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A Grand Unified model with Q_6 as the flavour symmetry

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Abstract. We present a non-minimal renormalizable SUSY $SU(5)$ model where the flavour sector exhibits a Q_6 flavour symmetry, and R-parity is conserved. We find that the CKM matrix has an NNI and a $Fritzsch$ texture for the down and up sectors respectively, both of which are known to be compatible with the experimental CKM values. At the same time, in the leptonic side, the model predicts a strong inverted hierarchy spectrum and a sum rule among the neutrino masses. As main results, the atmospheric ($\theta_{23}^{\ell th} = 46.18^{+0.66}_{-0.65}$) and solar ($\theta_{12}^{\ell th} = 36.62 \pm 4.06.$) mixing angles are found to be consistent with the experimental data. However, the reactor mixing angle value, $\theta_{13}^{\ell th} = 3.38^{+0.03}_{-0.02}$, is small and not in good agreement with the global experimental fits, but it is consistent with the MINOS experiment and fairly large in comparison to the tribimaximal scenario.

1. Introduction

The Q_6 flavour symmetry group has been proposed as responsible for the textures in the quark as well as in the leptonic sectors [1–5]. This appealing flavour symmetry allows the appearance of the NNI textures in the quark mass matrices [6,7], so that the mixing can be accommodated in good agreement with the experimental data [1,5]. The rich phenomenology Q_6 provides in supersymmetry (SUSY) scenarios is remarkable, such as prohibiting the dangerous terms that mediate fast proton decay [2,3]. Thus, an immediate question that arises is how to combine the Q_6 flavour symmetry with a GUT framework, in particular, with the SUSY $SU(5)$ model. Here we will address this problem and we will see that it is possible to accommodate the masses and mixings of fermions in such a framework.

2. SUSY $SU(5) \otimes U(1) \otimes Q_6$ MODEL

The assigned matter content under SUSY $SU(5) \otimes U(1) \otimes Q_6$ is displayed on Table 1. Let us comment on our notation and the matter content: ϕ and $\bar{\phi}$ are singlet scalars under the gauge group, the former gives mass to the right-handed neutrinos and the latter is introduced in order to cancel anomalies in the $U(1)$ abelian group. In addition, ϕ breaks the SUSY $SU(5)\otimes U(1)\otimes Q_6$ gauge group into SUSY $SU(5) \otimes Q_6$; Φ^a_b stands for the **24** adjoint scalar representation which breaks the SUSY $SU(5)$ gauge group to the MSSM; there are three families of Higgs type H^u_i and H^d_j. On the other hand, we do need to include H_{45} and $H_{\overline{45}}$ scalar representations so as to

	$SU(5)$	Q_6	$U(1)$
(H_1^d, H_2^d)	$\bar{\mathbf{5}}$	$\mathbf{2}_1$	$-x$
H_3^d	$\bar{\mathbf{5}}$	$\mathbf{1}_{+,2}$	$-x$
(H_1^u, H_2^u)	$\mathbf{5}$	$\mathbf{2}_1$	x
H_3^u	$\mathbf{5}$	$\mathbf{1}_{+,2}$	x
(F_1, F_2)	$\bar{\mathbf{5}}$	$\mathbf{2}_2$	$\frac{3x}{2}$
F_3	$\bar{\mathbf{5}}$	$\mathbf{1}_{-,3}$	$\frac{3x}{2}$
(T_1, T_2)	$\mathbf{10}$	$\mathbf{2}_2$	$-\frac{x}{2}$
T_3	$\mathbf{10}$	$\mathbf{1}_{-,3}$	$-\frac{x}{2}$
(N_1^c, N_2^c)	$\mathbf{1}$	$\mathbf{2}_2$	$-\frac{5x}{2}$
N_3^c	$\mathbf{1}$	$\mathbf{1}_{-,1}$	$-\frac{5x}{2}$
ϕ	$\mathbf{1}$	$\mathbf{1}_{+,2}$	$5x$
$\bar{\phi}$	$\mathbf{1}$	$\mathbf{1}_{+,2}$	$-5x$
$H_{\bar{45}}$	$\bar{\mathbf{45}}$	$\mathbf{1}_{+,2}$	$-x$
H_{45}	$\mathbf{45}$	$\mathbf{1}_{+,2}$	x
Φ	$\mathbf{24}$	$\mathbf{1}_{+,0}$	0

Table 1. Matter content in the SUSY $SU(5) \otimes U(1) \otimes Q_6$ model.

fix the incorrect relation $\mathbf{M}_d = \mathbf{M}_e^T$, this can also be achieved by means of higher dimensional operators [8–10]. Regarding the fermion sector, N_i^c denotes the right-handed neutrino, which is a singlet under the $SU(5)$ gauge group; F_i and T_j stand for the **5**-plets and the **10** antisymmetric-plet, respectively. Here, a, b, c are $SU(5)$ indices, and i, j are family indices. More explicitly,

$$\begin{aligned} F_{ia} &= (d^c, L)\,; \quad T_j^{ab} = \frac{1}{\sqrt{2}}\,(u^c, q, \ell^c)\,; \quad L = \begin{pmatrix} \nu_\ell \\ \ell \end{pmatrix}; \quad q = \begin{pmatrix} u \\ d \end{pmatrix}, \\ H^{ua} &= \begin{pmatrix} \mathtt{H}^u \\ \mathbf{H}^u \end{pmatrix}; \quad H_b^d = \begin{pmatrix} \mathtt{H}^d \\ \mathbf{H}^d \end{pmatrix}; \quad \mathbf{H}^u = \begin{pmatrix} h^{+u} \\ h^{0u} \end{pmatrix}; \quad \mathbf{H}^d = \begin{pmatrix} h^{0d} \\ h^{-d} \end{pmatrix}. \end{aligned} \tag{1}$$

Here, $\mathtt{H}^u$ and $\mathtt{H}^d$ are the coloured triplet scalars; $\mathbf{H}^d$ and $\mathbf{H}^u$ are identified as the weak doublets of the MSSM group. Since we are only interested in studying the masses and mixings, we do not dwell on the details of the full gauge symmetry breaking from SUSY $SU(5) \otimes U(1) \otimes Q_6$ to the MSSM group. We assume that the coloured Higgs triplets are very heavy, in order to avoid fast proton decay. Such subtle issues are part of a wider study of this particular model, that is still in progress. For the scalars fields, we use the following vacuum expectation values (vev's)

$$\begin{aligned} \langle H^u \rangle &= \begin{pmatrix} \mathbf{0} \\ \langle \mathbf{H}^u \rangle \end{pmatrix}; \quad \left\langle H^d \right\rangle = \begin{pmatrix} \mathbf{0} \\ \langle \mathbf{H}^d \rangle \end{pmatrix}; \quad \langle H_{45} \rangle_\alpha^{\alpha 5} = v_{45}; \quad \langle H_{45} \rangle_4^{45} = -3v_{45}, \\ \langle H_{\bar{45}} \rangle_{\alpha 5}^{\alpha} &= v_{\bar{45}}; \quad \langle H_{\bar{45}} \rangle_{45}^{4} = -3v_{\bar{45}}; \quad \langle \phi \rangle = v_s; \quad \left\langle \bar{\phi} \right\rangle = \bar{v}_s; \quad \alpha, \beta = 1, 2, 3. \end{aligned} \tag{2}$$

The superpotential, invariant under the Q_6 discrete group, is

$$\begin{aligned} W &= \sqrt{2} y_1^d (F_1 T_2 - F_2 T_1) H_3^d + \sqrt{2} y_2^d (F_1 T_3 H_2^d - F_2 T_3 H_1^d) + \sqrt{2} y_3^d F_3 (T_1 H_2^d - T_2 H_1^d) \\ &+ \sqrt{2} y_4^d F_3 T_3 H_3^d + \frac{y_1^u}{4} (T_1 T_2 - T_2 T_1) H_3^u + \frac{y_2^u}{4} (T_1 T_3 H_2^u - T_2 T_3 H_1^u) + \frac{y_3^u}{4} T_3 (T_1 H_2^u - T_2 H_1^u) \\ &+ \frac{y_4^u}{4} T_3 T_3 H_3^u + \sqrt{2} Y_1 (F_1 T_2 - F_2 T_1) H_{\bar{45}} + \sqrt{2} Y_2 F_3 T_3 H_{\bar{45}} + \sqrt{2} \tilde{Y}_1 (T_1 T_2 - T_2 T_1) H_{45} \\ &+ \sqrt{2} \tilde{Y}_2 T_3 T_3 H_{45} + y_1^n (N_1^c F_2 - N_2^c F_1) H_3^u + y_2^n (N_1^c F_3 H_2^u - N_2^c F_3 H_1^u) \\ &+ y_3^n N_3^c (F_1 H_2^u + F_2 H_1^u) + y_1^m (N_1^c \phi N_1^c + N_2^c \phi N_2^c) + y_2^m N_3^c \phi N_3^c. \end{aligned} \tag{3}$$

There is a missing term which is invariant under SUSY $SU(5)\otimes U(1)\otimes Q_6$, namely $y_{ij}N_i^c\bar{\phi}N_j^c$, however, it is not allowed by the $U(1)$ symmetry. From the above superpotential, using the scalar vev's given in Eq.(2), one must obtain the mass terms; working in the $SU(5)$ basis the Lagrangian for the mass terms is

$$\begin{array}{l}\mathcal{L}^q = -\bar{d}_{iR}\left(\mathbf{M}_d\right)_{ij} d_{jL} - \bar{u}_{iR}\left(\mathbf{M}_u\right)_{ij} u_{jL} + h.c.\,,\\ \mathcal{L}^l = -\bar{\ell}_{iR}\left(\mathbf{M}_\ell\right)_{ij} \ell_{jL} - \bar{N}_{iR}\left(\mathbf{M}_D\right)_{ij}\nu_{jL} - \frac{1}{2}\bar{N}_{iR}\left(\mathbf{M}_R\right)_{ij} N^c_{jR} + h.c.\,,\end{array}\tag{4}$$

where the fermion mass matrices have the following structures:

$$\begin{array}{rcl}\mathbf{M}_d &=& \begin{pmatrix} 0 & y_1^d h_3^{0d} + 2Y_1 v_{\bar{45}} & y_2^d h_2^{0d} \\ -y_1^d h_3^{0d} - 2Y_1 v_{\bar{45}} & 0 & -y_2^d h_1^{0d} \\ y_3^d h_2^{0d} & -y_3^d h_1^{0d} & y_4^d h_3^{0d} + 2Y_2 v_{\bar{45}} \end{pmatrix};\ \mathbf{M}_u = \begin{pmatrix} 0 & -2\tilde{Y}_1 v_{45} & \bar{y}^u h_2^{0u} \\ 2\tilde{Y}_1 v_{45} & 0 & -\bar{y}^u h_1^{0u} \\ \bar{y}^u h_2^{0u} & -\bar{y}^u h_1^{0u} & y_4^u h_3^{0u} \end{pmatrix}\\ \mathbf{M}_\ell &=& \begin{pmatrix} 0 & -\left(y_1^d h_3^{0d} - 6Y_1 v_{\bar{45}}\right) & y_3^d h_2^{0d} \\ y_1^d h_3^{0d} - 6Y_1 v_{\bar{45}} & 0 & -y_3^d h_1^{0d} \\ y_2^d h_2^{0d} & -y_2^d h_1^{0d} & y_4^d h_3^{0d} - 6Y_2 v_{\bar{45}} \end{pmatrix};\ \mathbf{M}_D = \begin{pmatrix} 0 & y_1^n h_3^{0u} & y_2^n h_2^{0u} \\ -y_1^n h_3^{0u} & 0 & -y_2^n h_1^{0u} \\ y_3^n h_2^{0u} & y_3^n h_1^{0u} & 0 \end{pmatrix}\end{array}\tag{5}$$

and $\bar{y}^u \equiv (y_2^u + y_3^u)/2$. At the same time, we get $\mathbf{M}_R = diag(M_{R_1}, M_{R_1}, M_{R_3})$. Therefore, after the type I see-saw mechanism one obtains the $\mathbf{M}_\nu = \mathbf{M}_D^T\mathbf{M}_R^{-1}\mathbf{M}_D$ effective neutrino mass matrix.

3. Masses and mixings

3.1. Quark and lepton masses

If we suppose that $h_2^{0u} = h_1^{0u} \equiv h^{0u}$ and $h_2^{0d} = h_1^{0d} \equiv h^{0d}$ in Eq. (5), then one realizes that the $\mathbf{M}_{(d,\ell)}$ and $\mathbf{M}_u$ mass matrices contain implicitly the NNI and Fritzsch textures respectively, which appear explicitly as follows: the above mass matrices are diagonalized by $\mathbf{U}_{f(R,L)}$ unitary matrices according to Eq.(4), thus one obtains, $\tilde{\mathbf{M}}_f = \mathbf{U}_{fR}^\dagger \hat{\mathbf{M}}_f \mathbf{U}_{fL}$, in general. Here, $\tilde{\mathbf{M}}_f = diag(\tilde{m}_{f_1}, \tilde{m}_{f_2}, 1)$ and $\hat{\mathbf{M}}_f = \mathbf{M}_f/m_{f_3}$ are dimensionless mass matrices.

Then, taking $\mathbf{U}_{f(R,L)} = \mathbf{U}_{\pi/4}\mathbf{u}_{f(R,L)}$, one can easily get $\tilde{\mathbf{M}}_f = \mathbf{u}_{fR}^\dagger \mathbf{m}_f \mathbf{u}_{fL}$ where

$$\mathbf{m}_f = \mathbf{U}_{\pi/4}^T \hat{\mathbf{M}}_f \mathbf{U}_{\pi/4} = \begin{pmatrix} 0 & \pm\tilde{A}_f & 0 \\ \mp\tilde{A}_f & 0 & -\sqrt{2}\tilde{B}_f \\ 0 & -\sqrt{2}\tilde{C}_f & \tilde{D}_f \end{pmatrix},\ \text{and}\ \mathbf{U}_{\pi/4} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}\tag{6}$$

Here, $\pm$ the upper (lower) sign corresponds to the $\mathbf{m}_d$ ($\mathbf{m}_\ell$ and $\mathbf{m}_u$) mass matrices. In addition, the $\mathbf{m}_f$ dimensionless coefficients can be read directly from Eq. (5), and we have to keep in mind that $\tilde{B}_u = \tilde{C}_u$ for the up quarks. The degeneracy in the vev's for two scalar fields modifies the effective neutrino mass matrix which is given explicitly as

$$\mathbf{M}_\nu = \begin{pmatrix} A_\nu^2 + B_\nu^2 & B_\nu^2 & A_\nu C_\nu \\ B_\nu^2 & A_\nu^2 + B_\nu^2 & A_\nu C_\nu \\ A_\nu C_\nu & A_\nu C_\nu & 2C_\nu^2 \end{pmatrix} \quad \text{with} \quad A_\nu = \sqrt{x} y_1^n h_3^{0u}, B_\nu = \sqrt{y} y_3^n h^u\ \text{and}\ C_\nu = \sqrt{x} y_2^n h^u \tag{7}$$

3.2. Quark and lepton mixings

We will describe briefly how to diagonalize the mass matrices, $\mathbf{m}_{(d,\ell)}$ and $\mathbf{m}_u$, respectively. Let us focus on the former one which has the NNI texture [6, 7]. We will follow the prescription applied to $\mathbf{m}_u$ in the case of Fritzsch mass textures [11, 12]. For a pedagogical review on how to diagonalize this type of matrices see ref. [13].

Going back to the expression $\tilde{\mathbf{M}}_f = \mathbf{u}_{fR}^\dagger \mathbf{m}_f \mathbf{u}_{fL}$, we are interested in obtaining the $\mathbf{u}_{fL}$ left-handed matrices necessary to construct the CKM matrix, for this we must build the bilineal form: $\tilde{\mathbf{M}}_f^\dagger \tilde{\mathbf{M}}_f = \mathbf{u}_{fL}^\dagger \mathbf{m}_f^\dagger \mathbf{m}_f \mathbf{u}_{fL}$. From this relation, we can factorize the CP phases that come from $\mathbf{m}_f^\dagger \mathbf{m}_f = \mathbf{Q}_f (\mathbf{m}_f^\dagger \mathbf{m}_f) \mathbf{Q}_f^\dagger$ see [13], such that, $\mathbf{Q}_f = \mathrm{diag}\,(1, \exp(-i\eta_{f_2}), \exp(-i\eta_{f_3}))$. Thus, we appropriately choose $\mathbf{u}_{fL} = \mathbf{Q}_f \mathbf{O}_{fL}$ where $\mathbf{O}_{fL}$ is the real orthogonal matrix that diagonalizes the $(\mathbf{m}_f^\dagger \mathbf{m}_f)$ matrix which we have not written here. Notice that $\mathbf{O}_{fL} = (\mid f_1\rangle, \mid f_2\rangle, \mid f_3\rangle)$, where the three eigenvectors have the following form

$$\mid f_i\rangle = N_{f_i} \begin{pmatrix} (\tilde{m}_{f_i}^2 - |\tilde{A}_f|^2 - |\tilde{C}_f|^2)|\tilde{A}_f||\tilde{B}_f| \\ (\tilde{m}_{f_i}^2 - |\tilde{A}_f|^2)|\tilde{C}_f||\tilde{D}_f| \\ (\tilde{m}_{f_i}^2 - |\tilde{A}_f|^2)(\tilde{m}_{f_i}^2 - |\tilde{A}_f|^2 - |\tilde{C}_f|^2) \end{pmatrix}, \mid f_3\rangle = N_{f_3} \begin{pmatrix} (1 - |\tilde{A}_f|^2 - |\tilde{C}_f|^2)|\tilde{A}_f||\tilde{B}_f| \\ (1 - |\tilde{A}_f|^2)|\tilde{C}_f||\tilde{D}_f| \\ (1 - |\tilde{A}_f|^2)(1 - |\tilde{A}_f|^2 - |\tilde{C}_f|^2) \end{pmatrix}. \quad (8)$$

Here, N_{f_i} $(i = 1, 2)$ and N_{f_3} stand for the normalization factors whose definition must be read directly from the above expression. Three dimensionless free parameters can be fixed in terms of the physical masses and $|\tilde{D}_f| \equiv y_f^2$, which is the only dimensionless free parameter [6, 7]. Explicitly, these are given by

$$|\tilde{A}_f| = \frac{q_f}{y_f};\ |\tilde{B}_f| = \sqrt{\frac{1 + P_f - y_f^4 - R_f}{2} - \left(\frac{q_f}{y_f}\right)^2};\ |\tilde{C}_f| = \sqrt{\frac{1 + P_f - y_f^4 + R_f}{2} - \left(\frac{q_f}{y_f}\right)^2}, \quad (9)$$

where

$$P_f = \tilde{m}_{f_1}^2 + \tilde{m}_{f_2}^2,\ q_f = \sqrt[4]{\tilde{m}_{f_1}^2 \tilde{m}_{f_2}^2},\ R_f = \sqrt{\left(1 + P_f - y_f^4\right)^2 - 4\left(P_f + q_f^4\right) + 8 q_f^2 y_f^2}. \quad (10)$$

Hence, there are two free parameters, y_f $(f = d, \ell)$, in the leptonic sector. So far, we have found the $\mathbf{U}_{(d,\ell)L}$ left-handed matrices.

Let us now focus on $\mathbf{m}_u$, where we must remember that $|\tilde{B}_u| = |\tilde{C}_u|$. In this case, one can fix the three free parameters in terms of the physical masses. Explicitly, we obtain

$$|\tilde{A}_u| = \sqrt{\frac{\tilde{m}_u \tilde{m}_c}{1 - \tilde{m}_c + \tilde{m}_u}},\ |\tilde{B}_u| = \sqrt{\frac{(1 - \tilde{m}_c)(1 + \tilde{m}_u)(\tilde{m}_c - \tilde{m}_u)}{1 - \tilde{m}_c + \tilde{m}_u}},\ |\tilde{D}_u| = 1 - \tilde{m}_c + \tilde{m}_u. \quad (11)$$

Following the analysis previously described, the $\mathbf{O}_{uL}$ orthogonal matrix that diagonalizes $(\mathbf{m}_u^\dagger \mathbf{m}_u)$ is fixed in terms of the above mentioned parameters. Thus, using the expression given in Eq. (8), we get

$$\mathbf{O}_{uL} = \begin{pmatrix} -\sqrt{\frac{\tilde{m}_c(1 - \tilde{m}_c)}{(1 - \tilde{m}_u)(\tilde{m}_c + \tilde{m}_u)(1 - \tilde{m}_c + \tilde{m}_u)}} & -\sqrt{\frac{\tilde{m}_u(1 + \tilde{m}_u)}{(1 + \tilde{m}_c)(\tilde{m}_c + \tilde{m}_u)(1 - \tilde{m}_c + \tilde{m}_u)}} & \sqrt{\frac{\tilde{m}_u \tilde{m}_c(\tilde{m}_c - \tilde{m}_u)}{(1 - \tilde{m}_u)(1 + \tilde{m}_c)(1 - \tilde{m}_c + \tilde{m}_u)}} \\ -\sqrt{\frac{\tilde{m}_u(1 - \tilde{m}_c)}{(1 - \tilde{m}_u)(\tilde{m}_c + \tilde{m}_u)}} & \sqrt{\frac{\tilde{m}_c(1 + \tilde{m}_u)}{(1 + \tilde{m}_c)(\tilde{m}_c + \tilde{m}_u)}} & \sqrt{\frac{(\tilde{m}_c - \tilde{m}_u)}{(1 - \tilde{m}_u)(1 + \tilde{m}_c)}} \\ \sqrt{\frac{\tilde{m}_u(1 + \tilde{m}_u)(\tilde{m}_c - \tilde{m}_u)}{(1 - \tilde{m}_u)(\tilde{m}_c + \tilde{m}_u)(1 - \tilde{m}_c + \tilde{m}_u)}} & -\sqrt{\frac{\tilde{m}_c(1 - \tilde{m}_c)(\tilde{m}_c - \tilde{m}_u)}{(1 + \tilde{m}_c)(\tilde{m}_c + \tilde{m}_u)(1 - \tilde{m}_c + \tilde{m}_u)}} & \sqrt{\frac{(1 + \tilde{m}_u)(1 - \tilde{m}_c)}{(1 - \tilde{m}_u)(1 + \tilde{m}_c)(1 - \tilde{m}_c + \tilde{m}_u)}} \end{pmatrix} \quad (12)$$

Therefore, the full left-handed unitary matrices are given, in general, by $\mathbf{U}_{fL} = \mathbf{U}_{\pi/4} \mathbf{u}_{fL}$, where $\mathbf{u}_{fL} = \mathbf{Q}_f \mathbf{O}_{fL}$. Then, the CKM matrix may be completely determined by $\mathbf{V}_{CKM} = \mathbf{U}_{uL}^\dagger \mathbf{U}_{dL} = \mathbf{O}_{uL}^T \mathbf{Q}_q \mathbf{O}_{dL}$, where we have defined $\mathbf{Q}_q = \mathbf{Q}_u^\dagger \mathbf{Q}_d$. In this way, the CKM matrix can be obtained either analytically or numerically. However, in here we are just interested in getting a numerical expression for it.

In the leptonic sector, we have that $\mathbf{U}_{\ell L} = \mathbf{U}_{\pi/4} \mathbf{Q}_\ell \mathbf{O}_{\ell L}$, but we need to obtain the neutrino mixing contribution in order to have a complete $PMNS$ matrix which is defined as $\mathbf{V}_{PMNS} = \mathbf{U}_{\ell L}^\dagger \mathbf{U}_{\nu L} \mathbf{K}$, we shall neglect the $\mathbf{K}$ Majorana phases. The $\mathbf{M}_\nu$ mass matrix given in Eq. (7) is diagonalized by $\mathbf{U}_\nu = u_{\pi/4} \mathbf{P}_\nu^\dagger \mathbf{O}_\nu$, that is, $\tilde{\mathbf{M}}_\nu = \mathrm{diag}\,(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) = \mathbf{O}_\nu^T \hat{\mathbf{M}}_\nu \mathbf{O}_\nu$ where $\mathbf{P}_\nu = diag(\exp i\eta_{\nu_1}, \exp i\eta_{\nu_2}, \exp i\eta_{\nu_1})$, and

$$u_{\pi/4} = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \end{pmatrix}; \quad \hat{\mathbf{M}}_\nu = \begin{pmatrix} |A_\nu|^2 + 2|B_\nu|^2 & \sqrt{2}|A_\nu||C_\nu| & 0 \\ \sqrt{2}|A_\nu||C_\nu| & 2|C_\nu|^2 & 0 \\ 0 & 0 & |A_\nu|^2 \end{pmatrix}. \quad (13)$$

A necessary condition to factorize the phases in a similar way to the quark sector, is that the A_ν^2 and B_ν^2 phases must be aligned [13]. On the other hand, $|A_\nu|$, $|B_\nu|^2$ and $|C_\nu|^2$ are determined by the physical masses

$$|A_\nu|^2 = m_{\nu_3};\ |B_\nu|^2_\mp = \frac{1}{4}\left(m_{\nu_2} + m_{\nu_1} - m_{\nu_3} \mp R_\nu\right);\ |C_\nu|^2_\pm = \frac{1}{4}\left(m_{\nu_2} + m_{\nu_1} - m_{\nu_3} \pm R_\nu\right), \quad (14)$$

where $R_\nu = \sqrt{\left(m_{\nu_2} + m_{\nu_1} - m_{\nu_3}\right)^2 - 4m_{\nu_2}m_{\nu_1}}$. A straightforward analysis on the inverted and normal hierarchies among the neutrino masses rules out the latter one. For the inverted case ($m_{\nu_2} > m_{\nu_1} > m_{\nu_3}$), $|B_\nu|^2_-$ and $|C_\nu|^2_+$ turn out to be real and positive, if and only if, m_{ν_3} is tiny. Actually, from R_ν we obtain the following sum rule, $m_{\nu_3} \leq (\sqrt{m_{\nu_2}} - \sqrt{m_{\nu_1}})^2$, where the equality in the above expression means an upper bound for the m_{ν_3} mass. Having fixed $|B_\nu|^2_-$ and $|C_\nu|^2_+$ in terms of the physical neutrino masses, the $\mathbf{O}_\nu$ matrix is well determined by them. Explicitly, we obtain

$$\mathbf{O}_\nu = \begin{pmatrix} \sqrt{\frac{m_{\nu_3}\left(m_{\nu_2} + m_{\nu_1} - m_{\nu_3} + R_\nu\right)}{\left(m_{\nu_2} - m_{\nu_1}\right)\left(m_{\nu_2} - m_{\nu_1} + m_{\nu_3} - R_\nu\right)}} & \sqrt{\frac{m_{\nu_3}\left(m_{\nu_2} + m_{\nu_1} - m_{\nu_3} + R_\nu\right)}{\left(m_{\nu_2} - m_{\nu_1}\right)\left(m_{\nu_2} - m_{\nu_1} - m_{\nu_3} + R_\nu\right)}} & 0 \\ -\sqrt{\frac{m_{\nu_2} - m_{\nu_1} + m_{\nu_3} - R_\nu}{2\left(m_{\nu_2} - m_{\nu_1}\right)}} & \sqrt{\frac{m_{\nu_2} - m_{\nu_1} - m_{3\nu} + R_\nu}{2\left(m_{\nu_2} - m_{\nu_1}\right)}} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (15)$$

Therefore, $\mathbf{V}_{PMNS} = \mathbf{U}_\ell^\dagger \mathbf{U}_\nu = \mathbf{O}_{\ell L}^T \mathbf{Q}_\ell^\dagger \mathbf{S}_{23} \mathbf{P}_\nu^\dagger \mathbf{O}_{\nu L}$, where $\mathbf{S}_{23} = \mathbf{U}_{\pi/4}^T u_{\pi/4}$. Comparing this matrix with the standard parametrization given in [14], we find that the reactor, atmospheric and solar mixing angles are completely determined as follows

$$|\sin\theta_{13}| = |O_{21\ell}|;\ |\sin\theta_{23}| = \frac{|O_{22\ell}|}{\sqrt{1 - |O_{21\ell}|^2}};\ |\tan\theta_{12}|^2 = \frac{|O_{11\ell}O_{12\nu} + O_{31\ell}O_{22\nu}\exp\left(i\bar{\eta}_{3e}\right)|^2}{|O_{11\ell}O_{11\nu} + O_{31\ell}O_{21\nu}\exp\left(i\bar{\eta}_{3e}\right)|^2}. \quad (16)$$

There is a remarkable coincidence between the above formulas and those presented in refs. [15,16].

4. Results

4.1. CKM mixing matrix

As we showed before, the CKM matrix is defined as $\mathbf{V}_{CKM} = \mathbf{U}_{uL}^\dagger \mathbf{U}_{dL} = \mathbf{O}_{uL}^T \mathbf{Q}_q \mathbf{O}_{dL}$, where the orthogonal matrices depend on the physical quark masses, to be more explicit, they depend on their ratios, that is on m_u/m_t, m_c/m_t and m_d/m_b, m_s/m_b for the up and down quarks, respectively. Then, knowing those ratios, we just need to tune the free parameters y_d and the two CP-violating phases to get a reliable CKM matrix. This numerical analysis is still in progress.

4.2. PMNS mixing matrix

In order to fix the y_e free parameter, we performed a χ^2 analysis using the theoretical expressions for the reactor and atmospheric angles given in Eq. (16) and the following experimental values: $\sin^2 2\theta_{13}^\ell = 0.076 \pm 0.068$ [17] and $\sin^2\theta_{23}^\ell = 0.52 \pm 0.06$ [18]. Also, we consider the following values for the charged lepton masses values: $m_e = 0.51099$ MeV, $m_\mu = 105.6583$ MeV and $m_\tau = 1776.82$ MeV [14]. As a result, we obtain that $y_e = 0.8478^{+0,0045}_{-0.0046}$ is the best fit at 1σ, moreover, the best theoretical values (at 1σ) for the reactor and atmospheric angles that come out from this analysis are

$$\theta_{23}^{\ell th} = 46.18^{+0.66}_{-0.65} \quad \text{and} \quad \theta_{13}^{\ell th} = 3.38^{+0.03}_{-0.02} \quad \text{where} \quad \chi^2_{bf} = 0.85 \quad (17)$$

The solar angle, which now depends on the neutrino masses and one Dirac phase, can be determined using the neutrino mass sum rule. This is written in terms of the observables $\Delta m^2_\odot$ and Δm^2_{ATM} as

$$m_{\nu_3} \leq \left(\sqrt[4]{m^2_{\nu_3} + \Delta m^2_\odot + \Delta m^2_{ATM}} - \sqrt[4]{m^2_{\nu_3} + \Delta m^2_{ATM}} \right)^2. \tag{18}$$

Using the experimental results on $\Delta m^2_\odot$ and Δm^2_{ATM} [18], we obtain, $0 \leq m_{\nu_3} \leq 4 \times 10^{-6}$ eV. As a result, the m_{ν_2} and m_{ν_1} neutrino masses are easily calculated taking $m_{\nu_3} = 3.9 \times 10^{-6}$ eV. Therefore, $m_{\nu_2} = 0.05080$ eV and $m_{\nu_1} = 0.04987$ eV.

Having done that, the neutrino masses are not any more free parameters in the solar mixing angle expression given in Eq. (16). As consequence, it is easy to calculate the solar mixing angle with a particular value for the $\bar{\eta}_{3e} = \pi$ Dirac phase and using the y_e value at 90% at C.L, we obtain

$$\theta^{\ell th}_{12} = 36.62 \pm 4.06. \tag{19}$$

Although, the atmospheric and solar angle are in good agreement with the experimental data, the reactor angle is not consistent with the global fits, although it is not completely negligible and it is large in comparison to the tribimaximal scenario, $\theta^{\ell th}_{13} = 3.38^{+0.03}_{-0.02}$. This value may be enhanced to the current experimental one by considering that the first and second right-handed neutrinos are not degenerate in mass, that is, $M_{R_1} \neq M_{R_2}$ in the right handed neutrino mass matrix. These results will be presented very soon.

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