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DISCUSSION OF SPACE CHARGE EFFECTS OF A BEAM TRAIN CONTAINING INFINITELY MANY BUNCHES*

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Abstract

In an electron linear accelerator, the continuous beam emitted by an electron gun will become an equally spaced beam train after passing through the bunching section. If the current of the beam is large, its expansion may be more intense than the case where only a single bunch is considered, resulting from the space charge forces between different bunches. In this article, using an algorithm capable of calculating the space charge effects of a beam train containing infinitely many bunches with uniform spacing, we compare bunch trains with different parameters to find the pattern of their space charge effects.

INTRODUCTION

RF linear accelerators are finding increasingly diverse applications in industry [1]. When electrons are ejected from a hot cathode electron gun, the beam resembles an infinitely long low-energy bunch. After the beam passes through a RF buncher, electrons are accelerated to a certain speed and form an equally spaced bunch train with an interval $\beta c/f$, where βc is the final speed of electrons, and f is the frequency of the RF buncher. During the subsequent transmission of the bunch train, the space charge forces between bunches may affect the bunches' phase space, making its changes different from the calculated results of a single bunch.

Based on MATLAB [2], We have developed an algorithm which is able to calculate the space charge effects of a beam train containing infinitely many bunches with uniform spacing. The algorithm uses the ring charge model to calculate space charge force and uses Particle-in-Cell method [3] to simplify the operation. Space charge effects are calculated in the Bunch System and then transformed back to the Laboratory System. The algorithm's results have been validated by ASTRA [4] in some special cases.

In this article, we calculate bunch trains with different parameters including bunches' spacing D, initial length $H0$, radius $R0$, Lorenz factor γ and charge Q. The space charge effects of the bunch train when different parameters are changed are discussed. For simplicity, all bunches used in this article are uniformly cylindrical in shape.

SPACE CHARGE EFFECTS OF A BUNCH TRAIN

The space charge forces of a bunch train with uniform spacing are shown in Fig. 1. In the transversal direction, space charge forces from different bunches superimpose, while in the longitudinal direction they cancel each other. As a result, when such bunch train drifts over a long distance, bunches' transversal size expands faster than the single bunch case, while the bunches' longitudinal expansion is suppressed. Furthermore, the space charge effects can be greatly affected by the spacing between bunches. When the spacing D tends to infinity, the bunch train model would degenerate to a single bunch. Under such situation, the space charge effects among bunches hardly exist. When the spacing D reaches its minimum value $H0$, which is the length of a single bunch, the bunch train would become an infinitely-long-bunch shown in Fig. 1.

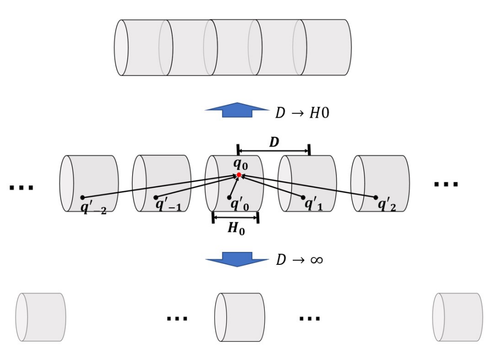

Figure 1: Space charge forces of a bunch train.

We used ASTRA to generate an example to illustrate such effects, as depicted by Fig. 2. The initial parameters of each single bunch in the bunch train is: charge $q = -2$ nC, radius $R0 = 50$ mm, length $H0 = 10$ mm, Lorenz factor $\gamma 0 = 8$, transport distance $L = 50$ m, spacing between bunches $D = 15$ mm. At first, the bunch train has 5 identical cylindrical bunches. After the transport, it can be seen that the central bunch has larger radius and shorter length compared to other bunches. When the number of bunches in the bunch train increases to infinity, each bunch would behave like this central bunch according to spatial symmetry. Different bunches' final size is listed in Table. 1. The color labeling in the first column corresponds to Fig. 1. The single bunch case is calculated separately for comparison.

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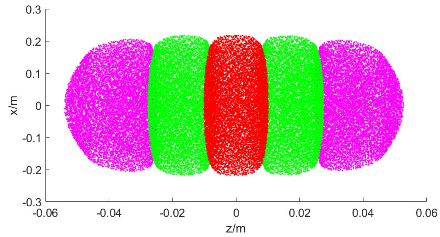

Figure 2: A calculation example of ASTRA's 5 bunch space charge effects.

Table 1: Parameters of Different Bunches in ASTRA's 5 Bunch Example

Bunch number	rms-radius/mm	rms-length/mm
0 (single bunch)	85.22	10.49
1 (red)	105.5	6.104
2 (green)	105.3	6.231
3 (purple)	96.22	7.981

COMPARISON AND DISCUSSION OF DIFFERENT BUNCH TRAIN'S RESULTS

In this section we will discuss the impact of different parameters on bunch trains' space charge effects. For charge Q and Lorenz factor γ, we will change them to observe the overall expansion degree of the bunch train. For bunches with different shape factors $A = R/(\gamma * H0)$, we will discuss the critical condition that makes the space charge effects among bunches insignificant. For simplicity, the illustrate and fitting of results only involve bunches' transversal expansion.

Bunch Trains with Different Charges

We take each bunch's parameters as: $R0 = 50\,\mathrm{mm}, H0 = 40\,\mathrm{mm}, L = 50\,\mathrm{m}, \gamma = 6$, and only change its charge Q. The calculation results are summarized in Fig. 3, where the circles are data points and the solid lines are the corresponding fitted smooth curves, which also applies to the following figures. As shown in Fig. 3(a), for each curve individually, when the spacing D increases, the bunches' final radius slowly decreases to close the case of a single bunch. When the charge Q changes, the shape of the curve does not change much. The charge Q changes the overall expansion of the bunch train, which holds for any value of spacing D. The larger the charge Q is, the more intense the bunches' expansion would be.

Figure 3(b) depicts the extra expansion percentage for each bunch train, compared to the single bunch case for each Q. The standard for normalization is the final Root Mean Square (RMS) radius of the single bunch case after expansion, which is also the value when $D \to \infty$. When $D/H0$ achieves 3 in Fig. 3(b), all 4 curves dropped to nearly 0, which indicates that the space charge effects among bunches are no longer significant when $D/H0 \geq 3$ for these cases. Furthermore, as shown with the purple line, when Q is larger, the bunch train's extra expansion percentage is also larger.

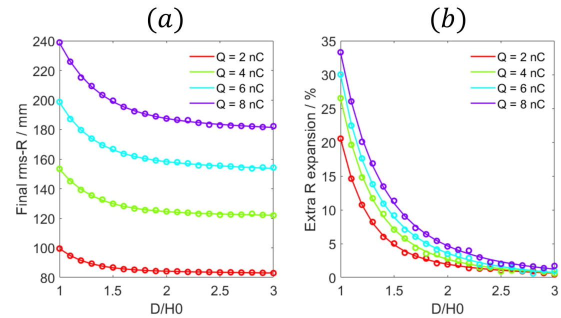

Figure 3: Comparison of bunch trains with different charges. (a) The final RMS radius of each bunch train. (b) The extra expansion percentage for each bunch train compared to the single bunch case.

Bunch Trains with Different Speeds

In this section, we take each bunch's parameters as: $Q = 8\,\mathrm{nC}, R0 = 50\,\mathrm{mm}, H0 = 40\,\mathrm{mm}, L = 50\,\mathrm{m}$, and only change its Lorenz factor γ. The calculation results are summarized in Fig. 4, which is similar to the case of different Q, except that when γ is smaller, the bunches' expansion would be more intense.

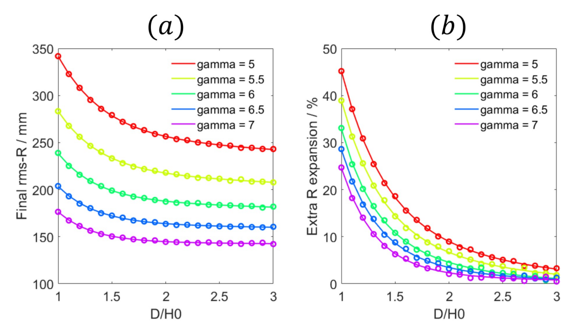

Figure 4: Comparison of bunch trains with different speeds. (a) The final RMS radius of each bunch train. (b) The extra expansion percentage for each bunch train compared to the single bunch case.

Bunch Trains with Different Shape Factors

In this section, we take each bunch's parameters as: $R0 = 50\,\mathrm{mm}, L = 50\,\mathrm{m}, \gamma = 6$, while changing $H0$ and Q in equal proportions. The calculation results are summarized in Fig. 5(a) and (b).

In Fig. 5(a), as the charge density of each single bunch remains the same, the longer the bunch is, the more intense its expansion would be. It should also be noted that when $D/H0 = 1$, the values of 4 curves all converge to the result of the infinitely-long-bunch. Accordingly, when each bunch is longer, the bunch train's extra expansion compared to the single bunch would be less significant, as shown in Fig. 5(b).

We decided to further explore that when the space charge effects among bunches would be insignificant. We take a

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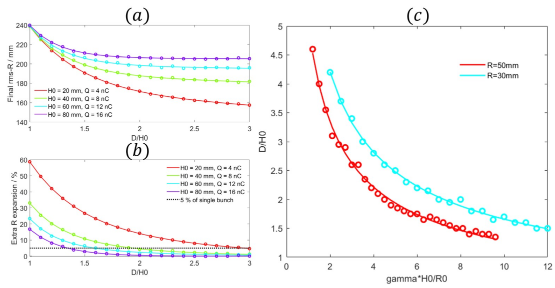

Figure 5: Comparison of bunch trains with different shapes. (a) The final RMS radius of each bunch train. (b) The extra expansion percentage for each bunch train compared to the single bunch case. (c) Fitting for the critical value of D/H0 when the extra expansion is at 5%.

criteria of 5%, which means that the bunch train's transversal expansion is 105% times of the corresponding single bunch, as shown in Fig. 5(b)'s black dashed line.

For instance, for the green line in Fig. 5(a) and (b) there is $H0 = 40$ mm. By taking the spacing $D = 3\times10^3$ to be a large value, we can get its corresponding single bunch's expansion result as $RMS(R) = 179.6$ mm, whose 105% is 188.6 mm. When $D/H0$ is respectively 1.8, 1.9 and 2.0, the bunch train's results are 191.1, 189.0 and 187.3 mm. Therefore, when $H0 = 40$ mm, its shape factor is $A = R0/(\gamma H0) = 0.208$, and its standard for insignificant space charge effects among bunches is $D/H0 = 1.9$. When the shape factor A is larger, the bunch would be shorter, so to achieve the same 5% criteria, $D/H0$ should be larger, as depicted by the red line in Fig. 5(b), whose intersection with the black line clearly corresponds to a larger $D/H0$.

According to the 5% standard, we iteratively calculated a series of data points for $D/H0$ and $1/A$, as shown by circles in Fig. 5(c). We found that their relationship can be well fitted by a power function $y = a * x^b$. The red curve is the fit of cases for $R = 50$ mm, whose expression is $y = 5.05 * x^{-0.5951}$ and the adjusted R^2 is 0.9946. For the blue curve, the expression is $y = 6.277 * x^{-0.5779}$ and the adjusted R^2 is 0.9967.

From Fig. 5(c) we can see that for the two cases adopted, when the reciprocal of shape factor A is 2, $D/H0$ should exceed 5 to make the space charge effects among bunches insignificant. When $1/A$ is greater than 6, the space charge effects among bunches would not be significant anymore once $D/H0$ reaches 2. Therefore, we can infer that when both $1/A$ and $D/H0$ are not too small, the space charge effects among bunches would be negligible.

CONCLUSION

Using a MATLAB algorithm, we calculated the space charge effects of beam trains containing infinitely many bunches with uniform spacing. Different parameters of the bunches are changed to demonstrate their impact on the beam expansion. The criteria when the space charge effects among bunches would be negligible for different shape of bunches is also discussed.

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