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Research Article

Appearance of a Minimal Length in e^+e^- Annihilation

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Experimental data reveal with a 5σ significance the existence of a characteristic minimal length $l_e = 1.57 \times 10^{-17}$ cm at the scale $E = 1.253$ TeV in the annihilation reaction $e^+e^- \rightarrow \gamma\gamma(\gamma)$. Nonlinear electrodynamics coupled to gravity and satisfying the weak energy condition predicts, for an arbitrary gauge invariant Lagrangian, the existence of spinning charged electromagnetic soliton asymptotically Kerr-Newman for a distant observer with the gyromagnetic ratio $g = 2$. Its internal structure includes a rotating equatorial disk of de Sitter vacuum which has properties of a perfect conductor and ideal diamagnetic, displays superconducting behavior, supplies a particle with the finite positive electromagnetic mass related to breaking of space-time symmetry, and gives some idea about the physical origin of a minimal length in annihilation.

1. Introduction

The concept of a minimal length suggested by path-integral quantisation [1], string theory [2, 3], black hole physics [4], and quantum gravity [5, 6] has been introduced into quantum mechanics and quantum field theory through generalised uncertainty principle which restricts an accuracy Δl in measuring a particle position by a certain finite minimal length scale l_m related to maximal resolution [7–9] (for a review [10]). In gravity, the limiting quantum length is the Planck length $l_p = \sqrt{\hbar G/c^3} = 1.6 \times 10^{-33}$ cm, the related energy scale $M_p \simeq 10^{16}$ TeV. However gravitational effects have only be tested up to 1 TeV scale [11] which corresponds to $l_m \simeq 10^{-17}$ cm [12]; therefore, minimal length could be in principle found within the range between l_p and l_m [12]. In models with extra dimensions, the Planck length can be reduced to $1/M_f$ with $M_f \simeq 1$ TeV, which results in modification of cross-sections of basic scattering processes $e^+e^- \rightarrow \mu^+\mu^-$, $e^+e^- \rightarrow \tau^+\tau^-$ ([13] and references therein).

In this paper, we summarize the results on working out experimental data on the annihilation reaction $e^+e^- \rightarrow \gamma\gamma(\gamma)$ motivated originally by looking for manifestations of the non-point-like behavior of fundamental particles.

The question of intrinsic structure of a charged spinning particle such as an electron has been discussed in the literature since its discovery by Thomson in 1897. In quantum field theory, a particle is assumed to point-like, and classical models of point-like spinning particles describe them by various generalizations of the classical lagrangian $(-mc\sqrt{\dot{x}\dot{x}})$ [14–29]. Another type of point-like models [30–40] goes back to the Schrödinger suggestion that the electron spin can be related to its Zittebewegung motion [41].

Second approach, which we apply in this paper, deals with extended particle models (for discussion and a review [42]).

To get evidence for an extended particle picture, we worked out available data of experiments performed to search for a non-point-like behavior, with focus on characteristic energy scale related to characteristic length scale of interaction region [43–48]. Experimental limits on size of a lepton [43–47] appear to be much less than its classical radius which suggests the existence of a relatively small characteristic length scale related to gravity [43–47].

Study of the pure electromagnetic interaction $e^+e^- \to \gamma\gamma(\gamma)$ using the data from VENUS, TOPAZ, ALEPH, DELPHI, L3, and OPAL puts the limit on maximal resolution at the scale $E = 1.253$ TeV by the characteristic length $l_e \simeq 1.57 \times 10^{-17}$ cm with the 5σ significance [48] (earlier, the 2.6σ effect was reported for the data on $e^+e^- \to e^+e^-(\gamma)$ reaction [49]. The increase of significance in our research is caused by the increase of statistics, since we used in our χ^2 test the most extensive available data set.) To find a possible generic physical mechanism responsible for appearance of the minimal length in annihilation related to gravity without appealing to extra dimensions, we apply the results obtained in nonlinear electrodynamics coupled to gravity (NED-GR) for generic features of spinning electrically charged soliton asymptotically Kerr-Newman for a distant observer.

Early electron models based on the concept of an extended electron, proposed by Abraham more than a hundred years ago [50, 51], encountered the problem of preventing an electron from flying apart under the Coulomb repulsion. Theories based on geometrical assumptions about the "shape" or distribution of a charge density were compelled to introduce *cohesive* forces of nonelectromagnetic origin (the Poincaré stress) [52].

The Kerr-Newman solution to the Einstein-Maxwell equations [53]

$$\begin{aligned} ds^2 &= -dt^2 + \frac{\left(2mr - e^2\right)}{\Sigma}\left(dt - a\sin^2\theta d\phi\right)^2 \\ &\quad + \left(r^2 + a^2\right)\sin^2\theta d\phi^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2; \\ A_i &= -\frac{er}{\Sigma}\left[1; 0, 0, -a\sin^2\theta\right], \end{aligned} \tag{1}$$

where A_i is associated electromagnetic potential, and

$$\Sigma = r^2 + a^2\cos^2\theta; \qquad \Delta = r^2 - 2mr + a^2 + e^2 \tag{2}$$

have inspired further search for an electromagnetic image of the electron since Carter [54] found that the parameter a couples to the mass m giving the angular momentum $J = ma$, and with the charge e giving an asymptotic magnetic momentum $\mu = ea$, which results in exactly the same gyromagnetic ratio $g = 2$ as predicted by the Dirac equation [54].

The Kerr-Newman solution belongs to the Kerr family solutions of the source-free Maxwell-Einstein equations with the only contribution to a stress-energy tensor coming from a source-free electromagnetic field [54]. It can represent the exterior fields of a spinning charged particle but cannot model a particle itself for the reason discovered by Carter: in the appropriate case, $a^2 + e^2 > m^2$, when there are no Killing horizons and the manifold is geodesically complete, the whole space is a single vicious set, in which any point can be connected to any other point by both a future and a past directed timelike curve, which means complete and unavoidable breakdown of causality [54].

The question of matching the Kerr-Newman exterior fields to an interior material source has been addressed in a lot of papers. The source models can be roughly divided into disk-like [55–57], shell-like [58–60], bag-like [61–67], and string-like ([68] and references therein) ones. Characteristic length in the equatorial plane is the Compton wavelength λ_e, and in bag-like models a characteristic thickness is of the order of the electron classical radius r_e. The problem of matching does not have a unique solution, since one is free to choose arbitrarily the boundary between the exterior and the interior [55].

On the other hand, nonlinear electrodynamics coupled to gravity and satisfying the weak energy condition (nonnegative density as measured along any timelike curve) predicts, for an arbitrary gauge invariant lagrangian, the existence of a spinning charged soliton (a regular finite-energy solution of the nonlinear field equations, localized in the confined region and holding itself together by its own self-interaction) asymptotically Kerr-Newman for a distant observer with the gyromagnetic ratio $g = 2$ [69].

In the spherically symmetric case and in the corotating frame in the axial symmetric case, a stress-energy tensor of any electromagnetic field has the algebraic structure such that

$$T_0^0 = T_1^1 \quad \left(p_r = -\rho\right). \tag{3}$$

Spherically symmetric space-time with a source term specified by (3) has obligatory de Sitter center [70], in which field tension goes to zero, while the energy density of the electromagnetic vacuum T_t^t achieves its maximal finite value which represents the de Sitter cut-off on the self-energy density [71].

A spherically symmetric electrically charged solution with de Sitter center [71] can be transformed by the Gürses-Gürsey algorithm based on the Newman-Trautman technique [72], into a spinning electromagnetic soliton with the Kerr-Newman behavior for a distant observer. Its internal structure includes the equatorial disk of a rotating de Sitter vacuum which has a perfect conductor and ideal diamagnetic properties, displays superconducting behavior [69], and supplies a particle with the finite positive electromagnetic mass related to interior de Sitter vacuum and breaking of space-time symmetry [69, 71, 73].

Nonlinear electrodynamics coupled to gravity provides actually an effective *cohesive* force related to negative pressure (unlike positive pressure which can, e.g., stop gravitational collapse if the mass of a collapsing body does not exceed a certain critical value). In the interior de Sitter region negative pressure $p = -\rho$ generates repulsive geometry which prevents formation of a singularity [74–76]. Interior de Sitter vacuum, as we will show, can be also responsible for the appearance of the minimal length in annihilation related to the distance of the closest approach of annihilating particles.

This paper is organised as follows. Experimental evidence is presented in Section 2. In Section 3, we outline basic features of electromagnetic soliton, and in Section 4, we apply them to find a possible physical origin of a minimal length in annihilation. In Section 5, we summarize and discuss the results.

2. Experimental Evidence

The purely electromagnetic interaction $e^+e^- \rightarrow \gamma\gamma(\gamma)$ proceeds via the exchange of a virtual electron in the t- and u-channels (the s-channel is forbidden) and is not interfered by the Z^0 decay. Differential cross-sections were measured at energies from $\sqrt{s} = 55$ GeV to 207 GeV using the data collected with the VENUS, TOPAZ, ALEPH, DELPHI, L3, and OPAL from 1989 to 2003 [77–85]. Comparison of the data with the QED predictions has been performed [86] to constrain models with an excited electron replacing the virtual electron in the QED process [87–89] and with deviation from QED due to an effective interaction with nonstandard $e^+e^-\gamma$ couplings and $e^+e^-\gamma\gamma$ contact terms [90–92]. The calculation of the differential cross-section QED-α^3 including radiative effects up to $O(\alpha^3)$ assumes a scattering center as a point. If the electron is an extended object, its structure would modify the QED cross-section if the test distances (corresponding to the CM-scattering energies) are smaller than its characteristic size.

A heavy excited electron of mass m_{e^*} could couple to an electron and a photon via magnetic interaction with an effective lagrangian [87–89]

$$\mathscr{L}_{\text{excited}} = \frac{e\lambda}{2m_{e^*}} \overline{\psi_{e^*}} \sigma_{\mu\nu} \psi_e F^{\mu\nu}. \tag{4}$$

Here λ is the coupling constant, $F^{\mu\nu}$ is the electromagnetic field, ψ_{e^*} and ψ_e are the wave function of the heavy electron and the electron, respectively, and λ and m_{e^*} are the model parameters. The modified equation reads

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{theo}} = \left(\frac{d\sigma}{d\Omega}\right)_{O(\alpha^3)} (1 + \delta_{\text{new}}). \tag{5}$$

If the center-of-mass energy $\sqrt{s}$ satisfies the condition $s/m_{e^*}^2 \ll 1$, then δ_{new} can be approximated as

$$\delta_{\text{new}} = \frac{s^2}{2} \frac{1}{\Lambda^4} \left(1 - \cos^2\Theta\right). \tag{6}$$

In this approximation, the parameter Λ is the QED cut-off parameter, $\Lambda^2 = m_{e^*}^2/\lambda$. In the case of arbitrary $\sqrt{s}$, the full equation of [87–89] was used to calculate $\delta_{\text{new}} = f(m_{e^*})$. The angle Θ is the scattering angle of two most energetic photons emitted with the angles Θ_1 and Θ_2,

$$|\cos(\Theta)| = \frac{1}{2} \left(|\cos(\Theta_1)| + |\cos(2\pi - \Theta_2)|\right). \tag{7}$$

The third order QED differential cross-section QED-α^3 is calculated numerically by generating a high number of Monte Carlo $e^+e^- \rightarrow \gamma\gamma(\gamma)$ events [48, 93, 94]. The angular distribution was fitted with a high order polynomial function

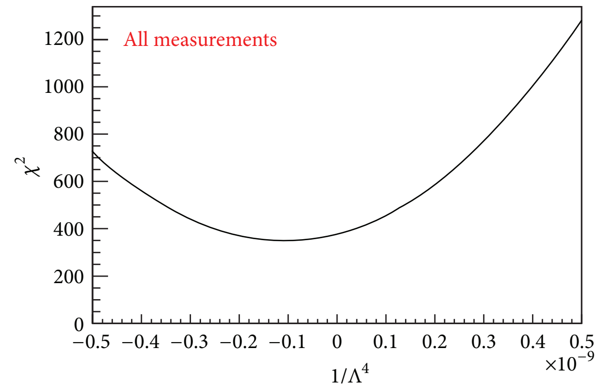

Figure 1: A minimum in the χ^2 fit with $P = 1/\Lambda^4$.

to get an analytical equation for the cross-section as function of Θ.

An overall χ^2 test was performed on the published differential cross-sections for energies between 55 GeV and 207 GeV. The result is $(1/\Lambda^4)_{\text{best}} = -(1.11 \pm 0.70) \times 10^{-10}\ \text{GeV}^{-4}$. The χ^2 overall fit shown in Figure 1 displays a minimum in χ^2.

The hypothesis used in (4) and (5) assumes that an excited electron would increase the total QED-α^3 cross-section and change the angular distribution of the QED cross-section. Contrary to these expectations, the fit displays a minimum with a negative fit parameter $1/\Lambda^4$ with a significance about 5σ.

Systematic errors arise from the luminosity evaluation, the selection efficiency, background evaluations, the choice of the QED-α^3 theoretical cross-section as the reference cross-section, and the choice of the fit procedure, of the fit parameter, and of the scattering angle in $|\cos\Theta|$ for comparison between data and theoretical calculation. The maximum estimated error for the value of the fit from the luminosity, selection efficiency, and background evaluations is approximately $\delta\Lambda/\Lambda = 0.01$. The choice of the theoretical QED cross-section was studied with 1882 $[e^+e^- \rightarrow \gamma\gamma(\gamma)]$ events from the L3 detector [93, 94]. In the worst case of scattering angles close to 90°, the $|\cos(\Theta)|_{\text{exper}} \sim 0.05$ would result in $(\delta\Lambda/\Lambda)_{\delta|\cos(\Theta)|} = 0.01$. The total systematic error is $\delta\Lambda/\Lambda \approx 0.015$. For a small sample of $e^+e^- \rightarrow \gamma\gamma(\gamma)$ events, the fit values were compared for χ^2, Maximum-Likelihood, Smirnov-Cramer von Misis, and Kolmogorov test, with and without binning [95, 96]. An approximate $\delta\Lambda/\Lambda = 0.005$ effect is estimated for the overall fit with the fit parameter $P = (1/\Lambda^4)$.

An effective contact interaction with nonstandard coupling is described by the Lagrangian [90–92]

$$\mathscr{L}_{\text{contact}} = i\overline{\psi_e}\gamma_\mu (D_\nu \psi_e) \left(\frac{\sqrt{4\pi}}{\Lambda_{C6}^2} F^{\mu\nu} + \frac{\sqrt{4\pi}}{\widetilde{\Lambda}_{C6}^2} \widetilde{F}^{\mu\nu}\right), \tag{8}$$

where Λ_{C6} is a cut-off parameter. This lagrangian involves an operator of dimension 6, the wave function of the electrons is ψ_e, the QED covariant derivative is D_ν, the tilde on $\widetilde{\Lambda}_{C6}$, and

$\widetilde{F}^{\mu\nu}$ stands for the dual fields. The corresponding differential cross-section involves a deviation term

$$\delta_{\text{new}} = \frac{s^2}{2\alpha}\left(\frac{1}{\Lambda_{C6}^4} + \frac{1}{\widetilde{\Lambda}_{C6}^4}\right)\left(1 - \cos^2\Theta\right). \quad (9)$$

The χ^2 fit for the hypothesis (4) was repeated for the hypothesis of the effective contact interaction (8) with using $(1/\Lambda_C^4)$ as the fit parameter with $\Lambda_C = \Lambda_{C6} = \widetilde{\Lambda}_{C6}$. An increase of the total QED-α^3 cross-section and a change of the angular distribution were expected. However, the best fit value of all data $(1/\Lambda_C^4) = -(4.05 \pm 0.73) \times 10^{-13}\,\text{GeV}^{-4}$ appeared negative at the minimum in the χ^2 test with the significance about 5σ. The results indicate decreasing cross-section of the process $e^+e^- \rightarrow \gamma\gamma(\gamma)$ with respect to that predicted by pure QED.

It is remarkable that for both hypotheses, of the excited electron and effective contact interaction, the χ^2 test leads to the best fit values for the complete data set with a 5σ significance. With the best value $(1/\Lambda)_C^4$, one can estimate the energy scale $E_\Lambda = (\Lambda_C)_{\text{best}} = 1.253$ TeV corresponding to a length scale of the interaction region $l_e \simeq 1.57 \times 10^{-17}$ cm which suggests the existence of a minimal length in annihilation.

3. Electromagnetic Soliton

In the nonlinear electrodynamics minimally coupled to gravity, the action is given by (in geometrical units $G = c = 1$)

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left(\mathscr{R} - \mathscr{L}(F)\right); \quad F = F_{\mu\nu}F^{\mu\nu}, \quad (10)$$

where $\mathscr{R}$ is the scalar curvature. The gauge-invariant electromagnetic Lagrangian $\mathscr{L}(F)$ is an arbitrary function of the field invariant F which should have the Maxwell limit, $\mathscr{L} \rightarrow F$, in the weak field regime.

The dynamic equations read

$$\nabla_\mu\left(\mathscr{L}_F F^{\mu\nu}\right) = 0; \qquad \nabla_\mu {}^*F^{\mu\nu} = 0, \quad (11)$$

where $\mathscr{L}_F = d\mathscr{L}/dF$ and an asterisk denotes the Hodge dual defined by

$$\begin{gathered} {}^*F^{\mu\nu} = \frac{1}{2}\eta^{\mu\nu\alpha\beta}F_{\alpha\beta}; \qquad {}^*F^{\mu\nu} = \frac{1}{2}\eta_{\mu\nu\alpha\beta}F^{\alpha\beta}; \\ \eta_{0123} = \sqrt{-g}. \end{gathered} \quad (12)$$

A stress-energy tensor of a spherically symmetric electromagnetic field

$$\kappa T_\nu^\mu = -2\mathscr{L}_F F_{\nu\alpha}F^{\mu\alpha} + \frac{1}{2}\delta_\nu^\mu\mathscr{L}, \quad (13)$$

where $\kappa = 8\pi G$, has the algebraic structure such that

$$T_t^t = T_r^r; \qquad p_r = -\rho. \quad (14)$$

The equation relating the tangential pressure $p_\perp = -T_\theta^\theta = -T_\phi^\phi$ with the density follows directly from (13) and reads

$$\kappa\left(p_\perp + \rho\right) = -F\mathscr{L}_F. \quad (15)$$

Electric field $E_r = F_{10}$, is given by [71, 97]

$$r^2\mathscr{L}_F E_r = e, \quad (16)$$

where e is a constant of integration identified as an electric charge by the asymptotic Coulomb behavior for a distant observer in the weak field limit.

From (15) and (16), we get

$$F = -\frac{\kappa^2\left(p_\perp + \rho\right)^2 r^4}{2e^2}; \qquad \mathscr{L}_F = \frac{2e^2}{\kappa\left(p_\perp + \rho\right)r^4}. \quad (17)$$

Symmetry of a source term (14) leads to the metric [74]

$$ds^2 = g(r)\,dt^2 - \frac{dr^2}{g(r)} - r^2 d\Omega^2. \quad (18)$$

The metric function is given by

$$g(r) = 1 - \frac{2\mathscr{M}(r)}{r} \;:\; \mathscr{M}(r) = \frac{1}{2}\int_0^r \rho(x)\,x^2 dx. \quad (19)$$

For the class of regular spherically symmetric solutions of the class (14), the weak energy condition leads inevitably to de Sitter asymptotic at $r \rightarrow 0$ [70, 74]

$$p_r = p_\perp = p = -\rho; \qquad g(r) = 1 - \frac{\kappa\rho_0}{3}r^2, \quad (20)$$

where $\rho_0 = \rho(r \rightarrow 0)$ is the density in the regular center.

The field invariant F, as well as the electric field E_r, vanishes for $r \rightarrow 0$ and for $r \rightarrow \infty$ (where they follow the Maxwell weak field limit); hence, the electric field achieves an extremal value somewhere in between [71].

The weak energy condition requires $\rho + p_\perp \geq 0$, which leads to $\rho' \leq 0$, so that the energy density of the electromagnetic vacuum achieves as $r \rightarrow 0$ its maximal value $T_t^t = \rho_0$ which represents the de Sitter cutoff on the self-interaction divergent for a point charge [71].

For a distant observer, the metric is asymptotically Reissner-Nordström

$$g(r) = 1 - \frac{r_g}{r} + \frac{e^2}{r^2}, \quad (21)$$

where $r_g = 2m$ is the Schwarzschild gravitational radius.

Spherically symmetric solutions specified by (14) belong to the Kerr-Schild class [67, 98]. By the Gürses-Gürsey algorithm [72], they can be transformed into regular solutions describing a spinning charged soliton. In the Boyer-Lindquist coordinates, the metric is [72]

$$\begin{aligned} ds^2 &= \frac{(2f - \Sigma)}{\Sigma}dt^2 - \frac{4af\sin^2\theta}{\Sigma}dt\,d\phi \\ &\quad + \left(r^2 + a^2 + \frac{2fa^2\sin^2\theta}{\Sigma}\right)\sin^2\theta\,d\phi^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2; \\ &\Sigma = r^2 + a^2\cos^2\theta; \qquad \Delta = r^2 + a^2 - 2f(r). \end{aligned} \quad (22)$$

The function $f(r)$ in (22) is given by

$$f(r) = r\mathscr{M}(r). \quad (23)$$

For NED-GR solutions satisfying the weak energy condition, $\mathcal{M}(r)$ is everywhere positive function growing monotonically from $\mathcal{M}(r) = 4\pi\rho_0 r^3/3$ as $r \to 0$ to m as $r \to \infty$. The mass m, appearing in a spinning solution, is the finite positive electromagnetic mass $m = \mathcal{M}(r \to \infty)$, generically related to interior de Sitter vacuum and to breaking of spacetime symmetry [69, 73]. Let us note that the condition of the causality violation [54] is never satisfied for this class of solutions due to nonnegativity of the function $f(r)$ [69].

For a distant observer, a spinning electromagnetic soliton is asymptotically Kerr-Newman, with $f(r) = mr - e^2/2$.

In the geometry with the line element (22), the surfaces $r = \text{const}$ are the oblate ellipsoids [54]

$$r^4 - \left(x^2 + y^2 + z^2 - a^2\right) r^2 - a^2 z^2 = 0 \tag{24}$$

which degenerate, for $r = 0$, to the equatorial disk

$$x^2 + y^2 \leq a^2, \quad z = 0\,(\Sigma = 0) \tag{25}$$

centered on the symmetry axis.

For $r \to 0$, the function $f(r)$ in (22) approaches de Sitter asymptotics

$$2f(r) = \frac{r^4}{r_0^2}; \quad r_0^2 = \frac{3}{\kappa\rho_0}. \tag{26}$$

The disk $r = 0$ is totally intrinsically flat, but with nonzero vacuum density ρ_0. Rotation transforms the de Sitter center $r = 0$ into the disk (25). The equation of state on the disk reads in the corotating frame

$$p_r = p_\perp = -\rho \tag{27}$$

and represents the rotating de Sitter vacuum spread over the disk [69].

Field components compatible with the axial symmetry are $F_{01}, F_{02}, F_{13}, F_{23}$. In terms of the field vectors defined as

$$\begin{aligned} \mathbf{E} &= \{F_{\alpha 0}\}; \qquad \mathbf{D} = \left\{\mathcal{L}_F F^{0\alpha}\right\}; \\ \mathbf{B} &= \left\{{}^{*}F^{\alpha 0}\right\}; \qquad \mathbf{H} = \left\{\mathcal{L}_F {}^{*}F_{0\alpha}\right\}. \end{aligned} \tag{28}$$

The dynamic equations (11) take the form of the Maxwell equations

$$\begin{aligned} \nabla\mathbf{D} = 0; &\qquad \nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}; \\ \nabla\mathbf{B} = 0; &\qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}. \end{aligned} \tag{29}$$

It follows that the electric induction $\mathbf{D}$ is connected with the electric field intensity $\mathbf{E}$ by

$$D_\alpha = \epsilon_{\alpha\beta} E_\beta, \tag{30}$$

where $\epsilon_{\alpha\beta}$ is the tensor of the dielectric permeability. The magnetic induction $\mathbf{B}$ is related to the magnetic field intensity $\mathbf{H}$ by

$$B_\alpha = \mu_{\alpha\beta} H_\beta, \tag{31}$$

where $\mu_{\alpha\beta}$ is the tensor of the magnetic permeability.

Symmetry of the oblate ellipsoid (24) gives two pairs independent nonzero eigenvalues

$$\begin{aligned} \epsilon_r &= \frac{\left(r^2 + a^2\right)}{\Delta} \mathcal{L}_F; \quad \epsilon_\theta = \mathcal{L}_F. \\ \mu_r &= \frac{\left(r^2 + a^2\right)}{\Delta} \frac{1}{\mathcal{L}_F}; \quad \mu_\theta = \frac{1}{\mathcal{L}_F} \end{aligned} \tag{32}$$

In the de Sitter region, $\epsilon_r = \epsilon_\theta = \mathcal{L}_F$; $\mu_r = \mu_\theta = \mathcal{L}_F^{-1}$.

In the corotating frame, we have [69]

$$\frac{\kappa}{2}\left(p_\perp + \rho\right) = \mathcal{L}_F F_{10}^2 + \mathcal{L}_F \frac{F_{20}^2}{a^2 \sin^2\theta} \tag{33}$$

which allows us to investigate the behavior of the fields on the disk where $p_\perp + \rho = 0$. The left hand side of (33) goes to zero by (27); hence, each component in the right hand side vanishes on the disk independently

$$\mathcal{L}_F \frac{F_{20}^2}{a^2} = 0; \qquad \mathcal{L}_F F_{10}^2 = 0. \tag{34}$$

The field invariant F and $\mathcal{L}_F$ derivative on the disk are given by [69]

$$\mathcal{L}_F = \frac{2e^2}{\Sigma^2 \kappa\left(p_\perp + \rho\right)}; \qquad F = -\frac{\kappa^2 \left(p_\perp + \rho\right)^2 \Sigma^2}{2e^2}. \tag{35}$$

The dielectric permeability $\epsilon_r = \epsilon_\theta = \mathcal{L}_F$ goes to infinity, and the magnetic permeability $\mu_r = \mu_\theta = \mathcal{L}_F^{-1}$ vanishes. The rotating de Sitter vacuum disk displays a perfect conductor and ideal diamagnetic behavior [69].

The magnetic induction $\mathbf{B}$ goes to zero on the disk independently on the magnetic permeability. Indeed, it follows from (34) and (28) that on the disk

$$\frac{2e^2 \left(B^r\right)^2}{\kappa\left(p_\perp + \rho\right)\left(r^2 + a^2\right)^2} = 0; \qquad \frac{2e^2 \left(B^\theta\right)^2}{\kappa\left(p_\perp + \rho\right) a^2} = 0, \tag{36}$$

so that $(B^r)^2$ and $(B^\theta)^2$ must vanish faster than $(p_\perp + \rho)$. On the intrinsically flat disk, we can apply the conventional definition of the surface current [99]

$$\mathbf{g} = \frac{(1 - \mu)}{4\pi\mu} [\mathbf{n}\mathbf{B}], \tag{37}$$

where $\mathbf{n}$ is the normal to the surface. The transition to a superconducting state corresponds formally to the limit $\mathbf{B} \to 0$ and $\mu \to 0$; the right hand side of (37) then becomes indeterminate, and there is no condition which would restrict the possible values of the current $\mathbf{g}$ [99]. The surface currents on the de Sitter disk can be any and amount to a nonzero total value [69].

4. Origin of a Minimal Length in Annihilation

In the asymptotically Kerr-Newman NED-GR models, there are two intrinsic length scales: the Compton wavelength $\lambda_e \simeq 3.9 \times 10^{-11}$ cm; the thickness of ellipsoid in a region of a maximal value of $|F|$ is of order of the classical electron radius $r_e \simeq 2.8 \times 10^{-13}$ cm. Experimental data on the annihilation reaction $e^+e^- \to \gamma\gamma(\gamma)$ reveal the existence of one more characteristic length scale, $l_e \simeq 1.57 \times 10^{-17}$ cm. It corresponds to the minimum in the χ^2 fit found with 5σ significance and represents the distance of the closest approach of annihilating particles which cannot be made smaller.

Generic features of electromagnetic soliton give some idea about the origin of the length l_e given by experiments. The definite feature of annihilation process is that, at its certain stage, a region of interaction is neutral and spinless. We can roughly model it by a spherical lump with de Sitter vacuum interior asymptotically Schwarzschild as $r \to \infty$.

For all structures with the de Sitter interior, there exists the characteristic zero gravity surface $r_s \simeq r_*$ at which the strong energy condition ($\rho + \sum p_k \geq 0$) is violated and beyond which the gravitational acceleration becomes repulsive [74, 75]. The related length scale $r_* \simeq (r_0^2 r_g)^{1/3}$ appears naturally in a geometry with the de Sitter interior and the Schwarzschild exterior [76, 100].

The gravitational radius of a lump at the characteristic energy scale $E \simeq 1.253$ TeV is $r_g \simeq 3.32 \times 10^{-49}$ cm. Adopting for the interior de Sitter vacuum the experimental vacuum expectation value at the electroweak scale $E_{EW} = 246$ GeV related to the electron mass [101], we get the de Sitter radius $r_0 = 1.374$ cm, where $r_0 = \sqrt{3/8\pi G\rho_0}$ and $\rho_0 = \langle T_0^0 \rangle = (E_{EW}/E_{Pl})^4 \rho_{Pl}$. Characteristic radius of zero gravity surface is $r_* \simeq 0.86 \times 10^{-16}$ cm, so that the scale $l_e \simeq 1.57 \times 10^{-17}$ cm fits inside a region where gravity is repulsive. The minimal length scale l_e can be thus understood as a distance at which electromagnetic attraction is stopped by the gravitational repulsion of the interior de Sitter vacuum.

Regular NED-GR solutions provide a characteristic de Sitter cut-off on electromagnetic self-energy whose numerical value depends on the choice of a density profile. Qualitatively it can be evaluated by [71]

$$\frac{e^2}{r_c^4} \simeq \kappa\rho_0 = \frac{3}{r_0^2}. \tag{38}$$

It gives, for the characteristic length scale at which electromagnetic attraction is balanced by de Sitter gravitational repulsion, $r_c \simeq 1.05 \times 10^{-17}$ cm, sufficiently close to the minimal length l_e for such a rough estimate.

5. Summary

In this paper, we analyze experimental data on the annihilation reaction $e^+e^- \to \gamma\gamma(\gamma)$. The global fit to the data is 5 standard deviations from the standard model expectation for the hypotheses of an excited electron and of contact interaction with nonstandard coupling. This corresponds to the energy scale $E_\Lambda = 1.253$ TeV and to related characteristic length scale $l_e \simeq 1.57 \times 10^{-17}$ cm. We interpret this effect as the existence of a minimal length in annihilation as a distance of the closest approach of annihilating particles related to generic properties of a NED-GR electromagnetic soliton.

Nonlinear electrodynamics coupled to gravity and satisfying the weak energy condition admits the class of regular solutions describing spinning charged soliton asymptotically Kerr-Newman for a distant observer with a gyromagnetic ratio $g = 2$. Due to algebraic structure of a stress-energy tensor for an electromagnetic field, electromagnetic soliton has obligatory de Sitter vacuum interior which provides a cut-off on self-interaction. The internal structure includes an equatorial disk of rotating de Sitter vacuum which has properties of a perfect conductor and ideal diamagnetic and displays superconducting behavior [69].

This behavior, found for an arbitrary gauge invariant lagrangian, is generic and suggests that a spinning particle dominated by the electromagnetic interaction might have de Sitter interior arising naturally in the regular geometry asymptotically Kerr-Newman for a distant observer. De Sitter vacuum supplies a particle with the finite positive electromagnetic mass related to breaking of space-time symmetry and provides a characteristic length scale at which electromagnetic attraction is balanced by the gravitational repulsion that can explain the existence of the minimal length in annihilation.

NED theories appear as low-energy effective limits in certain models of string/M-theories [102–104]. The above results apply to the cases when the relevant electromagnetic scale (l_e in our case) is much less than the Planck scale l_p.

Let us note that the NED-GR connection of a lepton mass with de Sitter vacuum and space-time symmetry can be responsible for mass-squared differences for sub-eV particles [105]. The key point is that, in the interaction region where particles are created, gravitational effects of massive gauge bosons may become important and then the interaction vertex is gravity-electroweak. If the weak energy condition holds and density is finite, the space-time group around a gravity-electroweak vertex can be de Sitter. Then particles participating in the vertex are described by the eigenvalues of Casimir operators in the de Sitter geometry. As a result sub-eV particles acquire difference in mass squares $\Delta m^2 = \hbar^2/(c^2 r_{deS}^2)$. For de Sitter radius r_{deS} related to the gravity-electroweak unification scale M_{GEW}, we get $M_{GEW} = [3/8\pi(\Delta m^2/M_p^2)]^{1/4} M_p$ which can be read off from atmospheric and solar neutrino data which predict a TeV scale for unification and dominantly bimaximal mixing for neutrinos [73, 105], in agreement with the ideas of gravity-electroweak unification [106–108].

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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