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ELECTRON BEAM'S CLOSED ORBIT IN THE CRAB CROSSING SCHEME OF FUTURE ELECTRON-ION COLLIDERS*

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Abstract

In crab-crossing collision geometry, the closed orbit of the electron beam will be altered by the beam-beam interaction and the tilted head and tail of the ion beam. We will present the linear model to determine the closed orbit and compare it with the simulation. Also, strong-strong simulation of the beam-beam effect is calculated to confirm that the change of the closed orbit due to crab crossing does not directly contribute to the luminosity degradation that observed in earlier studies.

INTRODUCTION

In the present design of Electron-Ion Colliders (EIC), the crab crossing scheme is adopted to achieve higher luminosity. The crab cavity is used to provide a linear kick to both colliding beams to compensate the geometric luminosity loss due to the crossing angle. However, the RF cavity always provides a sinusoidal kick to the beam. The beam only receives an approximately linear kick when its bunch length is much smaller than the RF wavelength. In EIC, the longer ion bunch will likely suffer the nonlinear kick. In the Lorentz boosted frame, the transverse offset as a function of the location away from its reference particle z:

$$\Delta x\left(z\right) \sim \frac{\theta_c}{k_c}\sin(k_c z) - \theta_c z \tag{1}$$

When the ion beam's bunch length is comparable to the crab cavity wavelength, the ion beam is tiled as shown in Figure 1 of [1] and observe luminosity degradation which may cause by the synchro-betatron resonance due to beam-beam interaction. Meanwhile, the closed orbit of the electron beam will be modified slightly by the tilted ion beam. In this paper, we will calculate the closed orbit of the electron beam in the crab crossing scheme of EIC and compare it with the strong-strong simulation. Also, we can control the closed orbit by introducing additional kick to the beam and study the pure effect of the closed orbit on the luminosity degradation.

ELECTRON'S CLOSED ORBIT

Let's first consider the reference particle in the electron beam, the motion with beam-beam interaction can be written as:

$$x_e''\left(s\right) + k_e x_e\left(s\right) = f_{bb}\left(x_e\left(s\right), \bar{x}_i\left(z_i, s\right), \rho_i\right) \tag{2}$$

where $\bar{x}_i$ represents the centroid of ion and beam. z_i is the coordinate of ions measured from the bunch center, which collides with the reference particle at $s = z_i/2$, ρ_i is ion beam distribution. At the presence of the imperfect crab kick, the centroid deviation of the ion beam $\bar{x}_{i,cc}$ is $\Delta x(z)$, as illustrated in Equation 1. Since the beam-beam parameter of the electron beam is small (≤ 0.1), the relation $\bar{x}_{i,cc} \gg x_e$ is expected.

We can estimate the closed orbit of the reference particle of the electron beam using the kick-drift model by slicing the ion beam into longitudinal slices. For each slice of the ion beam, we calculate the dipole kick and the focusing effect of the beam-beam force using the Bassetti and Erskine formula:

$$\begin{aligned}\Delta x' = -\Re\Bigg\{\frac{i2\sqrt{\pi}N_s r_e}{\gamma\sqrt{2\left(\sigma_x^2-\sigma_y^2\right)}}\Bigg[w\left(\frac{x+iy}{\sqrt{2\left(\sigma_x^2-\sigma_y^2\right)}}\right)\\ -\exp\left(-\frac{x^2}{2\sigma_x^2}-\frac{y^2}{2\sigma_y^2}\right)w\left(\frac{\frac{x\sigma_y}{\sigma_x}+i\frac{y\sigma_x}{\sigma_y}}{\sqrt{2\left(\sigma_x^2-\sigma_y^2\right)}}\right)\Bigg]\Bigg\}\end{aligned} \tag{3}$$

where $w\left(z\right) = \exp\left(-z^2\right)\operatorname{erfc}\left(-iz\right)$ is the complex error function, σ_x and σ_y are the transverse beam sizes and N_s is the number of ions within this slice. As the ion slice has offset $\bar{x}_{i,cc} \gg x_e$, we can use Taylor expansion to expand the beam-beam kick at the vicinity of the reference axis ($x = 0$, $y = 0$):

$$\Delta x'(x_e - \bar{x}_{i,cc}, 0) = \Delta x'\left(-\bar{x}_{i,cc}, 0\right) + \frac{d\Delta x'}{dx}\left(-\bar{x}_{i,cc}, 0\right)x_e \tag{4}$$

We adopt the 3×3 matrix method to calculate the closed orbit by multiplying the linearized beam-beam interaction matrix of each ion slice

$$M_{bb} = M_{s,k}\cdot M_{s,k-1}\cdots M_{s,2}\cdot M_{s,1} \tag{5}$$

here k is the total slice number. and the interaction matrix with i^{th} slice reads:

$$M_{s,i} = M_{-d,i}\cdot M_{bb,i}\cdot M_{d,i} \tag{6}$$

where $M_{d,i}$ is the drift matrix from IP to the i^{th} slice; $M_{-d,i}$ is the drift matrix from the i^{th} slice back to IP and $M_{bb,i}$ is

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the beam-beam kick matrix which reads:

$$M_{bb,i} = \begin{pmatrix} 1 & 0 & 0 \\ m_{21} & 1 & m_{23} \\ 0 & 0 & 1 \end{pmatrix} \tag{7}$$

$$m_{21} = \frac{d\Delta x'}{dx}\left(-\bar{x}_{i,cc}, 0\right)$$

$$m_{23} = \Delta x'\left(-\bar{x}_{i,cc}, 0\right)$$

both m_{21} and m_{23} is easy to evaluate at $y = 0$, since the complex error function becomes:

$$w(z = x + 0i) = e^{-x^2} + \frac{2i}{\sqrt{\pi}} F(x) \tag{8}$$

where $F(x) = e^{-x^2} \int_0^x e^{y^2} dy$ is the Dawson's integral. Since the derivative of $F(x)$ is $F'(x) = 1 - 2xF(x)$, we have:

$$\frac{d}{dx} w(x + 0i) = \frac{2i}{\sqrt{\pi}} - 2xw(x) \tag{9}$$

The total matrix starting at IP is then calculated by multiplying the one turn matrix with the beam-beam interaction matrix:

$$M_t = M_{oneturn} M_{bb} \tag{10}$$

$$\equiv \begin{pmatrix} M_\beta & \begin{pmatrix} m_{13} \\ m_{23} \end{pmatrix} \\ 0 & 1 \end{pmatrix} \tag{11}$$

Then the closed orbit of the electron reference particle at IP can be calculated as:

$$\begin{pmatrix} x_{co} \\ x'_{co} \end{pmatrix} = \left(I - M_\beta\right)^{-1} \begin{pmatrix} m_{13} \\ m_{23} \end{pmatrix} \tag{12}$$

In the following studies, we use the eRHIC design parameter as an example in the strong-strong beam-beam simulation code BeamBeam3D [2]. The designed repetition frequency is about 112.6 MHz. The first four harmonics of the frequency are used as the crab cavity frequency for this study. Figure 1 illustrates the horizontal trajectory of the electron's reference particle inside the ion bunch with different crab cavity frequency. The top figure shows the trajectory of a fresh electron beam, which as zero offsets before the beam-beam interaction; while the bottom figure shows the closed orbit of the electron beam inside the ion beam. The radiation damping will damp the trajectory to the closed orbit within the duration of several damping time.

Due to the nonlinearity of the beam-beam interaction, the centroid of the electron beam will behave differently than that of the reference particle of the electron beam, which is a single particle experiencing the linearized beam-beam field. We anticipate that there exists a factor that can approximate the behavior of the electron beam centroid by scaling from the reference particle. The factor will be the function of the frequency of the crab cavity. Figure 2 shows the comparison of closed orbit and its slope at IP of the electron's reference particle using the linear model and that from the beam centroid calculated from the nonlinear beam-beam effect using

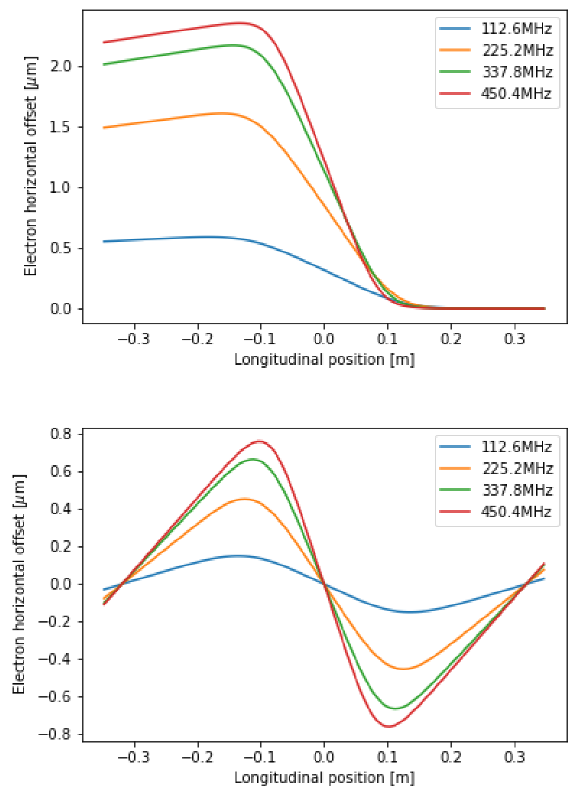

Figure 1: Trajectories of electron reference particle with different ion crab cavity frequency. Top: fresh electron with on-axis orbit before beam-beam interaction. Bottom: electron reference particle's closed orbit.

Table 1: Scaling Factor Needed for the Linear Model

Frequency (MHz)	Factor
112.6	0.61
225.2	0.68
337.8	0.74
450.4	0.77

strong-strong simulation. The closed orbits and it's slope calculated from the linear model have to be scaled by a factor κ, which is a function of the frequency, to match the results from the strong-strong simulation. The factors required are shown in the table 1.

EFFECT ON THE LUMINOSITY DEGRADATION

We observed that luminosity degradation is a function of the crab cavity frequency. The closed orbit of the electron beam is the zeroth order effect when colliding with a long tilted ion bunch. The betatron motion of both beams due to the tilted tail is the first order effect. It is important to prove that the zeroth order effect alone will not cause the

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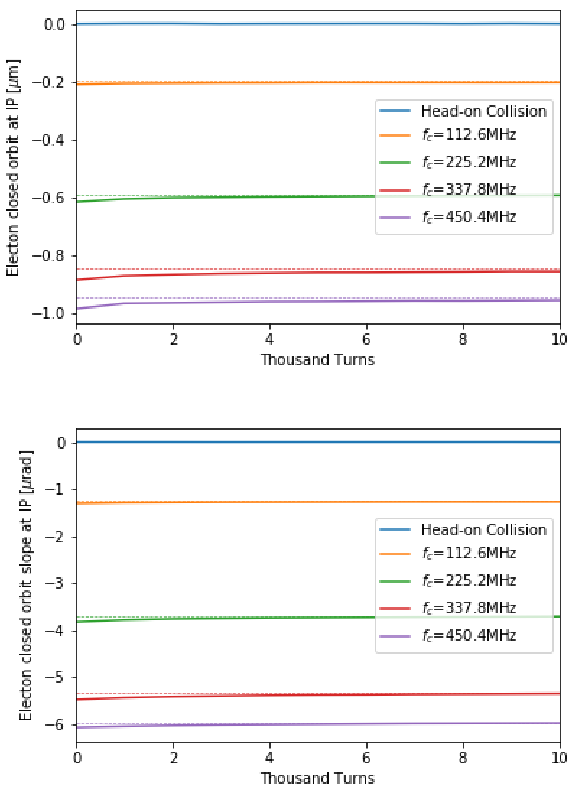

Figure 2: Comparison of the closed orbit (top figure)/ orbit slope (bottom figure), calculated from the linear model and from the strong-strong simulation code Beam-Beam 3D. The result from linear model is scaled by factor $\kappa(f)$ and represented by dotted lines .

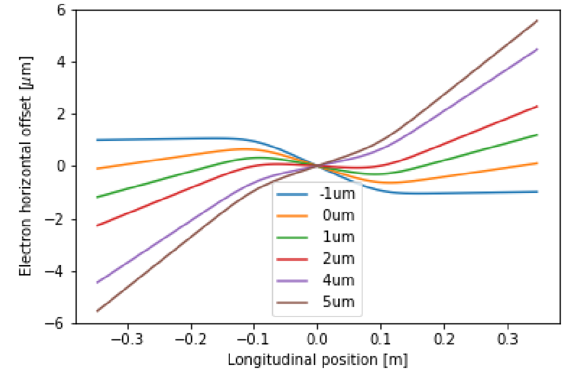

Figure 3: Electron beam closed orbit with different external control.

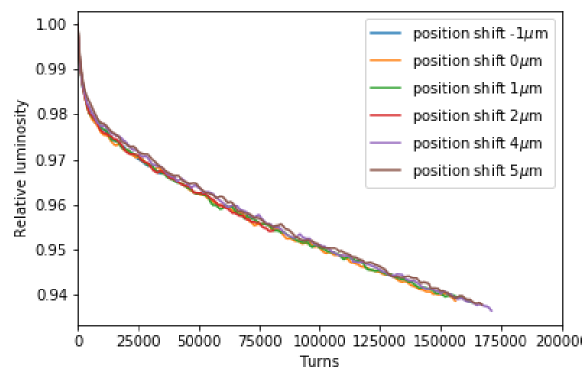

Figure 4: Luminosity degradation of various closed orbit via external control.

luminosity degradation. We may use the analysis in the previous section to add additional control of the electron closed orbit in the beam-beam simulation. The additional control can either be a position shift or an angle change of the electron beam at IP, before and after the beam-beam collision. To keep the anti-symmetric property of the closed orbit, the position shift should be symmetric before and after IP; while the angle change should be anti-symmetric.

In the following examples with crab cavity 337.8 MHz, we add a symmetric position shift on the electron beam with the amplitude from -1 µm to 5 µm, which leads to a significant change of the electron beam closed orbit at IP, as shown in Fig. 3. The 0 µm corresponds to the green curve of the bottom figure in Fig. 1. When the position shift is 4 µm, the closed orbit of the electron beam has the opposite slope at IP than that of the 0 µm case. The strong-strong simulation (Fig. 4) shows that no significant difference in the luminosity degradation can be observed in this range.

The study reveals that the luminosity degradation does not depend on the zeroth order orbit change of the electron beam. Therefore the luminosity degradation can only be caused by the electron 's betatron motion, which affects the ion beam by the beam-beam interaction. Next step, we have to explore the beam dynamics from the coupling of the betatron motion of both beams and the synchrotron motion of the ion beam to explain the luminosity degradation and find mitigation methods.

SUMMARY

This study calculated the closed orbit of the electron beam in the future EIC, due to the tilted ion beam generated by the finite ion bunch length and the wavelength of the crab cavity. For the design frequency of the crab cavity, the electron beam has closed orbit less than1 micron, which is 1% of the rms beam size at the interaction point.

The study also revealed that, although both the closed orbit and luminosity degradation are created by the tilted ion beam, the closed orbit does not directly affect the degradation of the luminosity. We have demonstrated that 5 times larger closed orbit does not change the luminosity behavior. Further studies have to be done to find the countermeasure of luminosity degradation.

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