Non-geometric Five-branes in Heterotic Supergravity

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Abstract.   We study non-geometric five-branes in heterotic supergravity where the first order α'-corrections are present. By performing the α'-corrected T-duality transformations of the heterotic NS5-brane solutions, we obtain the exotic 5_2^2-brane solutions associated with the symmetric, the neutral and the gauge NS5-branes. We find that the symmetric 5_2^2-brane, and the neutral 5_2^2-brane at $\mathcal{O}(\alpha')$, are T-folds while the gauge 5_2^2-brane is not a T-fold. This proceeding is based on the work [1] where the details that are omitted in this manuscript are found.

1. Introduction

Extended objects such as branes play an important role in superstring theories. The U-duality [2], which makes non-trivial connections among consistent superstring theories, relates various branes in each theory. It was shown that M-theory compactified on T^d has the U-duality symmetry group $E_{d(d)}(\mathbb{R})$ in lower dimensions [3, 4, 5]. BPS branes in superstring theories form lower-dimensional multiplets under the U-duality group. For example, when we consider M-theory compactified on T^8, we have the $E_{8(8)}(\mathbb{R})$ BPS point particle multiplet. The higher dimensional origin of parts of these point particles is the ordinary branes wrapped/unwrapped on cycles in T^8. Here, the ordinary branes are waves, F-strings, D-branes, NS5-branes, Kaluza-Klein (KK) branes. However, there are states whose higher dimensional origin does not trace back to the ordinary branes. These states are called exotic states and their higher dimensional origin is known as exotic branes [6]. Among other things, an exotic brane in type II string theories known as the 5_2^2-brane, has been studied intensively in the past. Type II string theories compactified on T^d has T-duality symmetry group $O(d, d, \mathbb{Z})$. The exotic 5_2^2-brane is obtained by performing the T-duality transformations along the transverse directions to the NS5-brane world-volume. A geometry whose monodromy is characterized by the T-duality symmetry group is known as a T-fold. The geometry associated with the 5_2^2-brane is given by the multi-valued supergravity fields and it has non-geometric nature. However, the 5_2^2-brane geometry has the $O(2, 2)$ monodromy. The multi-valuedness of the geometry is interpreted as the O(2,2) T-duality transformation and it is a consistent solution in string theory. This is an example of the T-fold.

The purpose of the work [1] is to study exotic 5_2^2-branes in heterotic supergravity. In the following, we introduce the explicit solutions of the 5_2^2-branes in heterotic supergravity and show their monodromy structures.

2. Heterotic supergravity

A remarkable property of heterotic supergravity is that the Yang-Mills gauge field contributes to the action as the first order α'-correction. The famous anomaly cancellation mechanism and the

supersymmetry completion result in the Riemann curvature square term which involves higher derivative corrections in the same order in α' [7]. The ten-dimensional heterotic supergravity action for the bosonic fields at the first order in α' is given by

$$S = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g}\, e^{-2\phi} \Big[R(\omega) - \frac{1}{3}\hat{H}_{MNP}^{(3)}\hat{H}^{(3)MNP} + 4\partial_M\phi\partial^M\phi$$
$$+ \alpha' \left(\mathrm{Tr}(F_{MN})^2 + (R_{MNAB}(\omega_+))^2 \right) \Big]. \tag{1}$$

Here we employ the convention such that $\frac{\kappa_{10}^2}{2g_{10}^2} = \alpha'$ where κ_{10} and g_{10} are the gravitational and the gauge coupling constants in ten dimensions. The Yang-Mills gauge field strength is denoted by F_{MN} and ϕ is the dilaton. The ten-dimensional metric g_{MN} in the string frame is defined through the vielbein as $g_{MN} = \eta_{AB}e_M{}^A e_N{}^B$ where the metric in the local Lorentz frame is $\eta_{AB} = \mathrm{diag}(-1, 1, \ldots, 1)$. The Ricci scalar $R(\omega)$ and the Riemann tensor $R_{MNAB}(\omega_+)$ are constructed from the spin connection $\omega_M{}^{AB}$. Here $M, N, \ldots, A, B, \ldots$ are the curved space-time and the local Lorentz indices. The modified spin connection is defined by

$$\omega_{\pm M}{}^{AB} = \omega_M{}^{AB} \pm \hat{H}_M^{(3)AB}, \tag{2}$$

where $\hat{H}_M^{(3)AB} = e^{NA}e^{PB}\hat{H}_{MNP}^{(3)}$. The modified H-flux $\hat{H}_{MNP}^{(3)}$ is defined by

$$\hat{H}_{MNP}^{(3)} = H_{MNP}^{(3)} + \alpha' \left(\Omega_{MNP}^{\mathrm{YM}} - \Omega_{MNP}^{\mathrm{L+}} \right). \tag{3}$$

Here $H_{MNP}^{(3)}$ is the ordinary field strength of the NS-NS B-field, $\Omega_{MNP}^{\mathrm{YM}}$, $\Omega_{MNP}^{\mathrm{L+}}$ are the Yang-Mills and the Lorentz Chern-Simons terms, respectively.

3. Five-brane solutions and Buscher rule in heterotic theory

The heterotic NS5-brane solutions satisfy the equation of motion derived from the action (1) at $\mathcal{O}(\alpha')$. There are three distinct NS5-brane solutions known as the symmetric, the neutral and the gauge types in heterotic supergravity [8, 9]. They are 1/2 BPS configurations and preserve a half of the sixteen supercharges. They satisfy the following ansatz:

$$ds^2 = \eta_{ij}dx^i dx^j + H(r)\delta_{mn}dx^m dx^n,$$
$$\hat{H}_{mnp}^{(3)} = -\frac{1}{2}\varepsilon_{mnpq}\partial_q H(r), \qquad \phi = \frac{1}{2}\log H(r),$$
$$r^2 = (x^1)^2 + (x^2)^2 + (x^3)^2 + (x^4)^2, \tag{4}$$

where the indices $i, j = 0, 5, 6, 7, 8, 9$, $m, n = 1, 2, 3, 4$ represent the world-volume and the transverse directions to the NS5-branes and ε_{mnpq} is the Levi-Civita symbol. The 1/2 BPS condition leads to the self-duality condition for the Yang-Mills gauge field:

$$F_{mn} = \frac{1}{2}\varepsilon_{mnpq}F_{pq}, \tag{5}$$

and other components A_i vanish. These heterotic NS5-branes are characterized by two charges. One is the topological charge k associated with the gauge instantons:

$$k = -\frac{1}{32\pi^2} \int \mathrm{Tr}[F \wedge F], \tag{6}$$

where the integral is defined in the transverse four-space. The other is the charge Q associated with the modified H-flux:

$$Q = -\frac{1}{2\pi^2\alpha'} \int_{S^3} \hat{H}^{(3)}. \tag{7}$$

Here S^3 is the asymptotic three-sphere surrounding the NS5-brane.

3.1. Symmetric solution

The most tractable NS5-brane solution in heterotic theory may be the so-called symmetric solution. The Yang-Mills gauge field which satisfies the self-duality condition (5) is given by an instanton solution in four-dimensions. The gauge field takes value in the $SU(2)$ subgroup of the gauge group $SO(32)$ or $E_8 \times E_8$ and the explicit solution is given by [9]:

$$A_m = -\frac{\sigma_{mn} x^n}{r^2 + n\alpha' e^{-2\phi_0}}, \quad n \in \mathbb{Z},$$

$$H(r) = e^{2\phi_0} + \frac{n\alpha'}{r^2}, \tag{8}$$

where σ_{mn} is the self-dual part of the $SO(4)$ Lorentz generator. The solution (8) is nothing but the BPST one-instanton in the non-singular gauge. The constant ϕ_0 is the asymptotic value of the dilaton. Compared with the general BPST instanton solution, the symmetric solution has a fixed finite instanton size $\rho = e^{-\phi_0}\sqrt{n\alpha'}$. This solution has charges $(k, Q) = (1, n)$. A remarkable fact about the symmetric solution is that the dilaton configuration, hence the harmonic function $H(r)$, is obtained through the standard embedding ansatz:

$$(A_m)^{ab} = \omega_{+m}{}^{ab}. \tag{9}$$

Here the $SU(2)$ indices a, b of the gauge field are identified with the indices of the $SU(2)$ subgroup of the local Lorentz group $SO(4)$ in the transverse directions. If the relation (9) holds, the Chern-Simons terms in the modified H-flux (3) cancel out. Therefore the H-flux only comes from the B-field and the Bianchi identity becomes $d\hat{H}^{(3)} = 0$. From the Bianchi identity and the relation between $H(r)$ and $\hat{H}_{mnp}^{(3)}$ in (4), the B-field is determined through the following condition:

$$\partial_m H = \frac{1}{2}\varepsilon_{mnpq}\partial_n B_{pq}. \tag{10}$$

3.2. Neutral solution

The neutral solution is the one with charges $(k, Q) = (0, n)$ [9]. The Yang-Mills gauge field becomes $A_M = 0$ and the harmonic function is given by

$$H(r) = e^{2\phi_0} + \frac{n\alpha'}{r^2}. \tag{11}$$

For this solution, the curvature term in the modified H-flux becomes a higher order in α' and it is neglected. Since the gauge field does not appear in the neutral solution, this is a solution to type II supergravities.

3.3. Gauge solution

The last is the so called gauge solution. The Yang-Mills gauge field is again given by the BPST instanton. As in the case of the neutral solution, the curvature term in the modified H-flux is neglected as it is a higher order in α'. However, the Yang-Mills gauge field still contributes to the H-flux in the gauge solution and the Bianchi identity becomes $d\hat{H}^{(3)} = \alpha'\mathrm{Tr}F \wedge F$. From the Bianchi identity, the harmonic function is determined to be

$$A_m = -\frac{\sigma_{mn} x^n}{r^2 + \rho^2},$$

$$H(r) = e^{2\phi_0} + 8\alpha'\frac{r^2 + 2\rho^2}{(r^2 + \rho^2)^2}. \tag{12}$$

Compared with the symmetric solution (8), the size modulus ρ is not fixed for the gauge solution and it has charges $(k, Q) = (1, 8)$.

4. Exotic five-branes and monodromies

We now compactify the transverse spaces to the five-brane world-volume to T^2 and perform the T-duality transformation of the solutions. Since the gauge field enters into the action (1) as the first order α'-correction, the T-duality transformation in the heterotic supergravity should be modified from the standard Buscher rule. The first order α'-corrections to the Buscher rule in heterotic supergravity have been written down in [10].

It is convenient to introduce the T-duality covariant quantity $\mathcal{H}$ known as the generalized metric. The generalized metric in heterotic theories is determined by utilizing the heterotic supergravity action (1) compactified on T^d [11]. This is given by

$$\mathcal{H} = \begin{bmatrix} G^{-1} & -G^{-1}B \\ BG^{-1} & G - BG^{-1}B \end{bmatrix},$$
(13)

where G is defined by

$$G_{\mu\nu} = g_{\mu\nu} + 2\alpha' \big[\mathrm{Tr}(\omega_{+\mu}\omega_{+\nu}) - \mathrm{Tr}(A_\mu A_\nu) \big],$$
(14)

Here μ, ν are the isometry directions. The generalized metric (13) takes the same form in type II supergravities but the second term in (14) is characteristic to heterotic theories. The spin connection term in (14) has been introduced as it enters into the action (1) in the same way as the Yang-Mills gauge field [11].

4.1. Symmetric solution

By performing the T-duality transformation along the transverse spaces, we obtain the following symmetric 5_2^2-brane solution:

$$ds^2 = \eta_{ij}dx^i dx^j + H \left[(dx^1)^2 + (dx^2)^2 \right] + \frac{H}{K} \left[(dx^3)^2 + (dx^4)^2 \right],$$

$$\phi = \frac{1}{2}\log\frac{H}{K}, \quad B_{34} = -\frac{\omega}{K}, \quad K = H^2 + \omega^2.$$
(15)

The harmonic function $H(r)$ in $\mathbb{R}^2 \times T^2$ is given by [6]:

$$H(r) = h_0 - \frac{\sigma}{2}\log\frac{r}{\mu},$$
(16)

where $\sigma = \frac{R^2}{2\pi\alpha'}$ and μ is a constant which specifies the region where the solution is valid. The constant h_0 diverges in the large cutoff scale limit $\Lambda \to \infty$. Since the harmonic function only depends on x^1, x^2, the NS-NS B-field does so. The only non-zero component of the B-field is determined as $B_{34} \equiv \omega = -\sigma \tan^{-1}\left(\frac{x^2}{x^1}\right)$. The gauge field $A_M{}^I$ is calculated as

$$A_1 = \frac{1}{2H}(\partial_2 H)T^3, \qquad A_2 = -\frac{1}{2H}(\partial_1 H)T^3,$$

$$A_3 = \frac{1}{2HK}\Big((H\partial_2 H + \omega\partial_1 H)T^1 - (H\partial_1 H - \omega\partial_2 H)T^2 \Big),$$

$$A_4 = -\frac{1}{2HK}\Big((H\partial_1 H - \omega\partial_2 H)T^1 + (H\partial_2 H + \omega\partial_1 H)T^2 \Big),$$

$$A_i = 0.$$
(17)

Here T^I are the $SU(2)$ generators. We find that (17) satisfies the vortex-like equation in two dimensions. We also find that the solution satisfies the standard embedding condition up to a gauge transformation:

$$\mathrm{Tr}F_{MN}F^{MN} = R_{MNAB}(\omega_+)R^{MNBA}(\omega_+).$$
(18)

The monodromy structure of the heterotic 5_2^2-brane is characterized by the generalized metric $\mathcal{H}(\theta)$. When we go around the center of the 5_2^2-brane and come back to the original point, namely if the angular position changes as $\theta = \arctan(x^2/x^1) = 0 \to 2\pi$, then the generalized metric for the symmetric 5_2^2-brane solution is evaluated to be

$$\mathcal{H}(2\pi) = O^t \mathcal{H}(0) O, \tag{19}$$

where

$$O = \begin{bmatrix} -i\tau_2 & 0 \\ 2\pi\sigma \mathbf{1}_2 & -i\tau_2 \end{bmatrix} \in O(2,2). \tag{20}$$

This implies that the metric is not single-valued (and non-geometric) but the monodromy for the solution is given by the $O(2,2)$ T-duality transformation. Therefore, we conclude that although the symmetric 5_2^2-brane solution is non-geometric, it is a T-fold and a consistent solution to heterotic string theories.

4.2. Neutral solution
As we have claimed before, the modified spin connection ω_+ is in the $\mathcal{O}(\alpha')$ for the neutral solution. Therefore it is negligible in G_{MN} in the heterotic Buscher rule. After some calculations, the metric, the dilaton and the NS-NS B-field are given by those for the type II 5_2^2-brane solution. This is a conceivable result since the neutral solution does not involve gauge field anymore and it is a solution in type II theory. We stress that although the symmetric solution is an exact solution, the neutral solution is a perturbative solution in the α' expansion. The generalized metric for the neutral solution is given that for the symmetric solution and the monodromy structure is the same with the symmetric case. Therefore we find that the neutral 5_2^2-brane is a T-fold at least at $\mathcal{O}(\alpha')$ in heterotic theories.

4.3. Gauge solution
After performing the T-duality transformation, we find the following gauge 5_2^2-brane solution:

$$ds^2 = \eta_{ij}dx^i dx^j + \mathcal{I}[(dx^1)^2 + (dx^2)^2] + \frac{\mathcal{I}}{e^{4\phi_0} + \Theta^2}[(dx^3)^2 + (dx^4)^2],$$

$$B_{34} = -\frac{\Theta}{e^{4\phi_0} + \Theta^2}, \quad \phi = \frac{1}{2}\log\left(\frac{\mathcal{I}}{e^{4\phi_0} + \Theta^2}\right), \quad \mathcal{I} = e^{2\phi_0} - \frac{\alpha'\tilde{\sigma}^2}{2r^2\left(\tilde{h}_0 - \frac{\tilde{\sigma}}{2}\log(r/\mu)\right)^2},$$

$$A_1 = \frac{-\tilde{\sigma}x^2}{4r^2\left(\tilde{h}_0 - \frac{\tilde{\sigma}}{2}\log(r/\mu)\right)}T^3, \quad A_2 = \frac{\tilde{\sigma}x^1}{4r^2\left(\tilde{h}_0 - \frac{\tilde{\sigma}}{2}\log(r/\mu)\right)}T^3,$$

$$A_3 = \frac{\tilde{\sigma}}{4r^2\left(\tilde{h}_0 - \frac{\tilde{\sigma}}{2}\log(r/\mu)\right)(e^{4\phi_0} + \Theta^2)}\left(e^{2\phi_0}(x^1T^2 - x^2T^1) + \Theta(x^1T^1 + x^2T^2)\right),$$

$$A_4 = \frac{\tilde{\sigma}}{4r^2\left(\tilde{h}_0 - \frac{\tilde{\sigma}}{2}\log(r/\mu)\right)(e^{4\phi_0} + \Theta^2)}\left(e^{2\phi_0}(x^1T^1 + x^2T^2) + \Theta(x^1T^2 - x^2T^1)\right),$$

$$A_i = 0, \tag{21}$$

where Θ, $\tilde{\sigma}$, $\tilde{h}_0$ are constants. The gauge field satisfies the vortex-like equation which is a reminiscent of the self-duality equation (5). In the symmetric and the neutral 5_2^2-brane solutions, there was the function ω which contains explicit angular coordinate on the x^1x^2-plane. In the gauge 5_2^2-brane solution, the B-field is governed by a parameter Θ instead of ω. However, since Θ is a constant parameter, the gauge 5_2^2-brane solution is completely determined by single-valued functions and it is a geometric solution. The function $\mathcal{I}$ becomes negative at a finite value of r.

IF-YITP GR+HEP+Cosmo International Symposium VI

IOP Publishing

IOP Conf. Series: Journal of Physics: Conf. Series **883** (2017) 012008

doi:10.1088/1742-6596/883/1/012008

This is the same situation in the case of the gauge KK5-brane and, unfortunately, its physical meaning is still obscure.

For the gauge 5_2^2-brane, the gauge field contributes to the generalized metric through (14) at $\mathcal{O}(\alpha')$. However, since all the fields in the gauge solution do not depend on the angle θ in the two-dimensional base space, they do not inherit multi-valuedness of the geometry. Therefore the monodromy becomes trivial. This can be seen by evaluating the generalized metric for example in $\Theta = 0$ gauge. In this gauge, we have

$$\mathcal{H} = \left[\begin{array}{cc} G^{-1} & 0 \\ 0 & G \end{array} \right], \qquad G = e^{-2\phi_0} \mathbf{1}_2. \tag{22}$$

This implies $\mathcal{H}(2\pi) = \mathcal{H}(0)$. Therefore we concludes that the gauge 5_2^2-brane does not exhibit non-geometric nature.

5. Conclusion

In this proceeding, we show the explicit solutions of the exotic 5_2^2-branes in heterotic supergravity. A specific feature of heterotic supergravity is the Yang-Mills gauge sector which appears in the linear order in the α'-corrections. There are also higher derivative corrections of the curvature square term in the same order in α'. The three different half BPS five-brane solutions in this α' order are known. They are the symmetric, the neutral and the gauge NS5-brane solutions. These are distinguished by the topological charge of instantons of Yang-Mills gauge field and the charge associated with the modified H-flux.

We then introduce $U(1)$ isometries to the heterotic five-brane solutions and perform the T-duality transformations. The resulting solutions are the exotic 5_2^2-branes in heterotic theory. For the symmetric solution, the metric, B-field and the dilation are given by that of the 5_2^2-brane in type II theory. The gauge field satisfies the vortex-like equation in two dimensions. We found that the standard embedding condition is satisfied up to a gauge transformation. For the neutral solution, we found that the neutral 5_2^2-brane is the same with the one in type II theory. They exhibit a non-geometric nature due to the multi-valuedness of the B-field. For the gauge 5_2^2-brane solution, we found that the fields do not show the multi-valuedness and they remain geometric. The monodromies of the three different 5_2^2-branes are also studied. We calculated the generalized metric for these solutions. We found that the symmetric and the neutral 5_2^2-branes have a monodromy given by the $O(2,2)$ T-duality transformation. Therefore they are T-folds. On the other hand, the gauge solution does not show the nature of a T-fold. Details that are not shown in this manuscript are found in the paper [1].

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