Eur. Phys. J. C (2021) 81:935
https://doi.org/10.1140/epjc/s10052-021-09736-y

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Mass spectroscopy and strong decays of excited open charm D_J mesons using relativistic Dirac formalism

Bhoomika Pandya[1,a], **Manan Shah**[2,b], **P. C. Vinodkumar**[1,c]

[1] Department of Physics, Sardar Patel University, Vallabh Vidyanagar, Gujarat 388120, India
[2] P. D. Patel Institute of Applied Sciences, Charusat, Changa, India

Received: 28 June 2021 / Accepted: 11 October 2021

Abstract The mass spectra for the heavy-light $(c\bar{q})$; $q = u$ or d charmed mesons are computed based on a relativistic framework. The low-lying $1P$ states are found to be in an excellent agreement with the PDG reported values. Using the computed mass spectra and following effective Lagrangian approach based on heavy quark and chiral symmetry, the OZI allowed two body strong decays are computed. The computed decay rates, ratios and branching fractions allow us to identify the proper spin-parity assignments of the newly observed charm states. Accordingly, we could identify $D_J(2560)$ as 2^1S_0, $D_J^*(2680)$ as 2^3S_1, $D_J(2740)$ as 1^3D_2, $D_J^*(2760)$ as 1^3D_3, $D_J^*(3000)$ as 2^3P_0, $D_J(3000)$ as 2^1P_1 and $D_2^*(3000)$ as 1^3F_2 open charm states. The effective coupling constants, g_T, $\tilde{g_H}$, g_Y, $\tilde{g_S}$ and g_Z extracted from the present study are found to be in accordance with the reported values. These coupling constants would be useful in further investigations. We found $D^{*+}\pi^-$ as a favorable channel for the experimental search of the missing 1^1F_3 state.

1 Introduction

During the past decade, numerous excited charmed states have been observed by the experimental groups $LHCb$ and $BABAR$ [1–3]. These achievements provoked great interest in experimental as well as theoretical studies of charmed mesons. In recent past $LHCb$ has employed the Dalitz Plot (DP) technique to analyse the contributing amplitudes in decay channel $B^- \rightarrow D^+\pi^-\pi-$, where charmed states were reconstructed through $D^+ \rightarrow K^-\pi^+\pi^+$ decay process. This analysis was based on data collected by the $LHCb$ detector during 2011 and 2012 when the pp collision center of mass energy was 7 TeV and 8 TeV, respectively. Their study summarises the resonant contribution coming from $D_2^*(2460)^0$, $D_1^*(2680)^0$, $D_3^*(2760)^0$ and $D_2^*(3000)^0$ states [1]. Their measured masses and widths are listed in Table 1.

In 2013, $LHCb$ has studied the $D^+\pi^-$, $D^0\pi^+$ and $D^{*+}\pi^-$ channel invariant mass spectra and enriched the spectrum of charmed mesons. They have reported the resonances $D_1^0(2420)$ in the $D^{*+}\pi^-$ final state and the resonance $D_2^*(2460)$ in the $D^+\pi^-$, $D^0\pi^+$ and $D^{*+}\pi^-$ final states [2]. Moreover, two natural parity resonances: $D_J^{0*}(2650)$ and $D_J^{0*}(2760)$ and two unnatural parity resonances: $D_J^0(2580)$ and $D_J^0(2740)$ were also observed in $D^{*+}\pi^-$ mass spectra. The $LHCb$ collaboration has tentatively assigned $D_J^{0*}(2650)$ as $2S(1^-)$, $D_J^0(2580)$ as $2S(0^-)$, $D_J^{0*}(2760)$ as $1D(1^-)$ and $D_J(2740)$ as $1D(2^-)$ states. The $D_J^0(3000)$ resonance was observed in $D^{*+}\pi^-$ final states with unnatural parity. The states $D_J^{0*}(3000)$ and $D_J^{+*}(3000)$ were recorded in the $D^+\pi^-$ and $D^0\pi^+$ spectra. The mass and width of these resonances are collected in Table 2.

The $BABAR$ collaboration has observed four resonances $D^0(2550)$, $D^{*0}(2600)$, $D^0(2750)$, $D^*(2760)$ and isospin partners $D^{0*}(2600)$, $D^{+*}(2760)$ in the inclusive production of the $D^+\pi^-$, $D^0\pi^+$and $D^{*+}\pi^-$ systems at SLAC PEP-II asymmmetric-energy collider, in 2010 [3]. After analysing the helicity distribution, the $BABAR$ collaboration suggested that $D^0(2550)$, $D^{0*}(2600)$ might be the radial excited states $2S$ and $D^0(2750)$, $D^*(2760)$ might belongs to D - wave states. Available details are collected in Table 3.

Although several D mesonic states are reported experimentally, their experimental spin-parity assignments are still an open problem in charm sector. However, analysis of their decay modes provides us a way out to extract the information regarding their internal structure and dynamics. In many cases, theoretical analysis of the mass spectra and decay properties are simultaneously required to assign their spin-parity (J^P) quantum numbers. Such a theoretical model also allows suitable testing ground for the newly observed states. The popular non-relativistic Schrödinger treatment is quite con-

[a] e-mail: bhumispandya@gmail.com (corresponding author)
[b] e-mail: mnshah09@gmail.com
[c] e-mail: p.c.vinodkumar@gmail.com

Published online: 22 October 2021

Table 1 Charmed resonances observed by $LHCb$ Collaboration in the DP analysis of $B^- \to D^+\pi^-\pi^-$ decays [1]

Resonance	Mass (in MeV)	Width (in MeV)	Spin-parity
$D_2^*(2460)^0$	$2463.7 \pm 0.4 \pm 0.4$	$47.0 \pm 0.8 \pm 0.9$	2^+
$D_1^*(2680)^0$	$2681.1 \pm 5.6 \pm 4.9$	$186.7 \pm 8.5 \pm 8.6$	1^-
$D_3^*(2760)^0$	$2775.5 \pm 4.5 \pm 4.5$	$95.3 \pm 9.6 \pm 7.9$	3^-
$D_2^*(3000)^0$	$3214 \pm 29 \pm 33$	$186 \pm 38 \pm 34$	2^+

Table 2 Charmed resonances observed from the LHCb analysis of inclusive $D^{(*)}\pi$ production [2]

Resonance	Mass (in MeV)	Width (in MeV)	Spin-parity
$D_J^0(2580)$	$2579.5 \pm 3.4 \pm 5.5$	$177.5 \pm 17.7 \pm 46.0$	UN
$D_J^{*0}(2650)$	$2649.2 \pm 3.5 \pm 3.5$	$140.2 \pm 17.1 \pm 18.6$	N
$D_J^0(2740)$	$2737.0 \pm 3.5 \pm 11.2$	$73.2 \pm 13.4 \pm 25.0$	UN
$D_J^{*0}(2760)$	$2761.1 \pm 5.1 \pm 6.5$	$74.4 \pm 4.3 \pm 37.0$	N
$D_J^{*0}(2760)$	$2760.1 \pm 1.1 \pm 3.7$	$74.4 \pm 3.4 \pm 19.1$	N
$D_J^{*+}(2760)$	$2771.7 \pm 1.7 \pm 3.8$	$66.7 \pm 6.6 \pm 10.5$	N
$D_J^0(3000)$	2971.8 ± 8.7	188.1 ± 44.8	UN
$D_J^{*0}(3000)$	3008.1 ± 4.0	110.5 ± 11.5	N

UN unnatural, *N* natural

Table 3 Charmed resonances observed by $BABAR$ Collaboration [3]

Resonance	Mass (in MeV)	Width (in MeV)	Spin-parity
$D^0(2550)$	$2539.4 \pm 4.5 \pm 6.8$	$130 \pm 12 \pm 13$	0^-
$D^{*0}(2600)$	$2608.7 \pm 2.4 \pm 2.5$	$93 \pm 6 \pm 13$	N
$D^0(2750)$	$2752.4 \pm 1.7 \pm 2.7$	$71 \pm 6 \pm 11$	
$D^{*0}(2760)$	$2763.3 \pm 2.3 \pm 2.3$	$60.9 \pm 5.1 \pm 3.6$	N

sistent for the quantitative study of quarkonium but not appropriate for mesonic systems consisting a heavy-light flavour quarks. For the hadrons consisting a heavy quark, QCD exhibits additional symmetries within the limit that heavy quark mass m_Q becomes infinite compared with the typical QCD scale [4]. This symmetry can be realised systematically within relativistic Dirac equation employing the equally mixed scalar plus vector potential. This framework allows to make satisfactory predictions on excited charmed mesons. Hence, for the present study, we consider relativistic Dirac equation with equally mixed four-vector plus scalar power-law confinement potential for the single particle bound state energy of the quark and anti-quark. Using these single particle energies along with their jj coupling, the massess of the charmed mesons are computed. We fix the potential parameters for the known ground state and predict the masses of the radial and orbital excited states.

Several models are available in literature dealing with decay properties of heavy-light mesons apart from their mass spectra [5–10]. The well known $^3P_0$ model or quark pair creation model is very effective in strong decay study of mesons [11–13]. This model was initially proposed by L. Micu [14] and later developed by Y. Le and collaborators [15–17]. The relativized quark model spectroscopy incorporating $^3P_0$ model for strong decays [5] supports the $D_J^0(2550)$ and $D_J^{0*}(2600)$ as the radial excitation states. In their study they also identified $D_1^{0*}(2760)$ and $D_J^{0*}(2760)$ as the 1^3D_1 and 1^3D_3 states. But quoted that identification of $D_J^0(2750)$ state requires further measurements. They concluded that the favourable assignment for states $D_J^{0*}(3000)$ and $D_J^0(3000)$ as $D_4^*(1^3F_4)$ and $D(3^1S_0)$, respectively. The $^3P_0$ is a simplified version of the complicated theory. So, predictability of this model is not accurate. Also,the oscillator parameter affects the shape of wave-functions significantly in $^3P_0$ model. More discussion regarding the analysis of uncertainties in $^3P_0$ predictions can be available in Refs. [18–20]. Another approach to predict the strong decays of heavy-light mesons is based on heavy quark effective theory where P. Colangelo et al. has proposed the framework to classify the hadrons in doublets involving the channels with a light final pseudoscalar mesons [21–24]. The work exploring the heavy quark effective theory for charmed mesons is covered in Refs. [25,26] by Wang. Very, recently Colangelo et al. [27] incorporated other channels with a light final vector mesons into their previous study. We believe that in the case of relativistic Dirac model mass predictions as an input, Colangelo et al.'s approach is more consistent as compared to $^3P_0$ model. In spite of the fact that Heavy quark effective theory contains many unknown phenomenological parameters, still HQET combined with chiral perturbation theory is a promising approach in the predictions of the strong decays of heavy-light hadrons [28]. This heavy quark symmetry impose constraints on the range of coupling constant between 0 and 1 for ground state charm mesons [29]. Thus, the present analysis for strong decays is carried out employing the Heavy quark effective theory at the leading order approximation where the mass and the spin degeneracy of the heavy hadrons are treated as approximate internal symmetry of the Lagrangian. The present study is organised as follows. In Sect. 2 we present our framework of relativistic Dirac model for heavy-light system and highlight the strong decays of heavy-light mesons within the framework proposed by Colangelo. Section 2 devoted to the results and analysis of the mass spectra and the strong decay widths. Finally, summary of the present study is included in Sect. 2.

2 Theoretical framework

2.1 The realtivistic Dirac model for heavy-light mesonic systems

The heavy-light mesons are composed of one heavy quark Q and other light quark $\bar{q}$, so the properties of heavy-light mesons are significantly influenced by relativistic effects. such systems can be systematically emphasised within relativistic Dirac framework. The form of the average flavour independent central potential for quarks and anti-quarks inside a meson is given as [30–32]

$$V(r) = \frac{1}{2}(1+\gamma_0)(\lambda r + V_0) \tag{1}$$

Here, r represents radial distance from the meson centre of mass. Both λ (strength of the potential) and V_0 (depth of the potential) are the phenomenological parameters which are fixed for the ground state.

We assume that Dirac equation is obeyed which can be derived from the Lagrangian density [30]

$$\mathcal{L}(x) = \bar{\psi}\left[\frac{i}{2}\gamma^{\mu}\overrightarrow{\partial_{\mu}} - V(r) - m\right]\psi(x) \tag{2}$$

The independent quark wave function $\psi(\vec{r})$ satisfies the Dirac equation [30,32]

$$[\gamma^0 E - \vec{\gamma}.\vec{P} - m - V(r)]\psi(\vec{r}) = 0. \tag{3}$$

where E is the individual quark binding energy.

The solution to the independent-quark wave function (normalised) can be written as [31,32,34,35]

$$\psi_{nlj}(r) = \begin{pmatrix} \psi_{nlj}^{(+)} \\ \psi_{nlj}^{(-)} \end{pmatrix} \tag{4}$$

where

$$\psi_{nlj}^{(+)}(\vec{r}) = N_{nlj}\begin{pmatrix} ig(r)/r \\ (\sigma.\hat{r})f(r)/r \end{pmatrix}\mathcal{Y}_{ljm}(\hat{r}) \tag{5}$$

$$\psi_{nlj}^{(-)}(\vec{r}) = N_{nlj}\begin{pmatrix} i(\sigma.\hat{r})f(r)/r \\ g(r)/r \end{pmatrix}(-1)^{j+m_j-l}\mathcal{Y}_{ljm}(\hat{r}) \tag{6}$$

and N_{nlj} is the overall normalization constant [31,32,34,36].

The normalized spin angular part is defined by [34]

$$\mathcal{Y}_{ljm}(\hat{r}) = \sum_{m_l,m_s}\langle l, m_l, \frac{1}{2}, m_s|j, m_j\rangle Y_l^{m_l}\chi_{\frac{1}{2}}^{m_s} \tag{7}$$

The two spinors $\chi_{\frac{1}{2}m_s}$ are the eigenfunctions of the spin operators and explicitly written as [34]

$$\chi_{\frac{1}{2}\frac{1}{2}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \ \chi_{\frac{1}{2}-\frac{1}{2}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{8}$$

The reduced radial parts $g(r)$ and $f(r)$ can be found to satisfy the following equations [6,31,32,35]

$$\frac{d^2g(r)}{dr^2} + \left[(E_D+m_q)[E_D-m_q-V(r)] - \frac{\kappa(\kappa+1)}{r^2}\right]g(r) = 0 \tag{9}$$

and

$$\frac{d^2f(r)}{dr^2} + \left[(E_D+m_q)[E_D-m_q-V(r)] - \frac{\kappa(\kappa-1)}{r^2}\right]f(r) = 0 \tag{10}$$

with the definition of quantum number κ as [34]

$$\kappa = \begin{cases} -(\ell+1) = -\left(j+\frac{1}{2}\right) & for\ \ j = \ell + \frac{1}{2} \\ \ell \quad\ \ = +\left(j+\frac{1}{2}\right) & for\ \ j = \ell - \frac{1}{2} \end{cases} \tag{11}$$

Taking the form of $V(r)$ as given in Eq. (1) and introducing a dimensionless variable $\rho = \frac{r}{r_0}$ with an arbitrary scale factor [31,32,36,37]

$$r_0 = \left[(m_q + E_D)\frac{\lambda}{2}\right]^{-\frac{1}{3}}, \tag{12}$$

Eqs. (9) and (10) reduces to the Schrödinger form [31,32,36–38]

$$\frac{d^2g(\rho)}{d\rho^2} + \left[\epsilon - \rho^{1.0} - \frac{\kappa(\kappa+1)}{\rho^2}\right]g(\rho) = 0 \tag{13}$$

and

$$\frac{d^2f(\rho)}{d\rho^2} + \left[\epsilon - \rho^{1.0} - \frac{\kappa(\kappa-1)}{\rho^2}\right]f(\rho) = 0 \tag{14}$$

Table 4 The input parameters for the D mesons

Quark mass (in GeV)	$m_{u,d}$ = 0.003 and m_c = 1.27
Potential strength (λ)	$0.0352(2n+l+1)^{0.5}$ GeV$^{\nu+1}$
V_0	−0.0001 GeV
σ ($j-j$ coupling strength)	0.0946 GeV^3

and ε represents dimensionless energy eigenvalue given by [31,32,36–38]

$$\epsilon = (E_D - m_q - V_0)(m_q + E_D)^{\frac{1}{3}} \left(\frac{\lambda}{2}\right)^{\frac{-2}{3}} \tag{15}$$

Here, E_D and m_q represent the Dirac confinement energy of the quark having its rest mass m_q. The numerical values of extracted Dirac confinement energy (E_D) are presented in Table 15 of Appendix A.

For proper choices of κ Eqs. (13) and (14) can be be solved numerically.

The solutions $g(\rho)$ and $f(\rho)$ are normalized to get

$$\int_0^\infty (f_q^2(\rho) + g_q^2(\rho))\, d\rho = 1. \tag{16}$$

The wavefunctions for heavy-light charmed meson now can be constructed as the symmetric combinations of the positive energy solution ψ_Q and the negative energy solution $\psi_{\bar{q}}$ of Eq. (5) and (6) and the corresponding mass of the $Q\bar{q}$ system can be written as

$$M_{Q\bar{q}} = E_D^Q + E_D^{\bar{q}} \tag{17}$$

where $E_D^{Q/\bar{q}}$ are obtained from (15) which include the centrifugal repulsion of the centre of mass also.

The mass of the state $M_{^{2S+1}L_J}$ is obtained by adding the contributions from spin–spin, spin–orbit and tensor interactions of the confined one gluon exchange potential (COGEP) between quark and anti-quark [39,40] to $M_{Q\bar{q}}$. Thus, we write

$$M_{^{2S+1}L_J} = M_{Q\bar{q}}\ (n_1 l_1 j_1, n_2 l_2 j_2) + \langle V_{Q\bar{q}}^{j_1 j_2}\rangle + \langle V_{Q\bar{q}}^{LS}\rangle + \langle V_{Q\bar{q}}^{T}\rangle \tag{18}$$

The spin–spin coupling part is defined from COGEP as [39,40]

$$\langle V_{Q\bar{q}}^{j_1 j_2}(r)\rangle = \frac{\sigma\ \langle j_1 j_2 J M | \hat{j}_1.\hat{j}_2 | j_1 j_2 J M\rangle}{(E_Q + m_Q)(E_{\bar{q}} + m_{\bar{q}})} \tag{19}$$

where σ represents the jj coupling constant. The angular brackets appearing in the term $\langle j_1 j_2 J M | \hat{j}_1.\hat{j}_2 | j_1 j_2 J M\rangle$ contains square of Clebsch–Gordan coefficients.

The tensor and spin–orbit parts of confined one-gluon exchange potential (COGEP) [39,40] have the form

$$V_{Q\bar{q}}^{T}(r) = -\frac{\alpha_s}{4} \frac{N_Q^2 N_{\bar{q}}^2}{(E_Q + m_Q)(E_{\bar{q}} + m_{\bar{q}})} \otimes \lambda_Q.\lambda_{\bar{q}} \left(\left(\frac{D_1''(r)}{3} - \frac{D_1'(r)}{3\,r} \right) S_{Q\bar{q}} \right) \tag{20}$$

where N_D, $N\bar{q}$ are the normalization constants of quark–antiquark wavefunctions, $S_{Q\bar{q}} = \left[3(\sigma_Q.\hat{r})(\sigma_{\bar{q}}.\hat{r}) - \sigma_Q.\sigma_{\bar{q}}\right]$ and $\hat{r} = \hat{r}_Q - \hat{r}_{\bar{q}}$ is the unit vector in the direction of $\vec{r}$ and

$$V_{Q\bar{q}}^{LS}(r) = \frac{\alpha_s}{4} \frac{N_Q^2 N_{\bar{q}}^2}{(E_Q + m_Q)(E_{\bar{q}} + m_{\bar{q}})} \frac{\lambda_Q.\lambda_{\bar{q}}}{2\,r} \otimes \left[\left[\vec{r} \times (\hat{p_Q} - \hat{p}_{\bar{q}}).(\sigma_Q + \sigma_{\bar{q}})\right] \left(D_0'(r) + 2D_1'(r)\right) + \left[\vec{r} \times (\hat{p_Q} + \hat{p}_{\bar{q}}).(\sigma_Q - \sigma_{\bar{q}})\right] \left(D_0'(r) - D_1'(r)\right)\right] \tag{21}$$

where α_s is the running strong coupling constant expressed as

$$\alpha_s = \frac{4\pi}{(11 - \frac{2}{3}\, n_f) \log\left(\frac{M_Q^2}{\Lambda_{QCD}^2}\right)} \tag{22}$$

n_f is number of flavours and for charmed mesons we take it as 3. The term $\langle \lambda_Q.\lambda_{\bar{q}}\rangle = -\frac{4}{3}$ is the color factor and independent of the flavour contents of the quarks [41].

The confined gluon propagators are given by [39,40,42] as

$$D_0(r) = \left(\frac{\alpha_1}{r} + \alpha_2\right) \exp(-r^2 c_0^2/2) \tag{23}$$

and

$$D_1(r) = \frac{\gamma}{r} \exp(-r^2 c_1^2/2) \tag{24}$$

with α_1 = 1.035, α_2 = 0.3977 GeV, c_0 = 0.3418 GeV, γ = 0.8639 and c_1 = 0.4123 GeV GeV as in the previous study [32,42,43]. Other model parameters employed in the present study are listed in Table 4. Results obtained from the present study are tabulated in Tables 5, 6 and 7. The present results are compared with the available average experimental values reported by PDG [33].

2.2 Strong decays of heavy-light mesons

The underlying dynamics of the hadrons involving single heavy quark can be understood systematically by considering the heavy quark mass limit tends to infinity ($m_Q \to \infty$) formalised in a heavy quark effective theory (HQET). This H_Q limit allows to classify heavy-light ($Q\bar{q}$) mesons based upon the decoupling of the heavy quark spin s_Q from the total angular momentum s_l of the light degrees of freedom. Here, Q corresponds to c quark while q refers to light quarks u and d. The heavy quark spin s_Q and total angular momentum of light d.o.f s_l are separately conserved in strong interactions. This leads to classification of heavy mesons in doublets as per the different value of s_l. Each doublet here has two states, spin partners having total spin $J = s_l \pm \frac{1}{2}$ and parity $P = (-1)^{l+1}$

Table 5 S-wave D meson ($c\bar{q}$, $q = u, d$) spectrum (in MeV)

State ($s_l J^P$)	$M_{Q\bar{q}}$	$\langle V_{j_1 j_2}\rangle$	Present	Experiment		
				PDG [33]	$LHCb$ [1]	$LHCb$ [2]
$1^3S_1(\frac{1}{2}1^-)$	1983.34	8.99	2002.33	$D^*(2010)(2010.26 \pm 0.05)$	...	...
$1^1S_0(\frac{1}{2}0^-)$	1899.66	−26.98	1872.68	D (1867.83 ± 0.05)	...	...
$2^3S_1(\frac{1}{2}1^-)$	2651.19	4.55	2655.75	$?D^*(2600)(2623 \pm 12)$	$?D^*(2680)(2681.1 \pm 5.6)$	$?D^*(2650)(2649.2 \pm 3.5)$
$2^1S_0(\frac{1}{2}0^-)$	2556.74	−13.67	2543.07	$?D(2550)(2564 \pm 20)$	...	$?D^*(2580)(2579.5 \pm 3.4)$
$3^3S_1(\frac{1}{2}1^-)$	3140.05	3.23	3143.28	...	...	$?D_J^*(3000)(3008.1 \pm 4.0)$
$3^1S_0(\frac{1}{2}0^-)$	3054.00	−9.69	3044.31	...	...	$?D_J(3000)(2971.8 \pm 8.7)$
$4^3S_1(\frac{1}{2}1^-)$	3639.83	2.43	3642.26	...	...	...
$4^1S_0(\frac{1}{2}0^-)$	3558.06	−7.29	3550.76	...	...	...
$5^3S_1(\frac{1}{2}1^-)$	4114.29	1.93	4116.22	...	...	...
$5^1S_0(\frac{1}{2}0^-)$	4036.00	−5.80	4030.19	...	...	...
$6^3S_1(\frac{1}{2}1^-)$	4569.18	1.59	4570.78	...	...	...
$6^1S_0(\frac{1}{2}0^-)$	4493.77	−4.79	4488.98	...	...	...

where l is the orbital angular momentum of light d.o.f. and $\vec{s}_j = \vec{l} + \vec{s}_q$, $\vec{s}_q$ being the spin of light antiquark.

For the lowest lying states $l = 0$ (S-wave states of the quark model), $s_l^P = \frac{1}{2}^-$ and doublet consists of two states along with spin-parity $J^P = (0^-, 1^-)$ represented by (P, P^*). For $l = 1$ (P-wave states) one can write $s_l^P = \frac{1}{2}^+$ and $s_l^P = \frac{3}{2}^+$. The two doublets can be denoted as $J^P = (0^+, 1^+)$ with the members as (P_0^*, P_1') and $J^P = (1^+, 2^+)$ with the members as (P_1, P_2^*). Similarly, $l = 2$ (D-wave states) corresponds to $s_l^p = \frac{3}{2}^-$ and $s_j^p = \frac{5}{2}^-$. Here, one doublet is expressed as $J^P = (1^-, 2^-)$ consisting members as (P_1^*, P_2) and another is $J^P = (2^-, 3^-)$ with (P_2', P_3^*). $l = 3$ (F-wave states) represents $s_l^p = \frac{5}{2}^+$ and $s_l^p = \frac{7}{2}^+$ and associated with doublets $J^P = (2^+, 3^+)$ (P_2^*, P_3) and $J^P = (3^+, 4^+)$ (P_3', P_4^*), respectively. This representation is for radial quantum number $n = 1$. These doublets can be characterized by effective fields (4×4 matrices) H_a, S_a, T_a, X_a, Y_a, Z_a and R_a [44].

$$H_a = \frac{1+\not{v}}{2}\{P^*_{a\mu}\gamma^\mu - P_a\gamma_5\} \tag{25}$$

$$S_a = \frac{1+\not{v}}{2}\{P^\mu_{1a}\gamma_\mu\gamma_5 - P^*_{0a}\} \tag{26}$$

$$T^\mu_a = \frac{1+\not{v}}{2}\left\{P^{*\mu\nu}_{2a}\gamma_\nu - P_{1a\nu}\sqrt{\frac{3}{2}}\gamma_5\left[g^{\mu\nu} - \frac{\gamma^\nu(\gamma^\mu - \nu^\mu)}{3}\right]\right\} \tag{27}$$

$$X^\mu_a = \frac{1+\not{v}}{2}\left\{P^{\mu\nu}_{2a}\gamma_5\gamma_\nu - P^*_{1a\nu}\sqrt{\frac{3}{2}}\gamma_5 \times\left[g^{\mu\nu} - \frac{\gamma_\nu(\gamma^\mu + \nu^\mu)}{3}\right]\right\} \tag{28}$$

$$Y^{\mu\nu}_a = \frac{1+\not{v}}{2}\left\{P^{*\mu\nu\sigma}_{3a}\gamma_\sigma - P^{\alpha\beta}_{2a}\sqrt{\frac{5}{3}}\gamma_5 \times\left[g^\mu_\alpha g^\nu_\beta - \frac{g^\nu_\beta\gamma_\alpha(g^\mu - \nu^\mu)}{5} - \frac{g^\mu_\alpha\gamma_\beta(\gamma^\nu - \nu^\nu)}{5}\right]\right\} \tag{29}$$

$$Z^{\mu\nu}_a = \frac{1+\not{v}}{2}\left\{P^{\mu\nu\sigma}_{3a}\gamma_5\gamma_\sigma - P^{*\alpha\beta}_{2a}\sqrt{\frac{5}{3}} \times\left[g^\mu_\alpha g^\nu_\beta - \frac{g^\nu_\beta\gamma_\alpha(\gamma^\mu + \nu^\mu)}{5} - \frac{g^\mu_\alpha\gamma_\beta(\gamma^\nu - \nu^\nu)}{5}\right]\right\} \tag{30}$$

$$R^{\mu\nu\rho}_a = \frac{1+\not{v}}{2}\left\{P^{*\mu\nu\sigma}_{4a}\gamma_5\gamma_\sigma - P^{\alpha\beta\tau}_{3a}\sqrt{\frac{7}{4}} \times\left[g^\mu_\alpha g^\nu_\beta g^\rho_\tau - \frac{g^\nu_\beta g^\rho_\tau\gamma_\alpha(\gamma^\mu - \nu^\mu)}{7} - \frac{g^\mu_\alpha g^\rho_\tau\gamma_\beta(\gamma^\nu - \nu^\nu)}{7} - \frac{g^\mu_\alpha g^\nu_\beta\gamma_\tau(\gamma^\rho - \nu^\rho)}{7}\right]\right\} \tag{31}$$

where H_a (P, P^*) describes S-wave mesons; S_a (P_0^*, P_1') and T_a (P_1, P_2^*) associated with P-wave mesons; X_a (P_1^*, P_2)

Table 6 P-wave D meson ($c\bar{q}, q = u, d$) spectrum (in MeV)

State ($s_l J^P$)	$M_{Q\bar{q}}$	$\langle V_{j1j2} \rangle$	$\langle V_T \rangle$	$\langle V_{LS} \rangle$	Present	Experiment		
						PDG [33]	$LHCb$ [1]	$LHCb$ [2]
$1^3P_2(\frac{3}{2}2^+)$	2458.43	−8.053	16.60	−1.84	2465.1	$D_2(2460)$ (2460.7 ± 0.4)	$D_2^*(2460)$ (2463.7 ± 0.4)	...
$1^3P_1(\frac{3}{2}1^+)$	2458.43	−26.84	−16.60	9.19	2424.2	$D_1(2420)$ (2420.8 ± 0.5)	...	...
$1^3P_0(\frac{1}{2}0^+)$	2458.43	−16.11	−33.20	−18.37	2390.7	$D_0(2300)$ (2349 ± 7)	...	...
$1^1P_1(\frac{1}{2}1^+)$	2353.82	65.26	0	0	2419.1	$D_1(2430)$ (2427 ± 40)	...	...
$2^3P_2(\frac{3}{2}2^+)$	2995.54	−5.32	11.99	−1.33	3000.90	...	...	?$D_J^*(3000)$(3008.1 ± 4.0)
$2^3P_1(\frac{3}{2}1^+)$	2995.54	−17.73	−11.99	6.64	2972.50	...	...	?$D_J(3000)$(2971.8 ± 8.7)
$2^3P_0(\frac{1}{2}0^+)$	2995.54	−10.64	−23.99	−13.28	2947.60	...	...	?$D_J^*(3000)$(3008.1 ± 4.0)
$2^1P_1(\frac{1}{2}1^+)$	2900.25	40.09	0	0	2940.13	...	...	?$D_J(3000)$(2971.8 ± 8.7)
$3^3P_2(\frac{3}{2}2^+)$	3498.17	−3.93	9.55	−1.06	3502.6	...	...	...
$3^3P_1(\frac{3}{2}1^+)$	3498.17	−13.11	−9.55	5.29	3480.8	...	...	...
$3^3P_0(\frac{1}{2}0^+)$	3498.17	−7.86	−19.09	−10.58	3460.6	...	...	...
$3^1P_1(\frac{1}{2}1^+)$	3409.38	28.67	0	0	3449.5	...	...	...
$4^3P_2(\frac{3}{2}2^+)$	3975.79	−3.09	7.99	−0.89	3979.8	...	...	...
$4^3P_1(\frac{3}{2}1^+)$	3975.79	−10.31	−7.99	4.43	3961.9	...	...	...
$4^3P_0(\frac{1}{2}0^+)$	3975.79	−6.18	−15.98	−8.87	3944.8	...	...	...
$4^1P_1(\frac{1}{2}1^+)$	3891.87	22.11	0	0	3914.0	...	...	...
$5^3P_2(\frac{3}{2}2^+)$	4433.77	−2.53	6.90	−0.77	4437.4	...	...	...
$5^3P_1(\frac{3}{2}1^+)$	4433.77	−8.43	−6.90	3.83	4422.3	...	...	...
$5^3P_0(\frac{1}{2}0^+0$	4433.77	−5.06	−13.80	−7.67	4407.3	...	...	...
$5^1P_1(\frac{1}{2}1^+)$	4353.51	17.85	0	0	4371.4	...	...	...

Table 7 D-wave and F-wave D meson ($c\bar{q}$, $q = u, d$) spectrum (in MeV)

State ($s_l J^P$)	$M_{Q\bar{q}}$	$\langle V_{j1j2} \rangle$	$\langle V_T \rangle$	$\langle V_{LS} \rangle$	Present	Experiment		
						PDG [33]	$LHCb$ [1]	$LHCb$ [2]
$1^3D_3(\frac{5}{2}3^-)$	2771.05	−30.46	28.03	−2.21	2766.4	?$D_3^*(2750)(2763.5 \pm 3.4)$	?$D_3^*(2760)(2775.5 \pm 4.5)$	?$D_3^*(2760)(2760.1 \pm 1.1)$
$1^3D_2(\frac{5}{2}2^-)$	2771.05	−27.29	−14.01	7.75	2737.5	?$D_J(2740)(2737.0 \pm 3.5)$	…	?$D_J(2740)(2737.0 \pm 3.5)$
$1^3D_1(\frac{3}{2}1^-)$	2771.05	−8.72	−42.04	−7.75	2712.53	…	…	…
$1^1D_2(\frac{3}{2}2^-)$	2671.12	54.22	0	0	2725.34	?$D_J(2740)(2737.0 \pm 3.5)$	…	?$D_J(2740)(2737.0 \pm 3.5)$
$2^3D_3(\frac{5}{2}3^-)$	3354.29	−20.83	21.55	−1.70	3353.3	…	…	…
$2^3D_2(\frac{5}{2}2^-)$	3354.29	−18.67	−10.77	5.96	3330.82	…	…	…
$2^3D_1(\frac{3}{2}1^-)$	3354.29	−5.97	−32.31	−5.96	3310.04	…	…	…
$2^1D_2(\frac{3}{2}2^-)$	3258.33	35.49	0	0	3293.82	…	…	…
$3^3D_3(\frac{5}{2}3^-)$	3835.63	−16.16	17.66	−1.40	3835.73	…	…	…
$3^3D_2(\frac{5}{2}2^-)$	3835.63	−14.48	−8.83	4.89	3817.21	…	…	…
$3^3D_1(\frac{3}{2}1^-)$	3835.63	−4.63	−26.48	−4.89	3799.62	…	…	…
$3^1D_2(\frac{3}{2}2^-)$	3746.04	22.78	0	0	3772.9	…	…	…
$4^3D_3(\frac{5}{2}3^-)$	4297.05	−13.14	15.04	−1.19	4297.74	…	…	…
$4^3D_2(\frac{5}{2}2^-)$	4297.05	−11.77	−7.51	4.17	4281.93	…	…	…
$4^3D_1(\frac{3}{2}1^-)$	4297.05	−3.76	−22.53	−4.17	4266.6	…	…	…
$4^1D_2(\frac{3}{2}2^-)$	4212.28	21.43	0	0	4233.71	…	…	…
$1^3F_4(\frac{7}{2}4^+)$	3230.46	−11.18	35.48	−2.18	3252.55	…	…	?$D_J^*(3000)(3008.1 \pm 4.0)$
$1^3F_3(\frac{7}{2}3^+)$	3230.46	−16.97	−11.81	6.54	3208.21	…	…	?$D_J(3000)(2971.8 \pm 8.7)$
$1^3F_2(\frac{5}{2}2^+)$	3230.46	−5.38	−47.26	−5.23	3172.58	…	…	?$D_J^*(3000)(3008.1 \pm 4.0)$
$1^1F_3(\frac{5}{2}3^+)$	3125.76	38.73	0	0	3164.49	…	…	?$D_J(3000)(2971.8 \pm 8.7)$

and Y_a (P_2', P_3^*) represents D-wave mesons; Z_a (P_2^*, P_3) and R_a (P_3', P_4^*) related with F-wave mesons. Here, $a = u, d$ or s is $SU(3)$ light flavour index and v corresponds to meson four-velocity. For radial quantum numbers $n = 1, 2, 3, 4, \ldots$, heavy mesons with the same heavy flavour holds identical parity, time-reversal, charge conjugation properties and only differ in the mass. So, it is possible to combine them into effective fields H_a, H_a', $H_a'' \ldots S_a$, S_a', $S_a'' \ldots$ where $\prime, \prime\prime, \prime\prime\prime$ indicates $n = 2, 3, 4 \ldots$ states. Following this analogy, for example, for radial excitation $n = 2$ one can represent the sates by tilde ($\tilde{P}$, $\tilde{P}^*$).

The definition $\xi = e^{\frac{i\mathcal{M}}{f_\pi}}$ introduce octet of light pseudoscalar mesons in the theory through the matrix

$$\mathcal{M} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^0 & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}.$$

containing light pseudoscalar π, K and η fields. We take $f_\pi = 130$ MeV for calculations.

At the leading order approximation in the heavy quark mass and light quark momentum, the interaction Lagrangian terms $\mathcal{L}_H$, $\mathcal{L}_S$, $\mathcal{L}_T$, $\mathcal{L}_X$, $\mathcal{L}_Y$,$\mathcal{L}_Z$ and $\mathcal{L}_R$ can be described as [44]

$$\mathcal{L}_H = g_H \; Tr\{\bar{H}_a H_b \gamma_\mu \gamma_5 A^\mu_{ba}\} \tag{32}$$

$$\mathcal{L}_S = g_S \; Tr\{\bar{H}_a S_b \gamma_\mu \gamma_5 A^\mu_{ba}\} + H.c. \tag{33}$$

$$\mathcal{L}_T = \frac{g_T}{\Lambda} \; Tr\{\bar{H}_a T^\mu_b \left(i D_\mu \not{A} + i \not{D} A_\mu\right)_{ba} \gamma_5\} + H.c. \tag{34}$$

$$\mathcal{L}_X = \frac{g_X}{\Lambda} \; Tr\{\bar{H}_a X^\mu_b \left(i D_\mu \not{A} + i \not{D} A_\mu\right)_{ba} \gamma_5\} + H.c. \tag{35}$$

$$\mathcal{L}_Y = \frac{1}{\Lambda^2} \; Tr\{\bar{H}_a Y^{\mu\nu}_b [k^Y_1 \{D_\mu, D_\nu\} A_\lambda + k^Y_2 \left(D_\mu D_\lambda A_\nu + D_\nu D_\lambda A_\mu\right)]_{ba} \gamma^\lambda \gamma_5\} + H.c. \tag{36}$$

$$\mathcal{L}_Z = \frac{1}{\Lambda^2} \; Tr\{\bar{H}_a Z^{\mu\nu}_b [k^Z_1 \{D_\mu, D_\nu\} A_\lambda + k^Z_2 \left(D_\mu D_\lambda A_\nu + D_\nu D_\lambda A_\mu\right)]_{ba} \gamma^\lambda \gamma_5\} + H.c. \tag{37}$$

$$\mathcal{L}_R = \frac{1}{\Lambda^3} \; Tr\{\bar{H}_a R^{\mu\nu\rho}_b [k^R_1 \{D_\mu, D_\nu, D_\rho\} A_\lambda + k^R_2 (\{D_\mu, D_\rho\} D_\lambda A_\nu + \{D_\mu, D_\rho\} D_\lambda A_\nu \{D_\nu, D_\rho\} D_\lambda A_\mu + \{D_\mu, D_\nu\} D_\lambda A_\rho)]_{ba} \gamma^\lambda \gamma_5\} + H.c. \tag{38}$$

where the definitions $D_\mu = \partial_\mu + V_\mu$; $\{D_\mu, D_\nu\} = D_\mu D_\nu + D_\nu D_\mu$; $\{D_\mu, D_\nu D_\rho\} = D_\mu D_\nu D_\rho + D_\mu D_\rho D_\nu + D_\nu D_\mu D_\rho + D_\nu D_\rho D_\mu + D_\rho D_\mu D_\nu + D_\rho D_\nu D_\mu$.

The vector and axial-vector currents are defined as

$$V_\mu = \frac{1}{2}(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger) \tag{39}$$

$$\mathcal{A}_\mu = \frac{1}{2}(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger) \tag{40}$$

The chiral symmetry breaking scale $\Lambda = 1$ GeV. g_H, g_S, g_T, $g_Y = k^Y_1 + k^Y_2$, $g_Z = k^Z_1 + k^Z_2$ and $g_R = k^R_1 + k^R_2$ are the strong coupling constants. These constants are involved in the strong interactions of higher excited charmed mesons to the ground state +ve and −ve parity charmed mesons emitting light pseudoscalar mesons (π, η, K) and can be fitted to experimental full widths.

The strong decay width to $D^{(*)}\pi$, $D^{(*)}\eta$ and $D^{(*)}K$ using the chiral Lagrangians $\mathcal{L}_H$, $\mathcal{L}_S$, $\mathcal{L}_T$, $\mathcal{L}_X$, $\mathcal{L}_Y$,$\mathcal{L}_Z$ and $\mathcal{L}_R$ [44]

$$\Gamma = \frac{1}{(2J+1)} \sum \frac{p_p}{8\pi \; M_i^2} \; |\mathcal{A}|^2$$

$$p_p = \frac{\sqrt{(M_i^2 - (M_f + M_p)^2)\;(M_i^2 - (M_f - M_p)^2)}}{2 \; M_i} \tag{41}$$

where $\mathcal{A}$ signifies scattering amplitude; M_i and M_f are the masses of initial and final charmed mesons,respectively; M_p and p_p denotes the mass and momentum of light pseudoscalar meson, respectively. Here, J is the total angular momentum of the initial heavy meson while σ shows the summation of all the polarization vectors of the total angular momentum $j = 1, 2, 3$ or 4.

Below we collect the explicit expression of the two body strong decay widths of heavy light mesons for different channels [21,24]:

1. Decaying S wave meson: $(0^-, 1^-)_{\frac{1}{2}} \rightarrow (0^-, 1^-)_{\frac{1}{2}} + p$

$$\Gamma\left(1^- \rightarrow 0^-\right) = \mathrm{C}_p \frac{g_H^2 M_f p_p^3}{6\pi f_\pi^2 M_i} \tag{42}$$

$$\Gamma\left(0^- \rightarrow 1^-\right) = \mathrm{C}_p \frac{g_H^2 M_f p_p^3}{2\pi f_\pi^2 M_i} \tag{43}$$

$$\Gamma\left(1^- \rightarrow 1^-\right) = \mathrm{C}_p \frac{g_H^2 M_f p_p^3}{3\pi f_\pi^2 M_i} \tag{44}$$

2. Decaying P wave meson: $(0^+, 1^+)_{\frac{1}{2}} \rightarrow (0^-, 1^-)_{\frac{1}{2}} + p$

$$\Gamma\left(0^+ \rightarrow 0^-\right) = \mathrm{C}_p \frac{4 g_S^2 M_f}{2\pi f_\pi^2 M_i} \left[p_p \left(M_p^2 + p_p^2\right)\right] \tag{45}$$

$$\Gamma\left(1^+ \rightarrow 1^-\right) = \mathrm{C}_p \frac{4 g_S^2 M_f}{2\pi f_\pi^2 M_i} \left[p_p \left(M_p^2 + p_p^2\right)\right] \tag{46}$$

3. Decaying P wave meson: $(1^+, 2^+)_{\frac{3}{2}} \rightarrow (0^-, 1^-)_{\frac{1}{2}} + p$

$$\Gamma\left(2^+ \rightarrow 0^-\right) = \mathrm{C}_p \frac{4 g_T^2 M_f p_p^5}{15\pi f_\pi^2 \Lambda^2 M_i} \tag{47}$$

$$\Gamma\left(1^{+}\rightarrow 1^{-}\right)=\mathrm{C}_{p}\frac{2g_{T}^{2}M_{f}\,p_{p}^{5}}{3\pi f_{\pi}^{2}\Lambda^{2}M_{i}} \tag{48}$$

$$\Gamma\left(2^{+}\rightarrow 1^{-}\right)=\mathrm{C}_{p}\frac{2g_{T}^{2}M_{f}\,p_{p}^{5}}{5\pi f_{\pi}^{2}\Lambda^{2}M_{i}} \tag{49}$$

4. Decaying D wave meson: $(1^{-},2^{-})_{\frac{3}{2}}\rightarrow(0^{-},1^{-})_{\frac{1}{2}}+p$

$$\Gamma\left(1^{-}\rightarrow 0^{-}\right)=\mathrm{C}_{p}\frac{4g_{X}^{2}M_{f}}{9\pi f_{\pi}^{2}\Lambda^{2}M_{i}}\left[p_{p}^{3}\left(M_{p}^{2}+p_{p}^{2}\right)\right] \tag{50}$$

$$\Gamma\left(1^{-}\rightarrow 1^{-}\right)=\mathrm{C}_{p}\frac{2g_{X}^{2}M_{f}}{9\pi f_{\pi}^{2}\Lambda^{2}M_{i}}\left[p_{p}^{3}\left(M_{p}^{2}+p_{p}^{2}\right)\right] \tag{51}$$

$$\Gamma\left(2^{-}\rightarrow 1^{-}\right)=\mathrm{C}_{p}\frac{2g_{X}^{2}M_{f}}{3\pi f_{\pi}^{2}\Lambda^{2}M_{i}}\left[p_{p}^{3}\left(M_{p}^{2}+p_{p}^{2}\right)\right] \tag{52}$$

5. Decaying D wave meson: $(2^{-},3^{-})_{\frac{5}{2}}\rightarrow(0^{-},1^{-})_{\frac{1}{2}}+p$

$$\Gamma\left(2^{-}\rightarrow 1^{-}\right)=\mathrm{C}_{p}\frac{4g_{Y}^{2}M_{f}}{15\pi f_{\pi}^{2}\Lambda^{4}M_{i}}|p_{p}|^{7} \tag{53}$$

$$\Gamma\left(3^{-}\rightarrow 0^{-}\right)=\mathrm{C}_{p}\frac{4g_{Y}^{2}M_{f}}{35\pi f_{\pi}^{2}\Lambda^{4}M_{i}}|p_{p}|^{7} \tag{54}$$

$$\Gamma\left(3^{-}\rightarrow 1^{-}\right)=\mathrm{C}_{p}\frac{16g_{Y}^{2}M_{f}}{105\pi f_{\pi}^{2}\Lambda^{4}M_{i}}|p_{p}|^{7} \tag{55}$$

6. Decaying F wave meson: $(2^{+},3^{+})_{\frac{5}{2}}\rightarrow(0^{-},1^{-})_{\frac{1}{2}}+p$

$$\Gamma\left(2^{+}\rightarrow 0^{-}\right)=\mathrm{C}_{p}\frac{4g_{Z}^{2}M_{f}}{25\pi f_{\pi}^{2}\Lambda^{4}M_{i}}\left[p_{p}^{5}\left(M_{p}^{2}+p_{p}^{2}\right)\right] \tag{56}$$

$$\Gamma\left(2^{+}\rightarrow 1^{-}\right)=\mathrm{C}_{p}\frac{8g_{Z}^{2}M_{f}}{75\pi f_{\pi}^{2}\Lambda^{4}M_{i}}\left[p_{p}^{5}\left(M_{p}^{2}+p_{p}^{2}\right)\right] \tag{57}$$

$$\Gamma\left(3^{+}\rightarrow 1^{-}\right)=\mathrm{C}_{p}\frac{4g_{Z}^{2}M_{f}}{15\pi f_{\pi}^{2}\Lambda^{4}M_{i}}\left[p_{p}^{5}\left(M_{p}^{2}+p_{p}^{2}\right)\right] \tag{58}$$

7. Decaying F wave meson: $(3^{+},4^{+})_{\frac{7}{2}}\rightarrow(0^{-},1^{-})_{\frac{1}{2}}+p$

$$\Gamma\left(4^{+}\rightarrow 0^{-}\right)=\mathrm{C}_{p}\frac{16g_{R}^{2}M_{f}\,p_{p}^{9}}{35\pi f_{\pi}^{2}\Lambda^{6}M_{i}} \tag{59}$$

$$\Gamma\left(3^{+}\rightarrow 1^{-}\right)=\mathrm{C}_{p}\frac{36g_{R}^{2}M_{f}\,p_{p}^{9}}{35\pi f_{\pi}^{2}\Lambda^{6}M_{i}} \tag{60}$$

$$\Gamma\left(4^{+}\rightarrow 1^{-}\right)=\mathrm{C}_{p}\frac{4g_{R}^{2}M_{f}\,p_{p}^{9}}{7\pi f_{\pi}^{2}\Lambda^{6}M_{i}} \tag{61}$$

The values of the coefficients C_p involved are different for different pseudoscalar mesons: $\mathrm{C}_{\pi^\pm}=\mathrm{C}_{K^\pm}=1$, $\mathrm{C}_{\pi^0}=\frac{1}{2}$, $\mathrm{C}_\eta=\frac{2}{3}$ [24]. The strong coupling constants and their notations depends upon the radial quantum number n. For transitions within $n_i=n_f=1$ notations are $g_{H,S,T,X,Y,Z,R}$ while for $n_i=2$ and $n_f=1$ they are $\tilde{g}^2_{H,S,T,X,Y,Z,R}$. Higher order loop corrections are eliminated to bypass the introduction of new coupling constants. For the present study we adopt the approximation $\mathcal{A}_\mu=i\frac{\partial_\mu\mathcal{M}}{f_\pi}$. If the momenta of the emitted light pseudoscalar mesons are not very small, the additional terms can be added to introduce unknown coupling constants. Moreover, the spin and the flavour violation correction having the order of $\mathcal{O}(\frac{1}{m_Q})$ to the heavy quark limit could be sizable, and in that case introduction of new coupling constants to the theory may not cancel out in the ratios of the decay widths. But we expect that their contribution would be much less than the leading order contributions. In decay rates the leading order unknown coupling constants can be theoretically predicted or can be evaluated from the available experimental data of decay widths. Successful predictions on coupling constants based on the QCD sum rules [21] and Lattice QCD [45] are found in literature. The numerical values of the meson masses used as input parameters for present calculations are $M_{D^{*+}}=2010.26$ MeV, $M_{D_S^{*+}}=2112.2$ MeV, $M_{D^{*0}}=2006.85$ MeV, $M_{D^+}=1869.65$ MeV, $M_{D_S^+}=1968.34$ MeV, $M_{D^0}=1864.83$ MeV, $M_{\pi^0}=134.97$ MeV, $M_{\pi^-}=139.57$ MeV, $M_{K^-}=493.67$ MeV, $M_\eta=547.86$ MeV [33].

3 Results and discussion

The mass spectra of the charmed mesons $(c\bar{q})$; $q=u$ or d are computed by employing the relativistic Dirac framework and listed in Tables 5, 6 and 7. The states D (1867.83 ± 0.05) and $D^*(2010)$ (2010.26 ± 0.05) are well-established ground state of D mesons. Our estimation of the mass of 1^1S_0 is 1872.68 MeV and for 1^3S_1 is 2002.33 MeV; which is found to be in close agreement with reported mass of D and $D^*(2010)$ [33]. However, if we incorporate the uncertainty of $\frac{\Delta m_c}{m_c}\approx 1.5\%$ as per $m_c=1.27\pm0.02$ GeV [33] results into just 1% variation in the predicted D meson mass for 1^3S_1 state and that for 1^1S_0 state. Here, as up and down quark mass and their uncertainty are negligible (few MeV only) we don't consider it in the uncertainty estimations. The other parameters λ, V_0 and σ are the optimized potential parameters which are fixed to yield the experimental ground state masses. The variation of the parameter λ (potential strength) of 5% and 10% keeping other two fixed leads to the uncertainty of less than 1% and 2% in mass of $1S$ state. Also, The variation of the parameter σ (coupling strength) of 5% and10% keeping other two fixed gives uncertainty of less than 0.07% and 0.09% in mass of $1S$

state. The third parameter V_0 variation results into negligible variations in the mass of $1S$ state. So, the variations are not much compared to the optimized value of these parameters. However, the value of λ is slightly more sensitive than that of V_0 and σ. The detailed results on the sensitivity of charm quark mass and these model parameters are presented Table 16 and 17 of Appendix B.

The states $D_2(2460)$, $D_1(2420)$, $D_0(2400)$ and $D_1(2430)$ are also well defined $1P$ multiplets. Our predictions for these states are in excellent agreement with PDG listed values [33]. With the successful description of $1S$ and $1P$ multiplets, we are able to assign proper J^P values of the newly observed states by $LHCb$ Collaboration [1,2] $D_J(2580)$, $D_J^*(2650)$, $D_J(2740)$, $D_J(2760)$, $D_J(3000)$, $D_J^*(3000)$ and $D_2^*(3000)$. Further to check the reliability of the present formalism in terms of the strong decays we have computed the ratio $\frac{\Gamma(D^+\pi^-)}{\Gamma(D^{*+}\pi^-)}$ for the well-established $D_2^*(2460)$ state. The transitions of the $1P_{\frac{3}{2}}$ multiplets $(1^3P_2, 1^3P_1)$ to ground state charm meson by emitting a pseudoscalar meson are described by the coupling constant g_T. The partial decay widths in terms of the g_T are listed in Table 8 for $D_2^*(2460)$. In Table 9 the computed ratio, $\frac{\Gamma(D_2^*(2460)^0\to D^+\pi^-)}{\Gamma(D_2^*(2460)^0\to D^{*+}\pi^-)}$ is compared with those reported by various experimental groups. The average experimental value found to be 2.35 ± 0.6 which is in good agreement with the value of 2.26 predicted by the present study. This indicates the usefulness of the present formalism in the prediction of strong decay widths. From the present study, we found $g_T = 0.40 \pm 0.003$ which is in excellent agreement with 0.43 ± 0.05 reported by in [46] and 0.43 ± 0.01 reported by [26]. The details of computing the uncertainty in the estimation of coupling constants are shown in Appendix C.

The $LHCb$ [2] and $BABAR$ [3] have observed states around the mass region 2550 MeV with unnatural parity and 2600 MeV with natural parity with different labels. But the mass and the widths of these states are so close making them to be considered having the same J^P values. Also, their properties and mass range make them suitable candidates for radial excited ($2S$) states of the charm mesons. In the case of $D(2550)$, the measured total width by $LHCb$ [2] and $BABAR$ [3] contain very large error bar. The mass difference of $LHCb$ [2] and $BABAR$ [3] is roughly 40 MeV (see Table 5). From our predictions the mass of 2^1S_0 is 2543.07 MeV which is in good agreement with the PDG average value of 2564 ± 20 MeV. Similarly for the $D(2600)$ our predicted mass is 2655 MeV. The mass and total width difference between LHCb [2] and BABAR [3] is around again 40 MeV and 47 MeV with large error bars, respectively. The latest effort of $LHCb$ has reported $D_1(2680)$ with 2681.1 ± 5.6 MeV mass and 186.7 ± 8.5 MeV width. Hence, we identify the $D(2580)$ and $D_J^*(2650)$ as 2^1S_0 and 2^3S_1; respectively. The partial decay widths in terms of the effective coupling constants are listed in Table 8 for $D_J(2580)$ and $D_J^*(2650)$ using our computed masses of 2^3S_1 and 2^1S_0. The total decay width $\Gamma(2580) = 1101.39\tilde{g}_H^2$ MeV and $\Gamma(2650) = 2627.78\tilde{g}_H^2$ MeV and can be compared with the experimentally measured width to extract the effective coupling constant. Considering $D_J(2580)$, $D_J^*(2650)$ as radial excited states the average effective coupling constant $\tilde{g}_H$ is deduced as 0.31 ± 0.017 which is close to 0.28 ± 0.01 reported by [26] but double the value of 0.14 ± 0.03 reported by Ref [46]. This, radial excited state can also decay to $1P$ states emitting the light pseudoscalar mesons (π, η, K). Their incorporation requires additional introduction of the unknown coupling constants into the theory. So, to avoid complexity we do not consider them here.

$BABAR$ and $LHCb$ also reported many states falling within the mass region between 2740 and 2800 MeV [2,3,47,48]. These observed states can be arranged into natural and unnatural parity states. The natural parity states are grouped with designation $D_J^*(2760)$ (2763.5 ± 3.4 MeV) and unnatural parity states are labeled as $D_J(2740)$ (2737 ± 3.5 MeV). This mass range is predicted to be close to $1D$ and $2P$ multiplets [5]. The quantum numbers of these states as $2P$ multiplets are found to be inconsistent with $LHCb$ [47,48] measurements. Assuming both of them to be spin partners of each other, the possible spin-parity assignments, for $D_J^*(2760)$ are $1^3D_1(\frac{3}{2}1^-)$ and $1^3D_3(\frac{5}{2}3^-)$ while for $D_J(2740)$ are $(1^1D_2)(\frac{3}{2}2^-)$ and $(1^3D_2)(\frac{5}{2}2^-)$. The predicted masses from the present study for 1^3D_3 (2766.4 MeV) and 1^3D_2 (2737.5 MeV) are close to reported massed of $D_J^*(2760)$ and $D_J(2740)$. The mass predicted for states 1^3D_1 (2712.53 MeV) and 1^1D_2 (2725.34 MeV) are also in good agreement with respect to reported mass of $D_J^*(2760)$ and $D_J(2740)$. For both the cases J^P value for $D_J(2740)$ is 2^-. The partial decay widths in terms of the effective coupling constants are listed in Table 8 for $D^*(2760)$ and $D_J(2740)$ for both possible assignments. The $D^+\pi^-$ mode found to dominant in both 1^3D_1 and 1^3D_3. The ratio, $\frac{\Gamma(D^+\pi^-)}{\Gamma(D^{*+}\pi^-}$ for 1^3D_1 is found to be 4.05 whereas for 1^3D_3 it is 1.93. These suggests that $D^+\pi^-$ mode is more dominant in 1^3D_1. The $BABAR$ Collaboration has observed $D^*(2760)$ signal in $D^+\pi^-$ mode very close in masss to the $D(2750)$ signal observed in $D^{*+}\pi^-$ [3]. The state $D_J^*(2760)$ was observed in both $D^{*+}\pi^-$ and $D^+\pi^-$ decay modes [2]. The LHCb has reported two D(2760) states: One assigned as 1^- [49] and other as 3^- [48] in different analysis. For the case of 1^- assignment, the reported mass and total width are $2781 \pm 18 \pm 11 \pm 6$ MeV and $177 \pm 32 \pm 20 \pm 7$ MeV respectively. The three quoted errors are statistical, experimental systematic and model uncertainties, respectively. While in the case of 3^- assignment ($m = 2798 \pm 7 \pm 1 \pm 7$ MeV ; $\Gamma = 105 \pm 18 \pm 6 \pm 23$ (Isobar) and $m = 2802 \pm 11 \pm 10 \pm 3$

Table 8 The strong decay widths (in MeV) for charmed resonance with possible spin-parity assignments. The ratio is calculated from $\frac{\Gamma}{\Gamma(D_J^* \to D^{*+}\pi^-)}$. Fraction (in%) represents the percentage of the partial decay width with respect to total decay width

Resonance	State ($s_l J^P$)	Decay mode	Width	Ratio	Fraction	Exp (in MeV)
$D_J(2580)$	2^1S_0 ($\frac{1}{2}0^-$)	$D^{*+}\pi^-$	725.18 $\tilde{g}_H^2$	1	65.84	
		$D_s^{*+}K^-$	…	…	…	
		$D^{*0}\pi^0$	371.29 $\tilde{g}_H^2$	0.51	33.71	
		$D^{*0}\eta$	4.92 $\tilde{g}_H^2$	0.00	0.44	
		Total	1101.39 $\tilde{g}_H^2$			177.5 ± 17.8 [2]
$D_J^*(2650)$	2^3S_1 ($\frac{1}{2}1^-$)	$D^+\pi^-$	631.90 $\tilde{g}_H^2$	0.78	24.04	
		$D_s^+K^-$	165.17 $\tilde{g}_H^2$	0.20	6.28	
		$D^0\pi^0$	321.17 $\tilde{g}_H^2$	0.39	12.22	
		$D^0\eta$	164.47 $\tilde{g}_H^2$	0.20	6.26	
		$D^{*+}\pi^-$	805.35 $\tilde{g}_H^2$	1	30.65	
		$D_s^{*+}K^-$	41.82 $\tilde{g}_H^2$	0.05	1.59	
		$D^{*0}\pi^0$	409.79 $\tilde{g}_H^2$	0.50	15.59	
		$D^{*0}\eta$	88.11 $\tilde{g}_H^2$	0.10	3.35	
		Total	2627.78 $\tilde{g}_H^2$			140.2 ± 17.1 [2]
$D_2^*(2460)$	1^3P_2 ($\frac{3}{2}2^+$)	$D^+\pi^-$	129.75 g_T^2	2.25	41.32	
		$D^0\pi^0$	67.78 g_T^2	1.18	23.66	
		$D^0\eta$	1.11 g_T^2	0.02	0.39	
		$D^{*+}\pi^-$	57.43 g_T^2	1	20.05	
		$D^{*0}\pi^0$	30.26 g_T^2	0.52	10.56	
		Total	286.39 g_T^2			47.0 ± 0.8 [1]
$D_J^*(2760)$	1^3D_1 ($\frac{3}{2}1^-$)	$D^+\pi^-$	1020.91g_X^2	4.05	38.06	
		$D_s^+K^-$	312.76 g_X^2	1.24	11.66	
		$D^0\pi^0$	522.46 g_X^2	2.07	19.47	
		$D^0\eta$	362.46 g_X^2	1.43	13.51	
		$D^{*+}\pi^-$	251.99 g_X^2	1	9.39	
		$D_s^{*+}K^-$	30.13 g_X^2	0.12	1.12	
		$D^{*0}\pi^0$	128.89 g_X^2	0.51	4.80	
		$D^{*0}\eta$	52.48 g_X^2	0.21	1.95	
		Total	2682.08 g_X^2			74.4 ± 3.4 [2]
$D_J^*(2760)$	1^3D_3 ($\frac{5}{2}3^-$)	$D^+\pi^-$	179.99 g_Y^2	1.93	38.59	
		$D_s^+K^-$	18.88 g_Y^2	0.20	4.04	
		$D^0\pi^0$	93.14 g_Y^2	1.00	19.97	
		$D^0\eta$	25.58 g_Y^2	0.27	5.48	
		$D^{*+}\pi^-$	93.04 g_Y^2	1	19.95	
		$D_s^{*+}K^-$	2.35 g_Y^2	0.02	0.50	
		$D^{*0}\pi^0$	48.13 g_Y^2	0.51	10.31	
		$D^{*0}\eta$	5.26 g_Y^2	0.06	1.12	
		Total	466.37 g_Y^2			74.4 ± 3.4 [2]
$D_J(2740)$	1^1D_2 ($\frac{3}{2}2^-$)	$D^{*+}\pi^-$	817.76 g_X^2	1	53.41	
		$D_s^{*+}K^-$	111.95 g_X^2	0.14	7.31	
		$D^{*0}\pi^0$	418.04 g_X^2	0.51	27.30	
		$D^{*0}\eta$	183.30 g_X^2	0.22	11.97	
		Total	1531.06 g_X^2			73.2 ± 13.4 [2]

Table 8 continued

Resonance	State ($s_l J^P$)	Decay mode	Width	Ratio	Fraction	Exp (in MeV)
$D_J(2740)$	1^3D_2 ($\frac{5}{2}2^-$)	$D^{*+}\pi^-$	127.92 g_Y^2	1	63.39	
		$D_s^{*+}K^-$	1.96 g_Y^2	0.01	0.97	
		$D^{*0}\pi^0$	66.31 g_Y^2	0.52	32.97	
		$D^{*0}\eta$	5.30 g_Y^2	0.04	2.63	
		Total	201.49 g_Y^2			73.2 ± 13.4 [2]

Table 9 The values of ratio $\frac{\Gamma(D_2^*(2460)^0\to D^+\pi^-)}{\Gamma(D_2^*(2460)^0\to D^{*+}\pi^-)}$ from various experiments and from present work

Present	2.26
BABAR	1.47 ± 0.03 ± 0.16
CLEO	2.2 ± 0.7 ± 0.6
CLEO	2.3 ± 0.8
ARGUS	3.0 ± 1.1 ± 1.5
ZEUS	2.8 $\pm 0.8^{+0.5}_{-0.6}$
Exp. average	2.35 ± 0.6

MeV ; $\Gamma = 154 \pm 27 \pm 13 \pm 9$ (K-matrix)), the error is less as compared to previous one [48]. Later, the state $D_J^*(2760)$ is confirmed by $LHCb$ [1] with J assignment as 3 in their latest attempt. So, we tentatively assign $D_J(2740)$ and $D_J^*(2760)$ as 1^3D_2 and 1^3D_3; respectively. The average value of effective coupling constant g_Y is obtained as 0.49 ± 0.039. This value is consistent with the 0.53 ± 0.13 [46] and 0.42 ± 0.02 [26]. At the same time, we found the ratio $\frac{\Gamma D(2760)^0\to D^+\pi^-}{\Gamma D^0(2740)\to D^{*+}\pi^-}$ equal to 1.40 which is larger than 0.42 ± 0.05 ± 0.11 reported by $BABAR$ [3]. Although the three states among the four $1D$ family are observed by experiments but one is still missing. It is very difficult to make precise measurements on the properties of the four overlapping states relying only upon their mass and widths. Also, $D^*\pi$ signal have large contribution from these overlapping states. So, further experimental efforts are required to resolve the above discrepancy and to understand these states more reliably.

The experimental value of the mass and the total decay width of $D_J^*(3000)$ is 3008.1 ± 4.0 MeV and 110.5 ± 11.5 MeV [2]. The measured mass of the state $D_J(3000)$ is 2971.8 ± 8.7 MeV and width is 188.1 ± 44.8 MeV [2]. From the present analysis we may assign these states as the $3S$, $2P$ or $1F$ multiplets. Considering both of them to be spin partners then the following five assignments of the J^P for natural parity state $D_J^*(3000)$ and unnatural parity state $D_J(3000)$ are possible.

$$D_J^*(3000) = 3^3S_1;\ D_J(3000) = 3^1S_0;\ (\tfrac{1}{2}1^-, 0^-)$$
$$D_J^*(3000) = 2^3P_0;\ D_J(3000) = 2^1P_1;\ (\tfrac{1}{2}0^+, 1^+)$$
$$D_J^*(3000) = 2^3P_2;\ D_J(3000) = 2^3P_1;\ (\tfrac{3}{2}2^+, 1^+)$$
$$D_J^*(3000) = 1^3F_2;\ D_J(3000) = 1^1F_3;\ (\tfrac{5}{2}2^+, 3^+)$$
$$D_J^*(3000) = 1^3F_4;\ D_J(3000) = 1^3F_3;\ (\tfrac{7}{2}4^+, 3^+)$$

Looking into the good agreement for $1S$, $2S$ and $1P$ masses, we believe that for higher excited states, our predictions are reliable. The predicted masses from the present study for 3^3S_1 (3143.28 MeV), 1^3F_2 (3172.58 MeV), 1^3F_4 (3252.55 MeV) and 3^1S_0 (3044.31 MeV), 1^1F_3 (3164.49 MeV), 1^3F_3 (3208.21 MeV) are high as compared to 3008.1 ± 4 MeV and 2971.8 ± 8 MeV for $D_J^*(3000)$ and $D_J(3000)$. However, the mass of 2^3P_0 (2947.60 MeV) and 2^1P_1 (2940.13 MeV) are comparable whereas mass of 2^3P_2 (3000.90 MeV) and 2^3P_1 (2972.50 MeV) are close to reported mass of $D_J^*(3000)$ and $D_J(3000)$. The partial decay widths in terms of the effective coupling constants are listed in Tables 10 and 11 for $D^*(3000)$ and $D_J(3000)$ respectively where we have used our computed masses. Among the five possibilities later two can be completely ruled out as 1^3F_2, 1^1F_3 resulting very small decay width and 1^3F_4, 1^3F_3 have very large decay width which is far from the experimentally predicted widths of the $D_J^*(3000)$ and $D_J(3000)$ [2]. The state $D_J^*(3000)$ decays to $D^+\pi^-$ whereas $D_J(3000)$ decays to $D^{*+}\pi^-$ final state [2]. If we identify $D_J^*(3000)$ as 3^3S_1 then the dominant mode is $D^{*+}\pi^-$ which is not favoured by experiment. The other modes such as $D^0\pi^0$, $D^0\eta$ are very small. However, the two remaining possibilities leads to the J^P of $D_J(3000)$ as 1^+. If we consider the $D^*(3000)$ 2^3P_0 then the $D^*\pi$ mode is completely forbidden and $D\pi$ mode is dominant. This fact is consistent with the experimental observation. Also, the $D_s^+K^-$ and $D^0\eta$ modes are considerably large which is supported by experimental analysis. Similarly if we assign $D_J(3000)$ as 2^1P_1, the $D_s^{+*}K^-$ and $D^{0*}\eta$ modes are sufficiently large as compared to 2^3P_1. With all such considerations, we identify $D_J^*(3000)$ as 2^3P_1 and $D_J(3000)$ as 2^1P_1. The previous study Ref. [50] supports this assignments for $D_J^*(3000)$ and $D_J(3000)$ states while the other studies [18] argues that $D_J^*(3000)$ could be 1^3F_4 or 1^3F_2 and

Table 10 The strong decay widths (in MeV) for resonance $D_J^*(3000)$ with possible spin-parity assignments

Decay mode	$3^3S_1(\frac{1}{2}1^-)$ (in $\tilde{\tilde{g}}_H^2$ MeV)	$2^3P_0(\frac{1}{2}0^+)$ (in $\tilde{g}_S^2$ MeV)	$2^3P_2(\frac{3}{2}2^+)$ (in $\tilde{g}_T^2$ MeV)	$1^3F_2(\frac{5}{2}2^+)$ (in g_Z^2 MeV)	$1^3F_4(\frac{7}{2}4^+)$ (in g_R^2 MeV)
$D^+\pi^-$	1918.58	4076.82	1959.61	2209.78	10139.80
$D_s^+K^-$	1262.91	3247.92	767.10	1048.74	3198.69
$D^0\pi^0$	966.27	2054.66	996.27	1125.38	5191.89
$D^0\eta$	950.14	2629.14	689.21	1005.76	3210.22
$D^{*+}\pi^-$	3140.96	…	1854.71	857.50	6552.91
$D_s^{*+}K^-$	1804.16	…	512.26	324.23	1521.38
$D^{*0}\pi^0$	1581.56	…	942.13	435.86	3347.20
$D^{*0}\eta$	1425.63	…	517.46	343.56	1708.31
Total	13050.21	12008.54	8238.75	7350.81	34870.40

Table 11 The strong decay widths (in MeV) for resonance $D_J(3000)$ with possible spin-parity assignments

Decay mode	$3^1S_0(\frac{1}{2}0^-)$ (in $\tilde{\tilde{g}}_H^2$ MeV)	$2^1P_1(\frac{1}{2}1^+)$ (in $\tilde{g}_S^2$ MeV)	$2^3P_1(\frac{3}{2}1^+)$ (in $\tilde{g}_T^2$ MeV)	$1^1F_3(\frac{5}{2}3^+)$ (in g_Z^2 MeV)	$1^3F_3(\frac{7}{2}3^+)$ (in g_R^2 MeV)
$D^{*+}\pi^-$	3822.46	3078.66	2742.03	2059.98	9000.8
$D_s^{*+}K^-$	1934.80	2179.07	692.41	768.04	1869.22
$D^{*0}\pi^0$	1926.77	1550.91	1393.86	1047.26	4603.02
$D^{*0}\eta$	1587.50	1892.95	718.41	818.25	2171.57
Total	9271.73	8701.59	5546.77	4693.52	17645.3

$D_J(3000)$ could be $1F_3$ or $2P_1'$. Ref. [7] suggests $D_J^*(3000)$ is a 1^3F_4 and $D_J(3000)$ is a $2P_1$ state. All these possibilities also can not be ignored and more precise measurements are needed from the experimental side to clarify the puzzles in $D(3000)$ states. Considering $D_J^*(3000)$ and $D_J(3000)$ as $2P(0^+,1^+)_{\frac{1}{2}}$ multiplets, the predicted value of the effective coupling constant $\tilde{g}_S$ found to be 0.10 ± 0.015 which would be useful for the future investigations on heavy-light systems. The numerical values of the decay widths and BRs emitting the light vector meson channels such as $D\rho$, D_sK^* and $D\omega$ can also be incorporated to make precise predictions based on the heavy quark effective theory framework [27]. Recently, the $LHCb$ group has observed a state labelled as $D_2^*(3000)$ with mass $3214 \pm 29 \pm 33$ MeV and total width $186 \pm 38 \pm 34$ MeV [1]. The labelling of this newly observed state in $B^- \to D^+\pi^-\pi^-$ decays and previously reported state $D_J^*(3000)$ are resembled to each other. However, the $LHCb$ collaboration assigned $J^P = 2^+$ for $D_2^*(3000)$ while $D_J^*(3000)$ has natural parity. The mass energy difference between $D_2^*(3000)$ and $D_J^*(3000)$ is 206 MeV which suggests that they both are different states [51]. The mass range $3214 \pm 29 \pm 33$ MeV likely to fall in the mass spectra of 2P, 3P and 1F multiplets. From the present study mass of 1^3F_4 is close to $3214 \pm 29 \pm 33$ MeV but the quantum numbers are not consistent with this assignment and for 3^3P_2 the mass difference is more than 240 MeV. So, the two most probable assignments for $D_2^*(3000)$ are 1^3F_2 or 2^3P_2. From the present study the mass of 1^3F_2 is 3172.58 MeV while the mass of 2^3P_2 is 3000.90 MeV. Allowed decay channels for $D_2^*(3000)$ are given in terms of coupling constant g_Z and $\tilde{g}_T$ in Table 12. The two different possibilities bring out different decay widths. The ratio $R_p^{(*)0}$ of various partial widths and branching fractions can be useful in the identification of the new signals with predicted states in experiments. Using the partial widths such ratios are defined as [27]

$$\mathbf{R}_\pi^0 = \frac{\Gamma(D_2^{*0}(3000) \to D^{*0}\pi^0) + \Gamma(D_2^{*0}(3000) \to D^{*+}\pi^-)}{\Gamma(D_2^{*0}(3000) \to D^0\pi^0) + \Gamma(D_2^{*0}(3000) \to D^+\pi^-)}$$

$$\mathbf{R}_K^0 = \frac{\Gamma(D_2^{*0}(3000) \to D_sK^-)}{\Gamma(D_2^{*0}(3000) \to D^0\pi^0) + \Gamma(D_2^{*0}(3000) \to D^+\pi^-)}$$

$$\mathbf{R}_K^{*0} = \frac{\Gamma(D_2^{*0}(3000) \to D_s^*K^-)}{\Gamma(D_2^{*0}(3000) \to D^0\pi^0) + \Gamma(D_2^{*0}(3000) \to D^+\pi^-)}$$

$$\mathbf{R}_\eta^0 = \frac{\Gamma(D_2^{*0}(3000) \to D^0\eta)}{\Gamma(D_2^{*0}(3000) \to D^0\pi^0) + \Gamma(D_2^{*0}(3000) \to D^+\pi^-)}$$

$$\mathbf{R}_\eta^{*0} = \frac{\Gamma(D_2^{*0}(3000) \to D^{*0}\eta)}{\Gamma(D_2^{*0}(3000) \to D^0\pi^0) + \Gamma(D_2^{*0}(3000) \to D^+\pi^-)}$$

The ratios for both the possibilities are presented in Table 13. It can be noted that the ratio R_K^0 has less sensitivity to the $2P$ and $1F$ identification while R_π^0 is highly sensitive. In both the cases $D^+\pi^-$ mode is found to be dominant. So, to get more insights we also considered the ratio $\frac{\Gamma D_2^*(3000) \to D^{*+}\pi^-}{\Gamma D_2^*(3000) \to D^+\pi^-}$ for 1^3F_2 and 2^3P_2 states. This ratio is found to be 0.95 for 2^3P_2 while it is 0.38 for 1^3F_2. This indicates that $D^{*+}\pi^-$ mode is more dominant in 2^3P_2 as compared to 1^3F_2. On the

Table 12 The strong decay widths (in MeV) for resonance $D_2^*(3000)$ with possible spin-parity assignments. The ratio is calculated from $\frac{\Gamma}{\Gamma(D_J^* \to D^{*+}\pi^-)}$. Fraction (in%) represents the percentage of the partial decay width with respect to total decay width

Decay mode	$2^3P_2(\frac{3}{2}2^+)$			$1^3F_2(\frac{5}{2}2^+)$		
	(in $\tilde{g}_T^2$ MeV)	Ratio	Fraction	(in g_Z^2 MeV)	Ratio	Fraction
$D^+\pi^-$	1959.61	1.06	23.78	2209.78	2.58	30.06
$D_s^+K^-$	767.10	0.41	9.31	1048.74	1.22	14.27
$D^0\pi^0$	966.27	0.53	12.09	1125.38	1.31	15.31
$D^0\eta$	689.21	0.37	8.36	1005.76	1.17	13.68
$D^{*+}\pi^-$	1854.71	1	22.78	857.50	1	11.66
$D_s^{*+}K^-$	512.26	0.28	6.22	324.23	0.38	4.41
$D^{*0}\pi^0$	942.13	0.51	11.44	435.86	0.50	5.92
$D^{*0}\eta$	517.46	0.28	6.28	343.56	0.40	4.67
Total	8238.75			12008.54		

Table 13 Various ratios for the two different identification of the state $D_2^*(3000)$

Ratios	$2^3P_2(\frac{3}{2}2^+)$	$1^3F_2(\frac{5}{2}2^+)$
R_π^0	0.95	0.38
R_K^0	0.26	0.31
R_K^{*0}	0.17	0.09
R_η^0	0.23	0.30
R_η^{*0}	0.17	0.10

other hand, experimentally $D^{*+}\pi^-$ mode is suppressed in $D_2^*(3000)$ state [1]. This fact is consistent with our predicted decay modes. So, we identify $D_2^*(3000)$ as 1^3F_2. As shown in Table 9, $D_2^*(3000)$ being 1^3F_2 candidate, its total decay width as 12008.54 g_Z^2 MeV results into, $g_Z = 0.12 \pm 0.012$.

After identifying the $D_2^*(3000)$ as $1^3F_2(\frac{5}{2}2^+)$ one can have further considerations on its spin partner $1^1F_3(\frac{5}{2}3^+)$. The information about them in literature is very less so it will be helpful for theorists and experimentalists for future investigations on these states. Our predicted mass of 1^1F_3 is 3164.49 MeV. The Okubo–Zweig–Iizuka (OZI) allowed decay channels for 1^1F_3 are listed in Table 14. The states 1^1F_3 can also decays to p-wave mesons through emission of light pseudoscalar mesons but due to small phase space those modes are neglected in the present work. If we consider the value g_Z =0.12 then the total decay width of 1^1F_3 found to be 67.15 MeV. Thus, the total width of 1^1F_3 is narrower than its spin partner 1^3F_2 ($\Gamma = 110.5 \pm 11.5$ MeV). The branching fraction 44.18% for the decay mode $D^{*+}\pi^-$ suggests as an appropriate channel for the experimental search of 1^1F_3. Finally we conclude that the predicted masses of several states of charmed mesons are in good agreement with the experimental values. And the mass spectroscopy is found to be consistent in predictions of strong decays of various D-mesonic states.

4 Summary

In this article, we obtained the mass spectra of the heavy-light charmed mesons within the framework of the relativistic formalism. Our predicted masses are in excellent agreement for the well established low-lying states of charmed mesons. Further, incorporating the heavy quark effective theory at the leading order approximation, we obtained the strong decays of the experimentally observed states and identified $D_J(2560)$ as 2^1S_0, $D_J^*(2680)$ as 2^3S_1, $D_J(2740)$ as 1^3D_2, $D_J^*(2760)$ as 1^3D_3, $D_J^*(3000)$ as 2^3P_0, $D_J(3000)$ as 2^1P_1 and $D_2^*(3000)$ as 1^3F_2 open charm excited states. The effective coupling constants g_T, $\tilde{g}_H$, g_Y, $\tilde{g}_S$ and g_Z are also computed. We found $D^{*+}\pi^-$ channel is the most suitable mode for experimental search for 1^1F_3. The computed decay widths and masses for higher excited states would be useful for the identification of future experimental observation of higher open charm states at $BABAR$, $LHCb$, $BESIII$.

Acknowledgements We acknowledge the partial support from DST-SERB, India through the major research project: (SERB/F/8749/2015-16).

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: All data generated or analyzed during this study are included in this published article and the paper has no associated data as it is theoretical work.]

Table 14 The strong decay widths (in MeV) for $1F_{\frac{5}{2}}3^+$. Fraction (in %) represents the percentage of the partial decay width with respect to total decay width

Decay channel	Decay width	Branching fraction
$1^1F_3(\frac{5}{2}3^+)$		
$D^{*+}\pi^-$	$2059.98\ g_Z^2$	44.18
$D^{*+}\pi^0$	$1032.15\ g_Z^2$	22.13
$D^{*+}\eta$	$803.36\ g_Z^2$	17.23
$D_s^{*+}K^-$	$767.57\ g_Z^2$	16.46
Total	67.15	

Funded by SCOAP[3].

Appendix A: Numerical single particle Dirac confinement energy of the quarks and antiquarks

The Dirac confinement energy (E_D) is extracted from Eq. (15) for quark(Q) and anti-quark($\bar{q}$) in accordance with Eqs. (5), (6) and (13), (14). These values for Q and $\bar{q}$ are tabulated Table 15.

Table 15 The Dirac confinement energy (in GeV) of Quark (c) and anti-quark (u or d) for few ground and excited states

State	Spin triplet		Spin singlet	
	E_D^Q	$E_D^{\bar{q}}$	E_D^Q	$E_D^{\bar{q}}$
1S	1.5261	0.4672	1.4498	0.3562
2S	1.8123	0.8389	1.7281	0.7342
1P	1.7264	0.7321	1.6348	0.6144
2P	1.9687	1.0268	1.8818	0.9232
1D	1.8664	0.9047	1.7765	0.7947
1F	2.0719	1.4726	1.9758	1.0351

Appendix B: Sensitivity calculation of the model parameters

We have used c quark mass = 1.27 GeV while the current quark mass for u and d as 0.003 GeV in the present study. This gives the mass of the D meson for 1^3S_1 = 2002.33 MeV and for 1^1S_0= 1872.68 MeV with other optimized model parameters as listed in Table 4. The PDG listed mass of c quark is 1.27 ± 0.02 GeV. While the mass of u and d quark is $2.16^{+0.49}_{-0.26}$ MeV and $4.67^{+0.48}_{-0.17}$ MeV; respectively. In order to test the sensitivity of the parameters, we have calculated the mass of $1S$ state incorporating error bars in mass parameter of charm quark only as up and down quark mass and their uncertainty are negligible (few MeV only).

Table 16 The effect on $1S$ mass (in MeV) due to uncertainty in charm quark mass (m_c)

m_c	% variation in m_c	State	Mass	% variation in mass
1.29	1.5	1^3S_1	2021.46	0.94
		1^1S_0	1892.56	1.05
1.25	1.5	1^3S_1	1983.75	0.93
		1^1S_0	1853.45	1.03

By considering the charm quark mass, $m_c = 1.27 + 0.02$ MeV as the upper bound and $m_c = 1.27 - 0.02$ MeV as the lower bound, the ground state masses of $D-$ mesons for (1^3S_1, 1^1S_0) states are computed. The results are given in Table 16. As one can see from Table 16 by recomputing the mass of $D-$ mesons by taking m_c = 1.29 GeV, the upper bound uncertainty of just 1.5 %, we find D meson mass for 1^3S_1 state as 2021.46 MeV and that for 1^1S_0 = 1892.56 MeV with a variation of about 1%. Similar computation using m_c = 1.25 GeV, (with the lower bound uncertainty of just 1.5 %) results into prediction of D meson mass for 1^3S_1 state as 1983.75 MeV and that for 1^1S_0 = 1853.45 MeV with a variation of about 1%.

The other model parameters λ, V_0 and σ are the optimized potential parameters which are fixed for known ground state. Below we present the effect on $1S$ mass due to an assumed 5% and 10% variations in the parameters keeping other two fixed. The results are shown in Table 17. One can see from Table 17 that the variations are not much compared to the optimized value of these parameters used in the present study. However, the value of λ is slightly more sensitive than that of V_0 and σ.

Table 17 The effect on 1S mass (in MeV) after % change in the potential parameters

	Change in λ keeping V_0 and σ fixed	% variation in mass	Change in V_0 keeping λ and σ fixed	% variation in mass	Change in σ keeping λ and V_0 fixed	% variation in mass
5%						
1^3S_1	1982.43	0.99	2002.32	0.49×10^{-3}	2001.88	0.02
1^1S_0	1854.37	0.97	1872.67	0.53×10^{-3}	1874.03	0.07
10%						
1^{3S}_1	1962.07	2.01	2002.31	0.99×10^{-3}	2001.43	0.04
1^1S_0	1835.58	1.98	1872.66	1.06×10^{-3}	1874.38	0.09

Appendix C:Illustration of uncertainty estimation in the coupling constants

In general the total decay width from the present study (say $X \times g_i^2$; $i = T, \tilde{H}, Y, \tilde{S}, Z$) can be compared with experimentally measured width (say Γ_{Exp}) to extract respective coupling constants.

The uncertainty in the estimations of the strong coupling constants as appear in Table 8 Column 4 due to uncertainty in the experimental width as shown in Table 8 Column 7 are computed as below.

$$X \times\ g_i^2 = \Gamma(Exp) \tag{C.1}$$

$$g_i^2 = \frac{\Gamma(Exp)}{X} \tag{C.2}$$

Taking the log and the derivative on both the sides we get,

$$\delta g_i = \frac{1}{2}\left[\frac{\delta\Gamma_{Exp}}{\Gamma_{Exp}} - \frac{\delta X}{X}\right] \times g_i \tag{C.3}$$

Here, the $\delta\Gamma_{ExP}$ is the uncertainty in Γ_{ExP} and δX that in X. In the present case $\delta X = 0$ results into

$$\delta g_i = \frac{1}{2}\left[\frac{\delta\Gamma_{Exp}}{\Gamma_{Exp}}\right] \times g_i \tag{C.4}$$

As a sample case, consider the total strong decay width $\Gamma(2460) = 286.39 g_T^2$ MeV from Table 8 and is equated to the experimentally measured width of $\Gamma(2460) = 47.0 \pm 0.8$ MeV, we get

Table 18 Value of various coupling constants obtained from the present study compared with other values available in the literature

	Present	[46]	[26]
g_T	0.40 ± 0.003	0.43 ± 0.05	0.43 ± 0.01
$g_{\tilde{H}}$	0.31 ± 0.017	0.14 ± 0.03	0.28 ± 0.01
g_Y	0.49 ± 0.039	0.53 ± 0.13	0.42 ± 0.02
$g_{\tilde{S}}$	0.10 ± 0.015	…	…
g_Z	0.12 ± 0.012	…	…

$$(282.39)\ g_T^2 = 47.0 \pm 0.8$$

$$g_T{}^2 = \frac{47.0}{286.39} \Rightarrow g_T = 0.40$$

and $\delta\Gamma_{Exp} = \pm\, 0.8$ MeV, then using Eq. (C.4)

$$\begin{aligned}\delta g_T &= \frac{1}{2}\left(\frac{\pm\, 0.8}{47.0}\right) \times 0.40 \\ &= \pm\, 0.0034 \\ g_T &= 0.40 \pm 0.003\end{aligned}$$

The same process is followed for g_z.

For computing the $g_{\tilde{H}}$ coupling constant we have two relations as shown in Table 8 . It follows that, the total decay width $\Gamma(2580) = 1101.39\tilde{g}_H^2 = 177.5 \pm 17.8$ MeV leads to

$$\begin{aligned}\delta g_{\tilde{H}} &= \frac{1}{2}\left(\frac{\pm\, 17.8}{177.5}\right) \times 0.40 \\ &= \pm\, 0.0200 \\ g_{\tilde{H}} &= 0.40 \pm\ 0.020\end{aligned}$$

we refer it here as, $g_{\tilde{H}1} = 0.40 \pm\ 0.020$

Similarly from the width of $\Gamma(2650) = 2627.78\tilde{g}_H^2 = 140.2 \pm 17.1$ MeV leads to

$$\begin{aligned}\delta g_{\tilde{H}} &= \frac{1}{2}\left(\frac{\pm\, 17.1}{140.2}\right) \times 0.23 \\ &= \pm\, 0.0140 \\ g_{\tilde{H}} &= 0.23 \pm 0.014\end{aligned}$$

We refer it as, $g_{\tilde{H}2} = 0.23 \pm 0.014$ Then the average g_H is obtained as

$$\begin{aligned}g_{\tilde{H}} &= \frac{g_{\tilde{H}1} + g_{\tilde{H}2}}{2} \\ &= 0.40 + 0.23 \\ &= 0.31\end{aligned} \tag{C.5}$$

and average uncertainty in this case is computed as the root mean square values

$$\delta g_{\tilde{H}} = \sqrt{\frac{(\delta g_{\tilde{H}1})^2+(\delta g_{\tilde{H}2})^2}{2}}$$
$$\delta g_{\tilde{H}} = \sqrt{\frac{(0.0200)^2+(0.0140)^2}{2}} = \pm 0.0172$$

Thus we obtain $g_{\tilde{H}} = 0.31 \pm 0.017$

The same method is followed for g_Y and $g_{\tilde{S}}$. The estimated uncertainty in g_i for $i = T, \tilde{H}, Y, \tilde{S}, Z$ are given in Table 18.

References

1. R. Aaij et al., Phys. Rev. D **94**(7), 072001 (2016)
2. R. Aaij et al., J. High Energy Phys. **2013**(9), 145 (2013)
3. P.M. del Amo Sanchez et al., Phys. Rev. D **82**(11), 111101 (2010)
4. N. Isgur, M.B. Wise, Phys. Lett. B **232**(1), 113 (1989)
5. S. Godfrey, K. Moats, Phys. Rev. D **93**(3), 034035 (2016)
6. M. Shah, B. Patel, P. Vinodkumar, Eur. Phys. J. C **76**(1), 36 (2016)
7. L.Y. Xiao, X.H. Zhong, Phys. Rev. D **90**(7), 074029 (2014)
8. X.H. Zhong, Q. Zhao, Phys. Rev. D **78**(1), 014029 (2008)
9. E.J. Eichten, C.T. Hill, C. Quigg, Phys. Rev. Lett. **71**(25), 4116 (1993)
10. M. Di Pierro, E. Eichten, Phys. Rev. D **64**(11), 114004 (2001)
11. Q.T. Song, D.Y. Chen, X. Liu, T. Matsuki, Phys. Rev. D **92**(7), 074011 (2015)
12. D.M. Li, P.F. Ji, B. Ma, Eur. Phys. J. C **71**(3), 1582 (2011)
13. F. Close, E. Swanson, Phys. Rev. D **72**(9), 094004 (2005)
14. L. Micu, Nucl. Phys. B **10**(3), 521 (1969)
15. A. Le Yaouanc, L. Oliver, O. Pene, J.C. Raynal, Phys. Rev. D **8**(7), 2223 (1973)
16. A. Le Yaouanc, L. Oliver, O. Pene, J.C. Raynal, Phys. Rev. D **9**(5), 1415 (1974)
17. A. Le Yaouanc, O. Pène, L. Oliver, J. Raynal, Resonant partial wave amplitudes in? n? n according to the naive quark pair creation model. Tech. rep., SIS-74-2600 (1974)
18. Y. Guo-Liang, W. Zhi-Gang, L. Zhen-Yu, M. Gao-Qing, Chin. Phys. C **39**(6), 063101 (2015)
19. H.G. Blundell, S. Godfrey, B. Phelps, Phys. Rev. D **53**(7), 3712 (1996)
20. H.G. Blundell, S. Godfrey, Phys. Rev. D **53**(7), 3700 (1996)
21. P. Colangelo, F. De Fazio, G. Nardulli, N. Di Bartolomeo, R. Gatto, Phys. Rev. D **52**(11), 6422 (1995)
22. P. Colangelo, F. De Fazio, S. Nicotri, Phys. Lett. B **642**(1–2), 48 (2006)
23. P. Colangelo, F. De Fazio, S. Nicotri, M. Rizzi, Phys. Rev. D **77**(1), 014012 (2008)
24. P. Colangelo, F. De Fazio, F. Giannuzzi, S. Nicotri, Phys. Rev. D **86**(5), 054024 (2012)
25. Z.G. Wang, Phys. Rev. D **83**(1), 014009 (2011)
26. Z.G. Wang, Phys. Rev. D **88**(11), 114003 (2013)
27. S. Campanella, P. Colangelo, F. De Fazio, Phys. Rev. D **98**(11), 114028 (2018)
28. A.F. Falk, T. Mehen, Phys. Rev. D **53**(1), 231 (1996)
29. I.W. Stewart, Nucl. Phys. B **529**, 62 (1998)
30. N. Barik, B. Dash, M. Das, Phys. Rev. D **31**(7), 1652 (1985)
31. M. Shah, B. Patel, P. Vinodkumar, Phys. Rev. D **93**(9), 094028 (2016)
32. T. Bhavsar, M. Shah, P. Vinodkumar, Eur. Phys. J. C **78**(3), 1 (2018)
33. P. Zyla et al., Prog. Theor. Exp. Phys. **2020**(8), 083C01 (2020)
34. W. Greiner et al., *Relativistic Quantum Mechanics*, vol. 2 (Springer, Berlin, 2000)
35. M. Shah, B. Patel, P. Vinodkumar, Phys. Rev. D **90**(1), 014009 (2014)
36. N. Barik, M. Das, Phys. Lett. B **120**(4–6), 403 (1983)
37. S. Jena, Pramana **21**(4), 247 (1983)
38. S. Jena, T. Tripati, Phys. Lett. B **122**(2), 181 (1983)
39. P. Vinodkumar, K. Vijayakumar, S. Khadkikar, Pramana **39**(1), 47 (1992)
40. S. Khadkikar, K. Vijayakumar, Phys. Lett. B **254**(3–4), 320 (1991)
41. F. Close, (Academic Press, New York, 1979)
42. A.P. Monteiro, K.B.V. Kumar et al., Nat. Sci. **2**(11), 1292 (2010)
43. B. Pandya, M. Shah, P. Vinodkumar, Eur. Phys. J. C **81**(2), 1 (2021)
44. A.F. Falk, M. Luke, Phys. Lett. B **292**(1–2), 119 (1992)
45. D. Becirevic, E. Chang, A.L. Yaouanc, arXiv preprint arXiv:1203.0167 (2012)
46. Z.G. Wang, Eur. Phys. J. Plus **129**(8), 186 (2014)
47. R. Aaij, Phys. Rev. D **93**, 119901 (2015)
48. R. Aaij, B. Adeva, M. Adinolfi, A. Affolder, Z. Ajaltouni, S. Akar, J. Albrecht, F. Alessio, M. Alexander, S. Ali et al., Phys. Rev. D **92**(3), 032002 (2015)
49. R. Aaij, B. Adeva, M. Adinolfi, A. Affolder, Z. Ajaltouni, S. Akar, J. Albrecht, F. Alessio, M. Alexander, S. Ali et al., Phys. Rev. D **91**(9), 092002 (2015)
50. Y. Sun, X. Liu, T. Matsuki, Phys. Rev. D **88**(9), 094020 (2013)
51. Z.G. Wang, Commun. Theor. Phys. **66**(6), 671 (2016)