Regular electrically charged objects in Nonlinear Electrodynamics coupled to Gravity

Irina Dymnikova [1] and Evgeny Galaktionov [1]

[1] A.F. Ioffe Physico-Technical Institute, Politekhnicheskaya 26, 194021 St Petersburg, Russia

E-mail: igd.ammp@mail.ioffe.ru, irina@uwm.edu.pl

Abstract. We present a brief review of the basic properties of regular electrically charged black holes and electromagnetic solitons, predicted by analysis of regular solutions to dynamical equations of Nonlinear Electrodynamics minimally coupled to Gravity (NED-GR). The fundamental generic feature of regular NED-GR objects is the de Sitter vacuum interiors and the relation of their masses to spacetime symmetry breaking from the de Sitter group. Regular spinning NED-GR objects have interior de Sitter vacuum disk with the properties of a perfect conductor and ideal diamagnetic. The disk is confined by the ring with the superconducting current which provides the non-dissipative source of the electromagnetic fields and of the intrinsic magnetic momentum.

1. Introduction

Nonlinear electrodynamics has been proposed by Born and Infeld as motivated by two goals: (i) to consider electromagnetic field and particles in the frame of one physical entity; (ii) to avoid divergences of physical quantities. Applying their theory to construct the model of the electron as an extended particle, Born and Infeld obtained the finite value for its electromagnetic energy, but cannot prevent the appearance of a singularity in geometry [1].

These goals can be achieved in a self-consistent way in the Nonlinear Electrodynamics minimally coupled to Gravity (NED-GR) [2, 3] (for a review [4, 5]). Regular compact NED-GR objects are related by electromagnetic and gravitational self-interaction and governed by the source-free NED-GR equations in such a way that their electromagnetic fields are described by source-free nonlinear Maxwell equations while their gravitational fields are described by the Einstein equations with the stress-energy tensors of their own nonlinear electromagnetic fields. Minimal coupling does not require introducing additional assumptions.

The fundamental general properties of regular electrically charged NED-GR objects follow from analysis of the NED-GR dynamical equations ([4] and references therein).

The algebraic structure of stress-energy tensors for electromagnetic fields is determined by

$$T_t^t = T_r^r \ (p_r = -\rho). \tag{1}$$

For the stress-energy tensors of this class satisfying the weak energy condition (WEC) which guarantees the non-negativity of density and mass, dynamical equations contain the class of regular spherically symmetric solutions with the de Sitter vacuum centers, which describe electrically charged regular black holes and electromagnetic solitons - non-singular non-dissipative particle-like structures bound by their self-interaction [2].

Regular spherical solutions to the Einstein equations with the stress-energy tensors specified by (1) are described by the metrics of the Kerr-Schild class [6] which can be transformed to the axially symmetric metrics, asymptotically Kerr-Newman for a distant observer [7], in the frame of the general model-independent approach developed by Gürses and Gürsey [8], which includes the Newman-Janis algorithm [9] usually applied for constructing the rotating solutions presented in the literature [10]-[19][1] (for a review [21, 4]). For axial solutions satisfying WEC the de Sitter vacuum center becomes the equatorial de Sitter vacuum disk [3].

For the electrically charged black holes and solitons satisfying WEC the de Sitter vacuum disk has the properties of a perfect conductor and ideal diamagnetic [3, 22]. It is confined by the ring with a superconducting current which replaces the ring singularity of the Kerr-Newman geometry, serves as a nondissipative source of the electromagnetic and gravitational fields [23, 24] and provides the origin of the intrinsic magnetic momenta of NED-GR regular objects [25].

The NED-GR regular objects satisfying WEC have positive masses of electromagnetic origin [2] generically related to breaking of spacetime symmetry from the de Sitter group, which is the fundamental property of all regular objects described by the metrics of the Kerr-Shild class [26]. This provides the intrinsic relation between gravity, spacetime symmetry and the Higgs mechanism for a particle mass generation [27].

Regular electrically charged black holes have two horizons or one horizon for an extreme black hole. An electromagnetic soliton as a regular structure without horizons replaces the naked singularity of the Kerr-Newman geometry, highly undesirable in General Relativity due to its unpredictable influence on outside observers what is protected by the weak cosmic censorship hypothesis which states that there can not exist a singularity visible for an observer at infinity - only singularities closed by the event horizon are allowed [28].

The basic properties of a spinning electromagnetic soliton offer the possible explanation for the appearance of a minimal length scale $l_e = 1,57 \times 10^{-17}$cm revealed with the 5σ significance in the annihilation reaction $e^+ + e^- \to \gamma\gamma(\gamma)$ at the energy E=1.253 TeV. The length l_e cm can be understood as characteristic distance at which the electromagnetic attraction is balanced by the gravitational repulsion of the de Sitter vacuum [29, 5].

In Section 2 we present the basic equations and spacetime structure of regular electrically charged NED-GR objects. Section 3 is devoted to electromagnetic dynamics which determines the electromagnetic source of the external fields and of the intrinsic magnetic momentum of spinning objects. Section 4 contains conclusions.

2. Basic equations and the spacetime structure for NED-GR regular objects

The spherical metrics from the Kerr-Schild class, describing the compact objects with the stress-energy tensors (1) have the form [6, 8, 30]

$$ds^2 = g(r)dt^2 - \frac{dr^2}{g(r)} - r^2(d\theta^2 + \sin^2\theta d\phi^2); \quad g(r) = 1 - \frac{\mathcal{M}(r)}{r}. \tag{2}$$

The mass function $\mathcal{M}(r)$ in (2) reads [30]

$$\mathcal{M}(r) = 4\pi \int_0^r \tilde{\rho(x)} x^2 dx \tag{3}$$

where $\tilde{\rho(r)}$ denotes the electromagnetic density for an original spherical solution applied to obtain a new axial solution.

The basic condition $T_t^t = T_r^r$ and conservation equation $T^{\mu\nu}_{;\nu} = 0$ define the radial pressure p_r and the transversal pressure $p_\perp$ as $p_r(r) = -\rho(r)$; $p_\perp(r) = -\rho(r) - r\rho'(r)/2$ [26].

[1] More general approach, based on the basic properties of metrics from the Kerr-Schild class, was applied for obtaining the axially symmetric solutions in the noncommutative geometry [20].

The axially symmetric Gürses-Gürsey metrics read [8]

$$ds^2 = \frac{2f-\Sigma}{\Sigma}dt^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 - \frac{4af\sin^2\theta}{\Sigma}dtd\phi + \left(r^2 + a^2 + \frac{2fa^2\sin^2\theta}{\Sigma}\right)\sin^2\theta d\phi^2 \quad (4)$$

in the Boyer-Lindquist coordinates and in the geometrical units $c = G = 1$, where

$$\Delta = r^2 + a^2 - 2f(r); \quad \Sigma = r^2 + a^2\cos^2\theta; \quad f(r) = r\mathcal{M}(r) \quad (5)$$

and a is the specific angular momentum. The basic condition $T_t^t = T_r^r$ is satisfied for the eigenvalues of a stress-energy tensor calculated in the co-moving frame, co-rotating with the angular velocity $\omega(r) = a/(r^2 + a^2)$ [31]. For $r \to \infty$ the metric (4) asymptotically approaches the Kerr-Newman metric with $f(r) = Mr - q^2/2$. The mass $M = \mathcal{M}(r \to \infty)$. The parameter q is the electric charge obtained as the constant of integration [7].

In the axially symmetric geometry the surfaces of constant r are the confocal ellipsoids $r^4 - (x^2 + y^2 + z^2 - a^2)r^2 - a^2z^2 = 0$ which degenerate, for $r = 0$, to the equatorial disk

$$x^2 + y^2 \le a^2, \quad z = 0, \quad (6)$$

bounded by the ring $x^2 + y^2 = a^2,\ z = 0$ [31]. The Cartesian coordinates x, y, z are related to the Boyer-Lindquist coordinates r, θ, ϕ by $x^2 + y^2 = (r^2 + a^2)\sin^2\theta; \quad z = r\cos\theta$.

An asymptotically Kerr-Newman rotating black hole can have at most two horizons defined by $\Delta(r_+, r_-) = r^2_{+(-)} + a^2 - 2f(r_{+(-)}) = 0$, where r_- and r_+ are the internal and event horizons, and two ergospheres [22] defined by $g_{tt} = r_e^2 + a^2\cos^2\theta - 2f(r_e) = 0$. The ergosphere which touches the event horizon (shown in Fig.1) confines the ergoregion where extraction of energy available for a distant observer, can occur due to $g_{tt} < 0$ ([32] and references therein). A soliton can have, dependently on the form of the original spherical density profile $\tilde{\rho}(r)$ which determines the master function $f(r)$ in (4), two ergospheres and ergoregion between them, one ergosphere confining the ergoregion involving the whole interior, or no ergospheres [22].

The density $\rho(r, \theta)$ and the transversal pressure $p_\perp(r, \theta)$ (eigenvalues of a stress-energy tensor in the co-rotating frame) are given by [3, 22]

$$\rho(r, \theta) = r^4\tilde{\rho}(r)/\Sigma^2; \quad p_\perp(r, \theta) = \left(r^4 - r^2\Sigma\right)\tilde{\rho}(r)/\Sigma^2 + r^2\tilde{p}_\perp(r)/\Sigma \quad (7)$$

where $\tilde{p}_\perp(r)$ refer to a related spherical solution.

The equation of state for the radial pressure and density in (7) follows from the algebraic structure of a stress-energy tensor (1), while the relation of the transversal pressure with the density is obtained from $T^\mu_{\nu,\mu} = 0$. This gives [3]

$$p_r(r, \theta) = -\rho(r, \theta); \quad p_\perp(r, \theta) = -\rho - \frac{\Sigma}{2r}\frac{\partial\rho(r, \theta)}{\partial r}. \quad (8)$$

In the limit $z \to 0$ the expression $r^2/\Sigma \to 1$ in (8) and we get in the equatorial plane [3]

$$\rho(r, \theta) = \rho(r) = \tilde{\rho}(r); \ p_\perp = -\tilde{\rho} - \frac{r}{2}\tilde{\rho}'. \quad (9)$$

For spherical solutions regularity requires $r\tilde{\rho}'(r) \to 0$ at $r \to 0$ [2]. By virtue of (9), the density on the disk (6) takes the de Sitter value $\rho(r \to 0) = \tilde{\rho}(0)$, while $p_\perp = -\tilde{\rho}$ in (9). Hence the equation of state on the disk (6) represents the de Sitter vacuum [3]

$$p_r = p_\perp = -\rho. \quad (10)$$

The de Sitter vacuum disk is the basic generic feature of interior for all regular compact objects in NED-GR determined by the spacetime structure [3] (for a review [21]).

The de Sitter disk is shown in Fig.1 [22] which presents the internal structure for a rotating black hole with two horizons, the internal and the event horizons r_- and r_+, and ergosphere which confines the ergoregion aroud r_+.

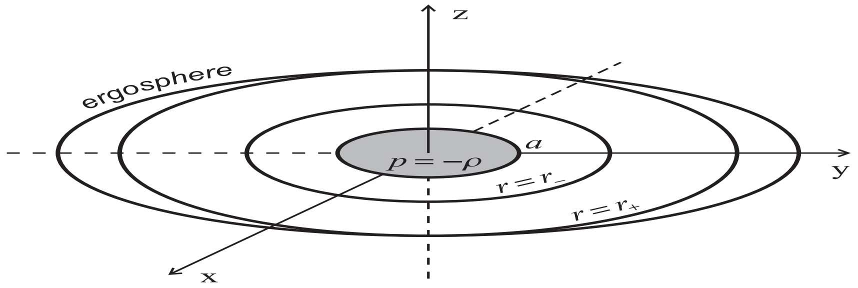

Figure 1. .Internal structure of a regular electrically charged black hole.

3. Electromagnetic fields, their source and the intrinsic magnetic momentum

The NED-GR spinning regular objects are made of a nonlinear electromagnetic field and governed by the source-free equations ([33, 2] and references therein)

$$\nabla_\mu \mathcal{L}_F F^{\mu\nu} = 0; \quad \nabla_\mu {}^*F^{\mu\nu} = 0 \tag{11}$$

where $F^{\mu\nu}$ are field intensities, F-the field invariant $F_{\mu\nu}F^{\mu\nu}$, ${}^*F^{\mu\nu} = \eta^{\mu\nu\alpha\beta}F_{\alpha\beta}/2$ where $\eta^{0123} = -1/\sqrt{-g}$ and $\mathcal{L}_F = d\mathcal{L}/dF$. These equations can be written in the Maxwell form

$$\nabla \cdot \mathbf{D} = 0; \ \nabla \times \mathbf{H} = \partial \mathbf{D}/\partial t; \ \nabla \cdot \mathbf{B} = 0; \ \nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t \tag{12}$$

for the field vectors $\mathbf{E} = F_{\beta 0}; \ \mathbf{D} = \mathcal{L}_F F^{0\beta}; \ \mathbf{B} = {}^*F^{\beta 0}; \ \mathbf{H} = \mathcal{L}_F {}^*F_{0\beta}$.

The electric induction **D** and magnetic induction **B** are related with the field intensities **E** and **H** by $D^\alpha = \epsilon^\alpha_\beta E^\beta; \ B^\alpha = \mu^\alpha_\beta H^\beta$, where ϵ^β_α and μ^β_α are the tensors of the electric and magnetic permeability, respectively. In geometry (4) their eigenvalues are given by [3]

$$\epsilon^r_r = (r^2 + a^2)\mathcal{L}_F/\Delta; \quad \epsilon^\theta_\theta = \mathcal{L}_F; \quad \mu^r_r = (r^2 + a^2)/(\Delta \mathcal{L}_F); \quad \mu^\theta_\theta = 1/\mathcal{L}_F. \tag{13}$$

Non-zero field components compatible with the axial symmetry are $F_{01}, F_{02}, F_{13}, F_{23}$, in geometry with the metric (4) they are related by

$$F_{31} = a \sin^2\theta F_{10}; \quad F_{23} = \frac{r^2 + a^2}{a} F_{02}. \tag{14}$$

The field invariant $F = F_{\mu\nu}F^{\mu\nu}$ in the axially symmetric case reduces, with taking into account (14), to

$$F = 2\left(\frac{F_{20}^2}{a^2 \sin^2\theta} - F_{10}^2\right). \tag{15}$$

The stress-energy tensor of a nonlinear electromagnetic field has the general form

$$\kappa T^\mu_\nu = 2\mathcal{L}_F F - \frac{1}{2}\delta^\mu_\nu \mathcal{L}. \tag{16}$$

The eigenvalues of the stress-energy tensor are given by $\rho = 1/2\mathcal{L} - 2\mathcal{L}_F(F_{01}F^{01} + F_{31}F^{31}); \ p_\perp = 2\mathcal{L}_F F_{2\alpha}F^{2\alpha} - 1/2\mathcal{L}$ [3] Taking into account (14), we get [3]

$$(p_\perp + \rho) = 2\mathcal{L}_F\left(F_{10}^2 + \frac{F_{20}^2}{a^2 \sin^2\theta}\right). \tag{17}$$

This equation provides the information about satisfaction of WEC by NED-GR electrically charged structures [22, 34].

For regular rotating objects WEC can in principle be violated. For NED-GR electrically charged objects the violation of WEC, $p_\perp + \rho < 0$, would lead to negative values of the electric permeability in (17) what is incompatible with the basic requirement of electrodynamics of continued media [35].

Dynamical equations (11) form the system of 4 equations for 2 independent functions (by virtue of (14)). These equations and the compatibility condition for this system which provides the necessary condition for the existence of solutions [22], are satisfied by the functions [3, 22]

$$\Sigma^2(\mathcal{L}_F F_{01}) = -q(r^2 - a^2\cos^2\theta); \quad \Sigma^2(\mathcal{L}_F F_{02}) = qa^2 r\sin 2\theta; \tag{18}$$

$$\Sigma^2(\mathcal{L}_F F_{31}) = aq\sin^2\theta(r^2 - a^2\cos^2\theta); \quad \Sigma^2(\mathcal{L}_F F_{23}) = aqr(r^2 + a^2)\sin 2\theta \tag{19}$$

in the linear regime, $\mathcal{L}_F = 1$, when the solutions (18)-(19) coincide with the solutions to the Maxwell-Einstein equations in the Kerr-Newman geometry [36, 37]. In the strongly nonlinear regime (18)-(19) satisfy the system (11) as the asymptotic solutions in the limit $\mathcal{L}_F \to \infty$ [22, 4]. In this regime the equations (17) and (18) yield $\mathcal{L}_F = 2q^2/(\Sigma^2(p_\perp + \rho))$ [3, 22], so that $\mathcal{L}_F \to \infty$ on the de Sitter disk (6). As a result $\mu^r_r = \mu^\theta_\theta = 1/\mathcal{L}_F \to 0$, $\epsilon^r_r = \epsilon^\theta_\theta = \mathcal{L}_F \to \infty$, and hence the disk (6) has the properties of a perfect conductor and ideal diamagnetic [3, 22].

At approaching the disk $r = 0$ and in the Maxwell weak field limit for $r \to \infty$, the field invariant (15) is determined by the solutions (18) and takes the form $F = -(p_\perp + \rho)/\mathcal{L}_F$. As a result F goes to -0 for both $r \to 0$ and $r \to \infty$. Non-monotonic behavior of the invariant F leads to the inevitable branching of an electromagnetic lagrangian $\mathcal{L}(F)$ [33, 2]. This requires description of the Lagrange dynamics by the non-uniform variational principle [38]

$$S = S_{int} + S_{ext} = \frac{1}{16\pi}\left[\int_{\Omega_{int}} (R - \mathcal{L}_{int}(F))\sqrt{-g}d^4x + \int_{\Omega_{ext}} (R - \mathcal{L}_{ext}(F))\sqrt{-g}d^4x\right]. \tag{20}$$

Variation with the action (20) results in the dynamical equations (11) in both Ω_{int} and Ω_{ext}, and in the proper internal conditions [38] on the boundary surface Σ_c where F has the extremum.

The surface current on the disk is defined as $4\pi j_k = [e^\alpha_{(k)} F_{\alpha\beta} n^\beta$ [10], where $e^\alpha_{(k)}$ are the base vectors related to the intrinsic coordinates on the disk t, ϕ, $0 \le \xi \le \pi/2$; the vector $n_\alpha = \delta^1_\alpha(1 + q^2/a^2)^{-1/2}\cos\xi$ is the unit normal to the disk, and the symbol [..] denotes a jump across its surface in the direction orthogonal to it [10]. Using on the disk the solutions (18)-(19) and taking into account that $\mu = 1/\mathcal{L}_F$ there, we obtain [23]

$$j_\phi = -\frac{q}{2\pi a}\sqrt{1 + q^2/a^2}\,\sin^2\xi\,\frac{\mu}{\cos^3\xi} = -\frac{q}{2\pi a}\sqrt{1 + q^2/a^2}\,\sin^2\xi U \tag{21}$$

where $U = \mu/\cos^3\xi$. The magnetic permeability $\mu = 0$ on the whole disk. The intrinsic coordinate ξ on the disk ranges within $0 \le \xi \le \pi/2$. As a result the current (21) is zero over the disk except the ring $\xi = \pi/2$, where the numerator and denominator in the second fraction vanish independently. This undeterminacy means that the ring current can be any and amount to a non-zero total value, and thus satisfies the general condition for transition to a superconducting state [35].

The electric field vanishes on the disk (6) [3, 24], the superconducting ring current (21) flows without resistance in the region of the perfect conductivity ($\epsilon \to \infty$), and represents a non-dissipative source of the exterior fields which can in principle provide a practically unlimited life time of an object [23, 24].

The ring current (21) produces a magnetic momentum $\mu_{in} = j_\phi S$ where S is the disk area. . NED-GR equations for electromagnetic field (11) are source-free, and hence this magnetic momentum is *intrinsic* for any regular rotating compact object in NED-GR [25]. When the magnetic moment of an object is known, the coefficient U can be restored which shall give the value of the superconducting ring current powering this regular electrically charged NED-GR object. For example, for the electromagnetic soliton with the parameters of the electron, $a = \hbar/(2m_e)$ [36] this gives $j_\phi = 79.277$ A [25] (for details [5]).

4. Conclusions

The basic generic features of regular electrically charged NED-GR black holes and electromagnetic solitons follow from general model-independent analysis of dynamical equations governing their behavior. This allows to come to the following conclusions:

(1) Descrption of regular NED-GR objects in the frame of the Lagrange dynamics requires the non-uniform variational principle.

(2) The common fundamental feature of all regular electrically charged NED-GR objects is the de Sitter vacuum interiors.

(3) Masses of objects are of electromagnetic origin and related to breaking of spacetime symmetry from the de Sitter group.

(4) Interiors of the spinning regular electrically charged NED-GR objects have the form of the de Sitter vacuum disk with the properties of a perfect conductor and ideal diamagnetic.

(5) The de Sitter disk is confined by a superconducting ring current which serves as the non-dissipative electromagnetic source of the external fields and of the intrinsic magnetic momentum for all regular electrically charged NED-GR objects.

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