Three-Nucleon Forces and Meson-Exchange Currents in the Trinucleon

The evidence for three-nucleon forces and meson exchange currents in the trinucleon systems is reviewed. The current status of theoretical calculations is described.

Much of the recent theoretical and experimental work in the three-nucleon subfield of nuclear physics has been directed at understanding certain low energy properties of the trinucleon systems. These fundamental quantities determine the size and energy scales of the trinucleon bound states (^{3}He and ^{3}H) and the scattering of neutrons on deuterons at zero energy. Although this most basic three-nucleon physics has been a source of study for decades, it has been only recently that our calculational abilities have reached a level where systematic investigation could be attempted. Indeed, the ability to calculate wavefunctions for the first set of nontrivial nuclei is the raison d'etre of this subfield.

Accurate calculation of energy eigenvalues and wavefunctions has proven exceptionally difficult. Only a few years ago different calculational methods applied to the same potential model gave different results. Recently, very different calculational methods applied to the same model have given virtually identical results.[1,2] This achievement has greatly increased the level of confidence in our calculational abilities.

Traditional nuclear physics attempts to describe nuclei as a collection of nucleons only, interacting by means of pairwise potentials. Recently this philosophy has been found to be inadequate, both conceptually and in achieving a quantitative understanding of experimental data. Two

Comments Nucl. Part. Phys.
1983, Vol. 11, No. 2, pp. 51–61
0010-2709/83/1102-0051/$18.50/0

new ingredients which we will discuss in this Comment are three-nucleon forces and meson exchange currents.

While pairwise nucleon–nucleon forces depend only on the relative spatial coordinates (as well as spin and isospin) of two nucleons, three-nucleon forces depend on the simultaneous positions of three nucleons. This concept is not new; it dates at least as far back as the seminal paper of Primakoff and Holstein[3] on both the three-electron force and the three-nucleon force. The primary thrust of that work was to extend to the three-nucleon case Yukawa's argument that virtual meson exchange between two nucleons was the genesis of the two-nucleon force. At the same time they derived the three-electron force: one electron simultaneously emits two virtual photons which are then absorbed by two different electrons. This latter force is very weak because it is proportional to $1/c^4$, where c is the speed of light and is large compared to any velocity scale in an atom. The three-nucleon force is similar and arises from simultaneous virtual meson exchanges; most theoretical attention has focused on virtual pion exchange, since this is the component of longest range and least theoretical uncertainty. It is expected to be proportionately larger than the corresponding atomic force.

Classically, the electric current is the sum of the separate currents of all moving charged particles in a system. Failure to keep track of all charges will obviously result in a lack of charge or current conservation. In a nucleus we must take into account the motion of virtual charged mesons as well as the nucleons. Since both charge and angular momentum can be exchanged between nucleons via the mesons, one expects on semiclassical grounds that the meson exchange currents will modify the static magnetic moments of ground states, as well as the transition magnetic moments between the ground and excited states. This is known to be important in the n–p system,[4] where a magnetic dipole transition can flip the spin of the deuteron from the $^3S_1$ state to the $^1S_0$ state. Meson exchange currents make a 10% contribution to this process near threshold. Similar mesonic effects can be expected in the three-nucleon system.

The trinucleon ground states are characterized by a number of attributes, several of which are listed in Table I. Recent calculations[1,2,5] using three "realistic' potential models are listed, as well as the corresponding experimental values. The rms radii are obtained from electron scattering data and include the intrinsic finite size of the nucleons themselves. These numbers characterize the extent of the charge dis-

TABLE I
Comparison of calculated trinucleon parameters vs experiment

Model	$<r^2>^{1/2}_{^3He}$ (fm)	$<r^2>^{1/2}_{^3H}$ (fm)	E_B (MeV)
RSC	2.09	1.83	7.2
SSCC	2.00	1.76	7.6
Paris	2.02	. . .	7.4
Expt	1.86(3)	1.69(5)	8.5

The three models correspond to the Reid Soft Core, Supersoft Core (C), and Paris potentials.

tribution, $\rho(r)$: $<r^2> \equiv \int r^2\rho(r)d^3r$. It is clear that the calculated binding energies E_B are systematically low while the radii are too large. These properties are consistent because more binding would be expected to shrink the system. What is the cause of this binding defect?

Several possibilities exist: (1) The so-called realistic nucleon–nucleon potentials are actually poor representations of the nucleon–nucleon physics and better potentials would elminiate the defect; (2) the Schrödinger equation is nonrelativistic and relativistic corrections are needed; (3) three-body forces are needed. This categorization is convenient but not unique; relativistic corrections, for example, can affect both the two-body forces and can generate three-body forces. While a definitive answer to our question above does not exist, there is no evidence that possibilities (1) or (2) solve our problem. The limited calculational evidence[6] that exists suggests that relativistic corrections may be small due to a cancellation between the separate kinetic and potential energy contributions. The consensus has been that possibility (3) is the most likely.

Before discussing three-nucleon forces in more detail, we mention one more set of experimental data which might be interpreted as being heavily influenced by such forces. Figure 1 depicts the ^{3}He charge density for point nucleons, the experimental data points with error bars and two theoretical calculations performed using the Reid Soft Core potential model,[7] with and without a Coulomb interaction between the two protons. The error bars are purely statistical and do not reflect the total uncertainty in the density; the point-nucleon density is obtained by taking out the finite size of the individual nucleons from the experimental form factor, and this procedure involves theoretical assumptions

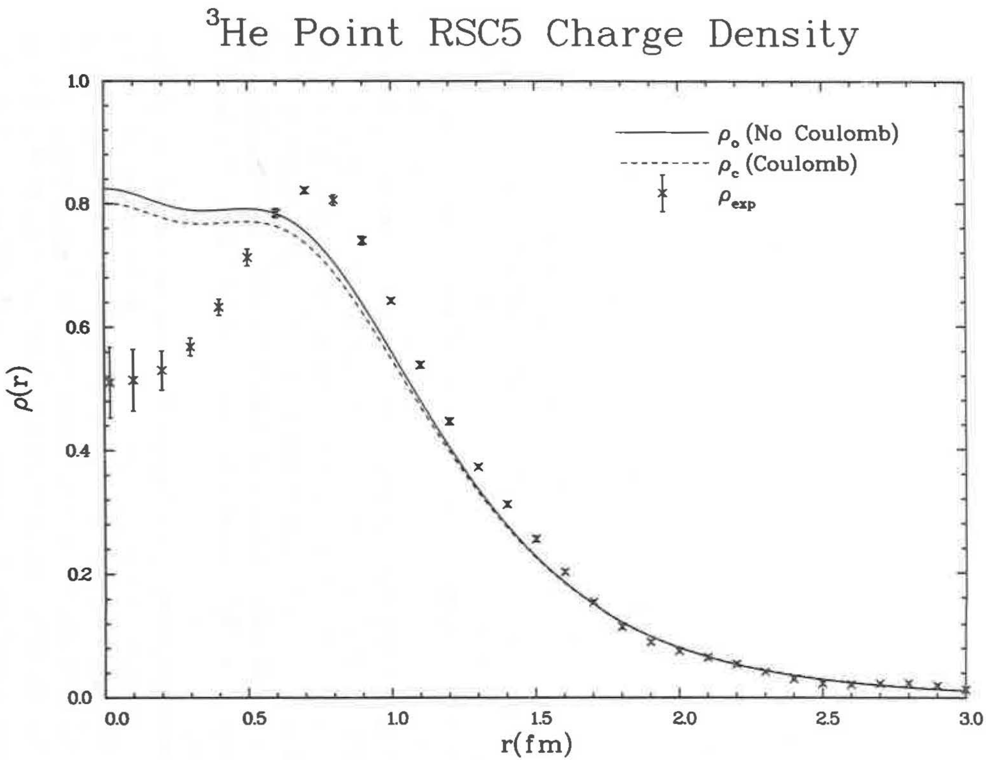

FIGURE 1 Calculated and experimental ^{3}He charge densities, $\rho(r)$, plotted vs r.

and extrapolations. Nevertheless, the comparison of theoretical and experimental densities is of qualitative, if not quantitative, importance. In addition to problems at large values of the radius r reflecting too little binding and too large a radius, there is an obvious discrepancy at the origin between theory and experiment. The size of this problem compared to the radius problem is somewhat misleading; a fairer comparison would be to multiply $\rho(r)$ by r^2. Somewhat less than 1% of the total charge of the nucleus is required to fill in the hole. This effect is less significant in the global sense than the radius problem.

The hole is simply another manifestation of an old problem: the lack of calculated strength in the secondary maximum of the ^{3}He form factor $F(q^2)$, or Fourier transform of the charge density. The latter relationship can be used to prove that

$$\rho(0) = \frac{2}{\pi}\int_0^\infty F(q^2)q^2dq. \quad (1)$$

Integration over the momentum transfer q is substantially weighted toward moderately large values of q because of the factor of q^2. The form factor has the value 1 at $q=0$, decreases to zero, becomes negative, and then achieves a secondary (negative) maximum. The negative contributions depress $\rho(0)$ in Eq. (1). It is the excess of experimental strength in the vicinity of the secondary maximum which causes a major part of the discrepancy in the charge density.

One possible way to explain this is to note the geometrical aspect of $r=0$ in the impulse approximation. This is indicated in Figure 2 which shows the three coordinates of the three-nucleon system: x, y, and θ. The center-of-mass of the planar system is indicated by a cross; the variable r is simply the distance from that cross to any one of the protons, which can be taken to be the bottom nucleon. Clearly, taking $r=0$ is equivalent to taking $y=0$ and this puts the nucleons in a linear configuration. For some currently not understood reason the theoretical probability of this configuration is too high. The energy on the other hand depends to a large extent on isosceles configurations, and these should lead to greater energy.

The geometric feature that has galvanized the attention of theoreticians is the fact that three-nucleon forces will depend on the angle θ. It is entirely possible, for example, that this force could be repulsive when θ is near 0 or π, thus reducing the linear configuration, while

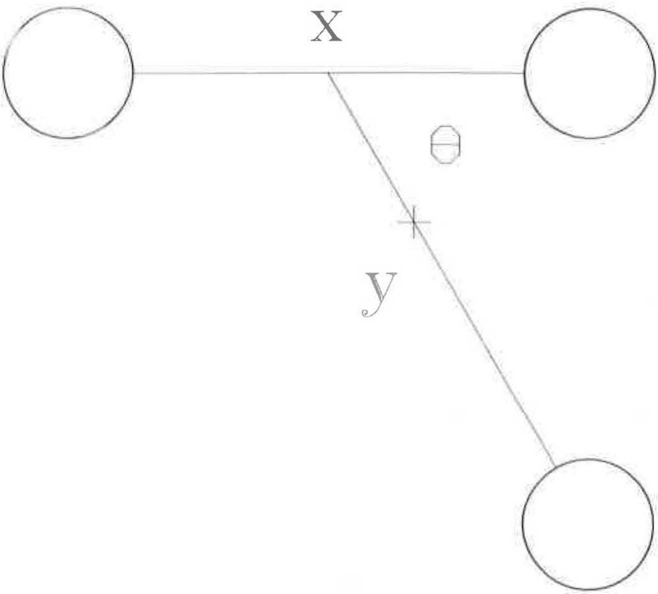

FIGURE 2 Coordinates used to describe the trinucleon system.

being attractive when θ is near π/2, thus increasing the binding. Simple theoretical arguments do indeed lead to such forces. The real question is whether more sophisticated forces which attempt to satisfy general principles and depend on the nucleon's spin and isospin possess the same general feature.

It is a regrettable fact that these forces must be theoretically constructed with little experimental input, which argues caution. How much progress would have been possible in the two-nucleon problem without two-nucleon scattering data? Nevertheless, at least the main features of the longest range part of the three-nucleon potential involving two pion exchanges should be explicable. This was the motivation of the Tucson force,[8] which is the most popular of the various three-nucleon potentials. From a technical viewpoint, this force was constructed with the most emphasis on a correct treatment of the particle physics aspects of off-shell pion–nucleon scattering, a major physical ingredient. At the present time there have been several calculations of three-body properties using the Tucson force. Regrettably, all of them differ somewhat on the two-body force model.[9–12] Results on the effect of the three-nucleon force differ drastically; one calculation gives little effect, while another similar calculation gives a large (~ 1.1 MeV) increase in binding. Most find little effect on ρ(0). Hopefully, this discrepancy will be resolved soon.

The previous argument presupposes that the entire structure of the trinucleon charge density is due to the structure of the wavefunction. Other effects such as meson exchange currents can also modify the charge density. Unfortunately, these exchange currents are relativistic corrections, and a consistent calculation requires the use of a wavefunction which also manifests relativistic effects.[6] Calculations that have been performed have not been consistent in this respect, and consequently are an unreliable indicator of the physics of the hole in ^3He.

Our previous discussions involved only the trinucleon ground states. Is there any evidence for three-body forces in scattering data? The evidence is lamentably slim. There is a strong correlation between values of the trinucleon bound state energies and the doublet scattering length. Scattering at zero relative energy of a deuteron and a nucleon is described by two different physical quantities: the doublet and quartet scattering lengths describing total angular momentum $J = 1/2$ and $J = 3/2$, respectively. The latter quantity is sensitive only to the (two-body) deuteron binding energy. The former quantity is plotted in Figure 3 versus trinucleon binding energy for several two-nucleon force models.[9,13] The resulting numbers are rather well fit by a straight line, the Phillips line,[14] which also passes through the (unfitted) experimental point describing the physical trinucleon system. This relationship would indicate that whatever physics is missing from our description of the bound state is also missing from the scattering problem. If three-body forces subsequently are found to account for the binding energy defect, they should be found to have a correspondingly large effect on the doublet scattering length.

Meson exchange currents is a topic which has come of age in nuclear physics. Not only are they needed for satisfying general principles such as current conservation, but also for a quantitative understanding of a variety of processes, including several in the three-nucleon systems. The experimental magnetic moments of ^3He and ^3H are -2.128 nm and 2.979 nm, respectively. To a reasonably good approximation, the two like nucleons in these nuclei pair off and make no contribution to the magnetic moment. Thus the magnetic moments of ^3He and ^3H look roughly like a free neutron and a free proton, respectively. For the Reid

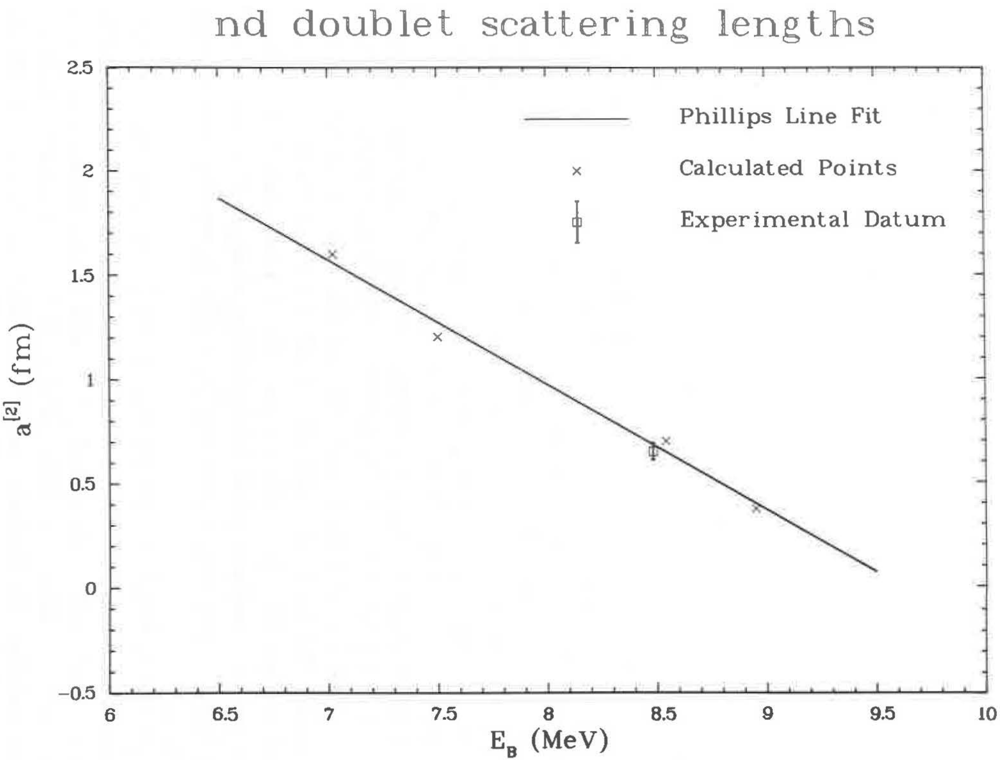

FIGURE 3 Doublet *nd* scattering lengths $a^{[2]}$ plotted versus ^3H binding energy.

Soft Core potential model a recent calculation[15] gives impulse approximation results of -1.732 nm and 2.521 nm for ^{3}He and ^{3}H. The differences between experiment and calculation are then given by -0.40 nm and 0.46 nm, respectively. The change in sign is symptomatic of an isovector process and indeed one expects the isovector pion exchange currents to play a dominant role. Exchange currents calculated in Ref. 15 give additional contributions of -0.41 nm and 0.42 nm, respectively. This has to be considered very satisfactory agreement.

A more sensitive test of our theoretical understanding is to elastically scatter electrons at backward angles from the trinucleons, which process determines the magnetic form factor $F_M(q^2)$, the Fourier transform of

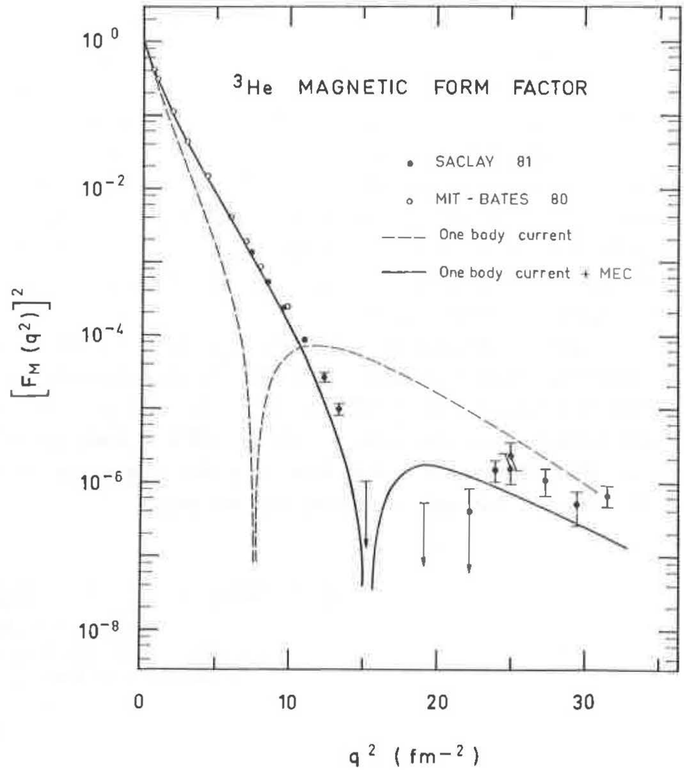

FIGURE 4 Square of ^{3}He magnetic form factor plotted vs square of momentum transfer q.

the spatial distribution of magnetization is these systems. The data from two recent experiments at Bates Linac (MIT) and Saclay[16] are shown in Figure 4 in comparison with two calculations: impulse approximation (dashed line) and impulse approximation plus meson-exchange currents (solid line). The magnitude of the exchange current effect is apparent, and agreement between experiment and theory has to be considered very good. Because the trinucleon ground states form an isodoublet, one can decompose the ground state properties into isoscalar and isovector parts. Unfortunately, this requires ^{3}H data and very few exist, because such targets are highly radioactive. Experiments which probe the ^{3}H ground state are highly desirable at this time, and the two that are planned at MIT (Bates) and Saclay are eagerly awaited by the theoretical community.

A somewhat different situation exists with the radiative capture of thermal neutrons on deuterons [$n + d \rightarrow {}^3\mathrm{H} + \gamma$]. The small cross section for this process in the impulse approximation is due to the fact that the dominant part of the ^{3}H ground state wavefunction is an eigenstate of the $M1$ operator and has no overlap with the initial (unbound) state. This accident causes meson exchange contributions to the process to be comparable to the impulse approximation. Calculations are currently underway by several groups. This should be a sensitive test of our understanding of meson currents and the nuclear force models used to calculate trinucleon wavefunctions.

Substantial theoretical progress has been made in recent years, and more is expected. New experiments on the ^{3}H ground state should provide new information. Hopefully we will soon have an indication of whether three-nucleon forces are needed in order to understand the trinucleons. This information could provide a strong impetus for extending three-body force calculations to heavier nuclei.

J. L. FRIAR and B. F. GIBSON
Theoretical Division,
Los Alamos National Laboratory,
Los Alamos, New Mexico 87545

and

G. L. PAYNE
Department of Physics and Astronomy,
University of Iowa,
Iowa City, Iowa 52242

References

1. J. L. Friar, E. L. Tomusiak, B. F. Gibson and G. L. Payne, Phys. Rev. **24,** 677 (1981).
2. Ch. Hajduk and P. U. Sauer, Nucl. Phys. A **369,** 321 (1981).
3. H. Primakoff and T. Holstein, Phys. Rev. **55,** 1218 (1939).
4. J. L. Friar, Lecture Notes in Physics **108,** 445 (1979) reviews this topic.
5. A. Laverne and C. Gignoux, Nucl. Phys. A **203,** 597 (1973).
6. J. L. Friar, Nucl. Phys. A **353,** 233C (1981) reviews this topic.
7. J. L. Friar, B. F. Gibson, E. L. Tomusiak and G. L. Payne, Phys. Rev. C **24,** 665 (1981).
8. S. A. Coon, et al., Nucl. Phys. A **317,** 242 (1979); S. A. Coon and W. Glöckle, Phys. Rev. C **23,** 1790 (1981).
9. J. Torre, J. J. Benayoun and J. Chauvin, Z. Phys. **300,** 319 (1981).
10. W. Glöckle, to be published and private communication.
11. Muslim, Y. E. Kim, and T. Ueda, Phys. Lett., to be published.
12. J. Carlson, V. R. Pandaripanda and R. B. Wiringa, Univ. of Illinois preprint, ILL-(NU)-82-10, and private communication.
13. G. L. Payne, J. L. Friar and B. F. Gibson, LANL preprint, LA-UR-82-1484.
14. A. C. Phillips, Rep. Prog. Phys. **40,** 905 (1977).
15. R. Bornais, thesis, University of Montreal (1981), unpublished.
16. J. M. Cavedon, et al., contribution to 9 ICOHEPANS, Versailles, July 1981, p. 215. The calculated values are taken from Ref. 15.