GOLDSTONE THEOREM AND POSSIBLE APPLICATIONS TO ELEMENTARY PARTICLE PHYSICS

H. P. Dürr

Max-Planck-Institut für Physik und Astrophysik, München, Germany

The Goldstone theorem states that under certain conditions, which will be stated later, in a dynamical theory which is invariant under a particular symmetry group and where this symmetry group is broken by the ground-state, e.g. the vacuum state in a relativistic field theory, there must exist particles of mass zero or—in the nonrelativistic case—excitation modes, the energy of which tends to zero with increasing wavelength. This theorem is of great interest for elementary particle physics for essentially two reasons:

(1) There are a number of symmetry groups in elementary particle physics, like $SU(3)$ or even higher symmetries, which are not exactly realized in nature in the sense that there exist one-particle states or resonances grouped into multiplets which are supposed to transform approximately according to an irreducible representation of this group but which do not have exactly the same mass, and secondly that in the interaction of these particles and resonances the conservation laws, related to this symmetry group by Noether's theorem, are only approximately obeyed. On the other hand, these symmetry violations do not seem to be connected in a direct or an indirect way with the appearance of mass zero particles. Therefore, *if* one wants to interpret this symmetry violation as a consequence of an asymmetrical vacuum as proposed for $SU(3)$ by Baker and Glashow[1], rather than as an asymmetry of the underlying dynamical law, one has to find some means to *invalidate* the Goldstone theorem.

(2) There do exist in nature a number of massless particles: the photon, the neutrinos and, probably, the graviton. In a general dynamical theory it is rather difficult to obtain such particles as particular solutions, except by chance or if they are introduced from the beginning. In this context the Goldstone theorem, it appears, could provide an interesting way to enforce their existence, because, in fact, all the known massless particles do occur in connexion with symmetry violations, the photon with isospin violation, the neutrinos with parity violation, etc., and the graviton

probably with a violation of the Poincaré group[2]. Unfortunately, however, it turns out that all these mass zero particles do *not* have the symmetry properties, as explicitly stated by the Goldstone theorem in its present mathematical formulation. Therefore, in order to uphold this conjecture, the present predictions of the Goldstone theorem on the symmetry properties of the massless particle must be generalized.

Consequently the validity of the conjecture that the observed symmetry violations in elementary particle physics arise from an asymmetry of the vacuum state, will decisively depend on an invalidation or—if we exclude $SU(3)$ and possible higher symmetries as fundamental symmetries—on a generalization of the Goldstone theorem. In contrast to this, in non-relativistic dynamics we know many systems where the Goldstone theorem holds in its present form. The magnons in the ferromagnet, the phonons in liquids and crystals are, for example, Goldstone modes connected with an asymmetry of the groundstate[3].

There exist many general proofs of the Goldstone theorem today. The first proofs were given by Goldstone, Salam and Weinberg[4] and by Bludman and Klein[5]. A proof on a much more rigorous basis using only the algebra of observables was recently given by Kastler, Robinson, and Swieca, and Ezawa and Swieca[6].

Let me roughly sketch the proof of the Goldstone theorem in order to indicate the basic assumptions. Let us assume there is a certain symmetry transformation which leaves the dynamics invariant. Formally, this may be expressed by the forminvariance of a Lagrangian density or the forminvariance of an equation of motion and the quantization condition. As an example we may just use a simple gauge transformation to simplify the discussion. As a consequence of the invariance there exists, according to the Noether theorem, a conserved current:

$$\partial_\mu j^\mu(x) = 0 \tag{1}$$

and a time-independent hermitean operator

$$Q = \int d\sigma^\mu j_\mu(x) = \int_{x^0=t} d^3x\, j^0(x) \tag{2}$$

which serves as a generator of the unitary representation of the symmetry group in the state space. The symmetry is *broken* by the translational invariant vacuum state, if for some field operator $\phi(x)$, which is not invariant under this symmetry group, i.e. which transforms as $\phi(x) \to \phi'(x)$, the vacuum expectation value changes

$$\langle 0|\, \phi(x)\, |0\rangle = \langle 0|\, \phi(0)\, |0\rangle \neq \langle 0|\, \phi'(x)\, |0\rangle = \langle 0|\, \phi'(0)\, |0\rangle \tag{3}$$

or, expressed differently, if for an infinitesimal symmetry transformation $\sim\delta\lambda$ we have

$$\frac{1}{\delta\lambda}\langle 0|\,\delta_{\mathrm{sym}}\phi(x)\,|0\rangle = -\mathrm{i}\,\langle 0|\,[Q, \phi(x)]\,|0\rangle = C \neq 0 \qquad (4)$$

(with C = constant). By introducing a complete set of intermediate states the latter may also be expressed as[7]

$$\sum_{0'\neq 0} 2\,\mathscr{Im}\,\langle 0|\,Q\,|0'\rangle\langle 0'|\,\phi(x)\,|0\rangle = C \neq 0. \qquad (5)$$

If we make use of the local form (2) of the generator one gets

$$-\mathrm{i}\int_{\text{(for all surfaces)}} d\sigma'^{\mu}\,\langle 0|\,[j_\mu(x'), \phi(x)]\,|0\rangle = C \neq 0 \qquad (6)$$

which leads to the local condition

$$\langle 0|\,[j_\mu(x'), \phi(x)]\,|0\rangle = \frac{\partial}{\partial z^\mu} f(z) \qquad (z = x - x') \qquad (7)$$

with

$$-\mathrm{i}\int d\sigma'^{\mu}\,\frac{\partial}{\partial z^\mu} f(z) = C.$$

Due to the locality requirement (vanishing of the commutator for space-like distances) $f(z)$ can be written as a superposition of causal functions $\Delta(z; m)$ with various masses m:

$$f(z) = \mathrm{i}C\int dm^2\rho(m^2)\,\Delta(z; m) \qquad \int dm^2\rho(m^2) = 1. \qquad (8)$$

As a consequence of the current conservation (1)

$$\partial_z^2 f(z) = 0 \quad \text{and hence} \quad \rho(m^2) = \delta(m^2) \qquad (9)$$

i.e. $\phi(x)$ must contain matrix elements leading to massless particles from the vacuum. This is the content of the Goldstone theorem.

From Eqn. (5) one merely deduces that there exists in the theory other states $|0'\rangle$ different from the vacuum state $|0\rangle$ which have the same energy (and momentum) as the vacuum state since Q is a (time independent) symmetry operator. We call these states $|0'\rangle$ spurion states. They are created from the vacuum by $\phi(x)$. Relation (9) which has made use of the local structure of the symmetry operator and hence contains more information, reveals that the spurions, in fact, are merely the infrared limit ($\mathbf{p} \to 0$) of massless particles generated by $\phi(x)$, the Goldstone particles, which we will shortly call 'zerons'. Roughly speaking the zerons are localized spurions.

In this connexion Heisenberg[2] has emphasized that any localized spurion connected with a nonlocalized spurion could again be a possible 'zeron'. Hence the symmetry character of the Goldstone zeron should in general not be immediately identified with the symmetry character of the spurions which is usually done, but states a separate problem.

I wish now to remark on the various steps of the rough (and partly inaccurate) derivation in particular to indicate the various assumptions of the Goldstone theorem.

The first assumption refers to the existence of a conserved current. This assumption is decisive because it expresses that there exists a symmetry of the dynamics, at all. In our proof above the existence of such a locally conserved current is necessary to conclude that the spurions are not isolated states $\delta(p^\mu)$ but can be localized to become mass zero particles. However, it is not important that $j^\mu(x)$ is really a local current. It is sufficient to require that for an arbitrarily large, but still finite volume V with the surface S, the change of the 'charge' $Q(t)$ with time within this volume is accompanied by a current $J_S(t)$ leaving through the surface, i.e.

$$\frac{d}{dt} Q_V(t) = -J_S(t). \tag{10}$$

The volume V e.g. may be a measurable region in a bubble chamber or even the volume of the bubble chamber itself. The requirement of a conserved local current $j^\mu(x)$ would mean, that this relationship holds for *any* volume, and hence also for the infinitely small volume element in which case we can write:

$$Q_V(t) = \int_V d^3x j^0(\mathbf{x}, t)$$

$$J_S(t) = \int_S \mathbf{ds} \cdot \mathbf{j}(\mathbf{x}, t) \tag{11}$$

If we have only the relationship (10), then upon a symmetry variation of $\phi(x)$ only within the volume V at time t', we would get

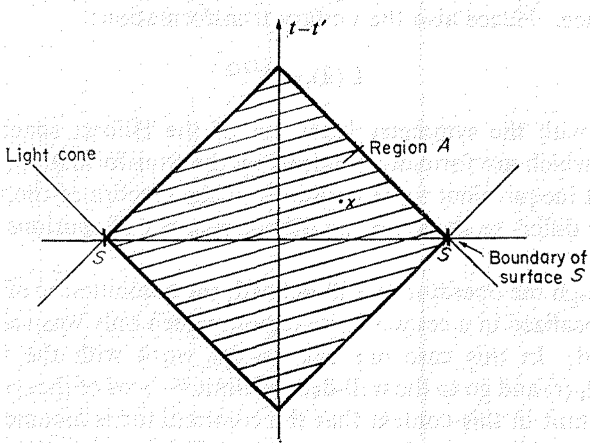
$$\frac{1}{\delta\lambda} \langle 0| \delta^{V_1 t'}_{\text{sym}} \phi(x) |0\rangle = -\mathrm{i} \langle 0| [Q_V(t'), \phi(x)] |0\rangle = C_V(\mathbf{x}; t - t') \neq 0 \tag{12}$$

with

$$\lim_{V \to \infty} C_V(\mathbf{x}; t - t') = C = \text{const.}$$

and hence due to (10)

$$\frac{1}{\delta\lambda}\frac{d}{dt}\langle 0|\,\delta_{\text{sym}}^{V_1t'}\phi(x)\,|0\rangle = -\mathrm{i}\,\langle 0|\,[J_S(t'),\,\phi(x)]\,|0\rangle = \frac{d}{dt}\,C_V(\mathbf{x};\,t-t')$$
$$= \begin{cases} 0 & \text{in region } A \\ \neq 0 & \text{outside.} \end{cases} \tag{13}$$



For a local theory this vanishes for x within a region A, bordered by light cones through the surface points S, i.e. for $t = t'$ if x is inside the volume. Condition (13) is sufficient to localize the spurion to a certain extent, and hence to prove the existence of a zero mass particle.

In a relativistic quantum field theory it is by no means trivial to establish the existence of a conserved current, because the construction of such currents usually involves products of field operators at the same space-time point which are rather singular objects. Consequently it is certainly not sufficient to establish the conservation laws simply on the basis of the usual (classical) substitution transformations. Nevertheless one has to keep in mind that the Goldstone theorem only breaks down if the weaker requirement (10) is violated.

An example of such an intrinsically broken symmetry is Schwinger's model of 2-dimensional quantum electrodynamics[8], where, despite of the formal γ_5-symmetry, the chirality current is not conserved. Some people call this case a locally broken symmetry[9]. However, such a definition seems only worthwhile if something is left of the symmetry at all, e.g. the weaker condition (10) which then would still imply a 'zeron'. Otherwise we should simply call this a 'no symmetry' case.

The next question refers to the existence of the generator Q. One can easily show that if the vacuum is *not* invariant under the symmetry group, i.e. if $Q\,|0\rangle \neq 0$, then the integral (2) will diverge. Physically speaking the 'charge' of the translational invariant vacuum state is either zero (invariant case) or infinite. In the latter case also all other states created from it by application of a finite number of field operators have also infinite charge. Hence Q itself is an infinite operator and no proper element of the Hilbert space. Hence also the unitary transformation:

$$U(\lambda) = e^{i\lambda Q} \tag{14}$$

connected with the symmetry leads out of the Hilbert space. Vacuum states $|0\rangle_\lambda$ which are formally obtained by the transformation (14) belong to different inequivalent representations of the algebra of the field operators. They differ, so to say, by an infinite number of spurions or infrared zerons.

Although the operator Q is ill-defined, the commutator of Q with any operator localized in a certain finite region, which only was used above, is well-defined. In this case one can always work with the well-defined operator $Q_V(t)$ and go to the well-defined limit $V \to \infty$ of the commutators. It is important in this context that the commutator is assumed to vanish for large space-like separations, a property which is required for all observables in a local relativistic theory.

It was pointed out by Higgs, Englert, and Brout[10], and also by Hagen, Guralnik, and Kibble[11] that the Goldstone zerons can be avoided if there are in addition long range interactions in the theory from the outset, however, as will be seen, at a dear price. That long range forces affect the Goldstone theorem, in fact, was recognized earlier by Anderson[12] for the nonrelativistic case in particular in connexion with superconductivity where the Goldstone modes are pushed up to become the plasmons. For the relativistic case, however, it was important to recognize that long range interactions occur in connexion with gauge fields, i.e. mass zero vector fields, where the nonlocal character of the interaction becomes apparent if one uses the non manifestly covariant Coulomb gauge related to a certain space-like surface

$$\partial A - (n\partial)(nA) = 0 \tag{15}$$

which involves the time-like vector n^μ the normal to this surface. Due to the long-range interaction (Coulomb type interaction) the commutator $\langle 0|\,[Q_V(t'), \phi(\mathbf{x}, t)]\,|0\rangle$ does *not* become time independent in the limit $V \to \infty$, since the surface current term does not decrease sufficiently fast.

At the same time the commutator is not manifestly covariant. In fact, the commutator (7) has now the more general form[10]

$$\langle 0|\,[j^{\mu}(x'),\,\phi(x)]\,|0\rangle = \frac{\partial}{\partial z_{\mu}}\,f(z) + (g^{\mu\nu}\partial^2 - \partial^{\mu}\partial^{\nu})n_{\nu}g(z) \qquad (16)$$

If long range interaction are involved, then we can assume $f(z) = 0$, i.e. that *no* mass zero particle occurs, since one has instead

$$g(z) = \mathrm{i}\,\frac{(n\partial)}{\partial^2 - (n\partial)^2}\,C\int dm^2\rho(m^2)\,\Delta(z;\,m) \qquad \int dm^2\rho(m^2) = 1 \qquad (17)$$

to take care of the condition (6), where the inverse operator $\partial^2 - (n\partial)^2 \rightarrow -\nabla^2$ indicates the nonlocal character of this case. In addition, the now massive Goldstone particle combines with the two massless vector bosons to form a normal 3 component massive vector field. So, no massless particles are left in the theory.

One may, of course, also use the manifestly covariant and local Gupta–Bleuler description using the Lorentz gauge, which, however, implies the introduction of an indefinite metric in the Hilbert space. In this case, the Goldstone theorem is valid and hence formally leads to a zeron, which, however, can be shown[13] to decouple completely from the physical states and eventually is eliminated, if one projects on the physical subspace of the Hilbert space. Hence also in this description there is eventually no physical zeron left.

However, it should be emphasized that the example of Higgs and others shows that a theory of this type is only causal, and hence physically acceptable, if the states created by the operator $\phi(x)$ from the vacuum can be separated completely from the physical states and suppressed. In this case one then has to check whether the symmetry, which is broken, still has a non-trivial meaning in the subspace after this projection. This does *not* seem to be the case, as we will shortly indicate in the model given by Higgs:

Higgs[10] starts out with a model originally suggested by Goldstone with the Lagrangian*

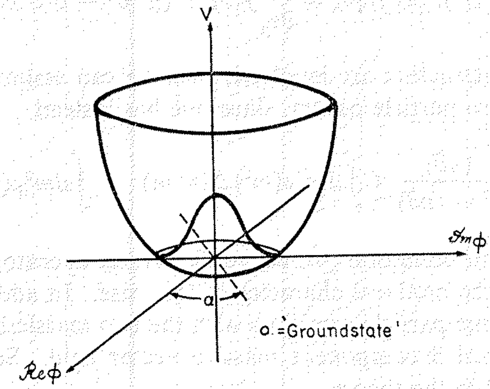
$$L_0 = \frac{1}{2}\left[\phi^{\mu *}\,\partial_{\mu}\phi + \phi^{\mu}\,\partial_{\mu}\phi^* - \phi^{\mu *}\phi_{\mu} - \frac{m_0^2}{4\eta^2}(\phi^*\phi - \eta^2)\right] \qquad (18)$$

for a scalar nonhermitean field operator $\phi(x)$, in which the potential has a

* The notation here is that essentially used by Kibble[13].

minimum for $|\phi|^2 = \eta^2 = \text{const.}$ In the groundstate hence

$$\langle 0|\, \phi\, |0\rangle = \eta e^{i\alpha} \neq 0 \qquad (\alpha = \text{const.}) \tag{19}$$



The time component of ϕ^μ are the canonical conjugate variable of $\phi(x)$. The Lagrangian is obviously invariant under the gauge transformations

$$\begin{aligned} \phi(x) &\to e^{ie\lambda}\phi(x) \\ \phi^*(x) &\to e^{-ie\lambda}\phi^*(x) \end{aligned} \tag{20}$$

which leads to a conserved current $\partial_\mu j^\mu = 0$ with

$$j^\mu = -ie[\phi^{\mu *}\phi - \phi^\mu \phi^*] \tag{21}$$

The symmetry (20) is broken by the groundstate, because

$$\frac{1}{\delta\lambda}\langle 0|\, \delta\phi\, |0\rangle = ie\langle 0|\, \phi(x)\, |0\rangle = ie\eta e^{i\alpha} \neq 0. \tag{22}$$

The Lagrangian may be rewritten by introducing the modular dependence $R(x)$ and the phase dependence $\theta(x)$ of the fields as new field variables:

$$\begin{aligned} \phi(x) &= R(x)e^{i\theta(x)} \\ \phi^*(x) &= R(x)e^{-i\theta(x)} \end{aligned} \tag{23}$$

with the corresponding canonically conjugate variables as the time components of the vectors $R^\mu(x)$ and $\theta^\mu(x)$. One obtains

$$\begin{aligned} L_0 &= R^\mu \partial_\mu R - \tfrac{1}{2}R^\mu R_\mu - \frac{m_0^2}{8\eta^2}[R^2 - \eta^2]^2 + \theta^\mu \partial_\mu \theta - \frac{\theta^\mu \theta_\mu}{2R^2} \\ &= L_R + L_\theta \end{aligned} \tag{24}$$

which upon variation besides the actual field equations leads to the algebraical relations

$$\begin{aligned} R_\mu &= \partial_\mu R \\ \theta_\mu &= R^2\,\partial_\mu\theta \end{aligned} \tag{25}$$

which may also be inserted into the Lagrangian without harm. The symmetry transformation (20) in the new variables is now simply

$$\theta(x) \to \theta(x) + e\lambda \tag{26}$$

with the other variables remaining unchanged, and the current (21) is

$$j^\mu = -e\theta^\mu(x) \tag{27}$$

The groundstate condition (19) is now expressed by

$$\begin{aligned} \langle 0|\, R(x)\, |0\rangle &= \eta \\ \langle 0|\, \theta(x)\, |0\rangle &= \alpha \end{aligned} \tag{28}$$

The first condition (28) does *not* indicate a symmetry violation, because $R(x)$ is invariant under the symmetry transformation. The symmetry breaking condition (22) arises solely from the second condition in a rather trivial fashion:

$$\frac{1}{\delta\lambda}\langle 0|\, \delta\theta(x)\, |0\rangle = e \neq 0 \tag{29}$$

From the latter it follows immediately by the Goldstone theorem that $\theta(x)$ is a massless field, in fact, the Goldstone zeron, which can also be directly seen from the Lagrangian, if we approximately replace $R(x) \approx \eta$. The $R(x)$ field on the other hand is connected with a particle of finite mass m_0, which can be deduced by introducing the new field operators

$$r(x) = R(x) - \eta \qquad r^\mu(x) = R^\mu(x) \tag{30}$$

One now introduces a massless gauge field $A^\mu(x)$ by the prescription ($F^{\mu\nu}n_\nu$ is the canonically conjugate variable of A^μ):

$$\begin{aligned} L &= -\tfrac{1}{2}F^{\mu\nu}(\partial_\nu A_\mu - \partial_\mu A_\nu) + \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + L_0(\text{with } \partial_\mu\theta \to \partial_\mu\theta + eA_\mu) \\ &= L_F + L_R + \theta^\mu(\partial_\mu\theta + eA_\mu) - \frac{\theta^\mu\theta_\mu}{2R^2} \end{aligned} \tag{31}$$

which obviously is invariant under the coordinate dependent gauge transformations:

$$\begin{aligned} \theta(x) &\to \theta(x) + e\lambda(x) \\ A^\mu(x) &\to A^\mu(x) - \partial^\mu\lambda(x) \end{aligned} \tag{32}$$

The vector θ_μ is here now obviously

$$\theta_\mu = R^2(\partial_\mu\theta + eA_\mu) = -\frac{1}{e}j_\mu(x) \tag{33}$$

The critical commutator for the Goldstone theorem is

$$\langle 0|\,[j^\mu(x'), \theta(x)]\,|0\rangle = -e\langle 0|\,[\theta^\mu(x'),\theta(x)]\,|0\rangle \tag{34}$$

which in the Coulomb gauge is nonlocal, and hence does not fulfil the requirements of the Goldstone theorem. In fact, one finds:

$$\mathrm{i}\langle 0|\,[\theta_\mu(x'), \theta(x)]\,|0\rangle = \frac{(g_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu)n^\nu}{\partial^2 - (n\partial)^2}(n\partial)\int \rho(m^2)\,dm^2\,\Delta(z; m)$$

$$z = (x' - x) \tag{35}$$

with $\rho(m^2) \approx \delta(m^2 - e^2\eta^2)$.

On the other hand one can also introduce the new field operator

$$B_\mu(x) = \frac{1}{eR^2}\theta_\mu = A_\mu(x) + \frac{1}{e}\partial_\mu\theta(x) \tag{36}$$

instead of $A_\mu(x)$ (the canonically conjugate variable is still $F^{\mu\nu}n_\nu$) and obtain the Lagrangian in the form

$$L = -\tfrac{1}{2}F^{\mu\nu}(\partial_\nu B_\mu - \partial_\mu B_\nu) + \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + \tfrac{1}{2}e^2R^2B_\mu B^\mu + L_R \tag{37}$$

which does not contain $\theta(x)$ any more as variable. The canonical momentum θ^μ did combine with the two transversal fields of A^μ to give a (3 component) massive vector field B^μ with mass $m_V^2 = e^2\eta^2$. If one restricts oneself to the state space produced by only applying the gauge invariant operators $F^{\mu\nu}(x)$, $B^\mu(x)$, $R(x)$ (but not $\theta(x)$!) on the vacuum, then the original symmetry no longer has meaning in this restricted state space, since all operators are (trivially) invariant under the gauge transformations. Hence this does not lead into contradiction with the general proof of Ezawa and Swieca[6].

The situation is similar, if one introduces non-Abelian groups as Kibble has shown[13]. Again the original Goldstone zerons become massive by a coupling to a corresponding gauge field by which procedure the gauge fields themselves become massive. At the same time the original symmetry transformations become meaningless in the gauge invariant observables. Only the mass zero gauge fields of the *unbroken* symmetries survive. However, their mass zero character has nothing to do with the Goldstone theorem. They are rather the leftover mass zero fields which were put in from the beginning.

To state the result more clearly, let us consider the isospin group $SU(2)$. If we break the symmetry around the x- and y-axis by the vacuum, one obtains in the simplest case of a scalar isovector field *without* gauge fields, a massive scalar boson S^0, and positively and negatively charged Goldstone zerons S^+, S^-, if for simplicity we identify the properties of the zerons with the properties of the corresponding spurions. We get other zerons by combining S^+ and S^- with positively and negatively charged spurions. If the gauge field is turned on, i.e. if one introduces a massless vector isotriplet A^+, A^0, A^-, the A^+ and A^- combines with S^+ and S^-, respectively, to give two massive vector bosons V^+, V^- and only the A^0 remains massless. Hence:

massive scalar $m = m_0$	S^0
massive vector $m = e\eta$	V^+, V^-
massless vector $m = 0$	A^0

The spectrum hence contains incomplete multiples. The remaining massless gauge field is connected with the nonviolated rotation symmetry around the 3-isospin-axis. If *all* symmetries are broken, there will be *no* massless field left.

If one applies this procedure to a $SU(3)$ invariant theory and subsequently breaks this symmetry by strong, electromagnetic and weak interactions, the only remaining mass zero particle is the photon[14]. Otherwise, however, the model has very unrealistic features, in particular again the original high symmetry has completely lost its meaning.

Perhaps one has to be somewhat more careful with the statement that these symmetries are physically meaningless because all observables are left invariant. Also the electric and baryon number gauge transformations are such symmetry transformations which leave the observables invariant, but nevertheless have important physical consequences. In fact, this invariance leads to the superselection rules. However, it appears, that the strict validity of these transformations is the decisive difference. A *broken* symmetry on the other hand can only be sensed, if the theory contains *some* observables which are *not* invariant.

One may believe that at least in the case of a non-Abelian symmetry group like $SU(2)$, there are some objects left in the theory (namely the incomplete multiplets), which do not have a trivial transformation character. According to the construction this, however, is not the case. For example, neither the V^+, V^- nor the S^0, nor the A^0 transform like an isotriplet any longer.

This is, in fact, a general deficiency of all multiplets which are broken by an asymmetrical vacuum, and is not characteristic of the Higgs case. It was Umezawa who particularly stressed this point[15]. Let us again consider the broken $SU(2)$ isospin symmetry case without the gauge fields. The vacuum in this case could be hypothetically imagined as a large isoferromagnet with nonvanishing magnetization in the z-direction. The energy levels of a particle with isospin $\frac{1}{2}$ will split up consequently into two nondegenerate mass components, depending on the isospin orientation. This, however, is not a correct description. If we imagine, for the moment, that the vacuum is very large but finite, we may characterize it by a large isospin I_v. The non-degenerate mass doublet is then described by states of total isospin $I = I_v \pm \frac{1}{2}$. i.e. as energy levels corresponding to *different* irreducible representations of the symmetry group. If one performs an isospin rotation the levels hence do *not* transform into each other, because they differ in energy, but rather transform into themselves plus spurion contributions. The spurions correspond to the infrared limit of the Goldstone zerons. The whole reaction to our symmetry transformation seems to consist in a rearrangement of the isospin in the groundstate. In particular, if we separate from the field operator a part which, like $R(x)$ described above, does not participate in the broken symmetry transformations and connect the physical particle with it, then the physical particle will behave like an isosinglet under the I_1 and I_2 rotations, i.e. its isospin degree of freedom will appear to be 'frozen'. The whole transformations only add terms to the rest fields, which, as Umezawa remarks, merely change the Bose–Einstein condensation of the Goldstone zerons. In this language all the components of a nondegenerate multiplet behave like singlets under the relevant transformations. The variant part, a spurion part, is disconnected and combines with the BE-condensation part of the Goldstone bosons.

This indicates that the original symmetry transformation no longer connects the components of a split multiplet. There may, however, exist *another*, although weakly time-dependent transformation, which *does* connect the components of a multiplet in the expected way. In our example, it will consist of the isospin transformation which rotates the isospin only of the particle and not the large isospin of the vacuum. Whether this new transformation is a sufficiently good approximate symmetry transformation, i.e. is sufficiently time-independent, will depend on the strength of the coupling of the isospin of the particle to the vacuum isospins relative to their mutual coupling. If the particle coupling is strong a description as an isosinglet or an isotriplet may be more appropriate. Biritz[16] has given an example of such anomalous multiplets in a model of a ferromagnetic chain. In connexion with the nonlinear spinor theory strange particles were interpreted as such anomalous isospin multiplets[17].

In the non-Abelian models of Higgs and Kibble the originally degenerate multiplets after breaking of the symmetry become nondegenerate and even partially incomplete. It is hard to see how in this case an approximately valid symmetry transformation can be found which would transform these objects as members of the original irreducible representation. Obviously, a theory which approximately retains *only* the multiplets, but not the corresponding Clebsch–Gordan coefficients and selection rules would be completely useless.

There are still a number of other questions which should be studied in detail. One question is, what happens to the description of the isospin rotations around the x- and y-axis in the state space, if the superselection rules for the electric charge are established, because then these transformations, even if performed locally, would connect different invariant sectors. An investigation of this question may perhaps reveal that Higgs' suggestion may be physically applicable, after all, in the case where a gauge symmetry is left intact, because the leftover massless gauge field would enforce a superselection rule.

The isospin symmetry in elementary particle physics has been conjectured long ago by Heisenberg and coworkers[2] to be a possible candidate for an exact dynamical symmetry which is broken by the groundstate, because its violation is accompanied by and phenomenologically attributed to the mass zero photon. Unfortunately, however, the Goldstone 'zerons' in connexion with violation of the $SU(2)$ isospin group are, as we have seen, scalar, and, in the usual description, charged objects. Intuitively they are the magnons, the Bloch spin waves, of an infinitely large isoferromagnet at zero temperature. On the other hand it is quite clear that the isoferromagnet would not be an adequate description for the physical situation, because such a vacuum state would also violate CPT-invariance. One can easily see this if we imagine e.g. a π-meson in such an isoferromagnet, which would naturally split up into a mass triplet and hence violate the requirement that π^+ and π^- are antiparticles. This indicates that in this case, one has to employ a more complicated way to break the $SU(2)$ symmetry.

A more appropriate model for the isospin violating vacuum state in elementary particle physics would be a model in which at every lattice point there is a particle and an antiparticle with their isospins pointing essentially in opposite directions. There would be no resultant polarization (charge), but the polarization of the particle and antiparticle subsystems would be very large and distinguish a certain direction. Under CPT such a system would transform into itself. A state of this type has formally some similarity with the groundstate of an antiferromagnet, where the spins of neighbouring particles try to be antiparallel. One obtains such a groundstate, if one introduces forces which tend to align the spins of like particles and antialign spins of unlike particles. The groundstate of such a system is a very complicated object. Biritz and Yamazaki[18] have recently started to investigate this model in an approximation where the antialigning forces between the particles and antiparticles are considered weak in comparison with the aligning forces between like particles. In lowest approximation one obtains essentially a double isoferromagnet, a particle- and antiparticle-isoferromagnet oriented in opposite directions. This describes a particular superposition of states with isospin $I = 0, 1 \cdots$ with zero charge, i.e. without net polarization in the 3 direction. With the particle–antiparticle forces fully switched on, the situation gets very complicated, because these forces tend to form local singlets which upset the double ferromagnetic ordering by flipping a certain number of particle-antiparticle pairs. The exact groundstate has not been worked out as yet. The model has to be studied in three dimensions, since, similar to the antiferromagnet[19], the zero-point fluctuations do not permit a long range ordering in one and two dimensions.

In this iso-antiferromagnetic model there seems to appear a new uncharged Goldstone mode which is connected with a localized flipping of a particle–antiparticle pair. There is a way to write this mode as a projection operator

$$|k\rangle \sim \sum_n \tfrac{1}{2}(1 + \tau_3)e^{ikn} |0\rangle$$

which senses localized flipped particle–antiparticle pairs and hence has some similarity with the photon. An interesting question, however, is whether one can avoid now the charged Goldstone modes which are suggested still to show up in the usual interpretation on the basis of a general proof of the Goldstone theorem in the algebraic approach.

One actually would suspect that such charged modes, at least in the usual interpretation, should arise in connexion with a rotation of the particle–antiparticle lattice as a whole. However, one realizes that a rotation of the lattice, which has vanishing magnetization, after the rotation leads to a state for which the expectation value of the magnetization in the z-direction is still zero. Hence one may expect that the charged modes do not occur in the same way as in the ferromagnet.

Of course, there would still remain the question how the Goldstone zeron can be endowed with spin without breaking the Lorentz group by the vacuum, which actually would be required in this case by the Goldstone theorem[20]. The hope is here, that actually the Coulomb force, which is scalar, is inferred by the Goldstone argument, and that the photons only follow indirectly from it by locality and Lorentz invariance. This all has still to be investigated.

The question whether the neutrino may follow from some kind of a Goldstone argument must be completely denied at present. This becomes particularly clear in Umezawa's argumentations, where the Bose–Einstein condensation of the zerons appear to be crucial.

Before closing I wish to make a few remarks about the possibility to directly observe the underlying dynamical symmetries. Up to now we have argued that the existence of a local conservation law can only be indirectly inferred through the appearance of the Goldstone zerons, at least, if no long range forces are present from the beginning. The question arises whether there is not a direct way to establish the local conservation law. After all, it states that for every given volume the time-change of the 'charge', e.g. the first or second component of isospin, must be connected with a corresponding current going through the surface of this volume. If we find in a bubble chamber experiment that in a certain process isospin is violated, then—in our interpretation—this can only mean that our book-keeping is incorrect, that some isospin must have leaked out of the chamber unaccounted. There are, in fact, two reasons why our conventional book-keeping could be wrong:

(1) mass zero particles carrying isospin with an energy smaller than our energy resolution may have escaped our observation (infrared problem);

(2) The interacting particles were erroneously assumed to be exact eigenstates of isospin. Already their mass splitting, however, indicates that this can be only approximately true. So e.g. the ω^0 has small admixtures of the quantum numbers of ρ^0, the π^0 those of η, etc.

I finally want to remark on the non-leptonic weak interactions which phenomenologically can be successfully formulated as isospin $\frac{1}{2}$-spurion emission processes. In a theory with an isospin degenerate vacuum this formal description may even have a realistic foundation, because, as Umezawa has indicated, such a spurion is connected with the Bose–Einstein condensation of the Goldstone boson. It indicates a transfer of intrinsic quantum numbers to the vacuum as a whole. Such a description seems to have some formal similarity with the Mössbauer effect, where apparently the recoil momentum of the γ-emitting nucleus is transferred to the crystal as a whole, and hence seems to be locally lost. Weisskopf[21] has demonstrated, due to the high zero-point fluctuations connected with a local measurement, that there is no observable violation of the causality principle.

In conclusion, I wish to stress that, to our knowledge, there exists no case in which a dynamically valid symmetry in a relativistic, causal theory can be broken by the groundstate in an observable way without involving mass zero particles. The main problem in our opinion in dealing with the Goldstone theorem in relativistic theories seems to be connected with the physical interpretation of the assumptions which go into it—in particular, what we mean by a symmetry and its violation—and the physical interpretation of the consequences we derive from it—in particular whether we retain an approximate symmetry for the non-degenerate one-particle states which originally belonged to a single multiplet of the symmetry group and what are the symmetry properties of the zerons. Probably all these questions can only be decided by actually carrying out dynamical calculations.

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Discussion on the report of H. P. Dürr

S. Weinberg. I would like to emphasize a little more strongly the applications of the Goldstone theorem to the real world. First, let me mention in passing, that there is one other classical example of a Goldstone mode besides the ones that are always quoted, i.e. the hose instability of a charged particle beam in a uniform plasma. Coming back to the strong interactions, I would say that the greatest triumph of the Goldstone theorem is that it gives a 'raison d'être' for the pion as an almost massless particle. From this point of view, it is not important whether the Goldstone theorem has been rigorously proved; the important thing is that it tells us how the strong interactions could keep the pion mass so small.

I was also very impressed with the suggestions of Higgs, Kibble, and others, and would like to point out that the ρ and A_1 mesons afford a good example of these ideas. Of course isotopic spin is not broken, so the ρ meson mass has to be put in at the beginning, and then chiral symmetry 'breaking' gives an additional mass to the A_1, which in fact agrees with experiment. (Because the ρ has a bare mass, the Goldstone bosons don't go away; they are just the pions.) We can also ask whether these ideas can be applied to unify the electromagnetic and the weak interactions. If we restrict ourselves to the observed electronic leptons, then there is just one way that this can be done: there must be a massless photon, a massive charged intermediate meson, and a heavier intermediate neutral meson. The neutral intermediate boson has observable effects, notably that in electron–neutrino scattering the axial-vector $(\bar{e}e)(\nu\bar{\nu})$ coupling is $\frac{3}{2}$ what we would expect from the old calculation of Feynman and Gell-Mann.

This raises a question that I can't answer: Are such models renormalizable? You start with a Yang–Mills type Lagrangian which is renormalizable, and re-order the perturbation theory by redefining the fields. I hope someone will be able to find out whether or not the resulting Lagrangian is a renormalizable theory of weak and electromagnetic interactions.

F. Englert. With regard to the renormalizability of gauge vector mesons in the presence of broken symmetry, Brout and I (*Phys. Rev. Letters* **13,** 9 (1964) and *Nuovo Cim.* **43,** 244 (1966)) showed that the propagator of the massy vector mesons is $[g_{\mu\nu} - q_\mu q_\nu/q^2](q^2 - \mu^2)^{-1}$, μ being the induced mass of the gauge field. The term in $(q_\mu q_\nu/q^2)$ having a singularity at $q^2 = 0$ is due to the Goldstone boson contribution to the vector meson propagator. The term enters in this way as a consequence of the Ward identity and is not therefore an effect due to an approximation. Therefore the answer to Weinberg's question is that these massy vector gauge fields constitute a renormalizable field theory.

H. P. Dürr. I wish to apologize that I haven't mentioned, at all, the theories where chirality is broken, which was first treated by Nambu. Of course, it appears that chirality is fundamentally broken by a slight amount such that the π-meson acquires a small mass. One can nevertheless, as Weinberg has pointed out, actually recover many relationships of the Goldstone situation.

F. E. Low. Does a Goldstone boson lie on a trajectory?

H. P. Dürr. I don't know, but I would suspect that the answer may depend on the particular dynamics one has to deal with.

R. Brout. At least in the Nambu model where the π-meson is an $(N\bar{N})$ bound state, it would appear unlikely that the π-meson reggeizes. When one turns on small bare mass, the matrix elements of $\partial_\mu j_{\mu 5}$ do not tend to zero at ∞ momentum transfer, but rather to the matrix elements of $m_0 \bar{\psi} \gamma_5 \psi$, m_0 being the bare mass. Thus there is no unsubtracted dispersion relation for the pseudoscalar form factor, as would be expected from reggeization of the pion and a no subtraction hypothesis. In the case that $m_0 = 0$ and $\partial_\mu j_{\mu 5} = 0$, the pion is the Goldstone pole and remains the pseudoscalar pole in the form factor, no matter how high the momentum transfer. This follows from the Ward identity. Again the behaviour is contrary to Regge behaviour.

R. E. Marshak. Since we are working in a rather speculative domain, I should like to make a remark which illustrates the different approaches to chirality symmetry taken by Weinberg and by some of us who have worked exclusively with fermion fields. I find it extremely strange and suggestive that we not only have three lepton fields in close analogy to the three quarks which seem to structure the hadrons, but we also have 3 symmetry breakings from the $SU(3) \otimes SU(3)$ level, down to $SU(3)$, down to $SU(2)$ and finally down to $SU(1) \times Y$. In a sense, the muon and electron are behaving like the quarks which produce the hypercharge and electromagnetic splittings respectively among hadrons. In a more serious vein, Weinberg is willing to tolerate a non zero mass pion as a sort of pseudo-Goldstone particle. One might have a chance to explain the 3 broken symmetries in terms essentially of lepton currents compounded out of the 3 objects we know with different masses: zero mass ν, electromagnetic mass e, and muon mass which is of the order of (hypercharge) $SU(3)$ breaking. Perhaps this conjecture can already be excluded by what is known about simultaneous symmetry breakings and the generalized Goldstone theorem.

H. P. Dürr. At present I cannot see how a connexion can be made between the Goldstone theorem and the leptons. Perhaps there might be a way to establish a connexion between weak currents and the breaking of symmetries if one uses some kind of a mechanism as suggested by Higgs and others which make use of long range forces in the original Lagrangian.

W. Heisenberg. In connexion with the Goldstone theorem I want to stress two points. The one concerns the interaction of a particle with the groundstate. If the breaking of the symmetry is seen experimentally in the splitting of mass levels in a multiplet, then in the Goldstone case, the splitting must be due to this interaction of the particle with the groundstate and its collective modes. Therefore if one were to try to explain the violation of $SU(3)$ by a Lagrangian exactly invariant under $SU(3)$ and an asymmetrical groundstate, there should not only exist the collective modes of mass zero in the groundstate, but these modes should also interact strongly with the particles—against existing experimental evidence. With respect to $SU(2)$ the situation is much better. There we have the Coulomb field and the photon, and their interaction is just of the correct order of magnitude for explaining the mass splitting in the iso-multiplets.

The second point concerns the projection operator in the definition of a Goldstone particle. These particles may be considered as localized spurions. Now a spurion, being the change from one vacuum to another vacuum, means some change which is—with equal probability—spread out over the whole space. If one then constructs a projection operator which picks out just those points in space where the change has occurred, then by means of this operator one can localize the spurion, i.e. construct a Goldstone particle. Therefore the occurrence of a projection operator like e.g. $\frac{1}{2}(Y + \tau_3)$ in the Gell-Mann–Nishijima rule is a strong indication for the Goldstone-character of the corresponding particle or field.

R. E. Marshak. How can you hope to demonstrate that the photon is the Goldstone particle in $SU(2)$ symmetry breaking when it has the wrong quantum numbers?

H. P. Dürr. There may be still some hope to change the internal quantum numbers of the Goldstone boson by coupling on spurions, as Heisenberg has suggested. The Lorentz properties would be all right if one could show that the Goldstone object is really the scalar Coulomb force from which then the photons would arise by a secondary step from Lorentz invariance.

Personally, I certainly would take the suggestion of Higgs and others as an attractive possibility, provided one can find some way out in the difficulties which seem to occur there.

G. Källén. If it is true that the π-meson is a Goldstone particle with approximately zero mass then I would like to ask: what is the situation for the other mesons usually classified to be in the same octet?

Do they all correspond to Goldstone particles, but with some of the masses more equal to zero than others? Alternatively, one could declare that the octet classification is an accident and that the π-meson is basically different from the other pseudoscalar mesons. If the first alternative is preferred by the official point of view my question is: Do all the pseudoscalar mesons correspond to the same broken symmetry or are they related to different broken symmetries? If so, which broken symmetry corresponds to which particle?

S. Weinberg. I don't know what the official view is, but the consensus seems to be that it is better not to think about the strange particles if you want to go on thinking you understand what is going on. Nevertheless, Glashow and I have looked at what happens to Goldstone's theorem if you include $SU(3) \times SU(3)$ symmetry breaking with a specific assumption as to how the symmetry breaking term transforms. The result is very weak, i.e. one inequality among the masses of the would-be Goldstone bosons (the pseudoscalar nonet and the unobserved kappa meson), which may be true. In addition, it should be noted that the calculations of KN and $\bar{K}N$ scattering lengths which use the idea of a partially conserved strangeness-changing current are in pretty good agreement with experiment.