

Dynamical behaviours of Chaplygin gas, cosmological and gravitational ‘constants’ with cosmic viscous fluid in Bianchi type V space-time geometry

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Abstract. This paper is devoted to study modified Chaplygin gas and cosmological ‘constant’ as candidates of dark energy in the presence of cosmic viscous fluid with reference to the Bianchi type V space-time geometry. To represent a more viable cosmological model, variation of gravitational ‘constant’ is also considered. Precise solutions of equations of field have been acquired, where scale factors expand as monomial functions of cosmic time. Further, by use of graphical representation, behaviours of various parameters are also examined.

Keywords: Bianchi type V, gravitational ‘constant’, cosmological ‘constant’, bulk viscosity and Modified Chaplygin gas.

1. Introduction

The well accepted mathematical model of the expanding universe is represented by Friedman-Robertson-Walker (FRW) space-time geometry. However, the literature survey of various cosmological observations suggests that during early stages of the evolution, universe might had gone through anisotropic phase resulting late time its isotropic behaviour. These results created interest of several authors to study cosmological models with anisotropic space-time geometry [1-17]. During early stages of universe Bianchi type V cosmological models are physically and geometrically more viable in structure than the standard FLRW model and hence are interesting to study the behaviour of the universe. Further, Bianchi type V models are natural generalization of open FRW cosmological models which are studied to understand low density of the universe. For more than three-decade researchers have studied Bianchi type V space time geometry to explain various dynamical behaviour of the universe under different conditions. A cosmological model with this space-time geometry and bulk viscous distribution was studied by Banerjee and Sanyal [18], while Coley [19] presented Bianchi type V spatially anisotropic and homogeneous imperfect fluid models for the equation of state $p = \gamma\rho, 0 \leq \gamma \leq 1$. A detailed review on Bianchi type I to X and Kantowski-Sachs cosmological models in Lyra geometry has been presented by Singh and Singh [20]. Recently a number of research papers have been published to present a suitable expanding Bianchi V model in general relativity and alternative theories of gravitation with different dynamical parameters [21-31].

On the basis of analysis of the Hubble diagram of distance type I a supernovae it has been established that the universe is currently undergoing an accelerated phase of expansion [32-33]. In order to know the accelerating manners of the universe, role of time changeable Λ is measured



as very important. A wide phase of observations compellingly suggests that the universe possesses a non zero cosmological constant [34]. However, there is vast differentiation between the lambda inferred from examination and the vacuum energy resulting from quantum field theory. This discrepancy is known as the cosmological constant problem. The first review of history of cosmological “constant” problem was presented by Weinberg [35]. As a solution of cosmological “constant” problem he proposed five different approaches viz. (i) Super symmetry, super gravity, superstring, (ii) Anthropic consideration, (iii) Adjustment mechanism, (iv) Changing gravity and quantum cosmology. Even today the issue of cosmological constant problem is one of the most challenging and controversial problem in cosmology. A phenomenological solution of this problem is suggested by considering Λ as a function of time ‘t’ so that it was large in the early universe and got reduced with the expansion of the universe [36-38]. Discussion on cosmological constant problem and consequences on cosmology with a time varying cosmological constant are presented by several authors [39-41].

In astrophysics time dependent G has many interesting consequences. Canuto and Narlikar [42] have shown that at present in available cosmological observations G-varying cosmology is consistent. Gravitational theories with variable G have been discussed in the literature in the context of induced gravity model where G is generated by means of non-vanishing vacuum expectation value of scalar field. A number of phenomenological models have been suggested to explain the changes in cosmological and gravitational constants. In an appealing approach, it is decided that the protection of the energy momentum tensor which accordingly renders G and Λ as coupled field, similar to the case of G in original Brans- Dicke theory. Singh and Kotambkar *et. al.* [43] have discussed higher dimensional cosmological models with varying G and Λ . Singh and Sorokhaibam [44] have considered FRW cosmological models with time changing G and Lambda. Singh *et al* [45] have argued perfect fluid Bianchi type I model with changeable G and Lambda. Chakraborty *et. al.* [46] have discussed in presence of time changing G and lambda bulk viscous anisotropic cosmological models. Some Robertson-Walker models with time dependent G and Lambda has been investigated by Tiwari [47]. Singh [48] has talked about some cosmological models with time changeable G and Lambda. Kotambkar *et. al.* [49] have discussed anisotropic BI cosmological models with Generalized Chaplygin gas and dynamical Gravitational and Lambda.

To consider practical model one should consider viscosity mechanism based on the common experience in fluid mechanism, We would expect that the concept of viscosity may be important in cosmology. Dissipative effect including bulk viscosity play a very important role in the evolution of the universe, hence it is a subject of growing importance.. For the description of these phenomena, the theories of Eckart [50], Landau and Lifschitz [51] were used. The Eckart theory was the most commonly used theory to describe the effect of bulk viscosity during evolution of the universe. Because of work of many researchers [52-56] it is obvious that the theory undergoes from severe drawbacks about causality and stability. In order to prevent non-causal and unstable behavior it is necessary to consider second order term. In case of isotropy and homogenous, the dissipative process can be modeled as a bulk viscosity within a thermo dynamical approach. The process of bulk viscosity leads to inflationary like solutions in general relativistic FLRW models have been pointed out by Padmanabhan and Chitre [57]. Grøn [58] and Maartens [59] have presented a detailed review of non-causal and viscous cosmological models. Number of researchers [60-64] have studied effect of bulk viscosity in different context. Kotambkar *et. al.* [65] have discussed bulk viscous anisotropic cosmological models with quintessence. Recently Prasanthi and Aditya [66] have investigated Kantowski-Sachs bulk viscous string cosmological model in f(R) theory of gravity.

Literature recommended that the universe has come in a new stage which is accelerating and dominated by some exotic dark energy. Hence in order to enlighten recent cosmic observations, researchers consider dark energy as a major cause of acceleration. As Chaplygin gas has positive

energy and negative pressure it is considered useful for describing dark energy and hence Chaplygin gas (CG) referred as exotic fluid. In explaining the evolution of the universe, CG is effective so number of authors have studied its effect to study the present accelerating expansion of the universe [67-69]. The equation of state for generalized Chaplygin gas (GCG), is given by

$$p = \frac{-B}{\rho^\alpha},$$

where B is positive constant and α lies between 0 and 1.[70-71]. From the equation of generalized Chaplygin gas one can see that at high density it corresponds to almost dust, which disagree completely with present universe. Thus, we have considered modified Chaplygin gas (MCG) explained by state of equation

$$p = \gamma\rho - \frac{B}{\rho^\alpha}.$$

Here γ takes the values in the interval $[0,1]$. This equation of state clearly indicate that the pressure and energy density are inversely proportional to each other. It may be point out here that the MCG is a single fluid model which unified dark matter and dark energy. The modified Chaplygin gas can explain the evolution of the universe from the radiation era to the Λ -CDM model. Several researchers considered cosmological models with MCG. [72-76].

2. Metric and Field Equations

The spatially homogeneous and anisotropic space-time metric is given by

$$ds^2 = -dt^2 + R_1^2 dx^2 + R_2^2 e^{2kx} dy^2 + R_3^2 e^{2kx} dz^2. \quad (1)$$

where R_1, R_2, R_3 are functions of cosmic time t only.

Einstein equations of field with time dependent Λ and G are given by

$$R_{ij} - \frac{1}{2} R g_{ij} = -8\pi G T_{ij} + \Lambda g_{ij}, \quad (2)$$

T_{ij} defined as

$$T_{ij} = (\rho + P)u_i u_j - (P) g_{ij}, \quad (3)$$

$$P = p + \Pi. \quad (4)$$

where p is equilibrium pressure, Π is bulk viscous stress together with $u_i u^i = 1$.

Einstein's equation of field (2) for the metric (1) obtain form

$$\frac{R_1}{R_1} + \frac{R_3}{R_3} + \frac{R_1 R_3}{R_1 R_3} - \frac{1}{R_1^2} = -8\pi G(p + \Pi) + \Lambda, \quad (5)$$

$$\frac{R_2}{R_2} + \frac{R_3}{R_3} + \frac{R_2 R_3}{R_2 R_3} - \frac{1}{R_1^2} = -8\pi G(p + \Pi) + \Lambda, \quad (6)$$

$$\frac{R_1}{R_1} + \frac{R_2}{R_2} + \frac{R_1 R_2}{R_1 R_2} - \frac{1}{R_1^2} = -8\pi G(p + \Pi) + \Lambda, \quad (7)$$

$$\frac{R_1 R_2}{R_1 R_2} + \frac{R_2 R_3}{R_2 R_3} + \frac{R_3 R_1}{R_3 R_1} - \frac{3}{R_1^2} = 8\pi G\rho + \Lambda, \quad (8)$$

$$2\frac{R_1}{R_1} - \frac{R_2}{R_2} - \frac{R_3}{R_3} = 0. \quad (9)$$

A combination of equations (5)-(8) yields

$$8\pi G\rho + \Lambda + 8\pi G \left[\rho + (\rho + p + \Pi) \left(\frac{R_1}{R_1} + \frac{R_2}{R_2} + \frac{R_3}{R_3} \right) \right] = 0. \quad (10)$$

The conservation of energy momentum ($T_{;j}^{ij} = 0$) suggest.

$$\rho + (\rho + p) \left(\frac{R_1}{R_1} + \frac{R_2}{R_2} + \frac{R_3}{R_3} \right) = 0, \quad (11)$$

using (11) in (10) we obtain

$$8\pi G\rho + \Lambda = -8\pi G\Pi \left(\frac{R_1}{R_1} + \frac{R_2}{R_2} + \frac{R_3}{R_3} \right). \quad (12)$$

Maartein[65] has suggested the causal evolution equation for bulk viscosity as

$$\Pi + \tau \dot{\Pi} = -\eta \left(\frac{R_1}{R_1} + \frac{R_2}{R_2} + \frac{R_3}{R_3} \right) - \frac{\varepsilon \tau \Pi}{2} \left(\frac{R_1}{R_1} + \frac{R_2}{R_2} + \frac{R_3}{R_3} + \frac{\tau}{\tau} - \frac{\eta}{\eta} - \frac{T}{T} \right). \quad (13)$$

here T and η are temperature and coefficient of bulk viscosity respectively which are always positive, τ indicate the relaxation time. For causality $\tau > 0$. When $\varepsilon = 0$, equation (13) give up evolution equation for truncated theory. For $\varepsilon = 1$ equation (13) reduces to full causal theory and for non-causal theory $\tau = 0$ (Eckart's theory).

3. Cosmological Solutions

In order get solution of system entirely we need three extra physically reasonable relations among $R_1, R_2, R_3, \rho, p, G, \Lambda$, & η as we have only five equations (5) - (9) with these eight unknowns

3.1. Case I: $\tau = 0$

For non causal solution $\tau = 0$, equation (13) reduce to

$$\Pi = -\eta \left(\frac{R_1}{R_1} + \frac{R_2}{R_2} + \frac{R_3}{R_3} \right) \quad (14)$$

For a closed system of equations number of unknowns and number of independent equations must be equal. Therefore we take physical plausible relations among parameters.

The well accepted relation between viscosity and energy density is

$$\eta = \eta_0 \rho^r, \quad (15)$$

where $\eta_0 \geq 0$ or is a real number.

In an exotic fluid background, the modified Chaplygin gas, is described by

$$p = \gamma \rho - \frac{B}{\rho^\alpha}, \quad (16)$$

where B is an arbitrary constant and $0 < \alpha \leq 1$.

For the deterministic situation of the universe, we have considered following power law relations

$$R_1 = a_0 t^a, \quad R_2 = b_0 t^b, \quad (17)$$

From equations (9) and (17), one can get

$$R_3 = c_0 t^c, \quad c = 2a - b, \quad (18)$$

Using equations (17) and (18), we get

$$\frac{R_1}{R_1} = \frac{a}{t}, \quad \frac{R_2}{R_2} = \frac{b}{t}, \quad \frac{R_3}{R_3} = \frac{c}{t}. \quad (19)$$

Using equation (19), equation (11) yields

$$\rho + (1 + \gamma) \left(\frac{C_1}{t} \right) \rho = \frac{B(C_1)}{t \rho^\alpha}, \quad (20)$$

Where $C_1 = a + b + c$

By solving equation (20), we get

$$\rho = \left[\frac{B}{1 + \gamma} + M t^{-k} \right]^{\frac{1}{\alpha + 1}} \quad (21)$$

Where $k = (1 + \gamma)(\alpha + 1)(C_1)$ and M is constant of integration.

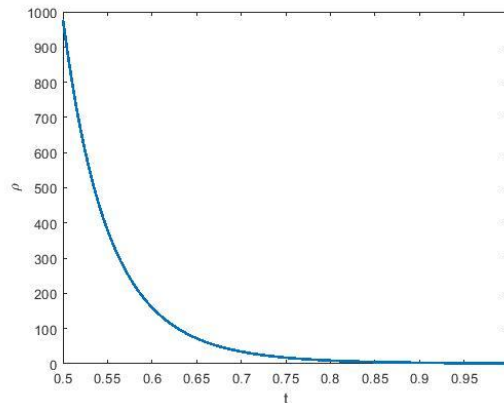


Figure1.Indicates deviation of the ρ against cosmic time t . In this case $B = 1, M = 1, a = 2, b = 1$ and $\alpha = 0.5, \gamma = 1/3$

From figure 1 we can observe that ρ is falling with cosmic time t .

After differentiating equation (21) with respect to cosmic time t , we have obtained

$$\rho = -\frac{M k}{(\alpha + 1)t^{k+1}} \left[\frac{B}{1 + \gamma} + M t^{-k} \right]^{\frac{-\alpha}{\alpha + 1}} \tag{22}$$

By using equations (17)-(19) and (21) in equation (8), we get

$$8\pi G\rho + \Lambda = \frac{C_2}{t^2} - \frac{1}{a_0^2 t^{2a}}, \tag{23}$$

Where $C_2 = ab + bc + ca$

which on differentiation yield

$$8\pi G\rho + 8\pi G\rho + \Lambda = \frac{-2(C_2)}{t^3} + \frac{2a}{a_0^2 t^{2a+1}}, \tag{24}$$

By use of equations (12), (14), (17)-(18) and (21), equation (24) takes the form

$$8\pi G \left[\rho + \eta \frac{(C_1)^2}{t^2} \right] = \frac{-2(C_2)}{t^3} + \frac{2a}{a_0^2 t^{2a+1}}, \tag{25}$$

With the help of (15), (21) and (22), equation (25), becomes

$$G = \frac{1}{8\pi} \left[\frac{B}{1 + \gamma} + M t^{-k} \right]^{\frac{-1}{1 + \alpha}} \left[\frac{-2C_2}{t^3} + \frac{2a}{a_0^2 t^{2a+1}} \right] \left[\frac{-M k}{(1 + \alpha)t^{k+1}} \left(\frac{B}{1 + \gamma} + M t^{-k} \right)^{-\alpha} + \frac{\eta_0 C_1^2}{t^2} \left(\frac{B}{1 + \gamma} + M t^{-k} \right)^{\alpha} \right]^{-1} \tag{26}$$

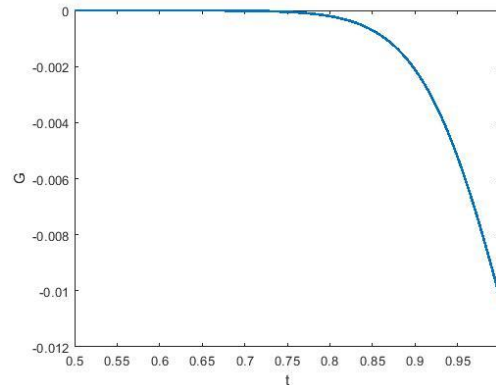


Figure 2. Indicates variation of gravitational constant G against cosmic time t . Here we have taken $B = 1, M = 1, a = 2, b = 1, \alpha = 0.5, \gamma = 1/3, r = 0.75,$ and $\eta_0 = 1$

From figure 2 it can be observe that G is growing with expansion of the universe.

with help of equations (21) and (26) , equation (23) yields

$$\Lambda = \frac{C_2}{t^2} - \frac{1}{a_0^2 t^{2a}} - \left[\frac{-2C_2}{t^3} + \frac{2a}{a_0^2 t^{2a+1}} \right] \left[\frac{-Mk}{(1+\alpha)t^{k+1}} \left(\frac{B}{1+\gamma} + Mt^{-k} \right)^{-\alpha} + \frac{\eta_0 C_1^2}{t^2} \left(\frac{B}{1+\gamma} + Mt^{-k} \right)^r \right]^{-1} \quad (27)$$

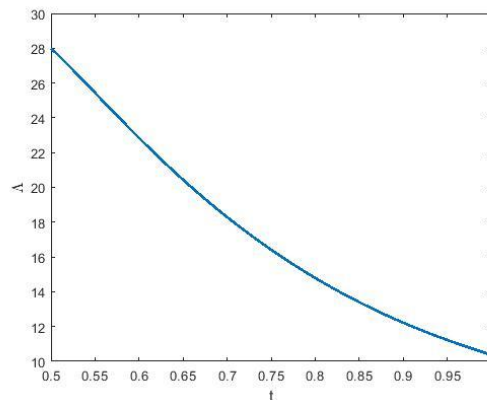


Figure 3. Presents deviation of Λ against cosmic time t . For this graph we have assumed $B = 1, M = 1, a = 2, b = 1, r = 0.75, \alpha = 0.5, \gamma = 1/3,$ and $\eta_0 = 1$

From Figure 3 one can observe that Λ is falling with the with age of the universe.

From equations (15) and (21),wehave obtained

$$\eta = \eta_0 \left[\frac{B}{1+\gamma} + Mt^{-k} \right] \frac{r}{\alpha+1} \quad (28)$$

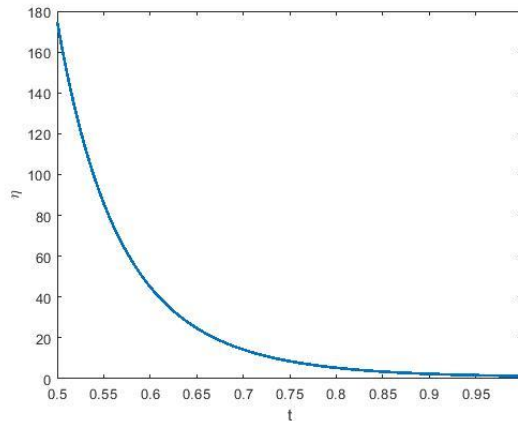


Figure 4.Indicates deviation of η against cosmic time t . For this graph $B = 1, M = 1, a = 2, b = 1, r = 0.75$ and $\alpha = 0.5, \gamma = 1/3$

Now line element (1) takes the form

$$ds^2 = -dt^2 + a_0^2 t^a dx^2 + b_0^2 t^b e^{2kx} dy^2 + c_0^2 t^c e^{2kx} dz^2 \tag{29}$$

The expression for deceleration parameter in terms of Hubble parameter is given by

$$q = -1 - \frac{H}{H^2},$$

The deceleration parameter for present model is given by

$$q = -1 + \frac{3}{C_1} \tag{30}$$

Deceleration parameter is negative for $C_1 = a + b + c > 3$

Cosmological parameters expansion scalar (Θ), shear coefficient (σ^2), relative anisotropy ($\frac{\sigma^2}{\rho}$) for this model are given below

$$\Theta = \frac{3H}{H} = \frac{-3}{t} \tag{31}$$

$$\sigma^2 = \frac{1}{2} \left[\left(\frac{R_1}{R_1} \right)^2 + \left(\frac{R_2}{R_2} \right)^2 + \left(\frac{R_3}{R_3} \right)^2 \right] - \frac{\Theta^2}{6}$$

$$\sigma^2 = \frac{D}{t^2} \tag{32}$$

Where $D = \frac{a^2 + b^2 + c^2 - 3}{2}$

Equation (32) suggest that shear rapidly vanishes with increase in cosmic time t

$$\text{Relative anisotropy} = \frac{\sigma^2}{\rho} = \frac{D}{t^2} \left[\frac{B}{1+\gamma} + M t^{-k} \right]^{\frac{1}{1+\alpha}} \quad (33)$$

The critical energy and the critical vacuum energy densities are defined by

$$\rho_c = \frac{3H^2}{8\pi G}, \quad \rho_v = \frac{\Lambda}{8\pi G}$$

for the present Bianchi type V model, these densities takes the form

$$\rho_c = \frac{\frac{1}{3} C_1^2 t^{-2}}{\left[\frac{B}{1+\gamma} + M t^{-k} \right]^{\frac{-1}{1+\alpha}} \left[\frac{-2C_2}{t^3} + \frac{2a}{a_0^2 t^{2a+1}} \right] \left[\frac{-Mk}{(1+\alpha)t^{k+1}} \left(\frac{B}{1+\gamma} + M t^{-k} \right)^{-\alpha} + \frac{\eta_0 C_1^2}{t^2} \left(\frac{B}{1+\gamma} + M t^{-k} \right)^r \right]^{\frac{-1}{r}}}, \quad (34)$$

$$\rho_v = \frac{\frac{C_2}{t^2} - \frac{1}{a_0^2 t^{2a}} - \left[\frac{-2C_2}{t^3} + \frac{2a}{a_0^2 t^{2a+1}} \right] \left[\frac{-Mk}{(1+\alpha)t^{k+1}} \left(\frac{B}{1+\gamma} + M t^{-k} \right)^{-\alpha} + \frac{\eta_0 C_1^2}{t^2} \left(\frac{B}{1+\gamma} + M t^{-k} \right)^r \right]^{\frac{-1}{r}}}{\left[\frac{B}{1+\gamma} + M t^{-k} \right]^{\frac{-1}{1+\alpha}} \left[\frac{-2C_2}{t^3} + \frac{2a}{a_0^2 t^{2a+1}} \right] \left[\frac{-Mk}{(1+\alpha)t^{k+1}} \left(\frac{B}{1+\gamma} + M t^{-k} \right)^{-\alpha} + \frac{\eta_0 C_1^2}{t^2} \left(\frac{B}{1+\gamma} + M t^{-k} \right)^r \right]^{\frac{-1}{r}}} \quad (35)$$

Mass density parameter

$$\Omega_M = \frac{\rho}{\rho_c},$$

and the density parameter of vacuum

$$\Omega_\Lambda = \frac{\rho_v}{\rho_c},$$

can be expressed as function of cosmic time t as

$$\Omega_M = -\frac{3t^2}{2C_2} \left[\frac{-2C_2}{t^3} + \frac{2a}{a_0^2 t^{2a+1}} \right] \left[\frac{-Mk}{(1+\alpha)t^{k+1}} \left(\frac{B}{1+\gamma} + M t^{-k} \right)^{-\alpha} + \frac{\eta_0 C_1^2}{t^2} \left(\frac{B}{1+\gamma} + M t^{-k} \right)^r \right]^{\frac{-1}{r}} \quad (36)$$

$$\Omega_\Lambda = -\frac{3t^2}{2C_2} \left[\frac{C_2}{t^2} - \frac{1}{a_0^2 t^{2a}} - \left[\frac{-2C_2}{t^3} + \frac{2a}{a_0^2 t^{2a+1}} \right] \left[\frac{-Mk}{(1+\alpha)t^{k+1}} \left(\frac{B}{1+\gamma} + M t^{-k} \right)^{-\alpha} + \frac{\eta_0 C_1^2}{t^2} \left(\frac{B}{1+\gamma} + M t^{-k} \right)^r \right]^{\frac{-1}{r}} \right] \quad (37)$$

The State finder parameters

$$r = \frac{R}{RH^3} \text{ and } s = \frac{r-1}{3\left(q - \frac{1}{2}\right)},$$

For this model

$$r = \left(1 - \frac{3}{C_1}\right) \left(1 - \frac{6}{C_1}\right), \quad (38)$$

$$s = \frac{2 - 2(C_1)}{(C_1) \left[1 - \frac{1}{2}(C_1)\right]}. \quad (39)$$

3.2. Case II: cosmological Solution in full causal theory

Considering equations (15) - (17), with ansatz given by Arbab[77]

$$\Lambda = \beta H^2. \quad (40)$$

The Hubble parameter H is

$$H = \frac{1}{3} \left(\frac{R_1}{R_1} + \frac{R_2}{R_2} + \frac{R_3}{R_3} \right), \quad (41)$$

with help of equation (19) it takes the form

$$H = \frac{a+b+c}{t} = \frac{C_1}{t}. \quad (42)$$

Using equations (17), (19), (40) and (42) in equation (8), we get

$$8\pi G\rho = \frac{N}{t^2} - \frac{3}{a_0^2 t^{2a}}, \quad (43)$$

From equations (21) and (43), one can easily obtain

$$G = \frac{1}{8\pi} \left[\frac{B}{1+\gamma} + M t^{-k} \right]^{\frac{1}{1+\alpha}} \left[\frac{N}{t^2} - \frac{3}{a_0^2 t^{2a}} \right] \quad (44)$$

where $N = ab+bc+ac - (a+b+c)^2 = C_2 - C_1^2$.

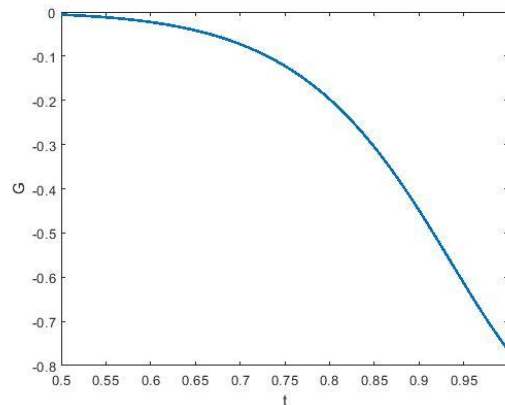


Figure 5. Demonstrates variation of gravitational constant (G) against t. Again we have taken $B = 1, M = 1, a = 2, b = 1, \alpha = 0.5, \gamma = 1/3, a_0 = 1$.

From figure 4 it can be seen that G is growing with expansion of the universe.

Using equations (17) - (19), (40) and (44) in equation (5), we have obtained

$$\frac{a(a-1)}{t^2} + \frac{c(c-1)}{t^2} + \frac{ac}{t^2} - \frac{1}{a_0^2 t^{2a}} - \frac{(a+b+c)^2}{t^2} = -8\pi G \left[\gamma \rho - \frac{B}{\rho^\alpha} + \Pi \right] \tag{45}$$

with help of equations (21) and (44), equation (45) yields

$$\Pi = \left[\frac{B}{1+\gamma} + M t^{-k} \right]^{\frac{1}{1+\alpha}} \left[\left(\frac{B}{1+\gamma} + M t^{-k} \right)^{-\alpha} - \gamma - \left(\frac{N_1}{t^2} - \frac{1}{a_0^2} \right) \left(\frac{N}{t^2} - \frac{3}{a_0^2 t^{2a}} \right)^{-1} \right]. \tag{46}$$

where $N_1 = a(a-1) + c(c-1) + ac - (a+b+c)^2$

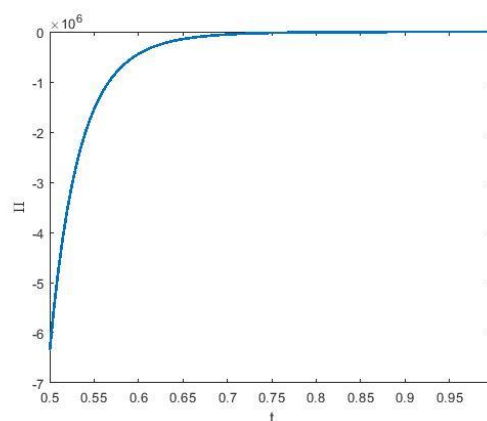


Figure 6. Indicates deviation of Π against t. similar to other cases we have taken $B = 1, M = 1, a = 2, b = 1, \alpha = 0.5, \gamma = 1/3, a_0 = 1$

3.2.1. Sub case (i): Effect of Bulk viscosity in Truncated Theory:

In this section we have considered $\epsilon = 0$. Under this consideration (13) takes the form

$$\Pi + \tau \Pi = -3\eta H \quad (47)$$

In order to find exact solution, we can consider the well accepted relation

$$\tau = \frac{\eta}{\rho} \quad (48)$$

Combination of equations (42), (46), (48) and equation (47) yields coefficient bulk viscosity as

$$\eta = \frac{\Pi}{3H - \frac{\Pi}{\rho}} \quad (49)$$

One can obtain the expression for bulk viscosity coefficient by using (21) and (46)

3.2.2 Sub case (ii). $\varepsilon = 1$

As in full causal theory $\varepsilon = 1$, hence equation (13) may be written as

$$\Pi + \tau \Pi = -3\eta H - \frac{\tau \Pi}{2} \left(3H - \frac{\tau}{\tau} - \frac{\eta}{\eta} - \frac{T}{T} \right) \quad (50)$$

The Gibb's integrability condition suggest that in case of barotropic fluid temperature also depends on the energy density and it may be considered as

$$T = T_0 \exp \int \frac{d\rho}{\rho + p}, \quad (51)$$

where T_0 is proportionality constant.

With the help of equation (21) and (51) one can obtain a relation between temperature and cosmic time t as

$$T = T_0 [B(2 + \gamma) + M(1 + \gamma)t^{-k}] \quad (52)$$

Further, using equations (21), (42), (48) and (52), the equation (50) yields

$$\Pi + \tau \Pi = -\eta \left(\frac{C_1}{t} \right) - \frac{\eta \Pi}{2\rho} \left[\frac{C_1}{t} - \frac{\rho}{\rho} - \frac{T}{T} \right] \quad (53)$$

On simplification of (53) we can obtain coefficient of bulk viscosity in the following form

$$\eta = \frac{-[\Pi + \tau \Pi]}{\left(\frac{C_1}{t} \right) + \frac{\Pi}{2\rho} \left(\frac{C_1}{t} - \frac{\rho}{\rho} - \frac{T}{T} \right)} \quad (54)$$

4. Conclusion

In this paper we have investigated modified Chaplygin gas model of the universe with reference to BV space- time geometry and dynamical bulk viscosity, G and Λ . In first part of the paper we have discussed non-causal cosmological models where as in the second part we have discussed causal cosmological models i. e. truncated theory and full causal theory, by considering scale factor as

monomial function of t . In the first part of the paper for $C_1 > 3$, we are getting accelerating model of the universe and shear dies out as t tends to infinity. In first part of the paper energy density, cosmological constant are decreasing with time, whereas gravitational constant is increasing which is consistent with observational results. In the second part bulk viscous pressure is decreasing with time whereas gravitational constant is increasing which is goes with observational results. One can easily see that all models, presented in this paper, produced overall consistent features with observational predictions.

5. References

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