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Non-Local String Theories on $AdS_3 \times S^3$ and Stable Non-Supersymmetric Backgrounds

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We exhibit a simple class of exactly marginal "double-trace" deformations of two dimensional CFTs which have AdS_3 duals, in which the deformation is given by a product of left and right-moving $U(1)$ currents. In this special case the deformation on AdS_3 is generated by a local boundary term in three dimensions, which changes the physics also in the bulk via bulk-boundary propagators. However, the deformation is non-local in six dimensions and on the string worldsheet, like generic non-local string theories (NLSTs). Due to the simplicity of the deformation we can explicitly make computations in the non-local string theory and compare them to CFT computations, and we obtain precise agreement. We discuss the effect of the deformation on closed strings and on D-branes. The examples we analyze include a supersymmetry-breaking but exactly marginal "double-trace" deformation, which is dual to a string theory in which no destabilizing tadpoles are generated for moduli nonperturbatively in all couplings, despite the absence of supersymmetry. We explain how this cancellation works on the gravity side in string perturbation theory, and also non-perturbatively at leading order in the deformation parameter. We also discuss possible flat space limits of our construction.

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1. Introduction

One interesting direction of research in string/M theory concerns novel phases of the theory. Examples include non-commutative Yang-Mills theory and non-geometrical phases of string compactifications. Although such phases may appear to be exotic, in some cases they are generic, in the sense that returning to more conventional backgrounds requires tuning a superselection parameter to a special value. These novel backgrounds are very much worth studying, both because of their intrinsic interest and because of the hope that their unconventional physics may play a role in solving open problems that remain in formulating and applying the theory (such as the cosmological constant problem).

In [1] we found strong evidence for a new type of perturbative string theory, non-local string theory (NLST), arising on the gravity side of AdS/CFT [2,3,4,5] dual pairs whose field theory side is deformed by a "multi-trace" operator[4]. In such theories, the "exotic" phase is generic, since it is obvious on the field theory side of the duality that one has to tune parameters in order to get back to the conventional theory, so the conventional string theory occupies a set of measure zero in the space of theories. These theories are gravitational, and have many intriguing features outlined in [1]. In a perturbative string description, the perturbative expansion in the deformation is reproduced by shifting the worldsheet action by a bilocal term of the general form

$$\delta S_{ws} = \sum_{I,J} \tilde{h}_{IJ} \int d^2 z_1 V^{(I)}[y(z_1)] \int d^2 z_2 V^{(J)}[y(z_2)], \tag{1.1}$$

where $V^{(I)}$ are some vertex operators in the string theory each including a factor of the string coupling g_s (in the examples of [1] the index I was continuous), and $y(z)$ are the embedding coordinates of the string worldsheet (or any other fields on the worldsheet). In [1] examples of double-trace deformations which were relevant or marginal in the dual CFT were exhibited. It was shown that these deformations could not be accounted for by

[4] We will use the names "single-trace" and "multi-trace" operators for any CFT which has a weakly-curved AdS dual, though the operators can only be represented in terms of traces in the case of four dimensional gauge theories. By a "single-trace" operator we will mean an operator which is dual to a single particle in string theory (for example, a KK mode of the graviton), while "multi-trace" operators will appear in the OPE of such operators. The distinction between these classes of operators is not always clear (see, e.g., [6]), but it can be made in an obvious way for operators of low dimension when the background is weakly curved (such "single-trace" operators correspond simply to supergravity fields) and this is all that we will use here.

local 10-dimensional supergravity, and that, in perturbation theory in the strength $\tilde{h}$ of the deformation, the changes in CFT correlators are formally reproduced by the shift (1.1) in the worldsheet action. This leads to a new type of diagrammatic expansion encoding the perturbation theory in both $\tilde{h}$ and g_s which has many interesting novel features. In particular, at a given order n in the g_s expansion, one has contributing diagrams which do not have the modular properties of genus n Riemann surfaces.

In these theories, some sectors are affected by the deformation at leading order in g_s (classically on the gravity side), while other sectors are not. For instance, exclusive graviton scattering along the AdS directions remains the same at tree level on the gravity side [1]. This parametric hierarchy between an approximately local sector and a completely non-local sector for small string coupling on the gravity side may potentially render these theories more viable as physical models than they would be otherwise.

The examples of [1] involved string theory in RR backgrounds, so it was difficult to make the formal expression (1.1) more explicit, due to the current limitations on our understanding of RR backgrounds in string theory. It is important to study more explicitly the conformal perturbation expansion around the undeformed background, in order to understand how divergences arising in conformal perturbation theory are regularized from the point of view of both sides of the duality, and in order to make progress on the larger questions regarding the consistency, degree of non-locality, and applications of the new theories.

In this paper, we present a rather explicit example of an interesting "double-trace" deformation in the Neveu-Schwarz version of AdS_3/CFT_2 arising from the low energy/near horizon limit of a system of Q_1 fundamental strings and Q_5 NS 5-branes [2]. In the dual CFT this deformation is of the form $\delta S_{CFT} \simeq \frac{\tilde{h}}{Q_1 Q_5} \int d^2x J(x)\tilde{J}(\bar{x})$ where J and $\tilde{J}$ are left and right moving global symmetry currents in the dual CFT. By using the explicit string theory description of undeformed AdS_3/CFT_2 that has been developed in recent years (see for example the comprehensive analysis in [7] and references therein) – in particular the formalism of [8,9] for vertex operators and correlation functions and the semiclassical analysis of [10] – we are able to analyze explicitly many aspects of this deformation. In particular, we check explicitly the absorption of divergences in conformal perturbation theory.

This deformation has an interesting physical property. It is exactly marginal but at the same time, if J and $\tilde{J}$ are $U(1)$ currents in the R-symmetry group, it breaks supersymmetry. Applying the basic relation between conformal invariance and AdS isometries [2] to nonsupersymmetric systems leads to an interesting element in the duality dictionary [11]. Namely, when there is a non-supersymmetric hypersurface of RG fixed points, a destabilizing potential for moduli is not generated along this hypersurface despite the absence of supersymmetry.

Our model provides for the first time an example realizing this possibility where the fixed surface exists for finite values of the string coupling. The price of this (which may end up being a positive feature) is that the fixed surface includes a "double-trace" deformation which controls the strength of supersymmetry breaking. Perturbatively in the string coupling g_s, and also non-perturbatively in g_s at first order in $\tilde{h}$, we find a simple cancellation mechanism that reproduces the cancellation of the moduli potential directly on the gravity side. For higher orders in $\tilde{h}$ we do not yet understand directly the way the cancellation occurs beyond string perturbation theory on the gravity side; this is a very intriguing prediction of the duality. The supersymmetry breaking in this model is "hard", in that the supersymmetry-breaking splittings of the masses (which are related to the splittings between the dimensions of corresponding operators in the dual CFT) grow with the masses. Unfortunately, the supersymmetry breaking effects are small – they disappear when we take the flat space limit, so that this does not yet provide a basis for a realistic theory of supersymmetry breaking. However, the cancellation of tadpoles for moduli is nontrivial in our model for finite AdS radius, since the (vanishing) moduli tadpoles are hierarchically smaller than the scale of supersymmetry breaking.

Given this prediction for stability after supersymmetry breaking, and more generally in the interest of clarifying the physics of NLST's, it is important to study the effects of the deformation on bulk physics on the gravity side of the correspondence.

The deformation has interesting effects on both the perturbative and non-perturbative sectors of the theory. The dimensions of operators corresponding to charged particles propagating in AdS are changed by the deformation. As far as the perturbative sector is concerned, because the "double-trace" deformation in this specific case involves vertex operators which are total derivatives on the worldsheet, we find semiclassically in Euclidean space that this causes the deformation of closed string diagrams to be localized near the boundary of AdS space. In Lorentzian space we do not expect this to be the case, and we present some indirect evidence (coming from the behavior of amplitudes in the flat space limit) that in Lorentzian space closed string amplitudes are affected in the bulk.

We also study explicitly the dynamics of D-branes. Diagrams involving D-branes have explicit bulk effects which are evident semiclassically in Euclidean space, and we explicitly compute the contribution of the deformation to bulk forces between D-branes.

We also discuss the deformation in the language of the low energy effective theory. The deformation we perform is by a product of currents, each of which is dual to a gauge field in the bulk with a Chern-Simons coupling at leading order in the low-energy expansion (see, for instance, [12]). The deformation of the dual CFT action by a product of chiral and antichiral currents can be identified with a local deformation of the boundary (surface) terms in the gravity-side $2+1$-dimensional Chern-Simons theory in a standard way [13,14,15]. This description is equivalent in this case to our description (1.1) (both descriptions lead to the same perturbation expansion involving insertions of bulk-boundary propagators), and leads equivalently to interesting bulk physics such as novel contributions to forces between D-branes. It is also worth emphasizing that even though the surface term is local in the $3d$ action on AdS_3, it is non-local in the $6d$ action on $AdS_3 \times S^3$, with a non-locality scale given by the AdS curvature radius. We will mostly use the formalism (1.1) which generalizes to other cases of NLSTs and "double-trace" deformations. It is interesting that in this simple case the NLST results obtained from a non-local shift in the worldsheet action can be reproduced by a change in the $3d$ local action involving boundary terms in spacetime.

The construction of a stable non-supersymmetric background in perturbative string theory (with flat moduli and maximal symmetry in the noncompact dimensions) provides one potential application of these theories. More generally, it is important to articulate the conditions for consistency of this type of theory directly in string theory language, in order to understand whether this phenomenon goes beyond the fascinating but somewhat esoteric realm of AdS spacetimes. In this work, we find that a particular scaling of the deformation leaves interesting effects in the flat space limit. It is not clear if this limit defines a consistent theory or not, but if it does then this may provide an avenue towards understanding more general realizations of NLST's[5].

The $3d$ boundary term which generates our deformation affects the bulk in AdS in two ways. One has to do with the analogy between AdS and a finite box – it takes some modes

[5] In a companion project [16], we are investigating the role of NLST's in describing squeezed states, such as those that occur in particle production processes in time dependent backgrounds, in perturbative string theory.

a finite time to reach the boundary. Another way in which the boundary can affect the bulk is via the fact that the boundary deformation existed for an infinite time in the past. The latter effect survives in the flat limit, along with severe non-locality felt by modes with momentum along the dimensions descending from the S^3.

This paper is organized as follows. In §2, we introduce the basic deformation on the field theory side and then translate it to the gravity side using the vertex operators of [9]. In §3, we study the effects of the deformation on closed string correlators. In §4 the description of the deformation in the low-energy effective theory in three dimensions is discussed. As mentioned above, this is simply given by a local boundary term in this case. Then, in §5, we calculate corrections to forces between D-branes (and to the instanton action of D-instantons) induced by the NLST deformation. Finally, in §6 we exhibit a scaling of the deformation parameter in which these effects survive in the flat space limit.

2. The Deformation

In this section we introduce the "double-trace" deformation we are turning on and calculate its effects on correlators on the CFT side. We then translate the deformation to the gravity side language using the vertex operators of [9]. In the subsequent sections we will calculate the effects of the deformation on physical quantities directly on the gravity side.

2.1. Field Theory Side

Consider an AdS_3 background of superstring theory which is dual to a two dimensional (super-)conformal CFT containing holomorphic and antiholomorphic $U(1)$ affine Lie algebras of level k generated by currents $J(x)$ and $\tilde{J}(\bar{x})$ (obeying $J(x)J(0) \sim k/x^2$). For example, in cases where the dual CFT has $\mathcal{N} = (4,4)$ supersymmetry, there is an $SU(2) \times SU(2)$ R-symmetry and we will be interested in a $U(1) \times U(1)$ subgroup of this. The dual CFT could also include sigma-models on circles (there are 8 such circles in the CFT which is dual to string theory on $AdS_3 \times S^3 \times T^4$, which is related by marginal deformations to the sigma-model on $[(T^4)^N/S_N \times T^4]$ [17]), in which case we can choose J and $\tilde{J}$ to be the generators of the corresponding isometries.

Our main interest is in the deformation of the dual CFT by

$$\delta S_{CFT} = h \int d^2x J(x)\tilde{J}(\bar{x}), \tag{2.1}$$

where h will be normalized shortly. This deformation is exactly marginal (as can be seen for example by bosonizing the currents). In the case that J and $\tilde{J}$ are part of the R-symmetry group of a superconformal theory, this deformation completely breaks the supersymmetry. This combination of exact marginality and SUSY breaking is very interesting, as it means for example that no destabilizing potential for moduli is generated in the dual string theory at all orders and nonperturbatively.

Many aspects of the effect of the deformation on the dual CFT can be calculated exactly, since the currents involved in the deformation (2.1) can be bosonized. It will be convenient to use such a bosonized description, in which we identify $J(x) = \sqrt{2k}\partial_x \eta(x,\bar{x})$ and $\tilde{J}(\bar{x}) = \sqrt{2k}\partial_{\bar{x}}\tilde{\eta}(x,\bar{x})$, where η and $\tilde{\eta}$ are canonically normalized scalar fields.

In the case of the CFT dual to the near horizon limit, $AdS_3 \times S^3 \times T^4$, of Q_1 fundamental strings and Q_5 NS5-branes on a T^4, the parameters of the CFT and those of the background are related as follows[6]. The central charge of the dual $\mathcal{N} = (4,4)$ SCFT is $c = 6Q_1Q_5$ (up to a correction of order one which we will ignore, since we will be interested in the perturbative weakly-curved limit of $Q_1 \gg Q_5 \gg 1$), and the level of its $SU(2)$ affine Lie algebra is $k = 2Q_1Q_5$. The gravity side AdS radius in string units is $\sqrt{Q_5}$, and the six-dimensional string coupling on $AdS_3 \times S^3$ is $g_6 = \sqrt{Q_5/Q_1}$. Therefore powers of g_6 correspond to powers of $1/\sqrt{Q_1}$; this will be important in comparing gravity side diagrams to the expansion of correlation functions on the field theory side.

Let us proceed with the analysis for the $U(1)$ currents coming from the $SU(2)$ R-symmetry, for definiteness. In this case we have

$$J(x)J(0) \sim \frac{2Q_1Q_5}{x^2}. \tag{2.2}$$

This scales as $1/g_6^2$, which is appropriate since it is related by the duality to a classical kinetic term for bulk gauge fields. In the bosonized language we can write our deformation in this case as

$$\frac{\tilde{h}(Q_5)}{Q_1Q_5}\int d^2x J(x)\tilde{J}(\bar{x}) = 4\tilde{h}(Q_5)\int d^2x \partial\eta\bar{\partial}\tilde{\eta}, \tag{2.3}$$

where we normalized the coefficient using the fact [1] that the deformation should scale

[6] In this case it was argued in [17] that the CFT which is dual to the perturbative string theory actually includes some specific terms of the form (2.1). So, in this case our discussion will refer to adding additional terms of this type beyond the terms which are already present in the "standard" string theory.

as g_s^2 in order to get a reasonable perturbation expansion[7], and we defined $h \equiv \tilde{h}/Q_1Q_5$, where apriori $\tilde{h}$ can have an arbitrary dependence on $Q_5 \sim L_{AdS}^2/l_s^2$. This normalization is natural from the dual CFT point of view, since at a generic point of the field theory moduli space Q_5/Q_1 plays no special role, but the central charge is always proportional to Q_1Q_5. On the string theory side a more natural choice might be $h \equiv \tilde{h}' g_6^2 = \tilde{h}' Q_5/Q_1$ which differs from the choice above by Q_5^2; we will see that indeed this choice will be more natural when we discuss the flat space limit in §6.

The operators of the dual CFT are of the form

$$\mathcal{O}_I = e^{i(p_I\eta + \tilde{p}_I\tilde{\eta})} P_I(\partial^n\eta, \bar{\partial}^{\tilde{n}}\tilde{\eta})\hat{O}_I, \tag{2.4}$$

where $P_I(\partial^n\eta, \bar{\partial}^{\tilde{n}}\tilde{\eta})$ denotes a polynomial in arbitrary derivatives of $\eta, \tilde{\eta}$, and where $\hat{O}_I$ is an operator in the coset obtained after dividing by the $U(1) \times U(1)$ bosonized by $\eta, \tilde{\eta}$. It is important to emphasize that there is a particular correlation between the coset part $\hat{O}_I$ and the free part $e^{i(p_I\eta + \tilde{p}_I\tilde{\eta})} P_I(\partial^n\eta, \bar{\partial}^{\tilde{n}}\tilde{\eta})$ encoded in the set of operators which exist in the CFT. In our main example where J and $\tilde{J}$ are part of the R-symmetry of the dual CFT, different components of the spacetime supermultiplets in the undeformed theory have different R-charges $q, \tilde{q}$. The deformation (2.3) breaks supersymmetry as it couples to these different components according to their charges. These R-charges are $SU(2)$ charges: we thus have $J(x)e^{ip\eta}(0) \sim qe^{ip\eta}(0)/x$ where q is the $SU(2)$ weight (integer or half integer) of the operator. This means that the charges $p, \tilde{p}$ which exist in the theory scale as

$$p \sim q/\sqrt{4Q_1Q_5}, \qquad \tilde{p} \sim \tilde{q}/\sqrt{4Q_1Q_5}. \tag{2.5}$$

The simplicity of our deformation (2.1) allows us to determine explicitly the effect of the deformation on correlation functions of the $\mathcal{O}_I$, starting from the basic Ward identities

$$\begin{aligned} J(x)J(0) &\sim \frac{2Q_1Q_5}{x^2}, \\ J(x)e^{ip\eta}(0) &\sim \frac{\sqrt{4Q_1Q_5}p}{x} e^{ip\eta}(0). \end{aligned} \tag{2.6}$$

One basic effect of the deformation is a shift in the dimension of charged operators of the form $Y_{p,\tilde{p}} \equiv \sqrt{Q_1} e^{i(p\eta + \tilde{p}\tilde{\eta})}$ (for which we chose an arbitrary normalization such that

[7] As just discussed, in $AdS_3 \times S^3$ with NS charges the only place Q_1 appears is in the string coupling, so counting powers of Q_1 is the same as counting powers of g_6.

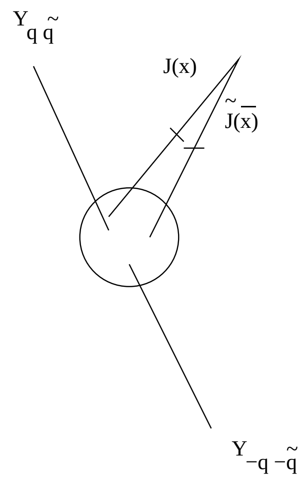

Figure 1: The leading contribution, at order $\tilde{h}g_6^0$, to the renormalization of the dimension of charged operators $Y_{\pm q,\pm\tilde{q}}$ (denoted by straight lines) by the "double-trace" deformation $J\tilde{J}$ (denoted by the slashed lines meeting at a boundary point x).

the 2-point function scales as $1/g_s^2$). A simple computation gives

$$\delta_{\tilde{h}}\langle Y_{p,\tilde{p}}(x,\bar{x})Y_{-p,-\tilde{p}}(0)\rangle = \\ Q_1\sum_{n=1}^{\infty}\frac{(4\tilde{h})^n}{n!}\int\prod_{i=1}^{n}d^2x_i\langle e^{ip\eta}(x)\prod_{i=1}^{n}\partial\eta(x_i)e^{-ip\eta}(0)\rangle\langle e^{i\tilde{p}\tilde{\eta}}(\bar{x})\prod_{i=1}^{n}\bar{\partial}\tilde{\eta}(\bar{x}_i)e^{-i\tilde{p}\tilde{\eta}}(0)\rangle. \tag{2.7}$$

This expression is a power series in the "double-trace" coefficient $\tilde{h}$ and in the string coupling $g_6^2 \sim 1/Q_1$ (the latter statement follows from the form of (2.7) combined with the scaling (2.5) of the charges). The corresponding diagrams on the gravity side are of effective genus ≥ 1, with the first contribution arising at $\mathcal{O}(\tilde{h}g_6^0)$ as depicted in figure 1.

Let us evaluate this explicitly at order $\tilde{h}$. Working out the correlators this reduces to

$$\frac{4Q_1\tilde{h}p\tilde{p}}{x^{p^2/2}\bar{x}^{\tilde{p}^2/2}}\int d^2x_1\left|\frac{1}{x_1-x}-\frac{1}{x_1}\right|^2. \tag{2.8}$$

This integral is logarithmically divergent when x_1 approaches the other operators Y at x and at 0 (the log divergence for large x_1 cancels among the different terms in (2.8)). Let us include a UV cutoff a, which cuts off the integrals such that for any other operator insertion at x_0, the range of x_1 is bounded by $|x_1-x_0|\geq a$. Doing the integral in (2.8), one then finds

$$\delta_{\tilde{h}}\langle Y_{p,\tilde{p}}(x,\bar{x})Y_{-p,-\tilde{p}}(0)\rangle = 8\pi Q_1\tilde{h}\frac{p\tilde{p}}{x^{p^2/2}\bar{x}^{\tilde{p}^2/2}}\log\frac{|x|^2}{|a|^2}. \tag{2.9}$$

The $\log |a|^2$ piece must be absorbed in a redefinition of the operators $Y_{p,\tilde{p}}$ as is standard in conformal perturbation theory [18] (see also the discussion of this in §3.1). Namely, here

$$Y_{p,\tilde{p}} \to Y_{p,\tilde{p}} + (8\pi\tilde{h}\ p\tilde{p}\ \log\ a)\ Y_{p,\tilde{p}}. \tag{2.10}$$

What remains amounts to a shift in the dimension of Y by

$$(-8\pi\tilde{h}p\tilde{p}, -8\pi\tilde{h}p\tilde{p}) \tag{2.11}$$

to first order in $\tilde{h}$. Taking into account the scaling (2.5) of the charges, this shift is of order $\tilde{h}g_6^2$ (for small charges). It is easy to generalize this to general correlation functions.

One can similarly work out changes to correlators involving currents (and their descendants) arising from our deformation. For example,

$$\delta_{\tilde{h}}\langle J(x)J(0)\rangle = \sum_{n=1}^{\infty}\left(\frac{\tilde{h}}{Q_1Q_5}\right)^n \frac{1}{n!}\int \prod_{i=1}^{n} d^2x_i \langle J(x)\prod_{i=1}^{n} J(x_i)J(0)\rangle\langle\prod_{i=1}^{n}\tilde{J}(\bar{x}_i)\rangle. \tag{2.12}$$

Here only even n contributions survive. All these contributions are (since they involve $n+1$ contractions of J's) at order $Q_1 \sim g_s^{-2}$, the same order as tree-level diagrams. This agrees with the set of diagrams that contribute to (2.12) on the gravity side, which involve $n+1$ disconnected spheres (connected by insertions of the deformation). The first contribution, at order $\tilde{h}^2$, is given by

$$4Q_1Q_5\tilde{h}^2\int d^2x_1 d^2x_2 \frac{1}{(\bar{x}_1-\bar{x}_2)^2}\left[\frac{1}{(x-x_1)^2}\frac{1}{x_2^2} + \frac{1}{(x-x_2)^2}\frac{1}{x_1^2} + \frac{1}{(x_2-x_1)^2}\frac{1}{x^2}\right]. \tag{2.13}$$

The last term here is related to a divergence in the vacuum amplitude,

$$\begin{aligned}\delta_{\tilde{h}}\langle 1\rangle &= \sum_{n=1}^{\infty}\left(\frac{\tilde{h}}{Q_1Q_5}\right)^n \frac{1}{n!}\int \prod_{i=1}^{n} d^2x_i \langle\prod_{i=1}^{n} J(x_i)\rangle\langle\prod_{i=1}^{n}\tilde{J}(\bar{x}_i)\rangle = \\ &= 2\tilde{h}^2\int d^2x_1 d^2x_2 \frac{1}{|x_1-x_2|^2} + \cdots,\end{aligned} \tag{2.14}$$

so it will cancel when we compute the properly normalized correlation function which involves dividing by $\langle 1\rangle$.

The first two terms in (2.13) give identical finite results, adding up to

$$\begin{aligned} 2\cdot 4Q_1Q_5\tilde{h}^2\int d^2x_1d^2x_2\frac{1}{(\bar{x}_1-\bar{x}_2)^2}\frac{1}{(x-x_2)^2}\frac{1}{x_1^2} &= \\ &= 8Q_1Q_5\tilde{h}^2\int d^2x_1d^2x_2\frac{\partial}{\partial\bar{x}_2}(\frac{1}{\bar{x}_1-\bar{x}_2})\frac{\partial}{\partial x_2}(\frac{1}{x-x_2})\frac{1}{x_1^2} = \\ &= 8Q_1Q_5\tilde{h}^2\int d^2x_1d^2x_2\frac{\partial}{\partial x_2}(\frac{1}{\bar{x}_1-\bar{x}_2})\frac{\partial}{\partial\bar{x}_2}(\frac{1}{x-x_2})\frac{1}{x_1^2} = \\ &= 32\pi^2Q_1Q_5\tilde{h}^2\int d^2x_1d^2x_2\delta^{(2)}(x_1-x_2)\delta^{(2)}(x-x_2)\frac{1}{x_1^2} = \\ &= \frac{32\pi^2Q_1Q_5\tilde{h}^2}{x^2}. \end{aligned} \tag{2.15}$$

If desired, one can always renormalize J by a multiplicative constant (depending on $\tilde{h}$) which will cancel this correction and keep the same form of $\langle J(x)J(0)\rangle$.

Another example is

$$\delta_{\tilde{h}}\langle J(x)\tilde{J}(0)\rangle = \sum_{n=1}^{\infty}\left(\frac{\tilde{h}}{Q_1Q_5}\right)^n\frac{1}{n!}\int\prod_{i=1}^{n}d^2x_i\langle J(x)\prod_{i=1}^{n}J(x_i)\rangle\langle\tilde{J}(0)\prod_{i=1}^{n}\tilde{J}(\bar{x}_i)\rangle. \tag{2.16}$$

Here only odd values of n contribute. For $n=1$, this is

$$\begin{aligned} 4Q_1Q_5\tilde{h}\int d^2x_1\frac{1}{(x-x_1)^2}\frac{1}{\bar{x}_1^2} &= -4Q_1Q_5\tilde{h}\int d^2x_1\frac{\partial}{\partial x_1}(\frac{1}{x-x_1})\frac{\partial}{\partial\bar{x}_1}(\frac{1}{\bar{x}_1}) = \\ &= -4Q_1Q_5\tilde{h}\int d^2x_1\frac{\partial}{\partial\bar{x}_1}(\frac{1}{x-x_1})\frac{\partial}{\partial x_1}(\frac{1}{\bar{x}_1}) = \\ &= 16\pi^2Q_1Q_5\tilde{h}\int d^2x_1\delta^{(2)}(x-x_1)\delta^{(2)}(x) = \\ &= 16\pi^2Q_1Q_5\tilde{h}\delta^{(2)}(x), \end{aligned} \tag{2.17}$$

which is just a shift in the contact term between J and $\tilde{J}$. We can swallow this by redefining the original contact term (the same will be true at higher orders as well).

By using exact formulas for correlators involving η and $\tilde{\eta}$, we can in principle calculate explicitly the effects of the deformation on all operators (2.4) of the theory, including the parts involving complicated descendants. It is worth emphasizing, however, that the set of operators (2.4) has a lot of structure. The AdS/CFT correspondence maps all states in global AdS to operators in the CFT, so operators of this form describe all possible bulk excitations on the gravity side. The CFT charge q maps to the charge under the corresponding gauge field on AdS_3 (given by the integral of the gauge field around the boundary of AdS_3 at fixed time in global coordinates). Clearly, there are many configurations with total charges $q, \tilde{q}$; the information about the distribution of this charge in the bulk of the spacetime is encoded in the details of the $P\hat{O}$ factors in the operator. It is interesting that the formula (2.11) implies that the change in the dimension of operators (and, therefore, the change in the energy of the corresponding states in global AdS) depends only on their charge. However, in order to understand the effects of our deformation on the dynamics of nontrivial distributions of charge in the bulk of the space, one needs to keep track of the "fine structure" in the operators.

In particular, in §5, we will be interested in forces between separated D-branes in the bulk of $AdS_3 \times S^3$. Pairs of $D0$-branes in the bulk of AdS_3 are not quite in stationary states, as there are forces between them (which are small for large L_{AdS}). Such a pair is therefore described by a combination of operators (2.4) which does not form an eigenstate of the dilatation operator in the dual CFT. This can be modeled by a sum of an operator of particular dimension plus $1/L_{AdS}$ times an operator or sum of operators of different dimension. After the deformation, the correlation functions of the different terms scale in different ways determined by their correlators with $J, \tilde{J}$ as in the simple examples worked out above. The force term is still multiplied by a small coefficient, $1/L_{AdS}$, but its magnitude will in general receive corrections. We will calculate this effect explicitly for some D-branes in §5, and reproduce this general structure predicted by the dual CFT.

2.2. The Gravity Side

The general formalism described in [1] implies that deforming the CFT by a "double-trace" operator of the form $h \int d^2x \mathcal{O}_1(x)\mathcal{O}_2(x)$ is described in string theory, at least to leading order in h, by deforming the worldsheet action by the non-local term $h \int d^2x \int d^2z_1 V_1(z_1; x) \int d^2z_2 V_2(z_2; x)$, where $V_{1,2}(z; x)$ are the vertex operators for $\mathcal{O}_{1,2}(x)$. In our case, as described in [9], the affine Lie algebra generated by $J(x)$ in the dual CFT is related to an affine Lie algebra generated by $k(z)$ on the worldsheet. An insertion of $J(x)$ into a CFT correlation function is equivalent to an insertion of $K(x)$ defined by

$$K(x) = -\frac{1}{\pi} \int d^2z k(z) \partial_{\bar{z}} \Lambda(z, \bar{z}; x, \bar{x}) \tag{2.18}$$

in the string worldsheet, where Λ is a particular operator such that $\partial_{\bar{z}}\Lambda(z, \bar{z}; x, \bar{x})$ is a primary operator of the worldsheet conformal algebra with dimension $(0, 1)$, and also a primary of the space-time conformal algebra with scaling dimension $(1, 0)$. We wrote down the vertex operator for the bosonic string; in the case of the superstring (which is the case we are interested in) there will be some additional terms in the expression above, but they do not change our discussion and our semi-classical computations below so we will not write them down explicitly.

If we choose coordinates on AdS_3 such that the string-frame metric is of the form $ds^2 = Q_5(d\phi^2 + e^{2\phi} d\gamma d\bar{\gamma})$ (where the curvature in string units is $-2/Q_5$), then we can write an expression for Λ in terms of the worldsheet fields $\phi(z,\bar{z}), \gamma(z,\bar{z})$ and $\bar{\gamma}(z,\bar{z})$, in the semi-classical approximation, of the form

$$\Lambda(z,\bar{z};x,\bar{x}) = -\frac{(\bar{\gamma}-\bar{x})e^{2\phi}}{1+|\gamma-x|^2 e^{2\phi}}. \tag{2.19}$$

The deformation of the worldsheet Lagrangian corresponding to (2.3) is given by

$$\delta S_{worldsheet} = \frac{\tilde{h}}{Q_1 Q_5 \pi^2} \int d^2x \int d^2z_1 \int d^2z_2 k(z_1)\partial_{\bar{z}_1}\Lambda(z_1,\bar{z}_1;x,\bar{x})\tilde{k}(\bar{z}_2)\partial_{z_2}\bar{\Lambda}(z_2,\bar{z}_2;x,\bar{x}). \tag{2.20}$$

The vertices (2.18) have many interesting properties that were analyzed in [9] and used there to derive the Ward identities for the current $J(x)$. Since $\partial_{\bar{z}} k(z) = 0$ except for delta function contributions at the locations of other vertices, we can integrate by parts and write (2.18) as a contour integral of $k\Lambda$ on contours surrounding the insertion points of vertex operators, and (if they exist) on boundaries of the worldsheet (note that there are no singularities when the vertex operators in $K(x)$ and $\tilde{K}(\bar{x})$ approach each other). In particular, the vertex operator $K(x)$ (2.18) can be written in the form

$$K(x) = \sum_{insertions,boundaries} \oint \frac{dz}{2\pi i} k(z)\Lambda(z,\bar{z};x,\bar{x}). \tag{2.21}$$

This leads [9] to the Ward identity for correlators of K with charged fields. Let $W_q(x)$ be the integrated vertex operator corresponding to a primary of the J affine Lie algebra with charge q, so that correspondingly it is a primary of the corresponding worldsheet affine Lie algebra with charge q. Then, one finds [9]

$$\langle K(x) \prod_i W_{q_i}(x_i,\bar{x}_i)\rangle = \sum_i \frac{q_i}{x-x_i}\langle \prod_i W_{q_i}(x_i,\bar{x}_i)\rangle \tag{2.22}$$

for closed string worldsheet correlation functions, reproducing the Ward identities of the dual CFT. Many interesting operators (including $J(x)$ itself) will not have this property of being primaries of charge q and then we will have more complicated expressions for their correlation functions, as discussed in §2.1.

3. Effect of the Deformation on Closed String Amplitudes

Now, let us take some correlation function of closed string vertex operators in the theory before the deformation, and consider the effect of the deformation on the correlation function. In perturbation theory the effect of the deformation is given by the insertion of some number of $K(x_i)$ and $\tilde{K}(\bar{x}_i)$ vertex operators into the correlation function, and integrations over x_i. If the correlation function involves only primary fields we can then easily compute it on the worldsheet using (2.22), and it is obvious that we reproduce the CFT computations of the same correlation functions (2.7)-(2.17) described in §2.1.

Our deformation is exactly marginal and affects physics at all scales on the field theory side, and we have introduced various changes to correlation functions of closed strings in AdS_3, so we might expect the bulk physics to be affected by the deformation, and perhaps to become non-local (with a non-locality scale much bigger than the string scale). For the case of a double-trace deformation in AdS_5 various arguments for bulk non-locality were given in [1]. However, in our case we need to be more careful because, as discussed above, the vertex operators we deform by are total derivatives on the worldsheet, so it is not clear that the deformation is really felt all over the worldsheet. Semiclassical worldsheets in Euclidean AdS_3 stretch all the way to the boundary, where the vertex operators describing external states in the Feynman diagrams are inserted [10]. It is straightforward to check, using the methods of [10], that the insertion of $K(x)$ does not change the shape of the saddle point configuration of the worldsheet near the vertex operator insertions at the boundary. The worldsheet path integral of course involves integration over all worldsheet shapes, but from [10] we see that the dominant (saddle point) contribution is one in which the W_q insertions are at the boundary. As discussed above, further insertions of $K(x)$ localize at the same points on the worldsheet. Thus, in this special case where the vertex operators we deform by are total derivatives, it seems that the only effect evident semiclassically on Euclidean closed-string amplitudes is localized at the boundary of AdS space.

The case of more physical interest on the gravity side is the Lorentzian case, where scattering events can take place in the bulk of the space. For the Lorentzian case we will provide an indirect argument in §6, based on features of the flat space limit, that the effects of our deformation are felt also in the bulk of the space and not just near the boundary.

The existence of non-supersymmetric shifts of charged closed string masses obtained from the shifted dimensions (2.11), combined with the exact stability of the model, raises the fascinating question of how to see the cancellation of the moduli potential directly on the gravity side of the correspondence. We will return to this question in §3.2 after considering the divergence structure of the deformation on the gravity side.

3.1. Regularization of Divergences

In studying marginal deformations of CFTs in conformal perturbation theory, one encounters divergences in calculating corrections to correlation functions, which can be consistently regularized and absorbed in rescalings of the operators (see e.g. [18]). The cutoff a we introduced in (2.9) and the rescaling (2.10) are an example of this procedure in our case on the field theory side. We would now like to illustrate how this regularization is described on the gravity side. This can be deduced by using the UV/IR correspondence.

On the gravity side, the first-order correction in a correlator like (2.7) is of the form

$$\frac{\tilde{h}}{Q_1Q_5}\int d^2x_1\langle W_{q,\tilde{q}}(x,\bar{x})W_{-q,-\tilde{q}}(0)K(x_1)\tilde{K}(\bar{x}_1)\rangle. \tag{3.1}$$

Anticipating that the result will be divergent, let us put an IR cutoff in space-time at a finite value of ϕ, leaving the region $\phi<\phi_c$, and use the semiclassical analysis of the worldsheet and of Λ. Taking into account the localization of K at the W insertions (2.21) and the fact that $\oint k(z)\frac{dz}{2\pi i}$ measures the charge, this becomes

$$\delta_{\tilde{h}}\langle WW\rangle=\frac{\tilde{h}}{Q_1Q_5}\int d^2x_1 q\tilde{q}\Big|\Lambda_1(x_1)-\Lambda_2(x_1)\Big|^2\langle WW\rangle, \tag{3.2}$$

where Λ_1 and Λ_2 refer to the semiclassical value of Λ at the positions of the two W insertions (cut off at ϕ_c). For large ϕ_c we find

$$\begin{aligned}\Lambda_1(x_1)&=-\frac{(\bar{x}-\bar{x}_1)}{e^{-2\phi_c}+|x-x_1|^2},\\ \Lambda_2(x_1)&=-\frac{(-\bar{x}_1)}{e^{-2\phi_c}+|x_1|^2},\end{aligned} \tag{3.3}$$

where we have replaced the γ coordinate of each insertion by its boundary value (x or 0 respectively) since the corrections to this value are subleading at large ϕ_c to the $e^{-2\phi_c}$ contribution we have included. Plugging (3.3) into (3.2) gives an x integral whose log divergence at large x_1 cancels among the various terms in (3.2) (just like in (2.8)). The leading divergent behavior when x_1 approaches the other insertions at x and 0, and as $\phi_c\to\infty$, is

$$\int d^2w\frac{|w|^2}{(e^{-2\phi_c}+|w|^2)^2}\sim-2\pi\log(e^{-2\phi_c})=4\pi\phi_c. \tag{3.4}$$

Now that we have expressed the cutoff divergence in terms of gravity side quantities, we can absorb this divergence into a rescaling of the vertex operators $W_{q,\tilde{q}}$, corresponding to the rescaling (2.10) we had on the field theory side. In string theory, we can further translate this cutoff into a short-distance cutoff on the worldsheet using [10]. The IR cutoff ϕ_c in the target space geometry corresponds to a cutoff

$$a_{worldsheet}(h) = e^{-\frac{\phi_c}{4h}} \tag{3.5}$$

on the worldsheet near an insertion of a vertex operator corresponding to a scalar operator of dimension $h(=\bar{h})$ in the dual CFT.

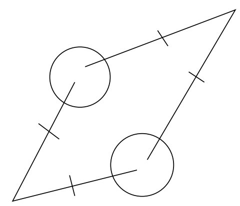

Figure 2: Vacuum diagram at order $\tilde{h}^2 g_s^0$. The insertions of the vertex operators in the "double-trace" deformation are indicated by the pair of lines with slashes joined at the boundary.

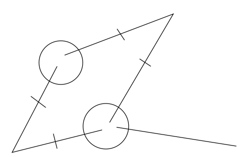

Figure 3: Modulus tadpole at order $\tilde{h}^2 g_s^0$. The insertion of the vertex operator for the modulus field is indicated by the plain line.

There are also formal divergences in contributions to the vacuum amplitude in the bulk. For example, the diagram in figure 2 has a logarithmic divergence (given by (2.14)). These diagrams by themselves are not physically observable – they map to $\langle 1 \rangle$ in the CFT which we should always choose to equal one. However, the ratio between any other diagram and the sum of vacuum diagrams is observable. For example, we can look at the same diagram probed by an external line as depicted in figure 3. This will be relevant for the moduli potential, which we turn to next.

3.2. The Moduli Potential in String Perturbation Theory

As discussed above, when we deform the CFT which is dual to string theory on (say) $AdS_3 \times S^3 \times T^4$ by a deformation (2.1) involving $U(1)_R$ currents, we explicitly break the space-time supersymmetry. From the space-time point of view we would naively expect to generate a moduli potential in such a case, such that not every point in the original moduli space would still give a stable background after the supersymmetry breaking. However, we know that this does not happen in our case since the deformation in the CFT is exactly marginal (independently of any of the other parameters of the CFT), so we expect to have an exact non-supersymmetric background after the deformation with the isometries of AdS_3 for any value of the other moduli of the theory. We are using a slight abuse of terminology here: since in general NLSTs do not have a local effective action, the notion of a *moduli potential* may not persist. However, we can still ask whether all the moduli of the original theory remain, and do not develop tadpoles even after we add the supersymmetry breaking deformation. We have moduli operators $\mathcal{O}^{(I)}_{modulus}(x,\bar{x})$ which are of dimension $(1,1)$, and the vanishing of a term of order m in the fields in the original "moduli potential" is manifested in the vanishing of the integrated correlation function of m of these operators in the CFT[8]. From the dual CFT it is clear that this must still be the case also after the deformation, and in this section we will see how this happens from the point of view of string perturbation theory in the bulk (which gives part of the contribution to the correlation functions in the full dual CFT).

In usual flat-space string theory, when we break supersymmetry we would expect to have a non-zero torus vacuum amplitude. There, this amplitude is proportional to the torus diagram with an insertion of the zero momentum dilaton, which is the worldsheet manifestation of the fact that the vacuum energy in perturbative string theory is really a potential for the dilaton. Our situation is different since the dilaton is a fixed scalar and therefore massive. Thus, we would expect to generate a potential only for the other moduli which actually correspond to massless fields on AdS_3. In any case the vacuum diagram by itself has no physical meaning, so we cannot use it to learn about supersymmetry breaking in the bulk; the physical effects of the vacuum energy are encoded in the diagrams with an external graviton or moduli line, which determine the curvature and moduli dynamics generated by the vacuum energy.

In the case we are interested in here, the moduli involve the T^4 part of the worldsheet

[8] The case $m = 2$ actually does not vanish; it is related to the propagator on the gravity side, and diverges after we integrate over x. The vanishing of the quadratic term in the "moduli potential" is accounted for by the dimension of the modulus operator, which corresponds to a massless field on the gravity side.

CFT; for most of the moduli the vertex operator corresponding to $\int d^2x \mathcal{O}_{modulus}(x,\bar{x})$ is simply $\int d^2z \partial X^i \bar{\partial} X^j$ (the others come from the RR sector and our argument in the next paragraph will apply to them as well). The leading correction to the moduli tadpole after the deformation comes from figure 3. It is easy to see that this vanishes, because the worldsheet correlation function on one of the spheres factorizes into a correlation function involving the T^4 directions and one involving the $AdS_3 \times S^3$ directions. The first factor is just of the form $\langle : \partial X^i \bar{\partial} X^j : \rangle$ where the X^i are embedding coordinates of the string in the T^4 directions. This vanishes.

Next, let us consider arbitrary diagrams contributing to the "moduli potential", at a general order in the perturbation theory in g_s and $\tilde{h}$. Such a digram would have various connected components, which are genus g surfaces with some number n of insertions of J, $\tilde{n}$ insertions of $\tilde{J}$, and m insertions of $\int d^2x \mathcal{O}^{(I)}_{modulus}(x,\bar{x})$ (where I labels the various moduli fields). This subdiagram is a correlator in the original undeformed theory, of the form

$$\langle J(x_1)\ldots J(x_n)\tilde{J}(\bar{x}_{n+1})\ldots \tilde{J}(\bar{x}_{n+\tilde{n}}) \int d^2x \mathcal{O}^{(1)}_{modulus} \cdots \int d^2x \mathcal{O}^{(m)}_{modulus}\rangle_{genus\ g}. \qquad (3.6)$$

If $n = \tilde{n} = 0$, the diagram is identical to a contribution to the "moduli potential" in the undeformed supersymmetric theory, which cancels[9]. For the other diagrams which feel the deformation and therefore the supersymmetry breaking, we note that the moduli of the torus (which are the scalar fields on AdS_3 we are discussing here) are uncharged under the $U(1)$ isometries generated by J and $\tilde{J}$, and have a non-singular OPE with the current operators. As discussed above, the vertex operators for J and $\tilde{J}$ are total derivatives on the worldsheet which can be written as integrals around the other insertion points, and (as in [9]) these integrals get no contributions near the moduli operators. Thus, ignoring the picture changing operators inserted on the Riemann surface at higher genus, which include terms from all sectors of the worldsheet CFT and can lead to additional singularities, one would find that the correlation function (3.6) factorizes into the part involving J and $\tilde{J}$ times the part involving the moduli, and the latter vanishes as argued above. This calculation of the $n + \tilde{n} + m$-point function can be done equivalently in the dual CFT description of the original theory, where it cancels by an exact factorization argument,

[9] More precisely, this subdiagram is a particular term in the expansion of the CFT "moduli potential" in powers of $g_s^2 = Q_5/Q_1$, but since the full correlation function vanishes every term in its expansion must vanish as well.

and one therefore deduces that the full calculation of the diagram including the picture changing operators still leads to a cancellation. Thus we see also on the string theory side that we do not produce a "moduli potential", despite the absence of supersymmetry.

One might worry that there could be moduli which have a singular OPE with the currents J or $\tilde{J}$. If we bosonize the currents as in section 2, then because the $\mathcal{O}^{(I)}_{modulus}$ are dimension $(1,1)$ operators in the dual CFT and they are uncharged under $J, \tilde{J}$, they could only depend on $\eta, \tilde{\eta}$ by a factor of $\partial\eta$ or $\bar{\partial}\tilde{\eta}$. So, we can write these operators generally as $\mathcal{O}^{(I)}_{modulus} = \mathcal{O}_0 + \partial\eta(x)\hat{\mathcal{O}}_R(\bar{x}) + \hat{\mathcal{O}}_L(x)\bar{\partial}\tilde{\eta}(\bar{x})$ where $\mathcal{O}_0$ has a non-singular OPE with the currents, $\hat{\mathcal{O}}_R$ is a dimension $(0,1)$ operator and $\hat{\mathcal{O}}_L$ is a dimension $(1,0)$ operator. Note that the last two terms are actually "double-trace" operators, since $\partial\eta$ is simply proportional to J, and they do not correspond to scalar fields on AdS_3. However, even for moduli of this "double-trace" form we can argue that no tadpoles are generated after our deformation. The same arguments above show that the effect of the deformation on the tadpole for these operators must be proportional to the value of $\langle\hat{\mathcal{O}}_R(\bar{x})\rangle$ or $\langle\hat{\mathcal{O}}_L(x)\rangle$ in the original theory, which obviously vanishes.

We can also give a direct space-time argument for the vanishing of the "moduli potential" after the deformation. On the gravity side, the vanishing of the "moduli potential" after our deformation corresponds to the statement that in the original theory before the deformation, the coupling of Chern-Simons gauge fields (which are the fields dual to $J, \tilde{J}$) to the moduli remains zero quantum mechanically. This follows by gauge invariance from the fact that the pure gauge modes $A = d\Lambda$ (whose field strength vanishes) do not couple only to each other or to the uncharged moduli fields at any order in perturbation theory in the original background.

In any case, the result is that in our diagrammatic expansion, in perturbation theory in $\tilde{h}$, the diagrams contributing potentially destabilizing contributions to the "moduli potential" cancel by virtue of the vanishing of corresponding diagrams in the original theory, which appear as subdiagrams in the deformed theory. It would be nice to gain a more intuitive understanding in the bulk spacetime of how the loop diagrams involving closed strings in the bulk, which have bose-fermi splitting (using (2.11), since the bosons and fermions have different charges under $U(1)_R$), manage to cancel in this theory. We will return to this in §5.2 after studying some bulk effects, including supersymmetry breaking effects, of D-branes in our theory in §5.

4. Effect of the Deformation on the Low-energy Action

In §3 we saw indications that when computing the n-point function in Euclidean space of any set of vertex operators on the worldsheet, the contribution of the "double-trace" deformation is localized at the boundaries of AdS. In this section we would like to discuss this in the context of the low energy effective description, and to clarify from this point of view where boundary terms arise. In the next section (§5) we will return to our analysis of the effects of the deformation in string theory and the stable supersymmetry breaking mechanism encoded in this model.

In general in a NLST, one would not expect a *local* gravity or supergravity action in the infrared. In our present case, which is based on Chern-Simons gauge fields in $2+1$ dimensions, some simplifications arise if we focus on the AdS_3 part of the geometry[10]. In particular, from [13,14,15] it follows that if we bosonize the currents as in section 2, then the bulk Chern-Simons gauge fields which are dual to the CFT operators J and $\tilde{J}$ are given by $A = \sqrt{4Q_1Q_5}d\eta$ and $\tilde{A} = \sqrt{4Q_1Q_5}d\tilde{\eta}$ away from sources (where η and $\tilde{\eta}$ are defined on all of AdS_3 and their boundary value is given by the objects defined in section 2). Then, one can realize our deformation $4\tilde{h}\int d^2x\partial\eta\bar{\partial}\tilde{\eta}$ by a boundary term in the CS theory

$$\delta S_{SUGRA} = \frac{\tilde{h}}{Q_1Q_5}\int_{\partial AdS_3} A\wedge\tilde{A}. \tag{4.1}$$

This prescription reproduces our perturbation expansion in $\tilde{h}$, as can be seen by regarding (4.1) as part of the interaction Lagrangian in the gravity-side theory. Bringing down powers of (4.1) in the path integral and contracting the boundary fields $A_\partial, \tilde{A}_\partial$ in (4.1) with bulk fields $A_b, \tilde{A}_b$ coming from insertions of interaction vertices from the bulk Lagrangian, one obtains the bulk-boundary propagators implicit in the vertex operators in (2.20). In particular, as we will see further in §5, we find significant effects of the deformation in the bulk arising from this. These come from the fact that the AdS_3 acts like a finite box for some modes, and more generally from the fact that the boundary term (4.1) is present throughout time. Note that (4.1) is not a local term in six dimensions, as each of the fields appearing in (4.1) is actually in a particular spherical harmonic on the S^3, so writing this term down in the six dimensional action entails performing two integrations over the S^3. Thus, in the full theory this term is manifestly non-local at the AdS curvature scale.

[10] We thank J. Maldacena for emphasizing this aspect.

In fact, writing the deformation in the form (4.1) is a special case of something we can do in general to describe deformations in AdS/CFT. Let us work in Euclidean AdS space with the standard coordinate system $ds^2 = (dr^2 + dx^\mu dx_\mu)/r^2$. In conformal perturbation theory, if we deform the Lagrangian by a "single-trace" operator $\mathcal{O}$ of dimension Δ which is dual to a SUGRA field $\phi(x,r)$, $\delta S_{CFT} = h\int d^dx\mathcal{O}(x)$, then we need to insert into the dual supergravity picture any number of boundary-to-bulk propagators of the field ϕ, each with a coefficient h. One way to do this is to deform the SUGRA action by a boundary term of the form $\delta S_{SUGRA} = \lim_{r\to 0} h\int d^dx\phi(x,r)r^{d-\Delta}$, which reproduces the same perturbation expansion because of the relation between the bulk-to-boundary and bulk-to-bulk propagators, if we add this term without changing the boundary conditions on the fields. However, usually this description is not very useful since the limit $r \to 0$ is singular so we do not get a local deformation of the action, except in the case $\Delta = d$ of marginal deformations. For marginal deformations the effect of the added term at first order in h is simply to change the bulk value of ϕ by a constant amount proportional to h, as in the usual description. However, this violates the usual boundary condition for a massless field (which sets its boundary value to a particular constant), so this formalism breaks down also in this case (leading to singular configurations). In any case, this illustrates that writing the deformation as a local boundary term does not preclude having large effects of the deformation in the bulk.

Similarly, also for "double-trace" deformations by a product of two scalar operators, of the form $\delta S_{CFT} = \tilde{h}\int d^dx\mathcal{O}_1(x)\mathcal{O}_2(x)$, we can reproduce the perturbation theory in $\tilde{h}$ by adding to the supergravity action $\delta S_{SUGRA} = \lim_{r\to 0}\tilde{h}\int d^dx\phi_1(x,r)\phi_2(x,r)r^{2d-\Delta_1-\Delta_2}$. Again, this is not very useful since the added term generally has no good $r \to 0$ limit, and in particular this happens in the marginal case of $\Delta_1 + \Delta_2 = d$. However, if we deform by vector fields instead of scalar fields, we get a power of $r^{2d-2-\Delta_1-\Delta_2}$ instead of the power we wrote above. In the case we are discussing in this paper (for which $d = 2, \Delta_1 = \Delta_2 = 1$) this power vanishes, so we simply reproduce the deformation (4.1), which is perfectly well behaved. Note that, as described for instance in the discussion around equation (A.19) of [15], we do not need to impose any boundary conditions on the fields $A, \tilde{A}$, since by adding appropriate boundary terms we can set the relevant currents to be chiral and anti-chiral by the equations of motion (the Euclidean action is of the form $\frac{k}{2\pi}\int_{AdS_3}(A\wedge dA - \tilde{A}\wedge d\tilde{A}) - \frac{ik}{4\pi}\int_{\partial AdS_3}(A\wedge *A + \tilde{A}\wedge *\tilde{A})$, where the $*$'s are taken in the boundary of AdS space). Thus, it is not necessary to change the boundary conditions after deforming by (4.1), and this term automatically reproduces the perturbation expansion in the CFT which we described in section 2.

5. D-branes: Bulk Effects and SUSY Structure

In section 3 we studied closed string amplitudes in which the operators $K(x), \tilde{K}(\bar{x})$ involved in our deformation localized to the boundary of AdS_3 (semiclassically). When the worldsheet has boundaries on D-branes, $K(x)$ gets additional contributions from these boundaries, and these do not have to be at the boundary of AdS_3. Thus, it seems that D-brane physics in the bulk could be manifestly different after the deformation, even in Euclidean space. Such physics could involve for instance D-instanton corrections to correlation functions, D-branes localized in the bulk, or D3-branes wrapping an $AdS_2 \times S^2$ cycle in $AdS_3 \times S^3$. D-branes in AdS_3 have been studied for example in [19,20,21,22,23,24,25,26,27,28,29,30].

Studying this requires us to be able to calculate correlation functions with (2.21) inserted along the boundary. In general we do not know how to treat $k(z)$ and Λ near the boundaries of the worldsheet. However, in certain circumstances, Λ approaches an x-dependent constant near the boundary, and we can calculate the effect of the deformation explicitly. One such circumstance involves worldsheets which can be treated semiclassically. In such a case we can simply replace Λ by the value of (2.19) at the locus in the target space where the boundary of the worldsheet is mapped. Another involves D-branes which preserve a diagonal subgroup of the $SL(2) \times SL(2) \times SU(2) \times SU(2)$ chiral algebra. In these cases the symmetries determine the behavior of Λ near the worldsheet boundaries. A third situation in which we have control is that of D-instantons on AdS_3, which freeze the worldsheet boundaries in all directions. Here again we can replace the worldsheet fields $\gamma, \bar{\gamma}, \phi$ appearing in (2.19) by their boundary values. We believe that a similar situation may also occur for D0-branes on AdS_3, at least with regard to emission of massless closed strings whose worldsheets intersect the D-branes at a point (up to string scale fluctuations, which may be canceled by ghosts, since they are just along the longitudinal time direction).

Our goal is to understand the effect of our deformation on the physics of the D-branes. This requires studying worldsheets with boundaries and insertions of (2.20). From the localization of K to a contour integral around each boundary, we see that in the above cases where Λ approaches some constant $\Lambda_i(x, \bar{x})$ at the i'th boundary, the expression for K reduces to $\sum_i \Lambda_i \oint_i \frac{dz}{2\pi i} k(z)$, where the sum goes over the disconnected boundaries of the worldsheet. The contour integral produces the charge q_i of the closed string channel state emitted by the D-brane. Thus, the effect of the deformation on a diagram with particular charges q_i floating through it is to multiply the diagram by

$$exp(\frac{\tilde{h}}{Q_1 Q_5}\int d^2x \sum_{i,j} q_i \Lambda_i(x,\bar{x}) \tilde{q}_j \bar{\Lambda}_j(x,\bar{x})). \tag{5.1}$$

Using the fact that the closed string vertices depend on q_i simply through a factor of $e^{iq_i\theta}$ (if we choose θ to be an angular variable along the isometry generated by J) and on $\tilde{q}_i$ similarly through a factor of $e^{i\tilde{q}_i\tilde{\theta}}$, one can show that (in the case of constant Λ) all string diagrams involving D-branes sitting at positions $(\theta_k, \tilde{\theta}_k)$ are multiplied by an insertion of the form

$$exp(-\frac{\tilde{h}}{Q_1 Q_5}\int d^2x \sum_{k,l} \Lambda_k(x,\bar{x}) \bar{\Lambda}_l(x,\bar{x}) \frac{\partial}{\partial\theta_k}\frac{\partial}{\partial\tilde{\theta}_l}), \tag{5.2}$$

where here the sum goes over the different D-branes in the background and we are assuming that none of the D-branes lie at fixed points of the isometries (since the θ's are ill-defined there).

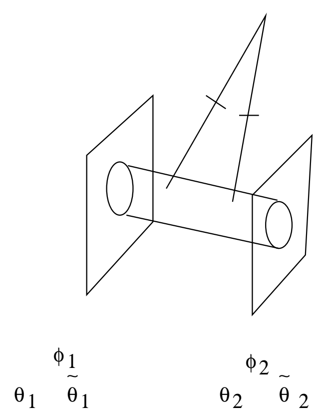

Figure 4: Annulus contribution to the force between D-branes at order $\tilde{h}g_s^2$.

For disk diagrams, with no charged closed string insertions, the deformation has no effect since no charge can be emitted by the boundary state (nothing can absorb it, and the contour integral above can be shrunk to zero size). Therefore, the leading contribution in all our calculable cases of D-brane interactions could arise from diagrams at order $g_s^2\tilde{h}$. One such contribution is the annulus with one insertion of the deformation operator, as depicted in figure 4. Other contributions at the same order come from diagrams where the "double-trace" wedge connects two otherwise disconnected annuli. We can calculate all these diagrams equivalently using (2.20) or (4.1).[11]

In some cases this contribution will vanish. For example, when Λ takes the same value on both boundaries of the annulus, then the sum over i (or over j) in (5.1) vanishes by charge conservation. This cancellation occurs for each closed string charge sector separately. The path integral involves a sum over all closed strings propagating between the two boundaries, and in particular a sum over all the possible closed string charges. Thus, another source of cancellation can arise (for example) when we deform by the $U(1)$ currents inside the $SU(2)\times SU(2)$, if the D-branes are not separated on the S^3, since then the sum over positive and negative q_i (and/or $\tilde{q}_j$) cancels (for a generic position of the D-branes which is not a fixed point of the isometries). If we separate the D-branes on the S^3 this cancellation is avoided by having different q_i and $\tilde{q}_j$-dependent spherical harmonics appearing in the closed string wavefunctions emanating from the separated branes. However, when these separated D-branes contribute to instanton effects, one integrates in spacetime over their positions on the S^3, yielding again a cancellation. In particular, this cancellation would occur in calculating instanton corrections to the "moduli potential" which we know from the dual CFT must cancel. We will discuss this further in §5.2.

We will mostly be interested in studying the effects of supersymmetry breaking on the bulk D-branes. In the original background, there are D-branes which break all the supersymmetry and therefore have 16 fermionic zero modes on their worldvolume from the broken supercharges, and there are other branes which break half the supersymmetry

[11] For example, we can use (4.1) to calculate the diagram in figure 4 as follows. Let us denote by $Q(y)$ the charged field propagating in the closed string channel, with charges q and $\tilde{q}$ under our two U(1)'s. The amplitude is

$$\langle B_1| \int d^3y q : A_\mu Q \partial^\mu Q(y) : \int d^3y' : \tilde{q}\tilde{A}_\nu Q\partial^\nu Q(y') : \frac{\tilde{h}}{Q_1 Q_5} \int_\partial : A\wedge \tilde{A} : |B_2\rangle, \tag{5.3}$$

where $|B_1\rangle$ and $|B_2\rangle$ are boundary states corresponding to the two D-branes, projected onto the sector with charges q and $\tilde{q}$, and where we have pulled down from the action three interaction terms: two cubic couplings between charged fields and the Chern-Simons gauge field, and the boundary term (4.1). All of the fields here can be contracted with each other (or in the case of two of the Q's, with the boundary states). The contraction between the bulk $A_\mu(y)$ and the boundary A_∂ gives the bulk-boundary propagator encoded in the vertex operator (2.18), and similarly for $\tilde{A}$. This yields the diagram in figure 4.

and have eight fermionic zero modes. We find that all these zero modes can be (and presumably are) lifted at order $\tilde{h}g_s^2$ from the diagram of figure 4. This is a local bulk signal of supersymmetry breaking, in contrast to the closed string sector where no such effect arose semiclassically in the Euclidean case. We will also study vacuum annulus diagrams, which indicate the effect of the deformation on forces between D-branes. The picture that emerges (at least at leading order in $\tilde{h}$) is that the D-branes do not sit in supermultiplets after the deformation, but because of the integration over spacetime collective coordinates, they do not contribute destabilizing instanton effects.

5.1. Localized Bulk D-branes

The $AdS_3 \times S^3 \times T^4$ background arises as the near-horizon limit of fundamental strings parallel to NS 5-branes wrapped on T^4. We can imagine putting in additional particle-like D-branes in this background – say, in type IIB, D1-branes or D3-branes wrapped around the 1-cycles and 3-cycles of the T^4. Before we took the near-horizon limit, these D-branes were attracted to the F1-NS5 system, and they could form a bound state whose energy was the square root of the sum of the energies squared of the separate systems (which is the BPS bound; the bound state is supersymmetric). If the F1-NS5 system is wrapped on a circle, the additional D-branes have a finite contribution to its energy, while if it is on a line they do not contribute to it. Thus, after taking the near-horizon limit, we find [17] that in Poincaré coordinates there is no lower bound on the mass of D-branes, but there is such a bound in global coordinates. This bound, which is proportional to the number of D-branes squared, appears even though the D-branes break all the supersymmetry; it is related to the original supersymmetries of the F1-NS5 system which are non-linearly realized. In any case, at weak coupling it is easy to see that such D-branes in $AdS_3 \times S^3 \times T^4$ have a mass which is much larger than the lower bound (this is fortunate since, for small D-brane number when we can ignore back-reaction, the mass grows linearly with the number of D-branes), they break supersymmetry completely, and one expects to have generic forces between them in the bulk (which at large distances arise from the exchange of massless particles). Moreover, these branes are not static in the bulk of AdS_3, but rather follow the geodesics for massive particles. In our coordinate system this means they are attracted towards smaller values of ϕ. This motion is insignificant at time scales much smaller than L_{AdS}, and in our discussion we will assume we are dealing with such time scales and we will ignore it. In addition to such branes which are D0-branes on AdS_3, we could also consider D-instantons on AdS_3, such as the type IIB D-instanton or Euclidean D-branes wrapped on cycles of the T^4. These also completely break the supersymmetry.

Let us consider the annulus contribution of figure 4 in the case that the two boundaries are localized on AdS_3. We place the D-branes, or the boundaries of the annulus, at positions $y_i = \{\gamma_i, \bar{\gamma}_i, \phi_i\}$ on AdS_3 and $\theta_i, \tilde{\theta}_i$ on the two circles on the S^3 corresponding to J and $\tilde{J}$, where $i = 1, 2$ labels the two branes. We will use the semiclassical equation for Λ,

$$\Lambda_i = \Lambda_{i,semiclassical} = -\frac{(\bar{\gamma}_i - \bar{x})e^{2\phi_i}}{1 + |\gamma_i - x|^2 e^{2\phi_i}}. \tag{5.4}$$

For D-instantons, the boundary of the worldsheet cannot fluctuate since there are Dirichlet conditions in all directions. In this case we also find that the semiclassical expression (5.4) agrees with the expression for Λ in [24], where it was found for a particular boundary condition that near the boundary an operator Φ_1, which is related to the operator Λ by $\partial_{\bar{x}}\Lambda = \pi\Phi_1$, goes to a constant times $1/(1 + |x|^2)^2$ as we approach the boundary[12]. This leads to $\Lambda \to \bar{x}/(1 + |x|^2)$, which exactly agrees with our expression above for an instanton positioned at $\gamma = \bar{\gamma} = \phi = 0$, which is the instanton corresponding to the boundary conditions discussed in [24] (other instantons can be generated from this by $SL(2)$ transformations). In the case of D0-branes, the boundary of the worldsheet can fluctuate in at most one (timelike) direction. We expect this longitudinal fluctuation to be cancelled by ghosts (and in the case of heavy winding mode exchange, to be suppressed regardless).

For simplicity let us take the two boundaries at $\gamma_i = \bar{\gamma}_i = 0$ and place the D-branes at points on S^3 which are not fixed points of the isometries corresponding to J and $\tilde{J}$. Note that by charge conservation along the diagram, $q_1 = -q_2 = q, \tilde{q}_1 = -\tilde{q}_2 = \tilde{q}$. Working at first order in $\tilde{h}$, plugging (5.4) into (5.1), we obtain a contribution of the form

$$\mathcal{A}_{q,\tilde{q}} = \frac{\tilde{h}}{Q_1 Q_5} \int d^2x q\tilde{q} \left| \frac{\bar{x}e^{2\phi_1}}{1 + |x|^2 e^{2\phi_1}} - \frac{\bar{x}e^{2\phi_2}}{1 + |x|^2 e^{2\phi_2}} \right|^2 G^{(0)}_{q,\tilde{q}}(\theta_i, y_i) \tag{5.5}$$

to the annulus amplitude arising from closed strings exchanged with particular $U(1) \times U(1)$ charges $(q, \tilde{q})$, where $G^{(0)}$ gives the annulus contribution without our "double-trace" insertion. The angular dependence of this contribution is of the form

$$e^{i(q_1\theta_1 + \tilde{q}_1\tilde{\theta}_1)} e^{i(q_2\theta_2 + \tilde{q}_2\tilde{\theta}_2)} = e^{iq(\theta_1 - \theta_2)} e^{i\tilde{q}(\tilde{\theta}_1 - \tilde{\theta}_2)}, \tag{5.6}$$

[12] In fact, in [24] various different possible boundary conditions were discussed, which give somewhat different behaviors of Φ_1 near the boundary. From an analysis of the symmetries of the problem it seems clear that the form of Φ_1 above must be the one corresponding to D-instantons, though this is not what is claimed in [24].

due to the wavefunctions of the closed strings at the two ends of the annulus. These contributions (5.6) explicitly break the symmetry which would otherwise exist between positive and negative values of $(q, \tilde{q})$. Note that in the absence of these contributions (for instance, if $\theta_1 = \theta_2$ or $\tilde{\theta}_1 = \tilde{\theta}_2$), the contributions from positive and negative $q, \tilde{q}$ in (5.5) would cancel when we sum over the different charge sectors.

The x integral in (5.5) can be performed, yielding the result

$$\mathcal{A}_{q,\tilde{q}} = \frac{\tilde{h} G^{(0)}_{q,\tilde{q}}}{Q_1 Q_5} q\tilde{q}\big(-2 + 2(\phi_1 - \phi_2)\coth(\phi_1 - \phi_2)\big). \tag{5.7}$$

For fixed nonzero separations $\theta_{12}, \tilde{\theta}_{12}$, this contribution survives the sum over $q, \tilde{q}$. This result constitutes a contribution to the force between D-branes (or in the D-instanton case, to the instanton action) which is present in the bulk of AdS. Because of the power of $1/L_{AdS}$ implicit in the ϕ_{12} contributions, with our current scalings this force disappears in the flat space limit $L_{AdS} \to \infty$, which is the same limit in which the AdS_3-induced tadpoles for the positions of the D-branes disappear. It therefore agrees nicely with the type of contribution expected from the CFT side. In the next section we will discuss another scaling for $\tilde{h}$ in which these contributions in fact survive in the flat space limit.

We can similarly calculate contributions from the other diagrams at order $\tilde{h} g_s^2$, involving two annuli connected by the deformation. For the D-instanton case, this leads to a similar contribution to (5.7); now we have four charges characterizing the diagram, $(q, \tilde{q})$ flowing through one annulus and $(q', \tilde{q}')$ flowing through the other, and the result is

$$\mathcal{A}_{q,\tilde{q},q',\tilde{q}'} = \frac{\tilde{h} G^{(0)}_{q,\tilde{q}} G^{(0)}_{q',\tilde{q}'}}{Q_1 Q_5}(q\tilde{q}' + q'\tilde{q})\big(-2 + 2(\phi_1 - \phi_2)\coth(\phi_1 - \phi_2)\big). \tag{5.8}$$

These contributions thus give different $\phi, \gamma, \bar{\gamma}, \theta, \tilde{\theta}$-dependence than the one we calculated above.

A very similar calculation predicts the lifting of the worldvolume fermion zero modes (Goldstinos) of the pair of D-branes. Before our "double-trace" deformation is turned on, space-time supersymmetry (in the absence of D-branes) is unbroken and the system of D-branes sits in a long multiplet and has 16 fermionic zero modes which are responsible for creating its superpartners. Let us denote the fermion zero modes on the i'th brane $\chi_i, \bar{\chi}_i$. Before the "double-trace" deformation, the quadratic terms for these fields on the worldvolume of the pair of branes are of the form

$$(\bar{\chi}_1 - \bar{\chi}_2)(\chi_1 - \chi_2), \tag{5.9}$$

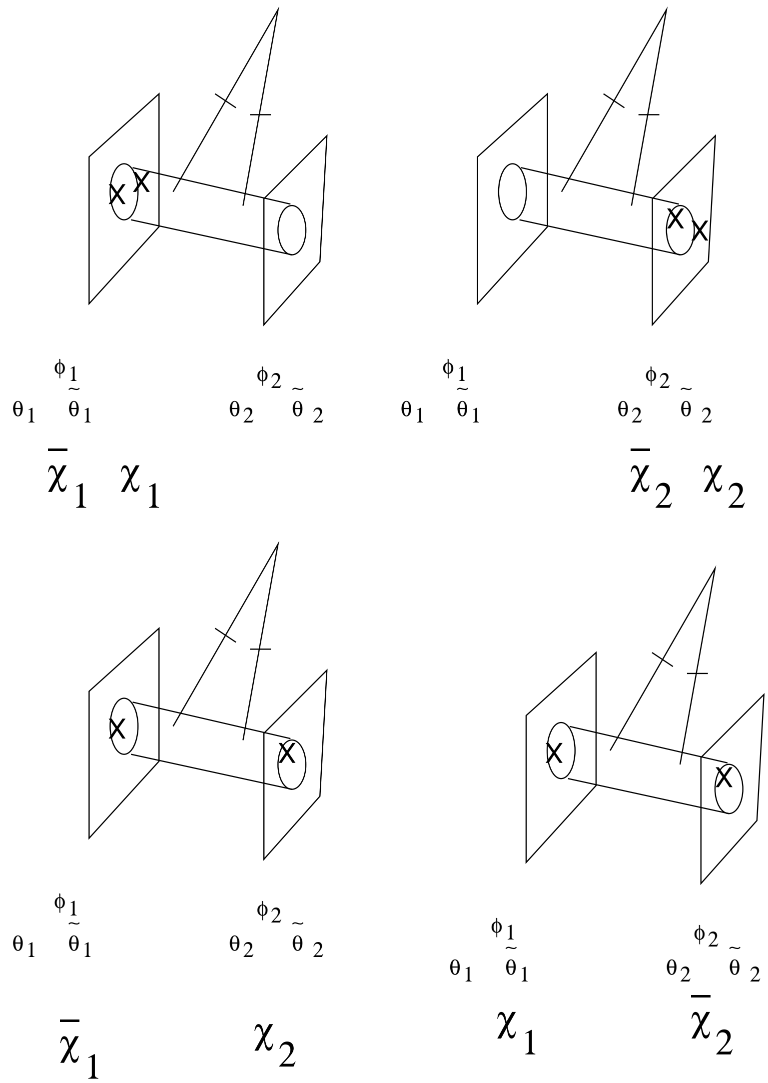

Figure 5: Annulus contribution to the mass of D-brane worldvolume fermions at order $\tilde{h} g_s^2$.

so that the overall combinations $\chi_1 + \chi_2$ are massless[13]. The issue is then whether the contributions in figure 5, which are the leading corrections to the fermion masses, produce the same combination of quadratic terms, preserving the masslessness of $\chi_1 + \chi_2$, or not. It is easy to convince oneself that there is no reason why the order $\tilde{h}$ amplitude should produce a result proportional to the combination (5.9). This is because the charges propagating in the closed string channel of the diagram are different for diagrams with one fermion on each boundary (which contribute masses $\bar{\chi}_1 \chi_2, \bar{\chi}_2 \chi_1$) relative to those with two fermions on a

[13] We are being schematic here, and ignoring the various indices of the fermions and the dependence of the massless combinations on the positions of the D-branes.

single boundary (which contribute masses $\bar{\chi}_1\chi_1, \bar{\chi}_2\chi_2$). The first two diagrams in figure 5, with two fermions inserted at a single boundary of the annulus, yield a contribution of the form (5.5) with a sum over integer $q, \tilde{q}$. The last two, with fermions on different boundaries, have fermionic closed strings propagating in the diagram, so (when the deformation involves the $U(1)_R$ currents) they involve a sum over half-integer $q, \tilde{q}$. Therefore, we do not expect the combination (5.9) where the two types of diagrams are weighted the same to persist at order $\tilde{h}$, and we expect all fermion zero modes to be lifted.

Thus, we have determined a bulk supersymmetry breaking effect of our NLST deformation in this system, at the level of forces between D-branes in the theory and their worldvolume action.

5.2. Nonperturbative Nonrenormalization in Nonsupersymmetric Non-local String Theory

As we explained above, an interesting feature of our deformation is that it breaks supersymmetry without introducing destabilizing tadpoles for moduli. From the field theory side, this is an exact statement. It is interesting therefore to explore how this phenomenon arises on the gravity side, given that we have just manifested bulk SUSY breaking effects in the D-brane sector.

In order to do this, there is a step remaining in the calculation. D-branes contribute to the "moduli potential" via virtual loops and instanton effects, which require a second quantized spacetime description. In such a calculation, (5.7) can represent a correction to the instanton action. The effect of the instanton on physical quantities in spacetime is obtained by a spacetime path integral including integrals over all the fermionic and bosonic zero modes. The fermionic zero modes, which before the deformation caused the amplitude to vanish, are now lifted. However, the bosonic zero modes, including the positions $\theta_i, \tilde{\theta}_i$, remain. Although we get a contribution for each value of $\theta_i, \tilde{\theta}_i$ as discussed above, the integral over these zero modes of (5.7) cancels due to the phases (5.6). Similarly, the diagrams we computed in (5.8) cancel after integration over the positions unless $q = -q'$ and $\tilde{q} = -\tilde{q}'$, and the remaining amplitudes cancel when we sum over the possible values of q because of a cancellation between positive and negative q's.

At this order, this provides a satisfying resolution to the problem of how the gravity side manages to avoid generating a "moduli potential" despite the supersymmetry breaking introduced by the deformation (and the absence of fermion zero modes). The D-branes experience non-local SUSY breaking forces in the bulk, but these effects cancel in computing their virtual and instantonic contributions to other physical observables via a cancellation in the integration over bosonic zero modes $\theta, \tilde{\theta}$.

We can apply this result from the D-brane sector to get more intuition, at least heuristically, for the cancellation of the "moduli potential" in the closed string sector discussed in §3.2. A diagram with charged closed strings running in loops would naively seem to contribute to the "moduli potential" once the deformation which splits their masses according to (2.11) is turned on. However, at the worldsheet level we have seen that semiclassically (in Euclidean space) the vertex operators $K, \tilde{K}$ localize on boundaries and charged vertex operator insertions, introducing factors of the form (5.1) into the contributions of individual worldsheets with charges $q, \tilde{q}$ propagating from boundaries or vertex operator insertions. The moduli are uncharged, so from the worldsheet point of view it is clear that the closed string "moduli potential" still cancels also after the deformation.

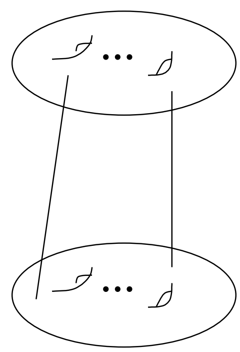

Figure 6: Degenerating Riemann surface contributing cancelling contributions to the "moduli potential".

However, we can dissect the closed string diagrams in a way that provides a little more intuition for how the naive spacetime intuition fails in this non-local theory. Consider a Riemann surface Σ which has degenerated into separate Riemann surfaces Σ_i connected by a set of thin tubes, as in figure 6. The ends of the tubes can be approximated by local operator insertions $T_{ij}(z, \bar{z})$ on the Σ_i. The K and $\tilde{K}$ insertions on each Σ_i then localize on the insertions T_{ij}, and for diagrams in which the closed strings propagating in the long thin tubes are charged, one gets a contribution from this.

Semiclassically, at order $\tilde{h}$, one therefore gets an insertion of the form (5.1) where the $\Lambda_i, \bar{\Lambda}_j$'s are the values of $\Lambda, \bar{\Lambda}$ at the positions of the ends of the long thin tubes. Generically, a semiclassical analysis will not be valid, but in some circumstances (such as when the strings propagating in the Σ_i are very heavy from say winding or momentum along the T^4) it will be. In any case it gives a useful heuristic picture of how cancellations might occur in spacetime similarly to the case of D-branes. Namely, the contribution to the "moduli potential" again involves integrating over the positions $\theta, \tilde{\theta}$ of the insertion points of the tubes, giving a cancellation at order $\tilde{h}$.

It is not obvious from the point of view described in this section what happens to the D-brane corrections to the "moduli potential" on the gravity side at higher orders in $\tilde{h}$ or in g_s. The field theory side again predicts no contributions to the "moduli potential". There are several diagrams at order $\tilde{h}^2$ which must therefore cancel if the duality is correct. These cancellations may be nontrivial, analogous to two and higher loop cancellations of protected quantities in supersymmetric theories which do not follow from any simple counting of Bose-Fermi degeneracies. In our case, the only symmetry principle we have so far identified to enforce the cancellation is the duality (namely, the exact marginality of the deformation on the field theory side), and it would be nice to obtain a more direct argument applicable for arbitrary $\tilde{h}$ on the gravity side.

6. The Flat Space Limit

It is interesting to contemplate NLST's arising in backgrounds other than AdS. One way to try to construct such backgrounds is to consider the flat space limit of the AdS realizations we have so far. It seems that we should not expect such a limit to make sense, since our deformation is maximally non-local on the S^3, and induces correlations at distances of the order of the AdS scale that go to infinity in the flat limit, leading to failure of the standard conditions for unitarity. This is related to the fact that in taking the flat limit one focuses on one small region of the S^3, and the other regions which are correlated with it in the original theory go off to infinity. In this section we will show that there is a scaling of $\tilde{h}$ which gives finite contributions when one takes the $L_{AdS}/l_s \to \infty$ flat space limit of the results derived in the previous section, and also gives a finite non-local deformation of the worldsheet action in the same limit. However, we have not been able to find sensible vertex operators in the resulting theory, so it is not clear if the flat space limit defines a sensible (unitary) NLST or not.

6.1. Definition of the Flat Space Limit

The flat space limit of AdS_3 backgrounds with NS-NS charges involves taking Q_1 and Q_5 to infinity with a fixed ratio $Q_5/Q_1 = g_6^2$. Since the AdS_3 string metric goes as

$ds^2 = Q_5(d\phi^2 + e^{2\phi}d\gamma d\bar{\gamma})$, the relation between ϕ and a flat space coordinate X_ϕ is of the form $\phi \simeq X_\phi/\sqrt{Q_5}$. Thus, if we wish to keep X_ϕ constant (which is the simplest possibility) we need to take $\phi \to 0$ when we take the flat space limit. Similarly, when we expand around some particular generic point on $AdS_3 \times S^3$, the angular coordinates on the S^3 are related to flat space coordinates by $\theta \simeq X_\theta/\sqrt{Q_5}$, $\tilde{\theta} \simeq X_{\tilde{\theta}}/\sqrt{Q_5}$, and the charges $q, \tilde{q}$ become momenta $p, \tilde{p}$ in the $X_\theta, X_{\tilde{\theta}}$ directions, where $p = q/\sqrt{Q_5}$, $\tilde{p} = \tilde{q}/\sqrt{Q_5}$.

Consider for example (5.7). In the flat space limit this result reduces to

$$\frac{\tilde{h}}{Q_1 Q_5} q_1 \tilde{q}_1 G_0 \frac{4}{3}(\phi_1 - \phi_2)^2 = \frac{\tilde{h}}{Q_1 Q_5} p_1 \tilde{p}_1 G_0 \frac{4}{3}(X_{\phi_1} - X_{\phi_2})^2. \tag{6.1}$$

Therefore if $\tilde{h}$ is constant, independent of Q_5, then this effect disappears in the limit (we are assuming that the amplitude G_0 before the deformation has a finite flat-space limit). We want the effect to actually depend in the flat space limit only on $g_6^2 = Q_5/Q_1$. Thus, we need to take $\tilde{h} \to \infty$ as

$$\tilde{h} = \tilde{h}_0 Q_5^2, \tag{6.2}$$

where $\tilde{h}_0$ is constant, and then we get a finite surviving contribution in this limit.

Let us denote the position of one brane by X and the other by Y. Then, because of the factors (5.6) and (6.1), the order $\tilde{h}$ contribution to the annulus diagram for a particular closed string s exchanged in the flat space limit is proportional to

$$\partial_{X_\theta} \partial_{X_{\tilde{\theta}}} D_s(X - Y), \tag{6.3}$$

where $D_s(X - Y)$ is the contribution of this mode to the exchange force and we only wrote down the dependence on $X_\theta, X_{\tilde{\theta}}$ (for a graviton exchange diagram D_s is the position-space propagator between the D-branes). In the flat space limit, the sum over charges $q, \tilde{q}$ turns into a continuous integral over momenta $p, \tilde{p}$ in the $X_\theta, X_{\tilde{\theta}}$ directions. This washes out the supersymmetry breaking effects, which arise from the distinction between sums over $q, \tilde{q} \in \mathbb{Z}$ and sums over $q, \tilde{q} \in \mathbb{Z} + 1/2$. So the force between flat space BPS branes will cancel when all the contributions are added in (since the added contributions will still be supersymmetric), but for branes and anti-branes the force discussed above will persist in the limit.

It is instructive to spell out more explicitly the form of the vertex $K(x)$ appearing in the deformation (2.20) in the flat space limit. Taking the limit as in (6.2), with $\tilde{h}$ scaling as Q_5^2, the deformation is

$$\delta S_{ws} = \tilde{h}_0 g_6^2 \int d^2x K(x) \tilde{K}(\bar{x}). \tag{6.4}$$

Taking the limit as above, one finds (from (2.18) and (2.19))

$$K(x) \to \frac{1}{\pi}\int d^2z \partial_z X_\theta \left[\frac{-2\bar{x}}{(1+|x|^2)^2}\partial_{\bar{z}}X_\phi + \frac{1}{(1+|x|^2)^2}\partial_{\bar{z}}X_{\bar{\gamma}} - \frac{\bar{x}^2}{(1+|x|^2)^2}\partial_{\bar{z}}X_\gamma\right], \quad (6.5)$$

where $X_\phi = \sqrt{Q_5}\phi, X_\gamma = \sqrt{Q_5}\gamma, X_{\bar{\gamma}} = \sqrt{Q_5}\bar{\gamma}$ are the flat space coordinates descending from the AdS_3 coordinates as discussed above, and similarly for $\tilde{K}(\bar{x})$. This linear combination of $\partial_{\bar{z}}X^\mu$ descends from a longitudinal (formally pure gauge) vector potential in AdS_3, and does not have fermionic pieces as a result[14]. In the flat space limit, $K(x)$ is an integrated physical vertex operator for a tensor field in spacetime at zero momentum.

Plugging (6.5) into (6.4) and performing the integral over x, we obtain

$$\begin{aligned}\delta S_{ws} \propto \tilde{h}_0 g_6^2 \int d^2z_1 \int d^2z_2 \Big[& 2\left(\partial_{z_1}X_\theta \partial_{\bar{z}_1}X_\phi\right)\left(\partial_{\bar{z}_2}X_{\tilde{\theta}}\partial_{z_2}X_\phi\right) \\ & + \left(\partial_{z_1}X_\theta\partial_{\bar{z}_1}X_{\bar{\gamma}}\right)\left(\partial_{\bar{z}_2}X_{\tilde{\theta}}\partial_{z_2}X_\gamma\right) + \left(\partial_{z_1}X_\theta\partial_{\bar{z}_1}X_\gamma\right)\left(\partial_{\bar{z}_2}X_{\tilde{\theta}}\partial_{z_2}X_{\bar{\gamma}}\right)\Big].\end{aligned} \quad (6.6)$$

Note that the coefficients in front of the three terms are exactly those which give an $SO(3)$ rotational invariance in the $X_\phi, X_\gamma, X_{\bar{\gamma}}$ directions, as expected in the flat space limit (in the Lorentzian case this will become $SO(1,2)$).

Thus we obtain a deformation of the general form (1.1) which persists in flat space. The deformation we have discovered is very simple: it consists of a sum of bilocal products of linear combinations of zero-momentum off-diagonal graviton and NS B-field vertex operators. Since they are total derivatives, these vertex operators localize to the boundaries of the worldsheet or to other operator insertions. The NS B-field decouples from closed strings, and the off-diagonal metric couples to modes with momentum along the X_θ and $X_{\tilde{\theta}}$ directions.

6.2. Observables in the Flat Space Limit ?

We would like to study whether the theory we obtain in this limit is sensible. To do so it is important to formulate and study the behavior of physical observables in this theory. Because of the relative simplicity of the theory (6.6), we can investigate this question rather explicitly. We will consider two types of candidate observables, using two techniques for analyzing the deformed theory. The first arises by considering familiar flat space vertex operators inserted into the path integral with the bilocal contribution to the action (6.6).

[14] We thank D. Kutasov for a discussion on this point.

The second, described in §6.3, arises by considering a different but equivalent presentation of the theory, in terms of a Lagrange multiplier which renders the action Gaussian, and considering a particular set of non-local insertions in the path integral which are natural in this formalism. In both cases, because of the non-locality of the underlying theory, we will find in the end no separately renormalizable constituents in a given amplitude; instead we will be left with a rather unpredictive situation in which each amplitude must be independently renormalized. This is presumably related to the problems one expects with unitarity when taking a limit which keeps only a region much smaller than the non-locality scale.

In the first approach we calculate correlation functions of vertex operators in the flat space limit by inserting powers of (6.6) to obtain the effect of our deformation, and we find that this leads to divergences. Consider for example a correlator of n vertex operators $V_{p_j} \sim e^{ip_j \cdot X}$. Let us compute the order $\tilde{h}_0$ correction to this correlator coming from the first term of our deformation (6.6). It is given by

$$\begin{aligned} \tilde{h}_0 \int \prod_{k=1}^{n} d^2 w_k d^2 z_1 d^2 z_2 \langle \prod_{j=1}^{n} e^{ip_j \cdot X(w_j, \bar{w}_j)} \partial X_\theta \bar{\partial} X_\phi (z_1, \bar{z}_1) \partial X_\phi \bar{\partial} X_{\tilde{\theta}} (z_2, \bar{z}_2) \rangle \sim \\ \sim \tilde{h}_0 \int \prod_{k=1}^{n} d^2 w_k d^2 z_1 d^2 z_2 \prod_{i,j=1}^{n} |w_{ij}|^{p_i \cdot p_j / 2} \cdot \\ \cdot \left[\sum_{i=1}^{n} \frac{p_i^\theta}{z_1 - w_i} \right] \left[\sum_{i=1}^{n} \frac{p_i^\phi}{\bar{z}_1 - \bar{w}_i} \right] \left[\sum_{i=1}^{n} \frac{p_i^\phi}{z_2 - w_i} \right] \left[\sum_{i=1}^{n} \frac{p_i^{\tilde{\theta}}}{\bar{z}_2 - \bar{w}_i} \right]. \end{aligned} \tag{6.7}$$

The last four factors come from contractions of the zero-momentum vertex operators in the deformation with those of the n vertex operators whose correlation function we are calculating. The integrals over z_1 and z_2 diverge when a zero-momentum vertex operator hits an e^{ipX} on the worldsheet. In ordinary flat space string theory, this divergence is a standard pole in the S-matrix arising from the fact that when a zero-momentum particle combines with a momentum p particle to produce a momentum p particle, the latter is still on-shell and gives a pole (this can be seen explicitly by continuing the zero momentum vertex operators to nonzero momentum q and expanding in small q). We would like to understand the meaning of this divergence in our application, where this correlator describes the shift of the correlation function of vertex operators V_{p_i} under the NLST deformation.

Let us first regularize this divergence. If we put a short-distance cutoff on the worldsheet analogous to (3.5) in the AdS case, namely letting other operators approach only to a distance a_j from V_j, we find that we need to redefine :

$$\left[\prod_j \int d^2w_j V_j(w_j)\right] \rightarrow \left[\prod_j \int d^2w_j V_j(w_j)\right]\left(1 - \sum_{l,k} \tilde{h}_0 g_6^2 p_l^\theta p_l^\phi log|\tilde{a}_l|^2 p_k^{\tilde{\theta}} p_k^\phi log|\tilde{a}_k|^2\right), \quad (6.8)$$

where $\tilde{a}$ is proportional to a and absorbs some subleading contributions. This shift cancels the divergence above at leading order in $\tilde{h}_0$. Note that the shift we need for the product of vertex operators is not equal to the product of the shifts we need for each vertex operator separately. This would not occur in a local worldsheet string theory. However, since in a NLST the worldsheet Lagrangian is non-local, it may be necessary to consider as observables the full set of multilocal excitations of the theory, since attempting to consider only local vertex operators would generically fail under quantum corrections.

Unfortunately, this prescription appears to render the theory unpredictive as far as these observables go, since one must renormalize separately each physical process rather than obtaining predictions for physical processes arising from a finite number of renormalizations of constituent fields and couplings. It is therefore unclear whether the theory is renormalizable in the appropriate sense, because each combination of vertex operators is a new multilocal operator in the theory and one therefore has to input an infinite amount of information to define the set of observables. Because of this issue, our results on the flat space limit are inconclusive (though we think intriguing) and we hope to improve our understanding of the proper physical constraints on this sort of theory in general backgrounds in future work.

We started with a theory in which the non-locality scale is of the order of the AdS curvature radius L_{AdS}, and this goes to infinity in the flat space limit. It would be very interesting to figure out what (if any) are the appropriate observables in such a non-local theory, that can give meaningful physical amplitudes. Of course it is worth emphasizing that with $\tilde{h}$ scaling independently of Q_5, we would obtain conventional flat space string theory in the limit. In usual flat space string theory we can define observables by S-matrix elements describing particles which are much farther from each other than the characteristic non-locality scale. These observables give well-defined correlation functions. In the flat space NLST's we constructed in this section we have seen that this fails, so some other types of observables are needed in order to get physical predictions. In the AdS case the consistency of our NLST constructions was guaranteed by the consistency of the dual conformal field theory, but it is not clear what are the consistency conditions for flat space

NLST's. Thus, in the absence of predictions for physical observables we cannot say if the theories we constructed in this section are consistent (e.g. if they are unitary) or not.

Although they may render the question of the existence of a useful flat space limit questionable, the above divergences do teach us something significant about the AdS_3 model that is our main focus in this paper. In §3, we saw that the vertex operators involved in Euclidean closed string amplitudes localize to the boundary of AdS. The nontrivial (divergent) answers we find in the closed string sector after taking the flat space limit here indicate that there was bulk physics in the closed string sector in AdS. In particular, as we have seen in some detail, the flat space limit does not leave us with a consistent S-matrix, which should have been the case if all of the effects of the deformation were at the boundary. This provides evidence that the effects of the deformation, and in particular the non-locality of the theory, permeate the bulk of AdS space, as expected from the marginality of the deformation, despite the fact that we can write the deformation as a boundary term (4.1).

Note from (6.7) that we see the non-local effects in the flat space limit only for correlators including vertex operators with nonzero momentum along what used to be the S^3 directions: $p^\theta \neq 0 \neq p^{\tilde{\theta}}$. This is consistent with our expectations from the form of the deformation (4.1) that the $6d$ theory is non-local even though the effect on the $3d$ action is a local boundary term.

6.3. Another Set of Non-local Operators in NLST

Despite the above complications, one might hope that the physics simplifies in terms of some other natural subset of observables. There is a way of presenting the theory (6.6) (and more generally the theories (1.1)) which simplifies the analysis considerably, and which suggests another set of multilocal operators in the theory.

Consider the worldsheet path integral for the theory (6.6), written as a Gaussian using Lagrange multipliers λ (and ignoring the fermionic fields which play no role) :

$$Z_{NLST} = \int d\lambda \int [DX] e^{-\int d^2z \partial X^\mu G_{\mu\nu}(\lambda) \bar{\partial} X^\nu} e^{-\frac{1}{2\tilde{H}}\lambda_{\theta\phi}\lambda_{\tilde{\theta}\phi} - \frac{1}{\tilde{H}}(\lambda_{\theta\bar{\gamma}}\lambda_{\tilde{\theta}\gamma} + \lambda_{\theta\gamma}\lambda_{\tilde{\theta}\bar{\gamma}})}, \qquad (6.9)$$

where $\tilde{H} \propto \tilde{h}_0 g_6^2$ and where

$$G_{\mu\nu}(\lambda) dx^\mu dx^\nu = \eta_{\mu\nu} dx^\mu dx^\nu + \lambda_{\theta\phi} dx^\theta dx^\phi + \ldots, \qquad (6.10)$$

where ... are other similar terms involving the other λ's. By integrating over λ one can see that equation (6.9) gives a description of the theory equivalent to the bilocal description of

(6.6), but now the worldsheet path integral is Gaussian. This is similar to what arises in wormhole physics [31,32,33] and it would be interesting to explore further the conceptual interpretation of this mathematical trick.

This method seems potentially useful, particularly in our flat space limit where it renders the partition function Gaussian. As discussed in section 5 of [1] one can also employ this method in the AdS/CFT case, but generically in AdS/CFT it is not trivial to implement either on the gravity side or on the field theory side of the correspondence. Naively it simplifies the analysis to one involving only "single-trace" deformations, but in fact this is complicated on both sides of the duality. On the field theory side, the "single-trace" operators in question are relevant operators, and one would be attempting to integrate over the corresponding scale-dependent couplings. This involves a sum over field theories with different parameters, whose physical interpretation is unclear. On the gravity side, these relevant perturbations deform the geometry dramatically. In terms of the worldsheet string theory, the BRST-invariance condition for vertex operators changes as a function of λ, an issue we will also encounter in our flat space analysis here.

Considering just the closed string sector, which feels only the symmetric part of $G_{\mu\nu}$, we can change variables to $Y^\mu(z,\bar{z}) \equiv E^\mu_\nu(\lambda)X^\nu(z,\bar{z})$, where the matrix E is defined by $E^\rho_\mu(\lambda)\eta_{\rho\sigma}E^\sigma_\nu(\lambda) = G_{\mu\nu}(\lambda)$. Then, the path integral becomes

$$\int d\lambda \int [DY] \prod_z [\det E(\lambda)]^{-1} e^{-\frac{1}{2H}\lambda_{\theta\phi}\lambda_{\bar{\theta}\phi} - \frac{1}{H}(\lambda_{\theta\bar{\gamma}}\lambda_{\bar{\theta}\gamma} + \lambda_{\theta\gamma}\lambda_{\bar{\theta}\bar{\gamma}})} e^{-\int d^2z \partial Y^\mu \eta_{\mu\nu}\bar{\partial}Y^\nu}. \tag{6.11}$$

Here the λ dependence is only in the Jacobian (and in the Gaussian), which is in this flat space situation independent of the embedding coordinates $Y(z,\bar{z})$.

Now let us consider observables (correlation functions of vertex operators). A new set of multilocal operators in the X variables are the simple operators

$$V_k[Y] \equiv e^{ik\cdot Y}. \tag{6.12}$$

In terms of X, these vertex operators vary with λ so as to preserve conformal invariance in the path integral for arbitrary λ. We can insert these into the integrand of (6.11), and divide by the vacuum path integral (6.11) to normalize. This reproduces the correlators of momentum modes for ordinary flat space string theory.

These momentum modes (6.12) of Y are highly non-local when expressed in terms of X (in the original formulation of the theory without λ). In general, when we map a product of the $V_k[Y]$ operators to the X variables it will not map into the product of the maps of these operators. So in terms of X there is still no S-matrix with amplitudes determined by renormalizations of operators describing separated excitations. However it is true here that there is a set of multilocal amplitudes (insertions of products of operators (6.12)) which are naturally determined by the standard renormalizations of (6.12) in the Y variables, and which produce results isomorphic to the flat space S-matrix.

A similar analysis using the (6.9) prescription can be performed in $AdS_3 \times S^3 \times T^4$, with similar results arising at leading order in $\tilde{h}$. The observables analogous to (6.12) there reproduce the standard AdS correlators in the original undeformed supersymmetric background. Again they are non-local and non-locally renormalized in terms of the physical variables $\phi, \gamma, \bar{\gamma}, \theta, \tilde{\theta}$. The meaning of these observables is unclear, since the physics of the CFT does seem to depend on $\tilde{h}$. It is tempting to speculate that these objects could realize a hidden non-local supersymmetry in the system which explains the vanishing of the "moduli potential", while as we have seen the physics in terms of the ordinary variables exhibits broken supersymmetry.

In general, it is important to clarify what are the conditions for physically consistent NLST models, both for conceptual interest and with regard to the potential for applications. In particular, it would be very interesting to develop more realistic models that have the exact stability after supersymmetry breaking that we have found in the AdS_3 backgrounds studied in this paper.

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