating expansion of the Universe points to a small positive vacuum energy density and negative vacuum pressure. A strong candidate is the cosmological constant in Einstein's equations of General Relativity. The vacuum dark energy density extracted from astrophysics is 10^{56} times smaller than the value expected from the Higgs potential in Standard Model particle physics. The dark energy scale is however close to the range of possible values expected for the light neutrino mass. We investigate this physics in a simple toy model where the chirality of the neutrino is treated by analogy as an Ising-like "spin" degree of freedom.

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1. Introduction

The vacuum energy density perceived by gravitation drives accelerating expansion of the Universe. Understanding of this vacuum energy is an important challenge for theory and connects the Universe on cosmological scales (the very large) with subatomic physics (the very small); for reviews see [1–5].

The physical world we observe today is built from spin- $\frac{1}{2}$ fermions interacting through the exchange of gauge bosons: massless spin-1 photons and gluons; massive W and Z bosons; and gravitational interactions. QED is manifest in the Coulomb phase, QCD is manifest in the confinement phase and the electroweak interaction is manifest in the Higgs phase. Further ingredients are needed to allow the formation of large-scale structures on the galactic scale and to explain the accelerating expansion of the Universe.

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These are the mysterious dark matter and dark energy, respectively. Current observations point to an energy budget of the Universe where just 5% is composed of atoms, 27% involves dark matter (possibly made of new elementary particles) and 68% is dark energy (the energy density of the vacuum perceived by gravitational interactions) [6].

The simplest explanation of this dark energy is a small positive value for the cosmological constant in Einstein's equations of General Relativity. Einstein's equations link the geometry of spacetime to the energy-momentum tensor

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\frac{8\pi G}{c^2}T_{\mu\nu} + \Lambda g_{\mu\nu}. \quad (1)$$

Here, $R_{\mu\nu}$ is the Ricci tensor which is built from the metric tensor $g_{\mu\nu}$ and its derivatives, R is the Ricci scalar and $T_{\mu\nu}$ is the energy momentum tensor. The left-hand side describes the geometry and the right-hand side describes the energy content of the gravitational system. Writing $\Lambda = 8\pi G\rho_{\text{vac}} + \Lambda_0$, we find that the cosmological constant tells us about the energy density of the vacuum ρ_{vac} perceived by gravitational interactions; Λ_0 is a possible counterterm.

The vacuum energy density receives possible contributions from the zero-point energies of quantum fields and condensates associated with spontaneous symmetry breaking. The vacuum is associated with various condensates. The QCD scale associated with quark and gluon confinement is around 1 GeV, while the electroweak mass scale associated with the W and Z boson masses is around 250 GeV. These scales are many orders of magnitude less than the Planck-mass scale of around 10^{19} GeV, where gravitational interactions are supposed to be sensitive to quantum effects. If the net vacuum energy is finite it will have gravitational effect. Being proportional to $g_{\mu\nu}$, a positive cosmological constant corresponds to negative pressure in the vacuum perceived by gravitational interactions. The vacuum energy density associated with dark energy is characterised by a scale around 0.002 eV, typical of the range of possible light neutrino masses, and a cosmological constant, which is 56 orders of magnitude less than the value expected from the Higgs condensate with no extra new physics. Why is this vacuum "dark energy" finite, and why so small?

The challenge presented by gravitation and the cosmological constant are fundamentally different from particle physics in that gravity couples to everything whereas other physics processes and experiments involve measuring the differences between quantities.

2. Vacuum energy and the cosmological constant

We next consider the zero-point and condensate contributions to the vacuum energy.

Quantization introduces zero-point vacuum energies for quantum fields and therefore, in principle, can affect the geometry through Einstein's equations. Before normal ordering, the zero-point energy of the vacuum is badly divergent, being the sum of zero-point energies for an infinite number of oscillators, one for each normal mode, or degree of freedom of the quantum fields [7]. Before interactions, the vacuum (or zero-point) energy is

$$\rho_{\text{vac}} = \frac{1}{2} \sum \{\hbar\omega\} = \frac{1}{2} \hbar \sum_{\text{particles}} g_i \int_0^{k_{\text{max}}} \frac{d^3k}{(2\pi)^3} \sqrt{k^2 + m^2} \sim \sum_i \frac{g_i k_{\text{max}}^4}{16\pi^2}. \quad (2)$$

Here, $\frac{1}{2}\{\hbar\omega\}$ denotes the eigenvalues of the free Hamiltonian and $\omega = \sqrt{k^2 + m^2}$, where k is the wavenumber and m is the particle mass; $g_i = (-1)^{2j}(2j+1)$ is the degeneracy factor for a particle i of spin j , with $g_i > 0$ for bosons and $g_i < 0$ for fermions. The minus sign follows from the Pauli exclusion principle and the anti-commutator relations for fermions. The vacuum energy density ρ_{vac} is quartically divergent in k_{max} .

What value should one take for k_{max} ?

Possible candidates are the energy-scale associated with electroweak symmetry breaking $\Lambda_{\text{ew}} = 2^{-1/4}G_{\text{F}}^{-1/2} = 246 \text{ GeV}$ and the Planck scale $M_{\text{Pl}} = \sqrt{\hbar c/G} = 1.2 \times 10^{19} \text{ GeV}$ where we expect quantum gravity effects to become important. Substituting $k_{\text{max}} \sim \Lambda_{\text{ew}}$ into Eq. (2) with no additional physics gives a cosmological constant

$$\Lambda_{\text{vac}} \sim 8\pi G \Lambda_{\text{ew}}^4 \quad (3)$$

or

$$\rho_{\text{vac}} = \frac{1}{2} \sum \hbar\omega \sim (250 \text{ GeV})^4. \quad (4)$$

This number is 56 orders of magnitude larger than the observed value

$$\rho_{\text{vac}} \sim (0.002 \text{ eV})^4. \quad (5)$$

What is more, summing over just the Standard Model fields in Eq. (2) gives a negative overall sign whereas the value of ρ_{vac} extracted from cosmology is positive. What dilutes the large particle physics number to the physical value measured in large scale astrophysics and cosmology? If we take $k_{\text{max}} \sim M_{\text{Pl}}$, then we obtain a value for ρ_{vac} which is 10^{120} times too big.

In quantum field theory (without coupling to gravity) the zero-point energy is removed by normal ordering so that the zero of energy is defined as the energy of the vacuum. This can be done because absolute energies here are not measurable observables. Only energy differences have physical meaning, *e.g.* in Casimir processes [2, 8], before we couple the theory to gravity.

Suppose we can argue away quantum zero-point contributions to the vacuum energy. One still has to worry about spontaneous symmetry breaking. Condensates that carry energy appear at various energy scales in the Standard Model, *e.g.* the Higgs condensate gives $\rho_{\text{vac}} \sim -(250 \text{ GeV})^4$ with negative sign. The QCD quark condensate gives about $-(200 \text{ MeV})^4$. These condensates form at different times in the early Universe, suggesting some time dependence to ρ_{vac} . If there is a potential in the vacuum it will, in general, correspond to some finite vacuum energy. Why should the sum of many big numbers (plus any possible gravitational counterterm) add up to a very small number?

Theoretical models commonly assume that particle physics contributions to the vacuum energy are cancelled by some (unknown) symmetry or gravitational counterterm. Then one introduces either a (time dependent) ultra-light scalar field with finite vacuum expectation value to describe the evolution of dark energy in the vacuum or, alternatively, modification of long range gravitation to describe the accelerating expansion of the Universe [9]. Each of these scenarios comes with its own theoretical and phenomenological challenges. Presently, cosmological observations are consistent with a time independent cosmological constant equation of state with $w = -1$ for the ratio of the vacuum pressure to energy density. General Relativity has proved very successful everywhere the theory has been tested. At short distances, recent torsion balance experiments [10] have found that Newton's Inverse Square Law holds down to a length scale of $56 \mu\text{m}$. Precision tests of General Relativity observables in the strong field regime of double pulsars have been verified at the level of 0.05% [11]. Studies of gravitational lensing from distant galaxies are also in very good agreement with General Relativity predictions [12]. If one introduces a new elementary scalar field, what protects its mass from quantum radiative corrections? Coupling a near massless scalar to Standard Model particles will introduce a "fifth force" (which is not gauged unlike the other forces of nature). There is presently no experimental evidence for any such interaction. Coupling to a time dependent scalar field may also induce time dependence in the fundamental constants [13]. There are strong constraints on the possible time dependence of the fine structure constant α and the ratio of the electron to proton masses μ_{ep} from precision quantum optics experiments (time = today) [14], from molecular clouds in space (μ_{ep} at time = 7.5 billion years ago) [15] and the cosmic microwave background (α at time when the Universe was 138 000 years old) [6] with no time variation observed in these experiments.

3. Seeking a possible explanation

It is interesting that the dark energy or cosmological constant scale in Eq. (5) is of the same order of magnitude that we expect for the light neutrino mass, *viz.* 0.002 eV [16–18]

$$\mu_{\text{vac}} \sim m_\nu \sim \Lambda_{\text{ew}}^2/M, \quad (6)$$

where $M \sim 3 \times 10^{16}$ GeV is logarithmically close to the Planck mass M_{Pl} and typical of the scale that appears in Grand Unified Theories and also the scale of inflation if the recent BICEP2 measurements of the tensor to scalar ratio in B modes in the cosmic microwave background [19] is interpreted as evidence of gravitational waves from the inflationary period. Further, the gauge bosons in the Standard Model which have a mass through the Higgs mechanism are also the gauge bosons which couple to the neutrino. Is this a clue? The non-perturbative structure of chiral gauge theories is not well understood¹.

The Higgs boson discovered at the LHC [20] is consistent with Standard Model expectations [21]. It is an open question whether at a deeper level this boson is elementary or of dynamical origin. Results from the LHC experiments ATLAS, CMS and LHCb are in good agreement with the Standard Model with (so far) no evidence of new physics. Recent precision measurements of the electron electric dipole moment are consistent with zero, constraining possible CP violation up to scales similar to or larger than those probed at the LHC [22]. One is led to the possibility that the Standard Model might describe particle physics up to close to the Planck mass.

Changing the external parameters of the theory can change the phase of the ground state. For example, QED in 3 + 1 dimensions with exactly massless electrons is believed to dynamically generate a photon mass [23]. In the Schwinger Model for 1 + 1 dimensional QED on a circle, setting the electron mass to zero shifts the theory from a confining to a Higgs phase [24]. In 1 + 1 dimensions the same result holds for SU(N) where all the dynamical fields are in the adjoint representation and play a physical role similar to that of transverse gluons in 3 + 1 dimensional theories plus massless adjoint

¹ We note previous investigations of the close value of the neutrino mass and the cosmological constant scale [17, 18]. Ideas include time varying scalar fields with possible coupling to neutrinos (with corresponding varying mass) [17] as well as possible neutrino condensates [18]. Neutrino condensates could be generated by introducing a new attractive 4-neutrino interaction into a BCS or Nambu–Jona-Lasinio-like model, induced by a new scalar or extra new physics since Z^0 exchange yields a repulsive vector interaction between left-handed neutrinos. In these models, one also needs additional new physics to remove the Higgs and QCD contributions to the net vacuum energy and to worry about possible fine tuning issues associated with couplings of the scalar field. Possible time dependence of the fundamental parameters in particle physics induced by time dependent dark energy is discussed in [13].

Majorana fermions [24]. Confinement gives way to the screening of fundamental test charges and Higgs phenomena if the fermion mass is set exactly to zero. Gross *et al.* [24] write “*The pure 4D Yang–Mills theory is expected to be confining. In view of what we learned from 1+1 dimensional examples we may wonder, however, whether instead it could be in the screening phase: certain gluonic excitations might be capable of screening fundamental test charges. This possibility seems to be experimentally ruled out, however, since no states of fractional baryon number have been observed*”. Changing the gauge group from SU(3) to SU(2), it is interesting to note that, unlike quarks in QCD, the electron and neutrino in the electroweak Standard Model are not confined. The $W^\pm$ and Z^0 gauge bosons which couple to the neutrino are massive and the QED photon and QCD gluons are massless. What happens to the structure of non-perturbative propagators and vacuum energies when we turn off the coupling of the gauge bosons to left- or right-handed fermions?

Assuming the theory is ultraviolet consistent, there are two issues to consider: the pure SU(2) sector and also its coupling to QCD. Pure Yang–Mills theory and Yang–Mills theory coupled to fermions are both confining theories but the mechanism is different for each. Confinement is intimately connected with dynamical chiral symmetry breaking [25]. Scalar confinement implies dynamical chiral symmetry breaking and a fermion condensate $\langle \bar{\psi}\psi \rangle < 0$ ². For neutrinos, this is absent if there is no right-handed neutrino participating in the interaction. Switching off the coupling of SU(2) gauge bosons to right-handed fermions must induce some modification of the non-perturbative propagators. Either confinement is radically reorganised or one goes to a Coulomb phase or to a Higgs phase whereby the Coulomb force is replaced by a force of finite range with finite mass scale and the issues associated with infrared slavery are avoided. Additionally, going further, QCD corrections dynamically break electroweak symmetry with Standard Model gauge interactions even with no Higgs condensate. The SU(2) gauge bosons couple to the quark axial-vector currents generating a small contribution to the mass of the SU(2) electroweak boson, about $gf_\pi \sim 30$ MeV [27], where g is the SU(2) gauge coupling and f_π is the pion decay constant. This QCD correction vanishes if the QCD coupling is set to zero.

We next suppose the confinement to Higgs transition applies and explore possible consequences for particle physics.

Suppose that some process switches off the coupling of right-handed neutrinos to the SU(2) gauge fields. In the electroweak Standard Model the electric charges of the quarks are fixed by the requirement of ultra-violet

² For example, in the Bag model of nucleon structure the Bag wall connects left- and right-handed quarks leading to quark–pion coupling and the pion cloud of the nucleon [26].

(axial-)anomaly cancellation in triangle diagrams involving three gauge boson legs when one sums over possible fermions in the triangle loop. Anomaly cancellation is required by gauge invariance and renormalisability. If some dynamical process acts to switch off left- or right-handed fermions, it will therefore have important consequences for the theory in the ultraviolet limit and should therefore be active there. If symmetry breaking is dynamical and hence non-perturbative it will appear with coefficients smaller than any power of the running coupling. Following Ref. [28], we suppose an exponentially small effect. Dynamical symmetry breaking then naturally induces a symmetry breaking scale Λ_{ew} which is much smaller than the high energy scales in the problem M_{cutoff} (which can be close to the Planck scale). If we take the mass scale M_{cutoff} to be very large, then the expression

$$\Lambda_{\text{ew}} = M_{\text{cutoff}} e^{-c/g(M_{\text{cutoff}}^2)^2} \ll M_{\text{cutoff}} \quad (7)$$

naturally leads to hierarchies. For example, the ratio of the weak scale Λ_{ew} to Planck mass is $\Lambda_{\text{ew}}/M_{\text{Pl}} \sim 10^{-17}$. For the mass scale in Eq. (6), $\Lambda_{\text{ew}}/M \sim 10^{-14}$. If symmetry breaking effects at very large scales are suppressed by the exponential $e^{-c/g(M_{\text{cutoff}}^2)^2}$, then Λ_{ew} is the mass scale appearing in the particle physics Lagrangian describing the energy domain relevant to practical experiments.

4. Spin model dynamics

To help understand the different physics, we next consider a phenomenological trick to parametrise the different scales in the problem.

Analogies between quantum field theories and condensed matter and statistical systems have often played an important role in motivating ideas in particle physics. Here, we consider a possible analogy between the neutrino vacuum and the Ising model of statistical mechanics where the “spins” in the Ising model are associated with neutrino chiralities.

The ground state of the Ising model exhibits spontaneous magnetisation where all the spins line up; the internal energy per spin and the free energy density of the spin system go to zero. For an Ising system with no external magnetic field, the free energy density is equal to minus the pressure

$$P = -\left(\frac{\partial F}{\partial V}\right)_T \quad (8)$$

— that is, the model equation of state looks like a vacuum energy term in Einstein’s equations of General Relativity, $\propto g_{\mu\nu}$.

The Ising model uses a spin lattice to study ferromagnetism for a spin system in thermal equilibrium. One assigns a “spin” ($= \pm 1$) to each site and introduces a nearest neighbour spin–spin interaction

$$H = -J \sum_{i,j} (\sigma_{i,j} \sigma_{i+1,j} + \sigma_{i,j+1} \sigma_{i,j}) . \quad (9)$$

Here, J is the bond energy and we consider zero external magnetic field. Physical observables are calculated through the partition function $Z = \sum_{\sigma_{ij}=\pm 1} \exp(-\beta H)$, where $\beta = 1/kT$, k is Boltzmann’s constant and T is the temperature. One can normalise the energy by adding a constant so that neighbouring parallel spins give zero contribution. Then, the only positive contribution to the energy will be from neighbouring disjoint spins of $2J$ and the probability for that will be $\exp(-2\beta J)$. Once a magnetisation direction is selected, it remains stable because of the infinite number of degrees of freedom in the thermodynamic limit. The Ising model has a second order phase transition. There is a critical coupling $(\beta J)_c$ so that for values of $(\beta J) \geq (\beta J)_c$ the system develops a net magnetisation per spin $\mathcal{M} = \pm 1$, the internal energy per spin and the free energy density each vanish modulo corrections with the leading-term starting as a power of $\exp(-2\beta J)$. The ground state “vacuum” energy drops to a value close to zero from a very large value in the phase transition which takes place close to the cut-off energy or temperature scale M_{cutoff} and is induced by the “spin” potential in the vacuum.

4.1. Spin model neutrinos

Can we construct a toy spin-model description for the neutrino vacuum?

First, the Ising-like interaction itself must be non-gauged, otherwise it will average to zero and there will be no spontaneous symmetry breaking and no spontaneous magnetisation [29].

Second, it is necessary to set a mass scale for J . If the spin model is to have connection with particle physics, it is important to note that the coupling constant for the “spin–spin” interaction is proportional to the mass scale J . It therefore cannot correspond to a renormalisable interaction suggesting that fluctuations around the scale J occur only near the extreme high-energy limit of particle physics near the Planck mass. We consider the effect of taking $J \sim +M$. The combination βJ is then very large making it almost certain that, if the spin model is applicable, the spontaneous magnetisation phase involving just left-handed neutrinos is the one relevant to particle physics phenomena. The exponential suppression factor $e^{-2\beta J}$ ensures that fluctuations associated with the Ising-like interaction are negligible in the ground state, thus preserving renormalisability for all practical purposes.

Setting the energy contribution of neighbouring parallel spins to zero in the Ising system is consistent here with zero net-vacuum energy in particle physics with just left-handed neutrinos, normal ordering, no Higgs condensate and no QCD contribution.

Next, suppose we start with a gauge theory based on $SU(3) \otimes SU(2) \otimes U(1)$ coupled to quarks and leptons with no chiral dependent couplings, unbroken local gauge invariance and no elementary scalar Higgs field. (Here the $SU(3)$ refers to QCD colour and $SU(2) \otimes U(1)$ is the electroweak gauge group.) We then turn on the spin model interaction coupled just to the neutrino in the upper component of the $SU(2)$ isodoublet with the coupling $J \sim M \gg \alpha_s, \alpha_{ew}, \alpha$ (the QCD, $SU(2)$ weak and QED couplings). The gauge sector with small couplings acts like an “impurity” in the spin system. It seems reasonable that the Ising interaction here exhibits the same two-phase picture with spontaneous magnetisation. Then, in the symmetric phase where $\beta J < (\beta J)_c$, the theory is symmetric under exchange of left- and right-handed neutrino chiralities and we have unbroken local gauge invariance. In the spontaneous magnetisation phase, the neutrino vacuum is “spin”-polarised, a choice of chirality is made and the right-handed neutrino decouples from the physics. Parity is spontaneously broken and the gauge theory coupled to the leptons becomes $SU(2)_L \otimes U(1)$. Following the discussion in Section 3, it seems reasonable to believe that the $SU(2)$ gauge symmetry coupled to the neutrino is now spontaneously broken.

4.2. Vacuum energy with spin model neutrinos

Weak interactions mean that we have two basic scales in the problem: $J \sim M$ and the electroweak scale Λ_{ew} induced by spontaneous symmetry breaking. For a spin model type interaction, the ground state with left-handed “spin” chiralities is characterised by vanishing energy density. Excitation of right-handed chiralities is associated with the large scale $2M$. Then the mass scale associated with the vacuum for the ground state of the combined system (spin model plus gauge sector) one might couple to gravity reads in matrix form as

$$\mu_{vac} \sim \begin{bmatrix} 0 & -\Lambda_{ew} \\ -\Lambda_{ew} & -2M \end{bmatrix} \quad (10)$$

with the different terms depending how deep we probe into the Dirac sea. Here, the first row and first column refer to left-handed states of the spin model “neutrino” and the second row and second column refer to the right-handed states. The off-diagonal entries correspond to the potential in the vacuum associated with the dynamically generated Higgs sector. Equation (10) looks like the see-saw mechanism [30] proposed to explain neutrino masses. Diagonalising the matrix for $M \gg \Lambda_{ew}$ gives the light mass

eigenvalue

$$\mu_{\text{vac}} \sim \Lambda_{\text{ew}}^2/2M \quad (11)$$

— that is, the phenomenological result in Eq. (6). Here, the electroweak contribution Λ_{ew} is diluted by the “spin” potential in the vacuum. The resultant picture is a Higgs sector characterised by scale Λ_{ew} embedded in the “spin” polarised ground state that holds up to the ultraviolet scale $2M$.

In conclusion, the cosmological constant puzzle continues to fascinate. Why is it finite, positive and so very small? What suppresses the very large vacuum energy contributions expected from particle physics? Understanding these vital questions will teach us much about the intersection of quantum field theory on the one hand, and gravitation on the other.

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