SLAC TRANS – 0216

TRANSLATION

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ENGLISH FROM GERMAN

ON ERROR PREDICTION FOR
THE UNDERGROUND GEODETIC
NETWORKS AT SLAC: A
FORTRAN PROGRAM PACKAGE

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On Error Prediction for the Underground Geodetic Networks at SLAC

A FORTRAN Program Package by

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Contents

1. Brief Description

The present program package consists of two individual programs:

-- NETZ22 Program

This program is for the free adjustment of the superordinate (above-ground) network or also for the free adjustment of the whole network (above and underground).

-- NETZ24 Program

This program is for the adjustment of the subordinate (underground) network with an unconstrained connection to 2 or more points of the superordinate, already adjusted network.

During adjustment with NETZ24, the constituent variance matrix of the connection points from the superordinate network adjustment should be given. This can be done automatically, if NETZ22 is used as a preparatory program for NETZ24.

The following observation values can be processed:

-- Length measurements

-- Direction measurements

-- Gyroscopic measurements, and

-- Offset measurements.

In both programs the following values are computed:

-- The degree of freedom

-- The confidence limits for m_o

-- The confidence level for S = 95% and

-- The estimated r.m.s. error ellipses for all points of the network.

In addition, the following can be computed with NETZ24 as desired:

-- The estimated r.m.s. relative error ellipses for given pairs of points, and

-- The estimated r.m.s. errors for given lengths and angles.

2. Computation Formulas

Only those computation formulas that are not already contained in "Adjustment of Geodetic Networks at SLAC" will be given here.

2.1. Unconstrained Network Connection

The connection points from the superordinate network are introduced as "observed unknowns" into the adjustment of the subordinate network.

The "observed unknowns" each receive the

coefficient 1; as weights, one applies the corresponding cofactors (including the mixed ones!) from the adjustment of the superordinate network.

In forming the normal equation matrix, the following constituents must be distinguished:

$N_L = A_L' P A_L$	Constituent of independent observations (lengths, directions, etc.)
$N_K = A_K' Q_K^{-1} A_K$	Constituent of "observed unknowns"

where

A_L	coefficient matrix (lengths, directions, etc.)
P	weight matrix " " "
A_K	coefficient matrix (observed unknowns)
Q_K	cofactor matrix " "

To calculate N_K, all rows and columns that are <u>not</u> connected to the observed unknowns are struck out, resulting in a correspondingly small computation system.

The total normal equation matrix N results from:

$$\boxed{N = N_L + N_K} \quad .$$

One works on the assumption that there are always

2 or more connection points from the superordinate network. The normal equation matrix N (2p,2p) is then regular and hence fully invertible.

2. Degree of Freedom

The degree of freedom f is computed as follows:

$$f = n_s + n_r + n_k + n_h + 2p_a - 2p - o + d$$

where

- n_s number of lengths
- n_r number of all directions at all stations
- n_k number of gyroscopic measurements
- n_h number of offset measurements
- p number of all points in network
- o number of orientation unknowns (= number of sets of directions)
- p_a number of "observed" connection points
- d defect in network
  - d = 3 in a free network adjustment <u>with</u> length measurements
  - d = 4 in a free network adjustment <u>without</u> length measurements
  - d = 0 for a connection to a superordinate network with 2 or more connection points

2.3. Confidence Limits for m_o

The confidence limits for m_o depend on the degree of freedom f and the statistical probability of error α; they are computed as follows:

$$S_u = m_o/\lambda_o \qquad \text{lower limit}$$

$$S_o = m_o/\lambda_u \qquad \text{upper limit}$$

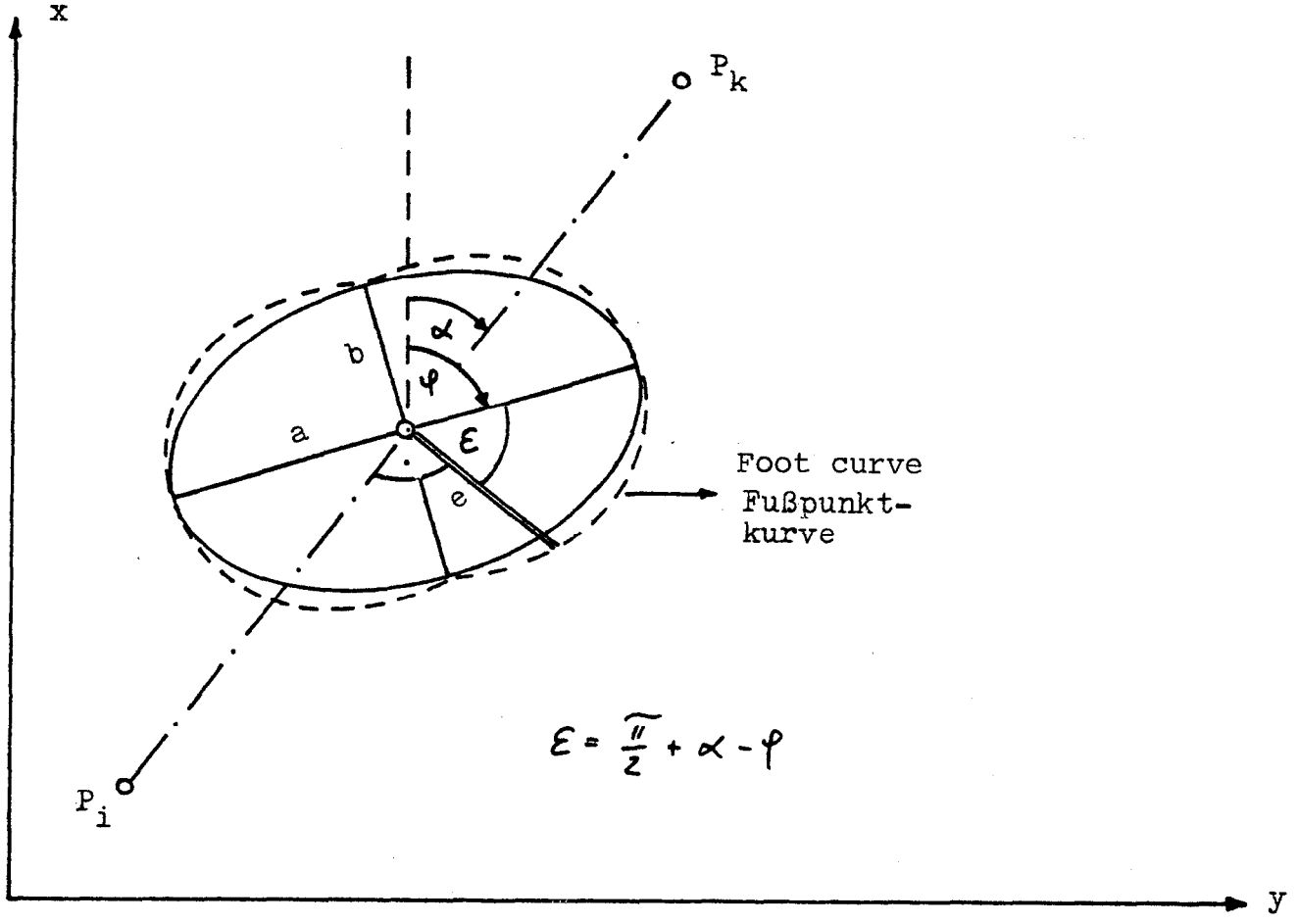
For α = 5%, the following values are drawn from the graph of the χ^2 distribution ("Zurmühl" textbook):

f	λ_u	λ_o
5	0.40	1.60
10	0.56	1.42
20	0.68	1.30
30	0.75	1.24
40	0.78	1.20
50	0.80	1.18

Because of the asymmetrical distribution of χ^2, the upward bounds are higher than the downward ones. The lower the degree of freedom is, the more these bounds differ from one another.

In the computer programs, λ_o and λ_u are automatically also computed for any desired degree of freedom as a fifth-degree polynomial. This polynomial approximation, however, applies only for networks with $f \geq 5$.

2.4. Relative Error Ellipses



α	grid bearing $P_i P_k$
a,b	semiaxes of ellipse
ϕ	grid bearing of the major semiaxis
ε	direction perpendicular to $\overline{P_i P_k}$
e	perpendicular component (<u>relative</u> perpendicular error $P_i P_k$)

From the network adjustment, one gets:

- m_o r.m.s. error of weight unit and
- Q cofactor matrix

With this the following values are computed:

$$D_{xx} = Q_{x_i x_i} + Q_{x_k x_k} - 2 \cdot Q_{x_i x_k}$$

$$D_{yy} = Q_{y_i y_i} + Q_{y_k y_k} - 2 \cdot Q_{y_i y_k}$$

$$D_{xy} = Q_{x_i y_i} - Q_{x_i y_k} - Q_{x_k y_i} + Q_{x_k y_k}$$

$$\lambda^2 = \sqrt{4 \cdot D_{xy}^2 + (D_{xx} - D_{yy})^2}$$

Finally one obtains the elements of the relative error ellipse from:

$$a = m_o \cdot \sqrt{0.5 \cdot (D_{xx} + D_{yy} + \lambda^2)}$$

$$b = m_o \cdot \sqrt{0.5 \cdot (D_{xx} + D_{yy} - \lambda^2)}$$

$$\varphi = 0.5 \cdot \text{arc tan } (2 \cdot D_{xy} / (D_{xx} - D_{yy}))$$

$$e = \sqrt{a^2 \cos^2 \varepsilon + b^2 \sin^2 \varepsilon}$$

2.5. Confidence Ellipses

The calculated error ellipses can be transformed into confidence ellipses (S = 95%) if their semiaxes are

multiplied by the factor

$$f_{95} = \sqrt{2 \cdot F_{2,f,0.95}}$$

f = degree of freedom

The direction of the semiaxes is maintained.

The factor $F_{2,f,0.95}$ can be drawn from the table for the distribution of F. In the present programs, it is automatically computed as a seventh-degree polynomial for any desired degree of freedom ≥ 5.

3. Directions for Use of Programs

3.1. NETZ22 Program

Input File:

	READ	FORMAT	Anz.[a] Zeilen	Erläuterungen[b]
1	TEXT	20A4	1	Überschrift [c]
2	NP	I5	1	Anz. Netzpunkte [d]
3	PP,Y,X	A8,2F15.5	NP	P_i , y_i^o, x_i^o
4	NS	I5	1	Anz. Strecken [e]
5	PPI,PPK,MS	2A8,F10.5	NS	P_i,P_k,m_s
6	NPR	I5	1	Anz. Richtungssätze [f]
7	PPI,NR	A8,I5	1 } x NPR	P_i, Anz. Richt. auf P_i [g]
8	PPK,MR	A8,F10.5	NR }	P_k , m_r
9	NK	I5	1	Anz. Kreiselmess.[h]
10	PPI,PPK,MK	2A8,F10.5	NK	P_i , P_k , m_k
11	NH	I5	1	Anz. Pfeilhöhen [i]
12	PPI,PPJ,PPK,MH	3A8,F10.5	NH	P_i , P_j , P_k , m_h
13	NPA	I5	1	Anz. Anschlußpunkte [j]
14	PPA	A8	NPA	P_a

Key: a. number of lines
b. explanations
c. heading
d. number of points in network
e. number of lengths
f. number of sets of directions
g. number of directions at P_i
h. number of gyroscopic measurements
i. number of offsets
j. number of connection points

Dimensioning in the Main Program

- PP(NP),Y(NP),X(NP),A(NA),ZA(NA),AA(NA,NA)
- G(NA,4),GG(4,4),QS(NA,NA),T(NA,NA)
- TEXT(20),KAN(NQK),QK(NQK,NQK)

where NA = 2 · NP and

NQK = 2 · NPA .

Read-In of File Names

File names ("Input File," "1st Output File," and "2nd Output File") are input interactively with the display screen. For the 2nd output file (connection point file), a valid file name should be given in each case, even if no connection points are to be prepared for a subsequent network adjustment with NETZ24 (i.e., if $n_{p_a} = 0$).

1st Output File

1)	Heading	
2)	Approximation coordinates	(Point, y_o, x_o)
3)	Length program (if $n_s \neq 0$)	(from, to, m_s)
4)	Direction program (if $n_{p_r} \neq 0$)	(from, to, m_r)
5)	Gyroscope program (if $n_k \neq 0$)	(from, to, m_k)
6)	Offset program (if $n_h \neq 0$)	(stat, left, rt, m_h)
7)	Statistical values	(m_o,f,S_u,S_o,f_{95}) †
8)	Error ellipses	(a, b, ϕ)

† m_o (a priori) = 0.0001 -- established by the program!
Dimensions: always (m) and (grads)!

2nd Output File

1) Number of connection points

2) Names of connection points

3) Cofactors of connection points

For $n_{p_a} = 0$, the second output file remains empty.

3.2. NETZ24 Program

First Input File:

	READ	FORMAT	Anzahl Zeilen [a]	Erläuterungen [b]
1	TEXT	20A4	1	Überschrift [c]
2	NP	I5	1	Anz. Netzpunkte [d]
3	PP,Y,X	A8,2F15.5	NP	P_i, y_i^o, x_i^o
4	NS	I5	1	Anz. Strecken [e]
5	PPI,PPK,MS	2A8,F10.5	NS	P_i, P_k, m_s
6	NPR	I5	1	Anz. Richtungssätze [f]
7	PPI,NR	A8,I5	1 } x NPR	P_i, Anz. Richt. auf P_i [g]
8	PPK,MR	A8,F10.5	NR }	P_k, m_r
9	NK	I5	1	Anz. Kreiselmessungen [h]
10	PPI,PPK,MK	2A8,F10.5	NK	P_i, P_k, m_k
11	NH	I5	1	Anz. Pfeilhöhenmess. [i]
12	PPI,PPJ,PPK,MH	3A8,F10.5	NH	P_i, P_j, P_k, m_h
13	NRE	I5	1	Anz. rel. Fehlerellipsen [j]
14	PPI,PPK	2A8	NRE	P_i, P_k
15	NFS	I5	1	Anz. Streckenfehler [k]
16	PPI,PPK	2A8	NFS	P_i, P_k
17	NFW	I5	1	Anz. Winkelfehler [l]
18	PPI,PPJ,PPK	3A8	NFW	P_i, P_j, P_k

Key: a. number of lines
b. explanations
c. heading
d. number of points in network
e. number of lengths
f. number of sets of directions
g. number of directions at P_i
h. number of gyroscopic measurements
i. number of offset measurements
j. number of relative error ellipses
k. number of length errors
l. number of angle errors

Second Input File

The second input file is automatically developed by the preparatory program NETZ22.

In case it must be developed by hand, the following conventions apply:

	READ	FORMAT	Anzahl Zeilen[a]	Erläuterungen[b]
1	NPA	I5	1	Anzahl Anschlußpunkte[c]
2	PPA	A8	NPA	Name Anschlußpunkt[d]
3	QK	5E14.6	(x)	Anteilige Q-Matrix[e]

Key: a. number of lines
b. explanations
c. number of connection points
d. name of connection point
e. constituent Q matrix

(x) The elements of the constituent Q matrix are input by rows:

QK(1,1),QK(1,2),QK(1,3), ... , QK(1,NQK),Q(2,1),Q(2,2), ... , QK(NQK,NQK)

where NQK = 2 · NPA.

-- One line contains 5 elements (5E14.6).

Dimensioning in the Main Program

- PP(NP),Y(NP),X(NP),A(NA),ZA(NA),AA(NA,NA),QK(NQK,NQK),KAN(NQK)
- TEXT(20)

where NA = 2 · NP and NQK = 2 · NPA .

Read-In of File Names

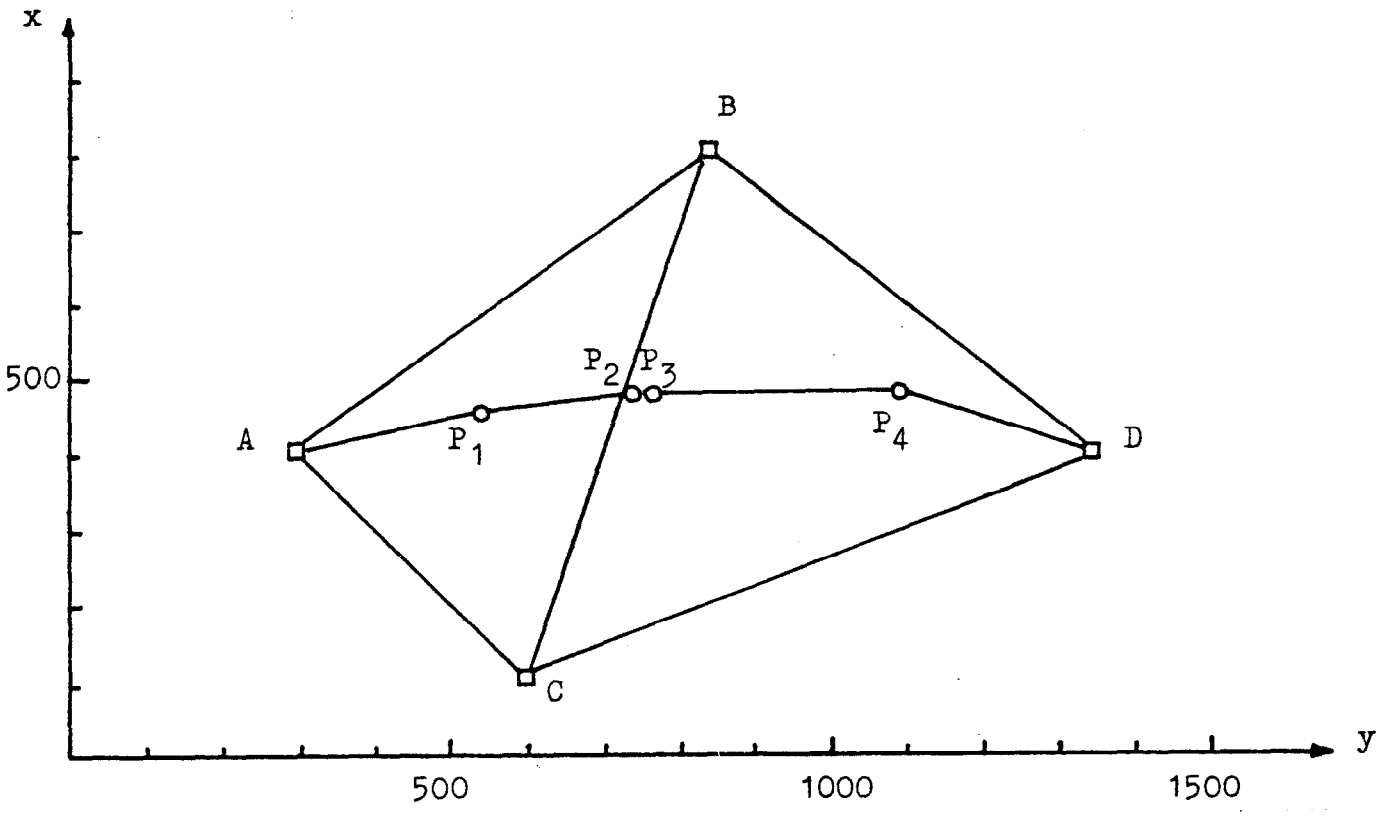
The file names ("1st Input File," "2nd Input File," "Output File") are input interactively with the display screen.

Output File

1)	Heading	
2)	Approximation coordinates	(point,y_o,x_o)
3)	Length program (if $n_s \neq 0$)	(from,to,m_s)
4)	Direction program (if $n_{p_r} \neq 0$)	(from,to,m_r)
5)	Gyroscope program (if $n_k \neq 0$)	(from,to,m_k)
6)	Offset program (if $n_h \neq 0$)	(stat,left,rt,m_h)
7)	Connection points	(point names)
8)	Statistical values	(m_o,f,S_u,S_o,f_{95})†
9)	Absolute error ellipses	(a,b,ϕ)
10)	Relative error ellipses (if $n_{re} \neq 0$)	(a,b,ϕ,e)
11)	Length error (if $n_{f_s} \neq 0$)	(from,to,$m_{\bar{s}}$)
12)	Angle error (if $n_{f_w} \neq 0$)	(stat,left,rt,$m_{\bar{w}}$)

† m_o(a priori) = 0.0001 ... established by program!
Dimensions: always (m) and (grads)!

4. Numerical Example



-- Superordinate network (above ground):.:A,B,C,D

-- Subordinate network (underground): A,P_1,P_2,P_3,P_4,D

-- Connection points (portal points): A,D

The underground network is observed from two sides:

-- From A in the direction of P_2 and

-- From D in the direction of P_3.

What is being sought (among other things) is the estimated relative perpendicular error (punch-through error) between the adjacent points P_2 and P_3.

Input File for NETZ22

```
 10 UEBERGEORDNETES NETZ  Superordinate network
 20      4    NETZPUNKTE  Points in network
 30 A         300.00           400.00
 40 B         850.00           800.00
 50 C         600.00           100.00
 60 D        1350.00           400.00
 70      5    STRECKENMESSUNGEN   Length measurements
 80 A        B        0.004
 90 B        D        0.004
100 D        C        0.004
110 C        A        0.004
120 B        C        0.004
130      4    RICHTUNGSSAETZE     Sets of directions
140 A            2
150 B        0.0005
160 C        0.0005
170 B            3
180 D        0.0005
190 C        0.0005
200 A        0.0005
210 D            2
220 C        0.0005
230 B        0.0005
240 C            3
250 A        0.0005
260 B        0.0005
270 D        0.0005
280      0    KREISELMESSUNGEN        Gyroscopic measurements
290      0    PFEILHOEHENMESSUNGEN    Offset measurements
300      2    ANSCHLUSSPUNKTE         Connection points
310 A
320 D
```

1st Output File of NETZ22

```
  1       UEBERGEORDNETES NETZ  Superordinate network
  2       NAEHERUNGSKOORDINATEN  Approximation coordinates
  3       A              300.00000       400.00000
  4       B              850.00000       800.00000
  5       C              600.00000       100.00000
  6       D             1350.00000       400.00000
  7       STRECKENPROGRAMM  Length program
  8       A         B          0.00400
  9       B         D          0.00400
 10       D         C          0.00400
 11       C         A          0.00400
 12       B         C          0.00400
 13       RICHTUNGSPROGRAMM   Direction program
 14       A         B          0.00050
 15       A         C          0.00050
 16       B         D          0.00050
 17       B         C          0.00050
 18       B         A          0.00050
 19       D         C          0.00050
 20       D         B          0.00050
 21       C         A          0.00050
 22       C         B          0.00050
 23       C         D          0.00050
 24       STATISTISCHE GROESSEN    Statistical values
 25       M0 = 0.00010  F =   6  SU = 0.00006  SO = 0.00023  F95 =  3.2
 26       FEHLERELLIPSEN    Error ellipses
 27       A            0.00205     0.00152     115.6
 28       B            0.00182     0.00172     181.8
 29       C            0.00173     0.00164     136.5
 30       D            0.00221     0.00142     102.6
```

2nd Output File of NETZ22 = 2nd Input File for NETZ24

(File of connection points A,D)

```
1      2
2 A
3 D
4    0.408081D+03 -0.441774D+02 -0.251808D+03 -0.258546D+02 -0.441774D+02
5    0.242155D+03  0.229332D+02  0.316268D+02 -0.251808D+03  0.229332D+02
6    0.489893D+03 -0.120044D+02 -0.258546D+02  0.316268D+02 -0.120044D+02
7    0.202121D+03
```

Input File for NETZ24

```
 10 UNTERGEORDNETES NETZ  Subordinate network
 20      6    NETZPUNKTE  Points in network
 30 A          300.00          400.00
 40 D         1350.00          400.00
 50 P1         550.00          450.00
 60 P2         750.00          480.00
 70 P3         751.00          480.10
 80 P4        1100.00          480.00
 90      5    STRECKENMESSUNGEN      Length measurements
100 A        P1       0.002
110 P1       P2       0.002
120 P2       P3       0.002
130 P3       P4       0.002
140 P4       D        0.002
150      4    RICHTUNGSSAETZE        Sets of directions
160 P1           2
170 A        0.0010
180 P2       0.0010
190 P2           2
200 A        0.0010
210 P1       0.0010
220 P4           2
230 D        0.0010
240 P3       0.0010
250 P3           2
260 D        0.0010
270 P4       0.0010
280      4    KREISELMESSUNGEN       Gyroscopic measurements
290 P1       A          0.0028
300 P2       P1         0.0028
310 P4       D          0.0028
320 P3       P4         0.0028
330      2    PFEILHOEHENMESSUNGEN   Offset measurements
340 P1       P2       A        0.0009
350 P4       D        P3       0.0009
360      1    RELATIVE FEHLERELLIPSE   Relative error ellipse
370 P2       P3
380      1    STRECKENFEHLER           Length error
390 A        D
400      1    WINKELFEHLER             Angle error
410 P3       D        A
```

Output File of NETZ24

```
 1      UNTERGEORDNETES NETZ     Subordinate network
 2      NAEHERUNGSKOORDINATEN    Approximation coordinates
 3      A              300.00000        400.00000
 4      D             1350.00000        400.00000
 5      P1             550.00000        450.00000
 6      P2             750.00000        480.00000
 7      P3             751.00000        480.10000
 8      P4            1100.00000        480.00000
 9      STRECKENPROGRAMM     Length program
10      A       P1           0.00200
11      P1      P2           0.00200
12      P2      P3           0.00200
13      P3      P4           0.00200
14      P4      D            0.00200
15      RICHTUNGSPROGRAMM    Direction program
16      P1      A            0.00100
17      P1      P2           0.00100
18      P2      A            0.00100
19      P2      P1           0.00100
20      P4      D            0.00100
21      P4      P3           0.00100
22      P3      D            0.00100
23      P3      P4           0.00100
24      KREISELPROGRAMM      Gyroscope program
25      P1      A            0.00280
26      P2      P1           0.00280
27      P4      D            0.00280
28      P3      P4           0.00280
29      PFEILHOEHENPROGRAMM  Offset program
30      P1      P2      A            0.00090
31      P4      D       P3           0.00070
32      ANSCHLUSSPUNKTE      Connection points
33      A
34      D
35      STATISTISCHE GROESSEN     Statistical values
36      M0 = 0.00010  F =   7  SU = 0.00007  SO = 0.00021  F95 =  3.1
37      ABSOLUTE FEHLERELLIPSEN   Absolute error ellipses
38      A            0.00185     0.00151     123.3
39      D            0.00198     0.00142     105.5
40      P1           0.00802     0.00241     188.8
41      P2           0.01412     0.00282     190.1
42      P3           0.01519     0.00251       1.7
43      P4           0.00654     0.00231       8.2
44      RELATIVE FEHLERELLIPSEN   Relative error ellipses
45      P2      P3       a= 0.02171   b= 0.00191   f= 194.3   0.02170 = e
46      STRECKENFEHLER       Length error
47      A       D              1050.00000    0.00322
48      WINKELFEHLER         Angle error
49      P3      D       A                180.34712    0.00369
```

Mean Punch-through error :	± 2.2 cm
Punch-through error for S = 95% (2.2 x 3.1)	± 6.7 cm