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A FINITE AND DISCRETE MODEL FOR SINGLE FERMION MASS RENORMALIZATION:
Derivation of the free particle Dirac equation*

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ABSTRACT

We assume that a single particle of mass m cannot be localized to better than $\pm h/2mc$. Using our understanding of finite and discrete measurement accuracy, the single particle transition in 1+1 space-time from $(0,0)$ to (x,t) can then be characterized by a scale factor N and two integers r, l defined by $x = N(r-l)h/mc$ and $ct = N(r+l)h/mc^2$. The average velocity over this finite interval is $v = \frac{r-l}{r+l}c$. The square of the average momentum is $p^2 = \frac{(r-l)^2}{4rl}mc^2$. We show that the solution of the free particle Dirac equation with these boundary conditions can be derived by assuming that the (unobserved) trajectories connecting the two endpoints are all constructed from Nr steps to the right and Nl steps to the left, with velocity $+c$ or $-c$ respectively; each single step has length h/mc. We attribute this *Zitterbewegung* to the emission and absorption of transverse photons to and from the background radiation, each of which necessarily flips the spin. We assert that the symmetry condition on the background radiation that this radiation be undetectable in free particle motion, plus the assumption that the starting and ending spin state must be the same, constitutes the essential requirement for successful single particle mass renormalization in our simple model. We then show that these requirements suffice to determine finite series which uniquely correspond to the (truncated) series solution of the corresponding free particle Dirac equation with the same boundary conditions. We sketch how to extend the model to 3+1 dimensions. The connection of our model to the derivation of Maxwell's equations from finite and discrete space-time measurement accuracy is briefly discussed.

1. Introduction

Relativistic quantum field theory can be interpreted as asserting that any attempt to localize a fermion of mass m to better than $\pm h/2mc$ requires the consideration of fermion-antifermion pairs as relevant degrees of freedom. If the attempted localization employs a probe with energy greater than $2mc^2$ and allows 3-momentum to be conserved in a process producing such a pair, experience shows that the pair will appear in the laboratory with finite probability. This probability can often be successfully computed. Renormalized field theory also allows the successful calculation of the indirect effects of these particle-antiparticle degrees of freedom in the Lamb shift and related phenomena. But these calculations often require the consideration of an infinite number of degrees of freedom and heuristic mathematical procedures to eliminate them, which are called "renormalization".

In this paper we make a preliminary examination of the shape "mass renormalization" could take for a single, free fermion of finite velocity in the context of a new fundamental theory[1–6] within which infinities cannot arise. The stimulus for this paper came from our success[7] in finding a rigorous mathematical context[8] for the Feynman proof[9] of Maxwell's equations starting from the non-relativistic quantum mechanical commutation relations and Newton's second law.[10] The extension of the proof to gravitation[11] has also been justified in terms of the discrete ordered calculus (DOC) by Kauffman.[12]

The precise mathematical description of the Feynman-Dyson-Tanimura derivation was preceded by several preliminary studies of the underlying physics in the context of discrete or "bit-string" physics using the concepts of scale invariance[13] and measurement accuracy.[14–16] It is hoped that this current exploration will also eventually lead to rigorous mathematical techniques that could properly be called "finite and discrete fermion mass renormalization".

Some of the exploratory work has already been reported.[17] This, in turn, grew out of an attempt by V.A.Karmanov, D.O.McGoveran, I. Stein and this author to convert the "random walk" derivation of the Dirac equation suggested by Feynman[18] and carried through by Jacobson and Schulman[19], into a new type of derivation valid in discrete physics. Unfortunately the four authors were never able to reach a consensus on what would constitute validity in that context. The problem which wrecked the collaboration was inability to agree on how to derive the factors $r^k/k!$ and $l^k/k!$ which (see below) multiplied together give the core form needed to construct the solution of the free particle Dirac equation in 1+1 space-time dimensions.

Here we adopt a different strategy, which we take from Galileo (see Acknowledgements). This is to start from the desired answer and the see if we can construct an argument which will lead to it. This raises the suspicion that the argument we develop below is heuristic rather than rigorous. We approach this difficulty by exploring some of the ingredients which might allow the replacement of this argument by a rigorous proof.

The next chapter reviews our basic operational concepts which relate current laboratory practice in high energy particle physics to the specific meaning we attribute to "event", "particle" and "conserved quantum number" in the new fundamental theory which provides the context for this paper. In Chapter 3 we apply these ideas to our understanding of how the transition of a fermion over a finite space interval with average velocity fixed by the boundary conditions and conserving spin can take place in the presence of background radiation which produces a *Zitterbewegung* with velocity $\pm c$ and step length h/mc. We derive in this way what turns out to be the usual solution of the free particle Dirac equation in 1+1 dimensions. We argue that this derivation can be thought of as a "mass renormalization". In chapter 4 we show how the concepts already introduced lead to the commutation relations needed to derive the Maxwell equations, and in Chapter 5, we summarize the derivation itself. In future work we intend to put these ideas together to make a finite and discrete theory of fermions and photons capable of generating at least some of the results achieved by renormalized QED.

2. The Counter Paradigm and Measurement Accuracy

2.1 Laboratory Counters

The basic device used to collect data in a high energy particle physics laboratory can be thought of as a "black box" attached to a recording clock. We call this device a "counter". The clock ticks away at a uniform rate, established and calibrated by standard techniques. We call the time interval between ticks Δt and measure it in seconds or some other time unit which we know how to relate to seconds. In addition to recording the time of each tick as the integer number of ticks since some known and recorded time, at each tick the counter records whether or not the black box "fired" during the interval $\pm\frac{1}{2}\Delta t$ centered on that tick. For example it may record a "1" if the box fires or a "0" if it does not fire. We call an ordered sequence of zeros and ones a *bit-string*, and if additional identifying information is provided with it a *labeled* bit-string.

The label for our counter data can, of course, itself be provided as a bit-string. For example, the time label in the example above could simply be a sequence of "1" 's with the same number of "1" 's as there have been ticks (sometime called "stone age binary"). More efficiently, we could record the integer number of ticks as a binary number. Still more efficiently, we could adopt the convention that for the bit-string with elements $b_s \in 0,1,\ s \in 1,2,...,S$, where "1" indicates that the counter fired and "0" that it did not fire, that the number of bits in the string (i.e. S) is simply the number of ticks of the clock since this record started.

If we go to that much compression of our data, one record will look just like another, and we will have to supply additional label information which might, for example, tell us *where* the recording counter is located. Explicitly, if one corner of the laboratory is the corner of a rectangular parallelepiped, we can give the perpendicular distances from the counter to the floor and to the two walls which form the vertical sides which start from that corner. Here our choice of units in which to measure our spacial intervals is no longer free. The *System International*, employed universally by physicists in reporting the results of measurement and establishing the meaning of "fundamental constants", takes time measurement to be primary and defines the unit of length:[20]

"The meter is defined to be the length of path traveled by light in vacuum in 1/299 792 458 s[econds]. See B.W.Petley, *Nature*, **303**, 373 (1983)."

Thus, following current practice, if our counter size (or "active volume" as it is sometimes called) is Δx in the three directions, and this is the minimum spacial resolution we can achieve (see next section for further discussion of this point), we must require that $\Delta x = c\Delta t$. Then the position of the counter (x_1, x_2, x_3) will be given by $x_i = n_i \Delta x$, $i \in 1, 2, 3$ with n_i *integer.* Our space resolution must be tied to our time resolution by the *scale invariant* requirement

$$\frac{\Delta x}{c\Delta t} = 1 \tag{2.1}$$

valid in *any* system of time units in which the velocity standard c and the unit of length are consistently derived from the SI *convention* that

$$c = 299\ 792\ 458\ meter/second \tag{2.2}$$

It should be obvious that our coordinates have been defined in such a way that they correspond to the time it takes a light signal to go from the appropriate side of the laboratory to the counter, divided by c. If we have two counters a and b lined up at positions $x_i^a = n_i^a \Delta x$ and $x_i^b = n_i^b \Delta x$ and we wish to compare the times at which they fire in a Lorentz invariant way, it is necessary for us to *synchronize* their recording clocks using the Einstein convention. Then, if a particle is emitted at time zero from a toward b at the same time that a light signal is sent toward a mirror on wall i and arrives at counter b in coincidence with the light signal reflected from the wall mirror, the velocity of the particle in direction i is *measured* to be

$$v^i_{ab} = \frac{n^b_i - n^a_i}{n^a_i + n^b_i} c \tag{2.3}$$

Note that this rational fraction velocity in units of c is always less than unity in absolute value for anything that can (so far as we know empirically) be identified as a particle with finite mass m. We have shown elsewhere[21] that rational fraction velocities so defined satisfy the relativistic velocity addition law *and* can be used to define finite and discrete Lorentz boosts in 1+1 dimensions. This discussion is reviewed below in Chapter 4, Section 2.

This description of laboratory practice is, of course, woefully inadequate for the purposes of an experimental high energy particle physicist. His "counters" or detectors are no longer simple black boxes which fire or don't fire when a particle goes through them, and record that fact. They consist of wire chambers, spark chambers, drift chambers, Cerenkov counters, multi-ton magnets, tons of iron or lead shielding, ... and other clever technical devices to assist in particle identification. Building these detectors is a major engineering enterprise costing millions of dollars, and collecting, storing and analysing the bit-string data they produce costs millions more. Nevertheless I claim that, conceptually speaking, the active ingredient in each of these millions and millions of "counter events" can be thought of as a volume of size Δx^3 which does or does not fire during a time interval Δt. As already noted, we can record any ordered sequence of these events as a bit-string.

We emphasize that our "counter events" are a far cry from the "space-time points" which are also called events in special relativity *and* from the happenings which are often called events in everyday language. In the next section we will put our definition of "event" on a more formal basis, but we have found that our usage often requires an introductory discussion of actual laboratory practice such as we have just given before the form it takes can be appreciated. In particular, we emphasize that the non-firing of a counter — recorded, for example, as a zero in a bit-string — can be just as important as the firing of the counter. In fact, in any experiment where the efficiency of the counters and the evaluation of the "background" is significant, this will be the part of an experimental presentation which evokes the strictest scrutiny and the liveliest discussion.

2.2 NO-YES EVENTS, PARTICLES, AND MEASUREMENT ACCURACY

My conceptual foundations for reconstructing relativistic quantum mechanics and physical cosmology start from three inextricably entwined technical terms: *event*, *particle* and *conserved quantum number.* I join them together in the following way:

An **event** *is a finite spacial region which particles enter and leave during a finite time interval. Both the spacial dimensions and the time interval are fixed in the context of a particular application of the definition.*

A **particle** *is a conceptual carrier of conserved quantum numbers between events. (Note that this definition is, in essence, due to Eddington.)*

The algebraic sum of the numerical values associated with each type of quantum number carried into the (event) region by the entering particles is individually equal to the algebraic sum of the numerical values for that type of quantum number carried out of the region by the leaving particles. This statement defines a (set of) **conserved quantum number(s)**. *Note that the number of particles entering the region need not equal the number of particles leaving the region; in other words, particle number is not necessarily conserved.*

N.B. **In this paper we will usually consider the restricted case in which only one particle enters and the "same" particle** (*i.e., carrying the same quantum numbers*) **leaves the event.** This allows us to talk about a "single particle trajectory", — a luxury usually denied to us in discussing relativistic quantum mechanics.

As indicated in the less formal discussion of the last section, the paradigm for an event we have in mind is a *counter firing* in which a counter of relevant spacial size Δx at a specified location in the laboratory does *not* fire during a time interval

Δt, which we call a *NO-event*, or *does* fire during that time interval, which we call a *YES-event*. We further assume that these NO-YES events can be *recorded*, using a clock at the counter (or calibrated in such a way that it can be thought of as "in" the counter) which has been synchronized to the laboratory clock using the Einstein *convention* in relation to spacial coordinates of the counter position *fixed* relative to the position of the laboratory clock ("origin") and three *fixed*, independent (in particular, non-coplanar) *directions*. This allows us to represent the record made by a single counter as an ordered sequence of two distinct symbols such as "0" and "1". When we have specified how two such ordered sequences of symbols of the same length combine, we will call them *bit-strings*.

The term "event" as used in special relativity (SR) has quite a different meaning than our usage spelled out above. Unfortunately, from our point of view, the term as used in SR got frozen nearly three decades before the "uncertainty principle" was invented and some of the implications of the uncertainty principle with regard to localization worked out in the context of the quantum theory of fields.[22] Consequently the meaning of the SR event ended up being practically indistinguishable from a space-time point in Minkowski space and the operational connotations of the term became lost along the way. This is one reason we have introduced the term *NO-YES event* in order to clearly distinguish our meaning from that in SR. Our discussion should make it clear that NO-YES events are ambiguous until the specific measurement accuracy context has been spelled out. As the last section indicates, this can turn out to be quite a complicated matter in practice. Here we abstract from that practice a concept of measurement accuracy which will often serve in place of a full blown discussion of the actual apparatus used. But the reader is warned that this abstraction is dangerous and should be thought through once again whenever we push our measurements to a new level of accuracy or into an unfamiliar application.

With this caveat understood, we take as our paradigm for *measurement accuracy* the smallest counter size Δx and time resolution Δt which we can either construct, or infer from the theory we are in the process of constructing. This is a very powerful and restrictive definition, because it prohibits us from considering *fractional* (indeed, more generally, all non-rational) space and time intervals. Once we have developed the theory far enough to give meaning to *interference*, as in optical interferometry, this assumption of a minimum distance *also* implies a maximum distance and time, which we can call the *event horizon.*

3. Derivation of the Solution of the Free Particle Dirac Equation in 1+1 Dimensions

3.1 Statement of the Space-Time and Momentum Boundary Conditions

The problem posed by this paper is to derive the solution of the free particle Dirac equation in 1+1 dimensions within the framework of finite and discrete measurement accuracy presented in the last chapter and then show that this way of deriving the solution can be viewed as a single fermion mass renormalization in our finite and discrete context. Although our derivation is carried out in space-time, we can think of it as a momentum space mass renormalization if we think of the two component solution $\Psi_1(x,t)$, $\Psi_2(x,t)$ as the propagation of the fermion from $(0,0)$ to (x,t) with average velocity $v = x/t$. Further, we assume that the particle enters and leaves the space-time region in question with this same average velocity. Then, the *unobservable* processes which go on within the space-time region traversed by trajectories which start at $(0,0)$ and end at (x,t) with this average velocity, and which are modeled in the course of our derivation, leave the velocity (and, since this is a single free particle, its momentum) unaltered. We argue in the last section of this chapter that this amounts to a "renormalization" of the fermion mass in the presence of the radiation background we introduce in order to account for the fact that the "trajectories" considered in the model are not simply a single Newtonian (or Minkowskian) straight line, but a relativistic *Zitterbewegung* at velocity $\pm c$ and fixed step length h/mc.

The reason we are at liberty to make this interpretation is that we have shown that in our discrete theory, once we have established a state with a rational fraction average velocity using a an appropriately specified "counter telescope" and fixed velocity resolution, subsequent measurement of the velocity will (over distances comparable to the length of the counter telescope or greater) yield the same velocity to good accuracy[23]. In appropriate circumstances, the farther the subsequent measurement is from the exit counter of the counter telescope, the better the measurement of the velocity. Our model reproduces this feature of experimental velocity measurement correctly. This basic fact about measurement accuracy distinguishes momentum space from space-time, and is the starting point for both Heisenberg's and Chew's S-Matrix theories. For them, momentum space is primary and the description of space-time at short distance is an artifact of Fourier transformation; it should have no ontological significance. The bit-strings used in our derivation of this aspect of our model are Bernoulli sequences ("random walks") concatenated to the sequences which model the initial action of the counter telescope. As we discuss in more detail in Ref.3, this calculation amounts to a derivation of Newton's First Law of Motion, appropriately restricted to our finite and discrete context.

As we will discuss in the next chapter, the ingredients of our finite and discrete theory so far developed are sufficient to underpin the discrete ordered calculus (DOC) we have used to derive the Maxwell equations from measurement accuracy. Here we need to go beyond the concept of *scale invariance bounded from below* by some arbitrary choice of Δt (recall that this fixes Δx by Eq. 2.1) used in underpinning the DOC to the *absolute* bound $\Delta t = \Delta t_{min} = h/mc^2$ for the measurement of a single fermion trajectory. This absolute lower bound is imposed by the experimental phenomenon of fermion-antifermion pair creation. Any relativistic quantum mechanics necessarily predicts that this phenomenon will occur with finite probability (unless prohibited by an exact conservation law) whenever we attempt to localize a fermion of mass m to better than $\pm h/2mc$. But the fermion in the pair so produced is *indistinguishable* from the fermion we are trying to localize. Hence the very *concept* of a fermion trajectory breaks down below this finite lower bound on space-time measurement. We therefor specify our finite and discrete model for *relativistic quantum mechanics* by taking the minimum time interval (and implied minimum space interval) for a single particle problem to be h/mc^2 (and h/mc) respectively.

Returning to the problem posed, this means that we must relate the space-time coordinates x, t in terms of which the Dirac equation is usually expressed to a scale factor N and two integers r, l by the definitions

$$x \equiv N(r-l)\frac{h}{mc}; \quad t = N(r+l)\frac{h}{mc^2} \Rightarrow v = \frac{r-l}{r+l}c \tag{3.1}$$

Note that this is compatible with a space-time picture in which the fermion takes Nr steps to the right (i.e. $+x$ direction) of length h/mc with velocity $+c$ and Nl steps to the left of the same length with velocity $-c$. This is the Stein "random walk" model[24–27] which was incorporated into the discrete physics program at an early stage.[28]

3.2 Spin Conservation in the Presence of Background Radiation

We intend to derive a two-component wave function $\binom{\Psi_1(x,t)}{\Psi_2(x,t)}$ in 1+1 space-time which can be interpreted in the usual way. For reasons that will become apparent, we will take these two amplitudes to be *real* (rather than complex or imaginary) numbers representing *non-normalized* probability *amplitudes*. That is, we assume that the probability of finding the fermion at (x, t) is proportional to $\Psi_1^2 + \Psi_2^2$. Further, if we have a coherent superposition of positive and negative spin states and take the expectation value of the spin operator $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, we find that $<\sigma_z> = \Psi_1^2 - \Psi_2^2$. Then probability conservation (particle number conservation in our case) requires that the first number be the same at $(0,0)$ and (x,t) while spin conservation requires that the second number be the same at $(0,0)$ and (x,t). We impose these boundary conditions in addition to our conserved average velocity boundary condition.

Our *model* for the *Zitterbewegung* attributed to interaction with some sort of bosonic background radiation which flips the spin at each interaction goes beyond the boundary condition to describe the sign of each amplitude as it evolves from $(0,0)$ to (x,t) in this *unobserved* region. We take the positive spin state to be in the $+x$ direction and the negative spin state to be in the $-x$ direction. We wish to calculate the probability amplitude $\Psi_1(x,t)$ that the spin will be in the positive x direction. Since we will have an ensemble of both positively and negatively aligned spins from which this amplitude is constructed, we take this amplitude to be the number of positively aligned cases *minus* the number of negatively aligned cases. Similarly we take Ψ_2 to be constructed from a second ensemble in which we take the number of negatively aligned cases minus the number of positively aligned cases. Constructed in this way, a positive (negative) value for Ψ_1 means that positively (negatively) aligned cases predominate, while a positive (negative) value for Ψ_2 means that negatively (positively) aligned cases predominate. We must use algebraically signed amplitudes, but do not need to uses complex or imaginary values. We also go over from the space-time variables to the integer parameters derived from measurement accuracy by writing any spin amplitude as

$$\Psi(x,t) = \Psi(N(r-l)h/mc, N(r+l)h/mc^2) = \psi(r,l) \tag{3.2}$$

The problem now posed is to construct a model which will allow us to *calculate* these amplitudes with these specific interpretations, and the coupling between them. Clearly this requires us to provide some *mechanism* for flipping the spin back and forth from one position to the other during the $(0,0) \rightarrow (x,t)$ *and* some way to *calculate* the four case counts needed to construct the two probability amplitudes described in the last paragraph. One obvious choice for a spin-flip mechanism is to assume that there are background photons which flip the spin. In a quantum field theory these would be "vacuum fluctuations". We do not have space here to develop the corresponding concept in discrete physics, but will not need any details in this paper other than those provided by obvious symmetry considerations.

Because our fundamental assumption is that we enter the region at $(0,0)$ and emerge from it at (x,t) with the same average velocity $\frac{r-l}{r+l}c$, the effect of the background photons must be such that they do *not* depend on r and l in such a way that we can detect their presence. Further, once we introduce some way to measure the spin direction, we must not allow the spin direction to be biased by the values of r and l, yet still have a way to fix the initial value of the spin (or distribution of values) and find that the transition $(0,0) \rightarrow (x,t)$ leaves this value *conserved* when the fermion emerges (though not necessarily at the *unobserved* positions within the interval). We can accomplish this by a careful balancing of the number of photons interacting with the fermion with the *number* of steps to the right and left independent of where they occur, as we now show. We will also have to include in the model the fact that we do not observe the initial and final spin, but that our wave function would allow it to be conserved if we did do so.

3.3 Case Counts for Spin Flip and No Spin Flip

In the Feynman derivation of the Dirac equation (Ref. 18) as articulated by Jacobson and Schulman (Ref. 19) using a "random walk" model with *imaginary* step lengths $i\epsilon h/mc$, the discussion is first carried out as if the step-lengths were real step-lengths with $\epsilon = 1$, and then the $\epsilon \rightarrow 0$ limit taken to give the result. Why this should give the usual result for a spin-$\frac{1}{2}$ remains part of Feynman's magic touch, to use Bethe's characterization of Feynman's physics. They note that the trajectories connecting $(0,0)$ to (x,t) can be separated into for classes: RR for which the first and last steps are to the right, LL for which the first and last steps are to the left, RL for which the first step is to the right and the second to the left, and LR for which the reverse holds. They also note that any RR trajectory has $k+1$ right moving segments, k left moving segments and $2k$ bends, while LL has the same number of bends but $k+1$ left moving and k right moving segments. RL and LR have k right and k left moving segments and $2k-1$ bends (cf. Ref. 19, p. 377, Fig. 1). Note that this requires k to be smaller than the lesser of r, l, which structures the way they take the limit $\epsilon \rightarrow 0$.

For us, the parameters r, l are fixed by the constant velocity (and momentum) boundary condition $v = \frac{r-l}{r+l}c$, and hence by some fixed *velocity resolution* $\Delta v = \frac{c}{r+l}$. For the same reason that we cannot define position to better than many compton wavelengths h/mc without implying some (at least indirect) way to take us out of the one particle space via pair creation, Δv will also be restricted. The result is that our distance $x = N(r-l)(h/mc)$ must be such that $N >> 1$. In the next chapter we will show that the scale factor N in the constant velocity case be interpreted as the number of repeating periods allowed between $(0,0)$ and (x,t). Here the most important fact about the scale factor is that it extends the allowed range of the parameter k, and hence the number of left or right moving segments to $k < Nr$ or $k < Nl$, allowing us to have many more bends for a given, fixed r, l than one might infer from the Jacobson-Schulman approach to counting trajectories.

A second difference in our calculation is that, as noted in the last section, we attribute both the spin-flip possibilities and the bends in the trajectories to the emission and absorption of photons. In our fundamental theory, we would invoke an actual bit-string model for the photon-fermion-fermion vertex. Here we use only general symmetry conditions, namely that we not be able to distinguish any asymmetry in the background radiation interacting with a *free* fermion whose average velocity is measurably constant to a velocity resolution $\Delta v = \frac{c}{r+l}$. Empirically, this condition is violated when one has sufficient accuracy to measure the dipole moment in the cosmic background ($2.7^o K$) radiation.

Consider first the case RR in which the first and last step are to the right, with $2k$ bends, and hence an even number of photons emitted and/or absorbed. Were we to include the radiation as a *dynamical* degree of freedom, as we would in a calculation that resembled more closely a conventional "self-energy" calculation, we would have to balance emission, absorption, momentum and energy. Here we need only to flip the spin at each bend. Since this implies that there an even number of spin-flips, we will end up with the same spin as that with which we started. There will be $k+1$ right-moving segments, which (thanks to the fact that, in contrast to the Jacobson-Schulman counting, we have Nr rather than r

steps) can start or end at any one of the r *indistinguishable* possibilities. Thus the relative number of cases is the same as that for placing $k+1$ indistinguishable bosons into r indistinguishable cells and is given by $\frac{r^{k+1}}{(k+1)!}$. Similarly, for the k left-moving segments we have $\frac{l^k}{k!}$ cases. Since the assignment of segment boundaries is independent, we must multiply these two independent case counts to get the total and hence obtain the result $[\frac{r^{k+1}}{(k+1)!}][\frac{l^k}{k!}]$ for the RR cases, given k.

We now must note that this result does *not* distinguish between whether we start with a spin to the right or to the left and hence does not provide enough structure to serve as our spin amplitude. We adopt the convention that we normalize to positive spin (as we discussed above) *and* to a first step to the right as a positive amplitude. Then, to obtain negative amplitudes corresponding negative spin, we make use of the fact that k can be even or odd, and subtract the odd cases. This is our prescription for *real* amplitudes, replacing the imaginary step length used by Feynman. Hence

$$\psi_{RR}(r,l) = \Sigma_{k=0,2,4,\ldots}\frac{r^{k+1}}{(k+1)!}\frac{l^k}{k!} - \Sigma_{k=1,3,5,\ldots}\frac{r^{k+1}}{(k+1)!}\frac{l^k}{k!} = \Sigma_k(-1)^k\frac{r^{k+1}}{(k+1)!}\frac{l^k}{k!} \tag{3.3}$$

Using the same normalization we can now write down the amplitude for RL as

$$\psi_{RL}(r,l) = \Sigma_{k=0,2,4,\ldots}\frac{r^k}{k!}\frac{l^k}{k!} - \Sigma_{k=1,3,5,\ldots}\frac{r^k}{k!}\frac{l^k}{k!} = \Sigma_k(-1)^k\frac{r^k}{k!}\frac{l^k}{k!} \tag{3.4}$$

Noting that $c^2t^2 - x^2 = 4rl(h/mc)^2$ and (for $(h/mc) = 1$) defining $z^2 = 4rl$, we have that

$$\psi_{RL}(r,l) = J_0(z) = \Sigma_k(-1)^k\frac{(\frac{z}{2})^{2k}}{(k!)^2} \tag{3.5}$$

where we have used a standard series expansion for the Bessel function of zero order called J_0.[29] Similarly

$$\psi_{RR}(r,l) = \Sigma_k(-1)^k\frac{r^{k+1}}{(k+1)!}\frac{l^k}{k!} = \frac{2r}{l}J_1(z) \tag{3.6}$$

and

$$\psi_{LL}(r,l) = \Sigma_{k=0,2,4,...}\frac{l^{k+1}}{(k+1)!}\frac{r^k}{k!} - \Sigma_{k=1,3,5,...}\frac{l^{k+1}}{(k+1)!}\frac{r^k}{k!} = -\frac{2l}{z}J_1(z) \qquad (3.7)$$

We conclude that we can take $\psi_1 = \psi_{RR} + \psi_{RL}$ while $\psi_2 = \psi_{LL} + \psi_{LR}$ for the two spin amplitudes generated in this way and hence that

$$\psi_1 = J_0(z) + \frac{2r}{z}J_1(z); \qquad \psi_2 = J_0(z) - \frac{2l}{z}J_1(z) \qquad (3.8)$$

It is straightforward to show that this representation is invariant under the combined action of time reversal and parity inversion.

One mystery in the Feynman derivation is that it nowhere refers to "spin". Of course this was also true of the original Dirac equation, whose formal "factorization" of the relativistic (second order) Schroedinger equation was interpreted as representing half-integral angular momentum components consistent with the fine structure spectrum of hydrogen (a spectrum which to that level of accuracy has alternative explanations)[30] only *after* the equation was in hand. We see that by introducing both spin and the background radiation explicitly, our derivation dissolves that mystery. We will pursue the topic of *why* finite measurement accuracy leads us to expect to find spin when we investigate lengths of order h/mc on another occasion.

3.4 Proof that We Have a Solution of the Dirac Equation

Having derived a form for the spin amplitudes which meets the average velocity boundary condition $v = \frac{r-l}{r+l}c$, distinguishes the two spin states, and guarantees that whatever distribution of spin states we start with at $(0,0)$ will reappear at (x,t) if we fit this distribution to the states in the usual way, our next step is to show that this representation is in fact a solution of the free particle Dirac equation.

The form of the Dirac Equation in 1+1 dimensions which Jacobson and Schulman derive (Ref. 19) is

$$-i\sigma_z\partial\psi/\partial x - m\sigma_x\psi = i\partial\psi/\partial t \tag{3.9}$$

where $h = 1 = c$, σ_x and σ_z are the Pauli spin matrices and ψ has two components. Their derivation required the introduction of the imaginary "i", which, following Feynman (Ref. 18), they do by using an imaginary step length $i\epsilon h/mc$. Their procedure also requires that they take the $\epsilon \to 0$ limit in order to achieve the desired result.

However, as Karmanov has pointed out[31], the equation can easily be written as a real two-component equation simply by using a different representation for the Pauli matrices. For example if we write (with m=1)

$$-i\alpha\partial\psi/\partial x + \beta\psi = i\partial\psi/\partial t \tag{3.10}$$

with $\alpha = \sigma_z, \beta = -\sigma_y$, i is a common factor which can be divided out and the equation is real. With $\psi = \binom{\psi_1}{\psi_2}$, the real equation we wish to model is

$$\psi_1 = (\partial/\partial t - \partial/\partial x)\psi_2;\ \psi_2 = -(\partial/\partial t + \partial/\partial x)\psi_1 \tag{3.11}$$

With $z^2 = t^2 - x^2 = 4r\ell$, this equation is solved by

$$\psi_1 = J_0(z) + \frac{2r}{z}J_1(z);\ \psi_2 = J_0(z) - \frac{2\ell}{z}J_1(z) \tag{3.12}$$

where J_0 and J_1 are the standard, real Bessel functions. As already noted (Ref. 29)

$$J_0(z) = \Sigma_{j=0}(-1)^j(z/2)^{2j}/(j!)^2 = \Sigma_{j=0}(-1)^j[\frac{r^j}{j!}][\frac{\ell^j}{j!}] \tag{3.13}$$

Further

$$J_1 = -J_0' = \Sigma_{j=1}j(-1)^{j+1}(z/2)^{2j-1}/(j!)^2 \tag{3.14}$$

Hence

$$\frac{2r}{z}J_1 = \Sigma_{k=0}(-1)^k \frac{r^{k+1}}{k!}\frac{\ell^k}{k!} \tag{3.15}$$

$$\frac{2\ell}{z}J_1 = \Sigma_{k=0}(-1)^k \frac{r^k}{k!}\frac{\ell^{k+1}}{(k+1)!} \tag{3.16}$$

This series representation then has the property needed to make $\binom{\psi_1}{\psi_2}$ a solution of the free particle Dirac equation in 1+1 dimensions, namely

$$J_1' = J_0 - \frac{1}{z}J_1 \tag{3.17}$$

This completes our proof that the derivation we have given in the last section by requiring macroscopic spin and velocity conservation in the presence of (unobservable) background radiation does in fact lead to the solution of the free particle Dirac equation in 1+1 dimensions.

To extend the derivation to 3+1 dimensions, we need only consider a 4-component wave function and use a *real* representation of the three α_i and the β matrices which Dirac invented to factor the Klein-Gordon equation. We need only satisfy the constraints

$$\alpha_i\beta + \beta\alpha_i = 0; \qquad \alpha_i\alpha_j + \alpha_j\alpha_i = 0, \qquad i \neq j$$

$$\alpha_i^2 = \mp 1, i \in 1,2,3; \qquad \beta^2 = \pm 1 \tag{3.18}$$

We also need to extend our constant velocity boundary condition to three components $r, l \rightarrow r_i, l_i$ where $i \in 1,2,3$. These can obviously be interpreted as right-left, forward-back, and up-down relative to the corner of the laboratory introduced in the first section in this chapter. Details are under investigation and will be presented elsewhere.

3.5 ARGUMENT THAT THIS DERIVATION IS EQUIVALENT TO MASS RENORMALIZATION

Why do we call this interpretation of the single, free particle Dirac equation a "mass renormalization of the single free fermion propagator"? We fix the boundary conditions on our solution of the Dirac equation by requiring spin conservation between $(0,0)$ and (x,t) and by requiring that the average velocity over this interval $v = \frac{r-l}{r+l}c$ be the same as that with which the fermion enters and leaves the region. Since we have a single free particle of mass m, fixing the velocity is the same as fixing the momentum, whose square is given by

$$p^2 = \frac{m^2v^2}{1-\frac{v^2}{c^2}} = \frac{(r-l)^2}{4rl}m^2c^2 \tag{3.19}$$

Clearly these are the same boundary conditions imposed on the free fermion propagator in quantum field theory.

In quantum field theory we start with a free particle Lagrangian with a "bare mass" m_0, calculate the effect of the vacuum fluctuations of the background radiation as a power series in the constant coupling the radiation to the fermion and require that the algebraic sum of the two terms yield the observed mass m. The problem is that both m_0 and the correction are infinite. It took a decade and a half before acceptable methods of performing the calculation were found by Tomonoga, Schwinger and Feynman, and shown to be equivalent by Dyson.[32,33]

In our calculation, we have no Lagrangian. Our time evolution is provided by *Program Universe*, as is explained in detail elsewhere (eg Refs. 3 and 6). All we need know about this program is that the class of bit-strings it generates to to represent the $(0,0) \rightarrow (x,t)$ transition are all Bernoulli sequences of length $S = N(r+l)$ which fit the constraint $x(mc/h) = N(r-l)$, i.e. which have Nr ones and Nl zeros. We take the mass m, and hence the step-length h/mc, to be given by the experimental mass, and then show that we can invoke symmetry constraints on the background radiation, which interacts by flipping the spin, in such a way that r, l and m are unaltered by the buffeting, and that the wave function which results is indeed a solution of the free particle Dirac equation with the *experimental* mass. However, if the symmetry conditions are destroyed, for example by an external point charge, or a constant magnetic field, we anticipate that there will be a *finite* change in the energy of the fermion. If this change can be shown to be equivalent to the Lamb shift in the first instance and to $g-2$ in the second, we will have proved our case. This problem is under active investigation, using the developments sketched in the next two chapters.

4. Derivation of the Commutation Relations from Measurement Accuracy

4.1 Constant Velocity Trajectories

As we will see in the next chapter, all we need to derive the Maxwell equations are the postulates $[X_i, \dot{X}_j] = const \cdot \delta_{ij}$, that the acceleration of a single particle testing the field is a function only of position, velocity and time, and the concept of a unit time shift along the particle trajectory. In this chapter we derive the commutation relation between position and velocity from our finite and discrete model of measurement accuracy.

Consider first the case when the velocity is the same whenever measured. Consistent with our finite measurement accuracy postulate, and taking Δx and Δt to be, respectively, the smallest space and time intervals we can measure between events, either directly or indirectly, any distance will be an integral multiple of Δx and any time an integral multiple of Δt. Then any velocity will be a rational fraction.

Confining ourselves to velocities that can always be interpreted as *particulate* velocities, these must then always be rational fractions less that unity. They must also always be greater than zero because zero is *not measurable.* With this understood, in the current context we can define $r = n_r^\beta \Delta x$, $d_r^\beta > n_r^\beta$ where n_r^β and

$d_r^\beta > n_r^\beta$ are integers with no common factor other than unity, and $\Delta x = c\Delta t$. Then the desired representation of a constant velocity in units of c can be taken to be

$$\beta \equiv \frac{n_r^\beta}{d_r^\beta} \tag{4.1}$$

Note that in this context the minimum time interval between measurements which can yield the velocity must be $d_r^\beta \Delta t$ and the minimum space interval between two such measurements — attributed to the firings of two counters produced by a particle of this velocity — must be $n_r^\beta \Delta x$.

In order to make it possible to define *larger* space and time intervals we now introduce the representation

$$r(N_r^\beta; n_r^\beta, d_r^\beta) \equiv N_r^\beta n_r^\beta \Delta x \tag{4.2}$$

This allows us to introduce negative as well as positive distances from the implied origin simply by keeping d_r^β positive and allowing the n_r^β to include negative integers with $|n_r^\beta| < d_r^\beta$. Clearly this also extends our velocity space to negative rational fractions between -1 and 0. We can now displace our origin by a *phase* defined by

$$\phi(n, \delta n) \equiv \frac{\delta n}{n}, \qquad \delta n \in -n+1, -n+2, ..., n-2, n-1; \qquad N_r \to N_r + \delta n \tag{4.3}$$

In keeping with our finite and discrete measurement restriction, we must also assign an *event horizon* for our counter array given by $R_{max} = N_{max}\Delta x$ and the requirement $N_r^\beta \pm \delta n < N_{max}$. Note that we will always keep N_r^β, the number of spacial periods of the counters which can be used to measure β, a positive integer. We also assume that the largest time interval we can measure is $T_{max} = 2\pi N_{max}\Delta t$, where "$\pi$" is for us a rational fraction known only to an accuracy consistent with the measurement context considered.[34] Ref. 34 includes a preliminary discussion of how Stillman Drake's analysis[35] of the experiment by which Galileo arrived at the "times squared law" can be viewed as a *dynamical* measurement of $\pi/2\sqrt{2}$.

We have been at some pains to make this representation consistent with the discussion of "measurement" in Ref. 7. For the constant velocity case at hand, we can envisage a "particle" traversing a counter array with counters spaced a fixed distance $n_r^\beta \Delta x$ apart which fire sequentially with a fixed time interval $d_r^\beta \Delta t$ between firings. We symbolize this sequence of firings by the sequence $R, R', R'', R''', ...$ For definiteness we consider positive velocity and take the first firing to be

$$R := r(N_r^\beta + \delta n; n_r^\beta, d_r^\beta) = (N_r^\beta + \delta n) n_r^\beta \Delta x$$

Here, consistent with Kauffman's notation, we take the symbol "R" to stand for the instruction *measure* R, and the symbol ":=" to indicate that the value on the right is the numerical value obtained by the measurement. Then

$$R' := r(N_r^\beta + \delta n + 1) n_r^\beta \Delta x$$

$$R'' := r(N_r^\beta + \delta n + 2) n_r^\beta \Delta x$$

$$R''' := r(N_r^\beta + \delta n + 3) n_r^\beta \Delta x \tag{4.4}$$

$$.....etc.$$

To measure velocity requires us to select *two* counter firings, measure the space and time intervals between them, and calculate the ratio; velocity measurement is intrinsically a more complicated process than the measurement of the spacial interval from an origin to the position R of some identified counter. To conform to Kauffman's usage, we assume that the " $'$ " is an *operator* which shifts us forward in the time sequence $R, R', R'', R'''...$ by one fixed time interval $d_r^\beta \Delta t$. The symbol $\dot{R}$ is to be interpreted as the evaluation of the interval between R' and R by *first* measuring R, *then* measuring R' and finally by dividing the difference between the two intervals by the (fixed) time interval for a single shift, namely $d_r^\beta \Delta t$. Clearly

$$\dot{R} := \frac{n_r^\beta \Delta x}{d_r^\beta \Delta t} = \beta c; \quad \Delta x = c\Delta t \tag{4.5}$$

In contrast to the notation in the Feynman-Dyson-Tanimura papers, which would seem to imply that position and velocity are measured at the *same* time, we trust that our notation and explicit model make it clear that the two measurements are made at *different* times, and hence can yield different results when made in the opposite order. Now let $d_r^\beta \Delta t = 1$; interpret $R\dot{R}$ as the *process* — measure $\dot{R}$, then measure R. We now have established in our context the fundamental relationship

$$R\dot{R} = R'(R' - R) := r'(r' - r) \tag{4.6}$$

However, if we first measure R and then measure $\dot{R}$, we obtain

$$\dot{R}R = (R' - R) := (r' - r)r \tag{4.7}$$

In any theory with finite space and time shifts these two values are necessarily *not* the same. In fact, by forming $R\dot{R} - \dot{R}R$ and adding and subtracting $-R(R' - R)$ we have that

$$R\dot{R} - \dot{R}R = (R' - R)^2 + [R, R']; \qquad [R, R'] \equiv RR' - R'R \tag{4.8}$$

Since for us, once we have removed the velocity operation symbol $\dot{R}$, measuring R and R' corresponds to ascertaining the positions of two counters which are fixed in the laboratory and can be measured as many times as we wish in any order without changing the result, we can take $[R, R'] \equiv RR' - R'R = 0$ and we have that

$$R\dot{R} - \dot{R}R \equiv [R, \dot{R}] = (R' - R)^2 := (r' - r)^2 = \kappa \tag{4.9}$$

where κ is an arbitrary *constant scalar* fixed by the measurement accuracy context. This establishes the desired commutation relation for a single fixed velocity and a single spacial direction. The extension to two different velocities and to more than one dimension will be pursued elsewhere. We examine the Lorentz invariance of the one dimensional situation in the next section.

4.2 Boosts in one direction

Consider three aligned counters $0, 1, 2$ at distances $s_\alpha = n_\alpha \Delta x$, $\alpha \in 0, 1, 2$, from a mirror and define

$$n_{\alpha\beta} = n_\alpha - n_\beta;\ d_{\alpha\beta} = d_\alpha + d_\beta;\ v_{\alpha\beta} = \frac{n_{\alpha\beta}}{d_{\alpha\beta}};\ \alpha \neq \beta \in 0, 1, 2 \tag{4.10}$$

As we saw already, these velocities correspond to particles which leave one counter and arrive at another in the time it takes to a light signal to go from the first counter to the mirror and be reflected back to the second. But then the velocities satisfy the usual SR velocity addition law, as we sketch below.

In Ref. 21, Sec. 1.3, we started from two positive integers n_0 and n_1 and defined position and time coordinates for the interval between two events by $x_{01} = n_0 - n_1 = -x_{10}$ and $t_{01} = n_0 + n_1 = t_{10}$ where the units are such that $c = 1$. Then the velocity $v_{01} = -v_{10}$ and square of the invariant interval $\tau_{01} = \tau_{10}$ are given by

$$v_{01} = \frac{n_0 - n_1}{n_0 + n_1}; \qquad \tau_{01}^2 = t_{01}^2 - x_{01}^2 = 4n_0 n_1 \tag{4.11}$$

and we find that we can define the usual Lorentz time dilation γ by

$$\gamma_{01}^2 \equiv \frac{t_{01}^2}{\tau_{01}^2} = \frac{(n_0 + n_1)^2}{4n_0 n_1} = \frac{1}{1 - v_{01}^2} \tag{4.12}$$

If we now introduce a third integer n_2 and generalize our definitions, $x_{01} + x_{12} + x_{20} = 0$ and a little algebra gives us the familiar result that

$$v_{02} = \frac{v_{01} + v_{12}}{1 + v_{01} v_{12}} \tag{4.13}$$

With the interpretation that v_{12} is the velocity of the Lorentz boost which takes

(x_{01}, t_{01}) to (x_{02}, t_{02}), a little more algebra suffices to show that

$$x_{02} = \gamma_{12}(x_{01} + v_{12}t_{01}); \qquad t_{02} = \gamma_{12}(t_{01} + v_{12}x_{01}) \tag{4.14}$$

Having shown how a trajectory consisting of two line segments with different constant velocities can be constructed in a Lorentz invariant way by Lorentz boosts, we could consider the same trajectory as due to the emission or absorption of radiation at counter 1. Then the same argument which produced a commutation relation between position and velocity measurement can be used to show that they do not commute in the more general case. Evaluating the minimal constant (which will be proportional to $\Delta x^2/\Delta t$ on dimensional grounds) and, by extending the discussion to finite rotations in 3 dimensions, evaluating this constant as $\kappa/2\pi$ requires a more extended discussion than we have space for here. Many of the ingredients which go into such a discussion have already been explored in Ref. 21.

5. Derivation of the Maxwell Equations

When I showed Ref. 7 to my colleague, M.Peskin, he noted that the "shift operator J" defined by Kauffman is, in our context of a single particle, isomorphic to the quantum mechanical operator $U = exp(-iHT)$ representing a finite time shift in the Heisenberg representation. Then the formal steps in Kauffman's rigorous version of the Feynman-Dyson-Tanimura "proof" go through easily. The difficulty with adopting Peskin's approach is that what *operational* context the Heisenberg formalism fits into is by no means obvious. In particular, the usual formalism requires U to have an inverse, while this is not needed in the DOC derivation. So, for mathematical and physical clarity, one needs to invoke the DOC and discuss the relationship between measurement accuracy and the DOC. I am indebted to Peskin[36] for allowing me to quote his shortened version of the Kauffman proof below.

Define

$$\dot{X} = XU - UX = [X, U] \tag{5.1}$$

where U is the time shift operator from X to X' in time Δt (eg $U = e^{-iH\Delta t}$).

Notice that

$$(AB)^{\cdot} = [AB, U] = [A, U]B + A[B, U] = \dot{A}B + A\dot{B} \tag{5.2}$$

as required.

Postulate:

$$1. [X_i, X_j] = 0; \qquad 2. \; [X_i, \dot{X}_j] = \kappa\delta_{ij}$$

Rewrite 2 as

$$[X_i, [X_j, U]] = -[X_j, [U, X_i]] - [U, [X_i, X_j]] \tag{5.3}$$

and noting that $[U, [X_i, X_j]] = [U, 0] = 0$ we find that

$$\kappa\delta_{ij} = [X_i, [X_j, U]] \; symmetric \; in \; i, j \tag{5.4}$$

Now *define*

$$H_l = \frac{1}{2\kappa}\epsilon_{jkl}[\dot{X}_j, \dot{X}_k] \tag{5.5}$$

Then

$$\nabla_l H_l = \frac{1}{2\kappa}\epsilon_{jkl}[[\dot{X}_j, \dot{X}_k], \dot{X}_l] \tag{5.6}$$

But this cyclic sum vanishes by the Jacobi identity. Thus

$$\nabla_l H_l = 0 \tag{5.7}$$

which is one of the two Maxwell equations we set out to derive.

Finally, *define*

$$E_i = \ddot{X}_i - \epsilon_{ijk} H_k \tag{5.8}$$

We wish to prove that

$$\frac{\partial H_i}{\partial t} + \epsilon_{ijk} \nabla_j E_k = 0 \tag{5.9}$$

First we need to *define* $\partial/\partial t$ by

$$\dot{H} = \frac{d}{dt} H = \frac{\partial H}{\partial t} + (\dot{X} \cdot \nabla) H \tag{5.10}$$

Then

$$\frac{\partial H_i}{\partial t} = \dot{H}_i - \dot{X}_j \nabla_j H_i$$

$$= \frac{1}{2\kappa} \epsilon_{ikl} ([\dot{X}_k, \dot{X}_l])^{\cdot} - \dot{X}_j \frac{1}{\kappa} [\frac{\epsilon_{ikl}}{2\kappa} [\dot{X}_k, \dot{X}_l], \dot{X}_j]$$

$$= \frac{1}{\kappa} \epsilon_{ikl} [\dot{X}_k, \ddot{X}_l] - \frac{1}{2\kappa^2} \dot{X}_j \epsilon_{ikl} [[\dot{X}_k, \dot{X}_l], \dot{X}_j] \tag{5.11}$$

$$\begin{aligned}
\epsilon_{ijk} \nabla_j E_k &= \epsilon_{ijk} \frac{1}{\kappa} \ \left[\left(\ddot{X}_k - \epsilon_{klm} \dot{X}_l H_m \right), \dot{X}_j \right] \\
&= \frac{1}{\kappa} \, \epsilon_{ijk} \left[\dot{X}_j \ddot{X}_k \right] \cdot (-1) - \epsilon_{ijk} \epsilon_{klm} \epsilon_{mab} \, \frac{1}{2\kappa^2} \left[\dot{X}_l \left[\dot{X}_a, \dot{X}_b \right], \dot{X}_j \right] \\
&= -\frac{1}{\kappa} \, \epsilon_{ijk} \left[\dot{X}_j, \ddot{X}_k \right] \\
&\quad - (\delta^{il} \delta^{jm} - \delta^{im} \delta^{jl}) \, \epsilon_{mab} \, \frac{1}{2\kappa^2} \left(\left[\dot{X}_l, \dot{X}_j \right] \ \left[\dot{X}_a, \dot{X}_b \right] + \dot{X}_l \left[\left[\dot{X}_a, \dot{X}_b \right] \dot{X}_j \right] \right) \\
&= -\frac{1}{\kappa} \, \epsilon_{ijk} \left[\dot{X}_j \ddot{X}_k \right] + \frac{1}{2\kappa^2} \, \epsilon_{iab} X_j \left[\left[\dot{X}_a \dot{X}_b \right], \dot{X}_j \right] \\
&\quad - \epsilon_{jab} \frac{1}{2\kappa^2} \ \left[\dot{X}_i, \dot{X}_j \right] \ \left[\dot{X}_a, \dot{X}_b \right]
\end{aligned} \tag{5.12}$$

now

$$\epsilon_{jab}\left[\dot{X}_i,\dot{X}_j\right]\left[\dot{X}_a,\dot{X}_b\right]=\left[\dot{X}_i,X_1\right]\left[X_2,X_3\right] + \left[\dot{X}_i,\dot{X}_2\right]\left[\dot{X}_3,\dot{X}_1\right]+\left[\dot{X}_i,\dot{X}_3\right]\left[\dot{X}_1,\dot{X}_2\right] \tag{5.13}$$

for $i=1$, eg

$$=\left[\dot{X}_1,\dot{X}_2\right]\left[\dot{X}_3,\dot{X}_1\right]+\left[\dot{X}_1,\dot{X}_3\right]\left[\dot{X}_1,\dot{X}_2\right]=0 \tag{5.14}$$

so

$$\begin{aligned}\epsilon_{ijk}\nabla_j E_k &= -\frac{1}{\kappa}\epsilon_{ijk}\left[\dot{X}_j,\ddot{X}_k\right]+\frac{1}{2\kappa^2}\epsilon_{iab}X_j\left[\left[\dot{X}_a,\dot{X}_b\right]\dot{X}_j\right]\\ &= -\frac{\partial H}{\partial t} \qquad QED\ .\end{aligned} \tag{5.15}$$

6. Conclusions

Once one accepts the idea that the random motion of a Dirac particle is due to interactions with the background radiation, which will correspond to "vacuum fluctuations" of the conventional theory at zero temperature, and models this by Program Universe bit-strings, these additional degrees of freedom lead to a different statistical counting than that envisaged by Feynman. Then a real, finite step-length h/mc leads directly to the known result. In spite of this background, which requires an infinite renormalization in the conventional approach, the symmetries of the problem keep the result finite in our case. This allows the mass in the Dirac equation to be interpreted as the observed mass of a free particle.

To demonstrate the consistency of this result with known experimental results conventionally explained by infinite renormalization will require a corresponding treatment of electromagnetism. A start on this has been made. Whether we can actually stitch these two approaches together and succeed in getting a good value for $g-2$, the Lamb shift, etc. remains to be seen. The continuing improvement of the fit between "bit-string physics" and conventional results in other contexts might be taken as a harbinger of ultimate success, but only the uncertain future could justify this optimism.

7. Acknowledgements

Our derivation was greatly assisted by knowing the result we wished to achieve. Rather than conceal this fact, we have left it in clear view. While this is not customary in the technical literature, we have good precedent for this practice. For example, Stillman Drake translates the following revealing passage[37] from Galileo's *Dialogue*

"SIMPLICICO: Aristotle first laid the basis of his argument a priori, showing the necessity of inalterable heavens by means of natural, evident and clear principles. He afterward supported the same a posteriori by the senses and by the tradition of the ancients.

"SALVIATI: What you refer to is the method he uses in writing his doctrine, but I do not believe it to be that with which he investigated. I think it certain that he first obtained it by means of the senses, experiments, and observations to assure himself as much as possible of his conclusions. Afterward he sought means to make those demonstrable. That is what is done for the most part in the demonstrative sciences ... *The certainty of a conclusion assists not a little in the discovery of its proof.*" [Italics supplied by HPN]

Since Salviati is known to express Galileo's own views in this work, we are confident that the founder of modern physics endorsed our strategy. Clearly our first acknowledgement must go to Galileo for developing and justifying our methodology — and also to Stillman Drake for making Galileo's intellectual development accessible to all of us.

This paper itself originated thanks to correspondence between V.A.Karmanov and I.Stein about the possibility of relating the Feynman-Hibbs suggestion to the Stein model, and a comment by D.O.McGoveran that an approximation suggested by Karmanov was already the exact result. Several drafts of a paper with the four of us as authors were made, but we never were able achieve consensus on the presentation, or approval of the mathematics by C.W.Kilmister. However, Kilmister's remark that McGoveran's formula for one basic ingredient in his transport operator within the *ordering operator calculus* (Ref.2), namely $r^k[\frac{S!}{k!(S-k)!}]/[\frac{S!}{(S-k)!}] = r^k/k!$, is reminiscent of the difference between Maxwell-Boltzmann and Bose-Einstein statistics, led by a tortuous route to the current paper. In the light of past experience, I have decided to go ahead with my own version as presented here, rather than try to make this a collaborative effort. But this paper never would have come into existence without my collaboration with all four of the scientists named above; its flaws are purely my own responsibility. I am also indebted to J.C. van den Berg for several clarifying discussions of this material.

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