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Suggested icosahedral states in ^{208}Pb

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Abstract. The known 6^+ yrast and 12^+ yrare states are recognized as members of an icosahedral rotational band. The 18^+ state at 8813 keV is newly identified as icosahedral rotor.

1. States in the heavy nucleus $^{208}_{82}$Pb$_{126}$.

Small particles with icosahedral symmetry are abundant in biology and astrophysics. Molecules may rotate with spins 6, 10, 12, 15, and larger but not with 17, 19, 23, and 29. Several low spins are disallowed by 2-, 3-, 5-fold symmetry. The complete sequence of spins from 0 to 30 allowed by icosahedral symmetry was recently detected in the fullerene molecule C_{60} [2]. In the heaviest stable nucleus ^{208}Pb six states are recognized as icosahedral rotors.

Isobaric analog resoncances (IAR) in heavy nuclei were discovered in 1964 as a surprise. In 1966 seven IARs in ^{209}Bi were identified [3]. Angular distributions and excitation functions for ^{208}Pb(p, p') were measured in 1968 using a scattering chamber equipped with 12 semiconductor detectors [4] (Fig. 1). The Q3D magnetic spectrograph constructed in 1973 was finished in 1999 with a final detector of 3 keV resolution after thirty years continuous development. From 2003 until 2020 ^{208}Pb(p, p') data for all known IARs in ^{209}Bi (and one new IAR) including corresponding ^{208}Pb(d, d') and ^{207}Pb(d, p) data were taken [7] [8].

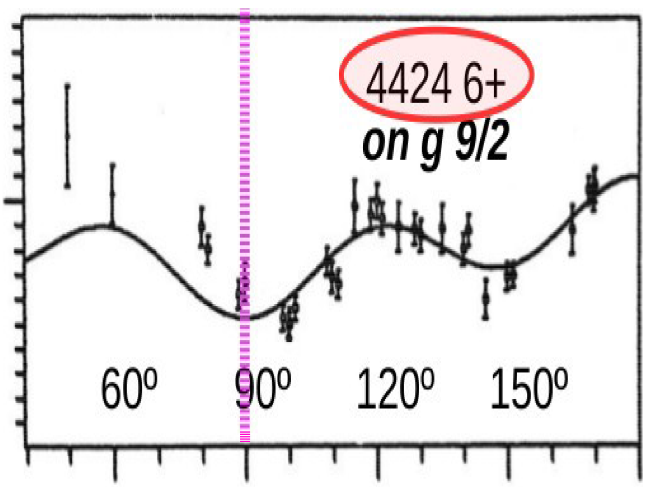

Figure 1. On an isolated IAR angular distributions for ^{208}Pb(p, p') should be symmetric around $\Theta = 90°$ (marked). On the $g_{9/2}$ IAR the 4424 6^+ yrast states shows a typical diffraction pattern, however. It is recognized as an icosahedral rotor [1].

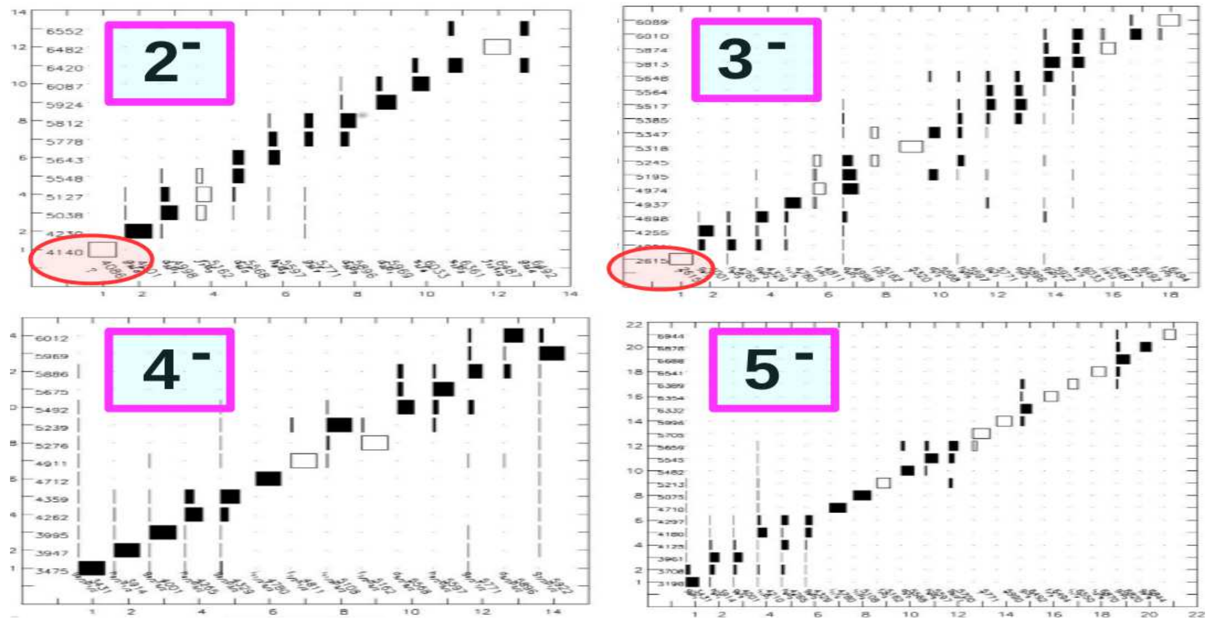

Figure 2. Orthogonal matrices with amplitudes of 2^-, 3^-, 4^-, 5^- states in ^{208}Pb [7]. Black rectangles denote strengths of observed configurations, white rectangles as determined by orthonormality and sum-rule considerations [6]. The 2^-, 3^- yrast states are tetrahedral configurations [9]. For 150 states at $E_x < 6.2$ MeV [7] most (for $E_x < 7.0$ MeV [8] many) amplitudes are larger than 0.8 (2/3 full strength). Many amplitudes are larger than 0.98 (missing strength less than 5 permil). Among two hundred SWM configurations in the essay (Sec. 2) order numbers of 1p-1h configurations are small ($1 \leq M \leq 7$) for all spins $I^{\pm}_M$ and $2 \leq M \leq 3$ for $I^\pi = 3^-$; no 1p-1h state with spin $0^- \leq I^\pi_M \leq 2^-$, $1^+ \leq I^\pi_M \leq 2^+$ is involved.

Three hundred neutron bound and many proton bound states in ^{208}Pb are rather completely identified with spin, parity, and dominant structure [5] [6] [7] [8]. Fig. 2 displays the strength of 1p-1h configurations for spins 2^-, 3^-, 4^-, 5^- [6] [7]. For most states at $E_x < 6.2$ MeV the strength is larger than 80% [6]. For many (nearly) stretched states [Eq. (2)] amplitudes from about 0.9 to 0.95 (98% strength) are observed. The amplitude of the dominant 1p-1h configuration ($g_{9/2}p_{1/2}$ as the lowest one) in the 5^-_1 state is still 0.8 (Table 4 in [9]).

It turns out that about ten percent of the bound states are described as *non-1p-1h* states. At least ten states are tetrahedral rotation-vibrations [9]. Eighteen states states are built by the coupling of 1p-1h states to the 2615 3^- yrast state [10]. Five states are recognized as icosahedral rotations [1]. A few states at $5 < E_x < 6$ MeV have an unknown structure [11].

2. Simplified weak coupling model (SWM).

Following an idea by de-Shalit [13] a simplified weak coupling model (SWM) is constructed from three constituents, (1) 1p-1h configurations, (2) tetrahedral rotation-vibrations, (3) icosahedral rotations. In general the description of coupling multiple spins needs 6j- and 9j-symbols [10]. Yet deep inelastic scattering [14] tends to excite states with couplings in stretched mode preferentially.

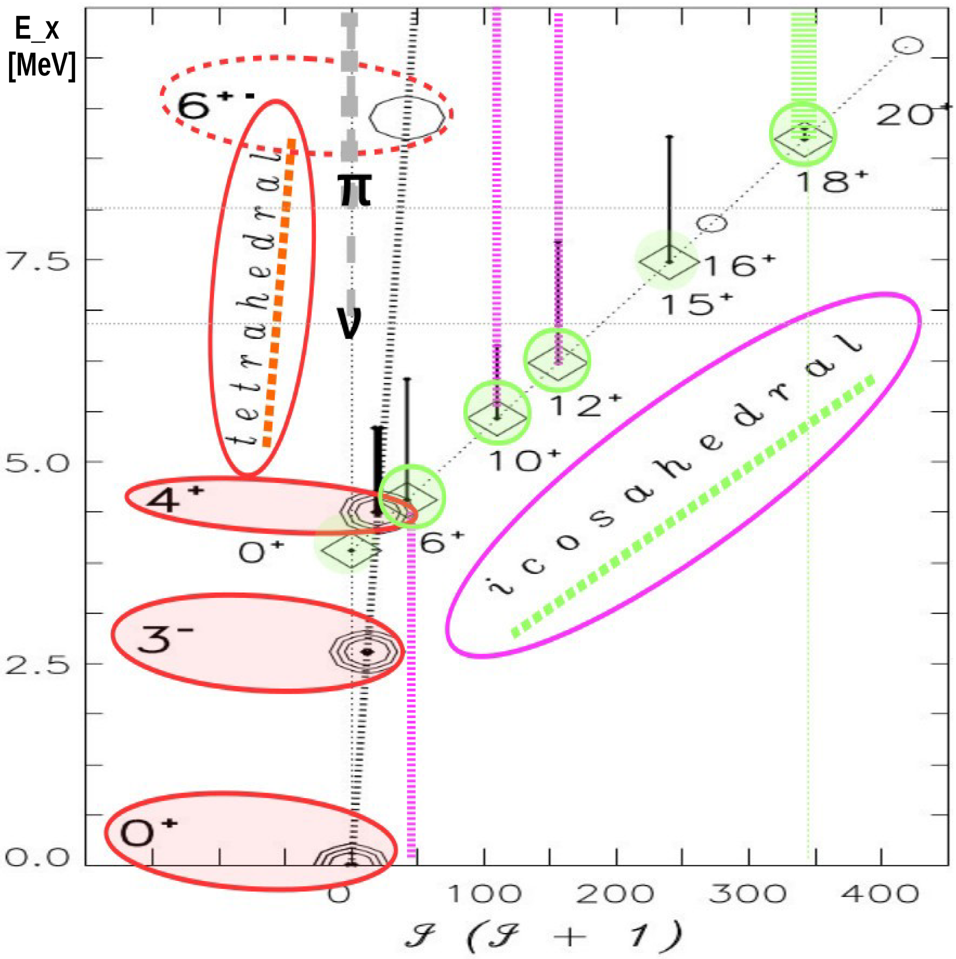

Figure 3. Tetrahedral (left) and icosahedral rotation bands in ^{208}Pb (right). ν and π mark neutron and proton threshold, $S(n) = 7368$, $S(p) = 8008$.

The SWM *assumes* stretched coupling within 1p-1h configurations and to collective configurations with tetrahedral and icosahedral symmetry. Hence spin and parity of the SWM state is calculated by adding the spins and multiplying the parities,

$$I_M^{SWM} = I_{M_1} + I_{M_2} + I_{M_{T_d}} + I_{M_{Y_h}}, \qquad \pi_M^{SWM} = \pi_{M_1} \times \pi_{M_2} \times \pi_{M_{T_d}} \times \pi_{M_{Y_h}}. \quad (1)$$

The coupling of the 2615 3^- yrast and 6101 12^+ yrare states with the double-intruder 6744 14^- state would generate spin 29^+ for a state at 15500. Shell model calculations tried to explain 40 observed levels within several γ-cascades. Spins $30 \lesssim I \lesssim 35$ are predicted for the highest observed level at 16362 [14]. (The unit keV is omitted.)

Depending on the Nordheim number [15] the yrast state in a 1p-1h multiplet has either the lowest or highest excitation energy. The multiplet $j_{15/2}i_{13/2}$ with two neutron intruders spans the largest range, $1^- \le I_M^\pi \le 14^-$. All members are known [15] except for the natural parity 9^-, 11^- states. The 6744 14^- state is stretched ($s = 0$), the 6449 13^-, 6435 12^- states are nearly stretched ($s > s_{min} - 3$),

$$s = -s_{min}, \cdots, \ -1, \ 0, \qquad s_{min} = J + j - \min(J - j)/2. \quad (2)$$

An essay [12] tried to explain the observed γ-transitions. Most levels are doublets within the 1 keV uncertainty of excitation energies, however.

80 states in forty observed levels [14] are described by nearly two hundred SWM configurations C_M^{SWM} [Eq. (6)] with spin I_M^{SWM} and parity π_M^{SWM} [Eq. (1)] and energy ${E_x}_M^{SWM}$ [Eq. (5)] More than 90% of the constituing 1p-1h configurations are in stretched or nearly stretched mode [Eq. (2)]. Almost all transition multipolarities among the 40 observed levels are low (maximal $E3$ or $M2$). The differences $E_x - {E_x}_M^{SWM}$ for the sum of the constituent's energies by the coupling of SWM configurations [Eq. (6)] are small, mostly less than 15 keV. The mean deviation is $|E_x - {E_x}_M^{SWM}| = 2$. (The third column in Fig. 4 shows deviations $E_x - {E_x}_M^{SWM}$ for a few states.)

Excitation energies of tetrahedral configurations are calculated by Eqs. (23), (27) in [16], see also Eq. (12) in [17]. For tetrahedral rotors rigidity is assumed,

$$ {E_x}^{T_d}[(v_1, v_2, v_3) I^\pi] = \overline{\kappa_{T_d}}\, I^\pi_m (I^\pi_m + 1), \quad \kappa_{T_d}(m,n) = \frac{I^\pi_m (I^\pi_m + 1) - I^\pi_n (I^\pi_n + 1)}{E_x^m - E_x^n}. \quad (3) $$

The rotation momentum $\overline{\kappa_{T_d}}$ is derived for spins $I^\pi_m = 0^+, I^\pi_n = 3^-$, $m = 1, n = 2$. The tetrahedron vibrates in addition to rotation in three modes $[(v_1, v_2, v_3) \neq (0\ 0\ 0)]$ [16] [17].

Excitation energies of rotating icosahedrons are calculated by assuming rigidity, too,

$$ {E_x}^{Y_h}(I^\pi) = \overline{\kappa_{Y_h}} I^\pi (I^\pi + 1), \quad \kappa_{Y_h}(m,n) = \frac{I^\pi_m (I^\pi_m + 1) - I^\pi_n (I^\pi_n + 1)}{E_x^m - E_x^n}. \quad (4) $$

The rotation momentum $\overline{\kappa_{Y_h}}$ is derived for spins $I^\pi_m = 6^+,\ I^\pi_n = 15^+$ with $m = 2,\ n = 5$. (For icosahedral vibrations no theory is available [1].) The excitation energy for state N_s with spin I_M^{SWM} and parity π_M^{SWM} is calculated by adding the energies,

$$ {E_x}_M^{SWM}(N_s) = {E_x}^{1p-1h}(LJ\,lj(1), I^\pi_{M_1}) + {E_x}^{1p-1h}(LJ\,lj(2), I^\pi_{M_2}) + {E_x}^{T_d}(I^\pi_{M_{T_d}}) + {E_x}^{Y_h}(I^\pi_{M_{Y_h}}). \quad (5) $$

The order number M is deduced by comparison to calculations with SDI [15]. The configuration is obtained by combining the wave functions, where J_i, j_i is the spin and L_i, l_i the orbital angular momentum named s, p, d, f-j for the particle and hole, respectively,

$$ C_M^{SWM} = |LJ\,lj(1), I^\pi_{M_1}\rangle \otimes |LJ\,lj(2), I^\pi_{M_2}\rangle \otimes |I^\pi_{M_{T_d}}\rangle \otimes |I^\pi_{M_{Y_h}}. \quad (6) $$

3. Tetrahedral and icosahedral rotations.

Fig. 3 shows tetrahedral and icosahedral rotation bands in ^{208}Pb. The energy difference $E_x - E_x^{SWM}$ between observation and SWM calculation is enhanced by a factor 30 (vertical drawn bar). The tetrahedral $6^\pm$ parity doublet and the icosahedral 16^+, 20^+, and higher members are unknown, The 0^+ g.s., 2615 3^- yrast and 4324 4^+ yrast members in the tetrahedral rotation band known since 1966 are recognized [9] by comparison to ^{16}O first predicted in 1937 [17]. The 4424 6^+ yrast (Fig. 3) and 6101 12^+ yrare members in the icosahedral rotation band are known since 1966 and 1990 [5], respectively.

The excitation energy of the 4424 6^+ yrast and 6101 12^+ yrare states cannot be explained by calculations with the shell model [7] [11]. The shell model predicts seven 6^+ states at $E_x < 6.2$ MeV but ten 6^+ states are identified. The 6^+ yrast state is 0.5 MeV lower than other predicted 6^+ states [11]. Two 12^+ states expected in the shell model are observed [5]. Yet another extremely low lying 12^+ state shows up [11]; the spin assignment was verified beyond any doubt [14]. A discussion with Halcrow and Manton [1] solved the puzzle with the low excitation energies of the 12^+ yrare state (Fig. 9 and Sec. IV B.3 in [7]) and the 6^+ yrast state. The spin sequence (0), 6, 10, 12, (15), 18 is explained by an icosahedral rotation in congruence to molecules [2]. The interpretation of the 5444 state known from ^{208}Pb(p,p') at $E_p = 35$ MeV [18] allows the assignment of spin 10^+. An orbital angular momentum $L \gg 6$ confirms it.

The observation of the 8813 (40) state [14] with spin 18^+ settles the identification of the 6, 10, 12, 18 members in the spin sequence of an icosahedral rotation band (Figs. 3, 4). The 0^+ state is tentatively identified by ^{208}Pb$(n, n'\gamma)$ interpreting the unplaced 1221 γ-ray [5] as the γ-transition from the 3836 0^+ yrare to the 2615 3^- yrast state.

4. The 18^+ icosahedral rotor.

Eighty states in 40 levels observed by deep inelastic heavy ion reactions [14] are identified with spin I_M, parity π_M, state number (in parentheses), and SWM configurations [Eqs. (1), (3)-(6)]. Fig. 4 describes the spin recoupling with $6j, 9j$-symbols for seven γ-transitions. The tetrahedral 3^- state is marked magenta (dashed frame), the icosahedral state blue (dotted frame), the transition multipolarity bold face (red circle). The 10342 18^+_2 (50) state feeds the 9394 15^-_3 (45) state, then the 9103 17^-_1 (44) state and proceeds to the 8813 18^+_1 (40) state (Fig. 4).

Solely the 8813 (40) 18^+_1 state has no tetrahedral or 1p-1h constituent. The 8813 state is a pure 18^+ icosahedral rotor. The 8724 (37) 15^-_2, 8027 (25) 13^-_2 states are entirely collective without 1p-1h constituents in contrast to other states at $E_x > 6.5$ MeV. They are couples of the 2615 3^- tetrahedral with the 12^+ and 10^+ icosahedral state. In the γ-cascade $18^+_1 \to 15^-_2 \to 15^+_1 \to 13^+_2$ the angular momentum for the icosahedral rotation decreases from 18 to 12 [Eq. (7 a)] and further to 10 by the presence of the tetrahedral 3^- or a pair of 1p-1h states (Fig. 4).

The 8813 (40) 18^+_1 state feeds the 8724 (37) state with the 3^- yrast state coupled to the 12^+ yrare state [Eq. (7 a)]. An intermediate γ-transition to the 8265 (25) state [Eq. (7 b)] feeds the 8027 (25) state with the 2615 3^- yrast state coupled to the 5444 10^+ state [Eq. (7 c)].

The γ-cascade from the 8813 18^+_1 state to states 37, 39, 25, 16 is described with $6j, 9j$-symbols,

$$\underset{\substack{-a-\\ 8724\,(37)\;15^-_2}}{\begin{Bmatrix} \mathbf{3} & 15 & \langle\mathbf{18}\rangle \\ 15 & \boxed{3} & \langle\mathbf{12}\rangle \end{Bmatrix}} \underset{\substack{-b-\\ 8265\,(30)\;15^-_1}}{\begin{Bmatrix} \boxed{3} & \langle\mathbf{12}\rangle & 15 \\ 5 & \mathbf{2} & 10 \end{Bmatrix}} \underset{\substack{-c-\\ 8027\,(25)\;13^-_2}}{\begin{Bmatrix} 15 & \mathbf{2} & 13 \\ 3 & 10 & 5 \\ 13 & \boxed{3} & \langle\mathbf{10}\rangle \end{Bmatrix}} \underset{\substack{-d-\\ 6744\,(16)\;14^-_1}}{\begin{Bmatrix} 15/2 & 13/2 & \mathbf{1} \\ 13/2 & 15/2 & 14 \\ \langle\mathbf{10}\rangle & \boxed{3} & 13 \end{Bmatrix}} \qquad (7)$$

The γ-cascade from 10342 (50) 18^+_2 to 6744 (16) 14^-_1 with seven γ-transitions terminates with 6744 (16) 14^-_1 built by coupling the neutron intruders $j_{15/2}$ and $i_{13/2}$ while switching $j_{15/2} i_{13/2}$ from squeezed to stretched mode [Eq. (7 d)].

5. Conclusion.

Only sporadic data for electromagnetic transitions in ^{208}Pb is available. Several large gaps are in the observations. In contrast particle spectroscopy gathered roughly ten times more experimental data during more than 50 years and is rather complete up to about 8.0 MeV. Several hundred amplitudes for configuration changes are known from the study of the proton decay of IARs in ^{209}Bi and from ^{209}Bi$(d, {}^3$He) (Fig. 2). Mainly four experiments on γ-spectroscopy were performed, ^{208}Pb$(d, p\,\gamma)$, ^{208}Pb$(n, n'\,\gamma)$, ^{207}Pb+nγ [5] in the 1990's, and deep inelastic scattering recently [14]. Among states at $E_x > 6.8$ MeV in ^{208}Pb γ-transitions were measured for high spin states ($I \geq 10$) essentially only. A single γ-transition crosses the γ-cascade from 16362 to the 6744 14^- state built by coupling the neutron intruders. Few states with spin $I \leq 10$ in γ-cascades to states at $E_x > 7.7$ MeV are identified, mostly rather tentatively. From the neutron to the proton threshold ($S(n) = 7368$, $S(p) = 8008$) almost all states have spins $I \leq 3$. At $E_x < 7.7$ MeV allmost all states have spins $I \leq 8$. Below 6.2 MeV one third of the γ-transitions is unplaced. However below 5.4 MeV twice more new γ-transitions are identified. In γ-spectroscopy knowledge about the recoupling of spins described by $6j, 9j$-symbols is limited.

Among about hundred observed γ-transitions the 8813 18^+ state sticks out as being identified as a pure icosahedral rotor. A γ-cascade with six γ-transitions to and from the 8813 18^+ state is described by the recoupling of two or three spins with $6j, 9j$-symbols.

γ-transitions between the 18^+ yrare and the 18^+ yrast state involve couples of two 1p-1h states with either $g_{9/2}$, $j_{15/2}$ neutron or $h_{9/2}$ proton particles and $p_{1/2}$, $f_{5/2}$, $i_{13/2}$ neutron or $d_{3/2}$, $h_{11/2}$ proton holes. γ-transitions from the 18^+ to the 14^- double intruder state involve the coupling of an icosahedral to a tetrahedral state, $T_d^{-3}\ Y_h^{+12}$ and $T_d^{-3}\ Y_h^{+10}$.

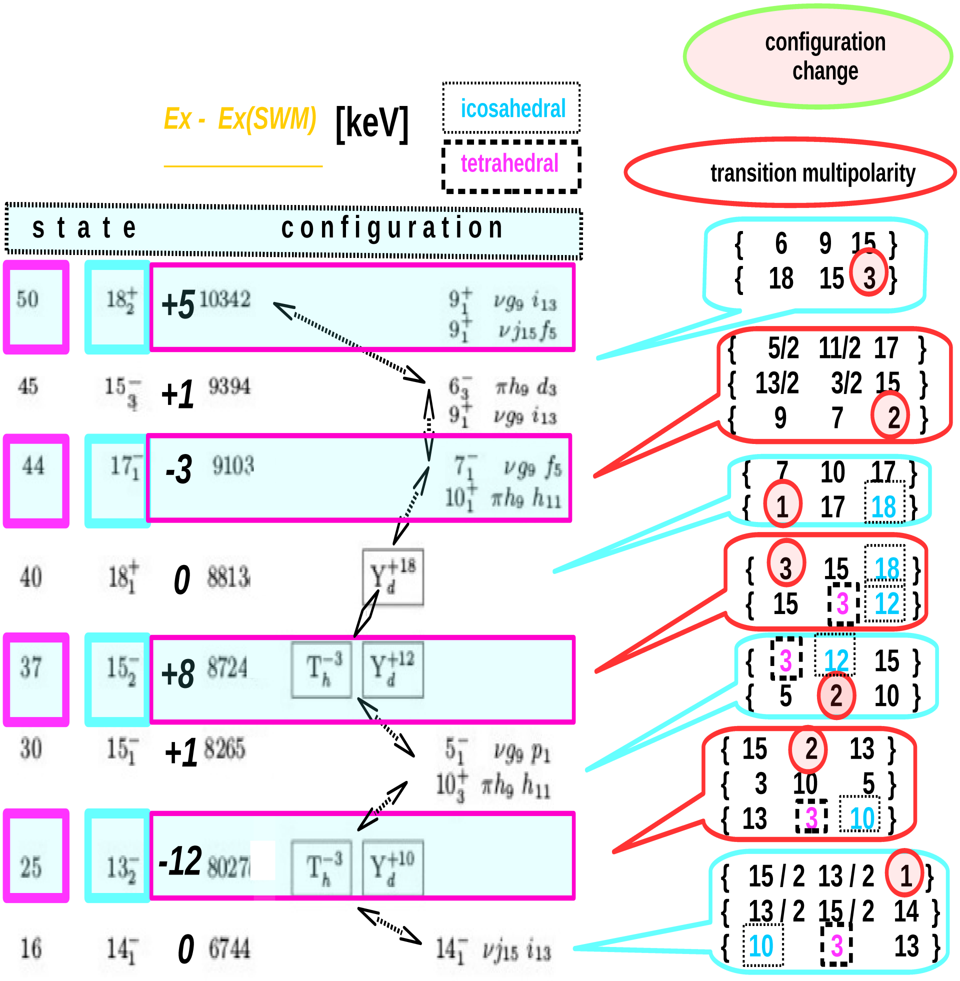

Figure 4. γ cascade 10342 (50) 18_2^+ $\longrightarrow$ 9394 (45) 15_3^- $\longrightarrow$ 9103 (44) 17_1^- $\longrightarrow$ 8813 (40) 18_1^+ $\longrightarrow$ 8724 (37) 15_2^- $\longrightarrow$ 8265 (30) 15_1^- $\longrightarrow$ 8027 (25) 13_2^- $\longrightarrow$ 6744 (16) 14_1^-

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