Hindawi Publishing Corporation
Advances in High Energy Physics
Volume 2013, Article ID 873236, 9 pages
http://dx.doi.org/10.1155/2013/873236

Review Article

Bruno Pontecorvo and Neutrino Oscillations

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Received 17 June 2013; Accepted 28 August 2013

Academic Editor: Vincenzo Flaminio

I discuss briefly in this review, dedicated to the centenary of the birth of the great neutrino physicist Bruno Pontecorvo, the following ideas he proposed: (i) the radiochemical method of neutrino detection; (ii) the μ-e universality of the weak interaction; (iii) the accelerator neutrino experiment which allowed to prove that muon and electron neutrinos are different particles (the Brookhaven experiment). I consider in some details Pontecorvo's pioneering idea of neutrino masses, mixing, and oscillations and the development of this idea by Pontecorvo, by Pontecorvo and Gribov, and by Pontecorvo and myself.

1. Introduction

Pontecorvo started his scientific work in 1932 in Rome as a student of E. Fermi. Later, he became a member of the Fermi group. He was the youngest "ragazzo di Via Panisperna." Pontecorvo took part in many experiments of the Fermi group, including classical experiments in which the effect of slow neutrons was discovered.

From 1936 till 1940, Pontecorvo worked on the investigation of nuclear isomers in Paris in the Joliot-Curie group. From 1940 till 1942, he worked in the USA. He developed and realized a method of neutron well logging for oil prospecting. This was the first practical application of neutrons. From 1943 till 1948, Pontecorvo worked in Canada, first in the Montreal Research Laboratory and then in the Chalk River Laboratory. He was the scientific leader of the project of the research nuclear reactor which was built in 1945 and was the first nuclear reactor outside the USA. In Canada, Pontecorvo started research in elementary particle physics.

Soon after the famous 1934 Fermi paper on the theory of β-decay [1], Bethe and Peierls [2] estimated the interaction cross section of neutrinos with nuclei. They showed that the cross section was extremely small ($\sigma < 10^{-44}$ cm^2). For many years, the neutrino was considered as an "undetectable particle."

Pontecorvo was the first who challenged this opinion. In 1946, he proposed the radiochemical method of neutrino detection [3]. The method was based on the observation of the decay of the daughter nucleus produced in the reaction $\nu + (A, Z) \rightarrow e^- + (A, Z+1)$. He discussed in details the reaction

$$\nu + {}^{37}\mathrm{Cl} \longrightarrow e^- + {}^{37}\mathrm{Ar}. \tag{1}$$

Pontecorvo considered the method of neutrino detection based on the reaction (1) as a promising one for the following reasons.

(i) C_2Cl_4 is a cheap, nonflammable liquid.

(ii) ^{37}Ar nuclei are unstable (K-capture) with a convenient half-life (34.8 days).

(iii) A few atoms of ^{37}Ar (rare gas), produced during the exposition time, can be extracted from a large detector.

The Pontecorvo Cl-Ar method was used by Davis Jr. in his pioneering experiment on the detection of solar neutrinos [4, 5] for which Davis Jr. was awarded the Nobel Prize in 2002.

The radiochemical method of neutrino detection based on the observation of the reaction

$$\nu_e + {}^{71}\mathrm{Ga} \longrightarrow e^- + {}^{71}\mathrm{Ge} \tag{2}$$

was used in the GALLEX-GNO [6, 7] and SAGE [8] solar neutrino experiments, in which ν_e's from all thermonuclear reactions in the sun including neutrinos from the main reaction $pp \to e^+\nu_e np$ were detected.

In Canada in 1948, Pontecorvo invented the low-background proportional counter that allowed to count very rare events. This counter was crucial for the detection of solar neutrinos in the Homestake, GALLEX, and SAGE experiments.

After the Conversi, Pancini, and Piccioni experiment [9], from which it followed that muons weakly interact with nuclei, Bruno Pontecorvo together with Hincks performed a series of brilliant pioneering experiments on the investigation of fundamental properties of the muon [10, 11].

In these experiments they showed that

(1) the charged particle emitted in μ-decay is the electron;

(2) the muon decays into three particles;

(3) the muon does not decay into electron and γ.

Pontecorvo was the first who paid attention to a deep analogy between the electron and the muon [12]. He compared the probabilities of the processes

$$\begin{aligned} \mu^- + (A,Z) &\longrightarrow \nu + (A, Z-1), \\ e^- + (A,Z) &\longrightarrow \nu + (A, Z-1) \end{aligned} \tag{3}$$

and came to the conclusion that the constants which characterize these two processes are of the same order of magnitude. On the basis of this observation, he came to the idea of the existence of a universal weak interaction which includes e-ν and μ-ν pairs. Later, the idea of μ-e universality was proposed by Puppi [13], Klein [14], and Yang and Tiomno [15].

In 1950, Pontecorvo moved to Russia. He started to work at Dubna where at that time the largest accelerator in the world was operating. He and his group performed several experiments on the investigation of the production of π^0 in neutron-proton and neutron-nucleus collisions, pion-nucleon scattering, and others.

In 1959, a project of a meson factory was under preparation in Dubna (for various reasons the project was not realized). Physicists started to plan different experiments which could be performed at such a facility. Pontecorvo thought about the feasibility of neutrino experiments with neutrinos from decay of pions and kaons which can be produced at high intensity accelerators. He came to the conclusion that experiments with accelerator neutrinos are possible [16] (Markov [17] and Schwartz [18] came to the same conclusion) and proposed the experiment which could allow to answer the question whether muon and electron neutrinos are the same or different particles. His proposal was realized in the famous Brookhaven experiment [19] (1962). In 1988, Lederman, Schwartz, and Steinberger were awarded the Nobel Prize for "the discovery of the muon neutrino leading to classification of particles in families."

In 1957, Pontecorvo came to the idea of neutrino oscillations.

2. First Ideas of Neutrino Oscillations (1957-1958)

We come now to the very bright idea of Bruno Pontecorvo, that of neutrino masses, mixing, and oscillations, which created a new field of neutrino research and a new era in neutrino physics. He proposed the idea of neutrino oscillations in 1957-1958 [20, 21] and pursued it over many years.

Pontecorvo was impressed by the possibility of $K^0 \rightleftarrows \overline{K}^0$ oscillations suggested by Gell-Mann and Pais [22]. This phenomenon was based on the following:

(1) K^0 and $\overline{K}^0$ are particles with strangeness +1 and −1, respectively. Strangeness is conserved in the strong interaction;

(2) weak interaction, in which strangeness is not conserved, induces transitions between K^0 and $\overline{K}^0$.

Pontecorvo raised the question [20], "... whether there exist other "mixed" neutral particles (not necessarily elementary ones) which are not identical to their corresponding antiparticles and for which particle $\leftrightarrows$ antiparticle transitions are not strictly forbidden."

He came to the conclusion that muonium (μ^+-e^-) and antimuonium (μ^--e^+) could be such a system. At that time, it was not known that ν_e and ν_μ are different particles. Pontecorvo wrote that μ^+-$e^- \leftrightarrows \mu^-$-$e^+$ transitions are allowed and "are induced by the same interaction which is responsible for μ-decay"

$$\left(\mu^+\text{-}e^-\right) \longrightarrow \nu + \overline{\nu} \longrightarrow \left(\mu^-\text{-}e^+\right). \tag{4}$$

In the paper [20], the following remark about the neutrino was made. "If the theory of the two-component neutrino is not valid (which is hardly probable at present) and if the conservation law for the neutrino charge does not hold, neutrino → antineutrino transitions in vacuum in principle be possible."

As it is well known according to the two-component neutrino theory [23–25], the neutrino is massless and for one neutrino type only a left-handed neutrino ν_L and a right-handed antineutrino $\overline{\nu}_R$ exist.

The subsequent paper on neutrino oscillations was published by Pontecorvo in 1958 [21]. He wrote in this paper, "... neutrino may be a particle mixture and consequently there is a possibility of real transitions neutrino → antineutrino in vacuum, provided that the lepton (neutrino) charge is not conserved. This means that the neutrino and antineutrino are *mixed* particles, that is, a symmetric and antisymmetric combination of two truly neutral Majorana particles ν_1 and ν_2." And later in the paper [21] he wrote, "... this possibility became of some interest in connection with new investigations of inverse β-processes."

Pontecorvo had in mind the following. In 1957, Davis performed a reactor experiment [26] in which he searched for the production of ^{37}Ar in the process

$$\text{“reactor antineutrino”} + {}^{37}\text{Cl} \longrightarrow e^- + {}^{37}\text{Ar}. \tag{5}$$

A rumor reached Pontecorvo that Davis had observed such events. Pontecorvo suggested that these "events" could be due to transitions of reactor antineutrinos into neutrinos in vacuum (neutrino oscillations).

In 1957-1958, only one neutrino type was known. Pontecorvo assumed that the transition $\bar{\nu}_R \rightarrow \nu_R$ (and $\nu_L \rightarrow \bar{\nu}_L$) was possible. Thus, he had to assume that not only the lepton number is not conserved but also that in addition to the standard right-handed antineutrino $\bar{\nu}_R$ and left-handed neutrino ν_L (quanta of the field $\nu_L(x)$) a right-handed neutrino ν_R and a left-handed antineutrino $\bar{\nu}_L$ quanta of the right-handed field $\nu_R(x)$ existed.

According to the two-component neutrino theory, which was confirmed by the experiment on the measurement of the neutrino helicity [27], only the field $\nu_L(x)$ enters in the weak interaction Lagrangian. Thus, from the point of view of this theory, ν_R and $\bar{\nu}_L$ are noninteracting "sterile" particles.

In order to explain the Davis "events," Pontecorvo had to assume that "a definite fraction of particles can induce the reaction (5)." Pontecorvo, however, pointed out (and this was the most important) that *in the inclusive experiment of Reines and Cowan Jr.* [28, 29] *due to neutrino oscillations a deficit of antineutrino events could be observed.* He wrote in the paper [20], "... The cross section of the process $\bar{\nu} + p \rightarrow e^+ + n$ with $\bar{\nu}$ from reactor must be smaller than expected. This is due to the fact that the neutral lepton beam which at the source is capable of inducing the reaction changes its composition on the way from the reactor to the detector."

It is impressive that already in 1958 Pontecorvo believed that neutrinos have small masses and oscillate. In [20], he wrote, "Effects of transformation of neutrino into antineutrino and vice versa may be unobservable in the laboratory but will certainly occur, at least, on an astronomical scale."

Let us go back to the Davis experiment. At a later stage of the experiment, the anomalous "events" (5) disappeared, and only an upper bound of the cross section of the reaction (5) was obtained in [26]. Pontecorvo soon understood that ν_R and $\bar{\nu}_L$ are sterile particles. The terminology "sterile neutrino," which is standard nowadays, was introduced by him in the next publication on neutrino oscillations [30].

3. The Second Pontecorvo Paper on Neutrino Oscillations (1967)

The subsequent paper on neutrino oscillations was written by Bruno Pontecorvo in 1967 [30]. At that time, the phenomenological V-A theory of Feynman and Gell-Mann [31], Sudarshan and Marshak [32] was well established, $K^0 \rightleftarrows \bar{K}^0$ oscillations had been observed, and it has been proven that (at least) two types on neutrinos ν_e and ν_μ existed in nature.

In [30] Pontecorvo wrote, "If the lepton charge is not an exactly conserved quantum number, and the neutrino mass is different from zero, oscillations similar to those in K^0 beams become possible in neutrino beams."

Pontecorvo discussed the transitions $\nu_e \rightleftarrows \bar{\nu}_{eL}$ and $\nu_\mu \rightleftarrows \bar{\nu}_{\mu L}$ which transform "active particles into particles, which from the point of view of ordinary weak processes, are sterile ..." In the paper [30], Pontecorvo considered also transition between active neutrinos $\nu_\mu \rightleftarrows \nu_e$. He pointed out that in this case not only the disappearance of ν_μ but also the appearance of ν_e can be observed.

In the 1967 paper [30], Pontecorvo discussed the effect of neutrino oscillations for solar neutrinos. "From an observational point of view the ideal object is the sun. If the oscillation length is smaller than the radius of the sun region effectively producing neutrinos, direct oscillations will be smeared out and unobservable. The only effect on the earth's surface would be that the flux of observable sun neutrinos must be two times smaller than the total (active and sterile) neutrino flux."

At that time, Davis Jr. prepared his famous solar neutrino experiment. When in 1970 the first results of the experiment were obtained [4, 5], it occurred that the detected flux of solar neutrinos was about (2-3) times smaller than the flux predicted by the standard solar model (this effect was called "the solar neutrino problem").

It was very soon commonly accepted that among the different possible astrophysical (and particle physics) explanations of the problem that of neutrino oscillations was the most natural one [33]. *Thus, Pontecorvo anticipated the solar neutrino problem.*

4. The Gribov-Pontecorvo Paper on Neutrino Oscillations (1969)

Gribov and Pontecorvo [34] considered a scheme of neutrino mixing and oscillations with four neutrino and antineutrino states: left-handed neutrinos ν_e, ν_μ and right-handed antineutrinos $\bar{\nu}_e$, $\bar{\nu}_\mu$, quanta of the left-handed neutrino fields $\nu_{eL}(x)$ and $\nu_{\mu L}(x)$. They assumed that there are no sterile neutrino states.

It was assumed in [34] that in addition to the standard charged current V-A interaction with the lepton current

$$j^\alpha = 2\left(\bar{\nu}_{eL}\gamma^\alpha e_L + \bar{\nu}_{\mu L}\gamma^\alpha \mu_L\right) \tag{6}$$

in the total Lagrangian enters *an effective Lagrangian of an interaction which violates L_e and L_μ*. After diagonalization of the effective Lagrangian, the following mixing relations were found:

$$\begin{aligned} \nu_{eL}(x) &= \cos\theta\chi_{1L}(x) + \sin\theta\chi_{2L}(x); \\ \nu_{\mu L}(x) &= -\sin\theta\chi_{1L}(x) + \cos\theta\chi_{2L}(x). \end{aligned} \tag{7}$$

Here, $\chi_{1,2}(x)$ are fields of the Majorana neutrinos with masses $m_{1,2}$, and θ is a mixing angle. All these parameters are determined by those of the effective Lagrangian.

The authors obtained the following expression for the $\nu_e \rightarrow \nu_e$ transition probability in vacuum (in modern notations):

$$P(\nu_e \longrightarrow \nu_e) = 1 - \frac{1}{2}\sin^2 2\theta\left(1 - \cos\frac{\Delta m^2 L}{2E}\right) \tag{8}$$

($\Delta m^2 = |m_2^2 - m_1^2|$) and applied the formalism developed to solar neutrino oscillations. They considered the possibility of the maximal mixing $\theta = \pi/4$ as the most simple and attractive one. In this case, the averaged observed flux of solar neutrinos is equal to 1/2 of that predicted.

5. The General Phenomenological Theory of Neutrino Mixing and Oscillations (Dubna, 1975–1987)

Pontecorvo and my work on neutrino masses, mixing, and oscillations started in 1975 [35, 36]. The first paper [35, 36] was based on the idea of *quark-lepton analogy*. It had been established at that time that the charged current of quarks has the form (the case of four quarks)

$$j_\alpha^{CC(\text{quark})}(x) = 2\left[\bar{u}_L(x)\gamma_\alpha d_L^c(x) + \bar{c}_L(x)\gamma_\alpha s_L^c(x)\right], \quad (9)$$

where

$$\begin{aligned} d_L^c(x) &= \cos\theta_C d_L(x) + \sin\theta_C s_L(x), \\ s_L^c(x) &= -\sin\theta_C d_L(x) + \cos\theta_C s_L(x) \end{aligned} \quad (10)$$

are Cabibbo-GIM mixtures of d and s quarks and θ_C is the Cabibbo angle.

The lepton charged current

$$j_\alpha^{CC(\text{lep})}(x) = 2\left[\bar{\nu}_{eL}(x)\gamma_\alpha e_L(x) + \bar{\nu}_{\mu L}(x)\gamma_\alpha \mu_L(x)\right] \quad (11)$$

has the same form as the quark charged current (same coefficients, left-handed components of the fields). In order to make the analogy between quarks and leptons complete, it was natural from our point of view to assume that $\nu_{eL}(x)$ and $\nu_{\mu L}(x)$ are also mixed fields:

$$\begin{aligned} \nu_{eL}(x) &= \cos\theta\nu_{1L}(x) + \sin\theta\nu_{2L}(x), \\ \nu_{\mu L}(x) &= -\sin\theta\nu_{1L}(x) + \cos\theta\nu_{2L}(x). \end{aligned} \quad (12)$$

Here, $\nu_1(x)$ and $\nu_2(x)$ are Dirac fields of neutrinos with masses m_1 and m_2 and θ is the leptonic mixing angle. We wrote in [35, 36], "... in our scheme ν_1 and ν_2 are just as leptons and quarks (which, may be, is an attractive feature) while in the Gribov-Pontecorvo scheme [34] the two neutrinos have a special position among the other fundamental particles."

If the mixing (12) takes place, the total lepton number $L = L_e + L_\mu$ is conserved and the neutrinos with definite masses ν_i $(i = 1, 2)$ differ from the corresponding antineutrinos $\bar{\nu}_i$ by the lepton number $(L(\nu_i) = -L(\bar{\nu}_i) = 1)$.

In 1975, after the success of the two-component theory, there was still a general belief that neutrinos are massless particles. It is obvious that in this case the mixing (12) has no physical meaning.

Our main arguments for neutrino masses were at that time the following.

(1) There is no principle (like gauge invariance in the case of γ-quanta) which requires that neutrino masses must be equal to zero.

(2) In the framework of the two-component neutrino theory, the zero mass of the neutrino was considered as an argument in favor of the left-handed neutrino field. It occurred, however, that in the weak Hamiltonian left-handed components of *all fields* enter (the V-A theory). It was more natural after that to consider the neutrino not as a special massless particle but as a particle with some mass.

We discussed in [35, 36] a possible value of the mixing angle θ. We argued that

(i) there is no reason for the lepton and Cabibbo mixing angles to be the same,

(ii) "it seems to us that the special values of the mixing angles $\theta = 0$ and $\theta = \pi/4$ (maximum mixing) are of the greatest interest."

Let us notice that probabilities of transitions $\nu_l \to \nu_{l'}$ are the same in the scheme of the mixing of two Majorana neutrinos [34] and in the scheme of the mixing of two Dirac neutrinos [33].

In the following paper [37], we considered the most general neutrino mixing. In accordance with gauge theories, we started to characterize neutrino mixing by *the neutrino mass term*. Three types of the neutrino mass terms are possible (we follow reviews [38, 39]).

5.1. Left-Handed Majorana Mass Term. Let us assume that in addition to the standard CC Lagrangian of the interaction of leptons and W-bosons

$$\begin{aligned} \mathcal{L}_{\mathcal{I}}^{\mathcal{CC}}(x) &= -\frac{g}{2\sqrt{2}} j_\alpha^{CC}(x) W^\alpha(x) + \text{h.c.}; \\ j_\alpha^{CC}(x) &= 2\sum_{l=e,\mu,\tau} \bar{\nu}_{lL}(x)\gamma_\alpha l_L(x) \end{aligned} \quad (13)$$

(and many other terms) in the total Lagrangian the following neutrino mass term enters

$$\mathcal{L}_L^M = -\frac{1}{2}\,\bar{\nu}_L\, M_L (\nu_L)^c + \text{h.c.} \quad (14)$$

Here,

$$\nu_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}, \quad (15)$$

$(\nu_L)^c = C(\bar{\nu}_L)^T$ is the conjugated field (right-handed component) (C is the matrix of the charge conjugation which satisfies the following relations $C\gamma_\alpha^T C^{-1} = -\gamma_\alpha$, $C^T = -C$), and M_L is a 3×3 symmetrical, complex matrix ($M_L = M_L^T$).

The mass term (15) is a generalization of the mass term considered by Gribov and Pontecorvo [34]. After the standard diagonalization of the matrix M_L, we find the following mixing relations

$$\nu_{lL}(x) = \sum_{i=1}^{3} U_{li}\nu_{iL}(x), \quad l = e, \mu, \tau. \quad (16)$$

Here, U is a unitary mixing matrix and $\nu_i(x)$ is the field of the Majorana neutrino with mass m_i. The field $\nu_i(x)$ satisfies the condition

$$\nu_i(x) = \nu_i^c(x) = C\left(\overline{\nu}_i(x)\right)^T. \tag{17}$$

Thus, if the neutrino mass term has the form (14), the flavor neutrino fields ν_{lL} ($l = e, \mu, \tau$), which enter into the standard charged current, are linear combinations of left-handed components of the Majorana fields with definite masses.

The mass term (14) does not conserve any lepton numbers. Thus, in the case of the mass term (13), there are no quantum numbers which allow to distinguish neutrino and antineutrino. This is the physical reason why ν_i are Majorana particles ($\nu_i \equiv \overline{\nu}_i$)

5.2. Dirac Mass Term. We will assume now that in addition to the standard CC Lagrangian of the interaction of leptons and W-bosons (13) in the total Lagrangian the following neutrino mass term enters

$$\mathscr{L}^D = -\overline{\nu}_L\, M^D\, \nu_R + \text{h.c.} \tag{18}$$

Here,

$$\nu_R = \begin{pmatrix} \nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R} \end{pmatrix}, \tag{19}$$

ν_L is given by (15), and M^D is a complex 3×3 matrix.

After the standard diagonalization of the matrix M^D, we obtain the following mixing relations

$$\nu_{lL}(x) = \sum_{i=1}^{3} U_{li}\nu_{iL}(x), \quad l = e, \mu, \tau. \tag{20}$$

Here, U is a unitary 3×3 mixing matrix, $\nu_i(x)$ is the field of the Dirac neutrinos with mass m_i.

The mass term $\mathscr{L}^D$ conserves the total lepton number L (which is the same for (ν_e, e), (ν_μ, μ), and (ν_τ, τ)). The Dirac neutrino ν_i and antineutrino $\overline{\nu}_i$ have the same mass m_i and differ by the lepton number ($L(\nu_i) = 1$, $L(\overline{\nu}_i) = -1$).

The scheme with the mass term $\mathscr{L}^D$ is the generalization of the scheme considered in [33].

5.3. Dirac and Majorana Mass Term. Let us assume that in addition to the standard CC Lagrangian of the interaction of leptons and W-bosons (13) in the total Lagrangian, the following neutrino mass term enters [37]

$$\mathscr{L}^{D+M} = \mathscr{L}_L^M + \mathscr{L}^D + \mathscr{L}_R^M. \tag{21}$$

Here, ${\mathscr{L}_L}^M$ is the left-handed Majorana mass term (14), $\mathscr{L}^D$ is the Dirac mass term (18), and the right-handed Majorana mass term ${\mathscr{L}_R}^M$ is given by the expression

$${\mathscr{L}_R}^M = -\frac{1}{2}\,\overline{(\nu_R)^c}\, M_R \nu_R + \text{h.c.}, \tag{22}$$

where M_R is 3×3 complex, symmetrical matrix.

After the diagonalization of the mass term (22), we find the following mixing relations

$$\begin{aligned} \nu_{lL}(x) &= \sum_{i=1}^{6} U_{li}\nu_{iL}(x), \\ (\nu_{lR})^c(x) &= \sum_{i=1}^{6} U_{\bar{l}i}\nu_{iL}(x), \quad l = e, \mu, \tau. \end{aligned} \tag{23}$$

Here, U is a unitary 6×6 mixing matrix and $\nu_i(x)$ is the field of the Majorana neutrino with mass m_i ($\nu_i(x) = \nu_i^c(x)$).

Thus, in the general case of the Dirac and Majorana mass term, the flavor neutrino fields $\nu_{lL}(x)$ are linear combinations of the left-handed components of six Majorana fields with definite masses. The same left-handed components of six Majorana fields with definite masses are connected with the conjugated right-handed sterile fields $(\nu_{lR})^c(x)$, which do not enter into the Lagrangian of the Standard electroweak interaction.

In the case of the Dirac and the Majorana mass terms, due to mixing only, transitions between flavor neutrinos $\nu_l \leftrightarrows \nu_{l'}$ are possible. In the case of the Dirac and Majorana mass term, not only transitions between flavor neutrinos but also transitions $\nu_l \leftrightarrows \overline{\nu}_{l'L}$ (sterile) are possible.

In 1977, we wrote a first review on neutrino oscillations [38] in which we summarized the situation of neutrino masses, mixing, and oscillations at the time when dedicated experiments on the search for neutrino oscillations had not started yet. This review attracted the attention of many physicists to the problem.

We assumed that neutrinos take part in the standard CC and NC interactions. (this assumption was based on the data of all existing experiments in which weak processes were investigated.) In the case of the neutrino mixing, $\nu_{eL}(x)$, $\nu_{\mu L}(x)$, and $\nu_{\tau L}(x)$ are not quantum fields but linear combinations of the fields of neutrinos with definite masses ν_{iL}. The first question was, *What are the QFT states of flavor neutrinos ν_e, ν_μ, and ν_τ (and flavor antineutrinos $\overline{\nu}_e$, $\overline{\nu}_\mu$, and $\overline{\nu}_\tau$), particles which are produced in weak decays, captured in neutrino processes, and so forth?*

By definition, the muon neutrino ν_μ is a particle, which is produced together with μ^+ in the decay $\pi^+ \to \mu^+ + \nu_\mu$, the particle which produces e^+ in the process $\overline{\nu}_e + p \to e^+ + n$ is the electron antineutrino $\overline{\nu}_e$, and so forth.

The case of mixing this definition is unambiguous if neutrino mass-squared differences can be neglected in matrix elements of neutrino production (and absorption) processes. In this case, the matrix element of a decay, in which ν_l is produced, is given by the standard model matrix element (with zero mass-squared differences) and *independently on the production process* the state of the flavor neutrino ν_l ($l = e, \mu, \tau$) is given by

$$|\nu_l\rangle = \sum_i U_{li}^* |\nu_i\rangle. \tag{24}$$

Here, $|\nu_i\rangle$ is the state of a neutrino with mass m_i, momentum $\vec{p}$, and energy ($E = p$ is the energy of neutrino at $m_i \to 0$)

$$E_i = \sqrt{p^2 + m_i^2} \simeq E + \frac{m_i^2}{2E}. \quad (25)$$

Thus, in the case of the mixing of neutrinos with small mass-squared differences, the state of a flavor neutrino is a coherent superposition of states of neutrinos (Dirac or Majorana) with definite masses. In [38], we formulated the following coherence condition

$$L_{ik} \gtrsim a. \quad (26)$$

Here, $L_{ik} = 4\pi(E/|\Delta m_{ik}^2|)$ $(i \neq k)$ is the oscillation length ($\Delta m_{ik}^2 = m_k^2 - m_i^2$) and a is the QM size of a source. Notice that for mass-squared differences determined from the data of modern neutrino oscillation experiments

$$\begin{aligned} \Delta m_{12}^2 &= \left(7.65_{-0.20}^{+0.13}\right) \cdot 10^{-5}\ \mathrm{eV}^2, \\ \Delta m_{23}^2 &= (2.43 \pm 0.13) \cdot 10^{-3}\ \mathrm{eV}^2, \end{aligned} \quad (27)$$

and neutrino energies $E \gtrsim 1\,\mathrm{MeV}$, the condition (27) is obviously satisfied.

The relation (24) *is basic for the phenomenon of neutrino oscillations.* In accordance with QFT, we assume that the evolution of states is determined by the Schrodinger equation

$$i\frac{\partial |\Psi(t)\rangle}{\partial t} = H|\Psi(t)\rangle. \quad (28)$$

From (28) it follows that if at $t = 0$ a flavor neutrino ν_l is produced at time t we have for the neutrino state

$$|\nu_l\rangle_t = e^{-iHt}|\nu_l\rangle = \sum_i |\nu_i\rangle e^{-iE_i t} U_{li}^*. \quad (29)$$

Thus, if a flavor neutrino is produced, the neutrino state at a time t is a superposition of states with different energies, that is, *nonstationary state*.

Neutrinos are detected via the observation of weak processes

$$\nu_{l'} + N \longrightarrow l' + X, \text{etc.}, \quad (30)$$

in which flavor neutrinos are participating. Expanding the state $|\nu_l\rangle_t$ over the flavor neutrino states, we find

$$|\nu_l\rangle_t = \sum_{l'} |\nu_{l'}\rangle \left(\sum_i U_{l'i} e^{-iE_i t} U_{li}^*\right). \quad (31)$$

Thus, the probability of the transition $\nu_l \to \nu_{l'}$ during the time t is given by the expression

$$P(\nu_l \longrightarrow \nu_{l'}) = \left|\sum_i U_{l'i} e^{-iE_i t} U_{li}^*\right|^2. \quad (32)$$

Analogously, for the probability of the transition $\bar{\nu}_l \to \bar{\nu}_{l'}$, we find

$$P(\bar{\nu}_l \longrightarrow \bar{\nu}_{l'}) = \left|\sum_i U_{l'i}^* e^{-iE_i t} U_{li}\right|^2. \quad (33)$$

The expression (33) has a simple interpretation: U_{li}^* is the amplitude of the probability to find in the flavor state $|\nu_l\rangle$ the state $|\nu_i\rangle$; the factor $e^{-iE_i t}$ describes evolution of the state with energy E_i; $U_{l'i}$ is the amplitude of the probability to find in the state $|\nu_i\rangle$ the flavor state $|\nu_{l'}\rangle$; because of the coherence of the flavor states, the sum over i is performed.

Taking into account the unitarity of the mixing matrix, we can rewrite the expression (33) for the $\nu_l \to \nu_{l'}$ transition probability in the following form:

$$\begin{aligned} P(\nu_l \longrightarrow \nu_{l'}) &= \left|\sum_{i \neq k} U_{l'i} e^{-i(\Delta m_{ki}^2 L/2E)} U_{li}^*\right|^2 \\ &== \left|\delta_{l'l} + \sum_{i \neq k} U_{l'i}\left(e^{-i(\Delta m_{ki}^2 L/2E)} - 1\right) U_{li}^*\right|^2, \end{aligned} \quad (34)$$

where k is a fixed index. In (34), we took into account that for the ultrarelativistic neutrinos

$$t \simeq L, \quad (35)$$

where L is the distance between the neutrino source and the detector.

The expression (34) became the standard expression for the transition probability. It is commonly used in the analysis of data of experiments on the investigation of neutrino oscillations.

We know now that three flavor neutrinos exist in nature. If the number of neutrinos with definite masses is also equal to three (there are no sterile neutrino states), the neutrino transition probabilities depend on two mass-squared differences Δm_{12}^2 and Δm_{23}^2 and on parameters which characterize 3×3 unitary mixing matrix (three angles and one phase).

It follows from analysis of the experimental data that $\Delta m_{12}^2 \ll |\Delta m_{23}^2|$ and one of the mixing angle (θ_{13}) is small. It is easy to show (see, e.g., [40]) that in the leading approximation oscillations observed in atmospheric and accelerator neutrino experiments there are two-neutrino $\nu_\mu \rightleftarrows \nu_\tau$ oscillations. For the ν_μ survival probability from (34), we find the following expression:

$$P(\nu_\mu \longrightarrow \nu_\mu) \simeq 1 - \frac{1}{2}\sin^2 2\theta_{23}\left(1 - \cos\frac{\Delta m_{23}^2 L}{2E}\right). \quad (36)$$

In the leading approximation, the disappearance of $\bar{\nu}_e$'s in the reactor KamLAND experiment is due to $\bar{\nu}_e \to \bar{\nu}_{\mu,\tau}$ transitions. The survival probability is given in this case by the expression

$$P(\bar{\nu}_e \longrightarrow \bar{\nu}_e) \simeq 1 - \frac{1}{2}\sin^2 2\theta_{12}\left(1 - \cos\frac{\Delta m_{12}^2 L}{2E}\right). \quad (37)$$

There exists at present a convincing proof that neutrinos are massive and mixed particles. The proof was obtained via the observation of neutrino oscillations in the Super-Kamiokande atmospheric neutrino experiment [41, 42], in the SNO solar neutrino experiment [43, 44], in the KamLAND reactor experiment [45], in K2K [46], MINOS [47], and T2K [48, 49] accelerator experiments, and in many other neutrino experiments.

The discovery of neutrino oscillations was a great triumph for Pontecorvo, who came to the idea of neutrino oscillations at a time when the common opinion favored massless neutrinos and no neutrino oscillations and who pursued this idea over decades.

6. Conclusion

We discussed here the pioneering Pontecorvo neutrino oscillations papers and the development of the idea of neutrino masses, mixing, and oscillations in Dubna at the end of the seventies.

First indication in favor of neutrino oscillations was obtained in the Davis Jr. solar neutrino experiment in 1970 [4, 5]. Additional indication in favor of oscillations was found in another solar neutrino experiment (Kamiokande) [50] and in the atmospheric neutrino experiment [51].

Evidence of the disappearance of the solar ν_e's was obtained in GALLEX [6, 7] and SAGE [8] solar neutrino experiments in which neutrinos from all solar thermonuclear reactions, including the main p-p reaction, were detected.

The first model independent evidence of neutrino oscillations was obtained in 1998 in the Super-Kamiokande atmospheric neutrino experiment [41, 42]. A few years later, a proof of disappearance of solar ν_e's and reactor $\bar{\nu}_e$'s, driven by neutrino oscillations, was obtained in the solar SNO experiment [43, 44] and in the reactor KamLAND experiment [45]. The Super-Kamiokande evidence of neutrino oscillations was confirmed in the K2K [45], MINOS [47], and T2K [48, 49] accelerator neutrino experiments. Additional evidence of neutrino oscillations was found in recent Daya Bay [52] and RENO [53] reactor neutrino experiments in which the small mixing angle θ_{13} was measured.

The discovery of neutrino oscillations was a great triumph for Bruno Pontecorvo who came to the idea of neutrino oscillations at a time when the common opinion favored massless neutrinos.

From my point of view, the history of neutrino oscillations is an illustration of the importance of analogy in physics. It is also an illustration of the importance of new courageous ideas which are not always in agreement with general opinion.

Small neutrino masses cannot be naturally explained in the framework of the standard model. Their explanation requires new physics beyond the SM. Many models were proposed. The most plausible and viable mechanism for the generation of neutrino masses is the seesaw mechanism [54–58], which connects the smallness of neutrino masses with a violation of the lepton number at a large scale.

In the most general form, the seesaw mechanism was formulated in the paper [59] in the framework of the effective Lagrangian approach. It was shown in [59] that the only dimension 5 effective Lagrangian is a lepton number violating $SU(2) \times U(1)$ invariant product of two lepton doublets and two Higgs doublets. After spontaneous violation of the electroweak symmetry, this effective Lagrangian generates Majorana mass term of the type considered first by Gribov and Pontecorvo [34]. In this approach, the scale of neutrino masses is determined by the parameter v^2/Λ, where $v = ((\sqrt{2}G_F)^{-1/2})^{-1} \simeq 246\,\text{GeV}$ is the parameter which characterizes electroweak breaking and Λ characterizes the scale of a new physics. From experimental data, it follows that $\Lambda \simeq 10^{15}$ GeV.

From the investigation of solar neutrinos in numerous solar neutrino experiments (Homestake [4, 5], GALLEX-GNO [6, 7], SAGE [8], Super-Kamiokande [60, 61], SNO [43, 44], and BOREXINO [62]), it was discovered that disappearance of the solar ν_e's is not only due to neutrino masses and mixing but also due to coherent scattering of neutrinos in matter (MSW effect [63, 64].)

In the LEP experiments, it was found that three flavor neutrinos ν_e, ν_μ, and ν_τ exist in nature. In the minimal scheme of the neutrino, mixing the number of neutrinos with definite masses ν_i is also equal to three. In this case, the unitary mixing matrix U is 3×3 matrix. Such matrix is characterized by three mixing angles θ_{12}, θ_{23}, and θ_{13} and CP phase δ. From analysis of data of neutrino oscillation experiments, it was found that in the very first approximation,

$$\sin\theta_{12} \simeq \frac{1}{\sqrt{3}}, \qquad \sin\theta_{23} \simeq \frac{1}{\sqrt{2}}, \qquad \sin\theta_{13} \simeq 0, \tag{38}$$

and the unitary matrix U has a tribimaximal form. This finding led to many papers in which possibilities of broken discrete symmetries in the lepton sector were thoroughly investigated (see, for example, the review [65]).

The pioneering papers of Pontecorvo on neutrino masses, mixing, and oscillations created a new field of research.

The investigation of neutrino oscillations, driven by small neutrino masses and neutrino mixing, raised new questions which need further investigation. The major problems are the following.

(1) Are neutrinos with definite masses ν_i Majorana or Dirac particles?

This problem can be solved via observation of the lepton number violating neutrinoless double β-decay of some even-even nuclei.

(2) Is the neutrino mass spectrum normal or inverted?

Existing neutrino oscillation data do not allow to distinguish the following two possibilities for the neutrino mass spectrum:

(i) normal spectrum $m_1 < m_2 < m_3$, $\Delta m^2_{12} \ll \Delta m^2_{23}$,

(ii) inverted spectrum (IS) $m_3 < m_1 < m_2$, $\Delta m^2_{12} \ll |\Delta m^2_{13}|$.

Future accelerator and reactor neutrino experiments will solve this problem.

(3) What is the value of the CP phase δ, the last unknown parameter of the neutrino mixing matrix?

This very challenging problem apparently will be also solved in future neutrino oscillation experiments.

(4) Are there transitions of flavor neutrinos ν_e, ν_μ, and ν_τ into sterile states?

This problem will be solved in short-baseline reactor and accelerator neutrino oscillation experiments. (some indications in favor of existence of the sterile neutrinos exist at present (see, e.g., [66])).

Independently from Pontecorvo in 1962, Maki et al. [67] came to the idea of neutrino masses and mixing. Their arguments were based on the Nagoya model in which neutrinos were considered as constituents of barions. In [67], the possibility of the transition ("virtual transmutation") $\nu_\mu \rightarrow \nu_e$ was discussed. To acknowledge the pioneer ideas of Pontecorvo and Maki and Nakagawa and Sakata, the neutrino mixing matrix is usually called the PMNS matrix.

Pontecorvo was one of the first who understood the importance of neutrinos for elementary particle physics and astrophysics. He felt and understood neutrinos probably better than anybody else in the world. Starting from his Canadian time, he thought about the neutrino for his entire life. He was never confined by narrow theoretical frameworks. He was completely open minded, without any prejudices, very courageous, and with very good intuition and scientific taste.

Pontecorvo was very bright, wise, exceptionally interesting, and a very friendly personality. People liked him, and he had many friends in Italy, Russia, France, Canada and many other countries. He participated in many conferences, seminars, and discussions. His clear laconic questions and remarks were very important for the clarification of many problems.

The name of Pontecorvo, the founder and father of modern neutrino physics, will be forever connected with neutrino. He will remain with us in our memory and our hearts as a great outstanding physicist, as a man of great impact and humanity.

Acknowledgment

This work is supported in part by RFBR Grant no. 13-02-01442.

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