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Mapping the frontier: a review of quantum and evolutionary game theory for complex decision-making

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Abstract

In the current context, where the productivity paradox coexists with growing concerns about inequality, quantum economics and quantum game theory are emerging as novel frameworks to address inherent uncertainty and complex strategic interactions. Quantum game theory applies the principles of quantum mechanics to model strategic interactions, providing new insights into decision-making, optimization and conflict resolution. In this article, we conduct a systematic review following the PRISMA methodology and a bibliometric analysis of the literature on quantum game theory to assess its development, methodologies and applications. We examine the growing interest in quantum technologies such as quantum computing, quantum communication and quantum simulation, which have had a significant impact on both theoretical models and practical economic and social systems. By mapping trends in the quantum game theory research landscape, this article aims to identify key advances, highlight important milestones and propose future directions for research in this emerging field. This comprehensive mapping not only elucidates the dynamic evolution and interdisciplinary convergence of quantum and evolutionary game theory, but also establishes

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a foundation for future research, fostering innovative approaches to complex decision-making in increasingly uncertain and interconnected environments.

Keywords Quantum theory · Game theory · Quantum economics · Systematic bibliometric analysis

1 Introduction

Quantum theory [1] is one of the most surprising expressions of modern physics, not only due to the extraordinary and counterintuitive nature of its postulates but also because of its disruptive potential in the development of new technologies as well as in the creation of new theoretical models that better describe certain aspects of reality. Over the past few decades, this theory has given rise to a wide range of quantum technologies including quantum computing [2, 3], quantum communications [4], and quantum sensors [5] and quantum simulation [6]. Among these, quantum computing stands out for its potential is undoubtedly quantum computing, which is divided into two distinct computing paradigms: adiabatic quantum computing [7], with its most popular expression being quantum annealing [8], and gate-based quantum computing, with its many physical implementations: superconducting circuits [9], trapped ions [10], diamond nitrogen-vacancy (NV) centres [11], photons [12], NMR [13], among others. These technologies have diverse applications, such as cryptography [14], quantum machine learning (QML) [15], optimization problems [16], quantum communication, which transmits secure information using quantum states, quantum sensors and metrology, which exploit quantum sensitivity for high-precision measurements in navigation, medicine and materials science; and quantum simulation, which uses controllable quantum systems to simulate other complex quantum systems, crucial for designing new materials and drugs [17, 18].

In recent years, there has been growing interest in applying quantum theory beyond physics, particularly in fields that involve uncertainty, complex dynamics, and strategic decision-making. One such area is economics, where quantum theory has inspired the development of quantum economics [19, 20] and, more specifically, quantum game theory (QGT). More generally, in their relation to economics, quantum technologies offer powerful tools for solving complex economic problems [21–23]. Quantum computing optimizes finance, logistics and energy. Quantum simulation improves market modelling and consumer behaviour. Quantum AI and machine learning accelerate model training for fraud prediction and detection. Quantum communication provides unconditional security for transactions and data. Specifically, quantum game theory analyses how quantum mechanics affects strategic interactions, enabling quantum strategies incorporating features as superposition and entanglement that improve equilibria and solve classical dilemmas [24]. Quantum information influences outcomes. In addition, new types of games could be formulated based on quantum phenomena, with potential economic applications in market design, negotiation, voting and secure communication.

From the perspective of the new economy, profoundly shaped by digital transformation and technological advancement, significant expectations for economic growth and increased productivity are observed. The invention of new machines or technologies, such as those associated with the digital revolution, holds the potential to generate far-reaching changes in the economy. It has been argued that technological innovations can drive growth along the trajectory of transitional dynamics.

However, this technological transition is not without its challenges. There is concern that automation driven by new technologies could affect the labour market [25]. While historically automation has been linked to the replacement of routine and arduous tasks, new waves of innovation, such as artificial intelligence (AI), are also impacting white-collar, executive, and professional jobs. Despite optimism about the potential of new technologies to increase productivity, a significant slowdown in productivity growth has been observed in recent decades, known as the productivity paradox [26]. One possible explanation is that time is needed for new technologies to be fully implemented and for businesses and the economy to restructure to maximize their potential, a phenomenon similar to what occurred with the diffusion of electricity.

The new economy also faces the challenge of managing the benefits and risks of technological innovations. There is concern that the benefits of new technologies may be concentrated in a few companies or individuals, which would limit their impact on overall productivity and could increase inequality. The regulation of the digital economy and the implementation of appropriate policies are crucial to prevent market concentration and ensure that the benefits of productivity growth are shared more equitably. AI is a key driver of these economic shifts, with significant implications for both economic growth and the future of work [25, 27, 28]. Understanding its impact is crucial in navigating the complexities of the modern economy.

In a global landscape marked by increasing digitisation, interconnectedness, and the emergence of disruptive technologies such as AI, the foundations of traditional economic theory are facing ever greater scrutiny [29]. Recent economic challenges and the growing complexity of today's world have highlighted the limitations of conventional models in offering comprehensive explanations and solutions [30]. This juncture has motivated the exploration of new theoretical and methodological perspectives that can better capture the dynamics of the digital age. In this sense, the Cobb-Douglas production function has been key to model the relationship between capital, labour and production, under assumptions of constant returns to scale and fixed elasticities. However, in the current context of digitization and accelerated technological change, both its usefulness and limitations have become apparent. Rather than explicitly incorporating new intangible or digital factors, recent debate has focussed on the model's ability to adequately reflect the role of technology and efficiency, which are often represented as an external multiplicative factor rather than as a separate input. These limitations have prompted critical revisions and proposed extensions to the model, seeking to better capture the growing importance of innovation and intangible assets in the contemporary economy [31].

In this context, innovative proposals are emerging, such as quantum economics, inspired by the principles of quantum physics, which advocates for a paradigm shift from the traditional macroeconomic approach towards a microeconomic vision that recognizes the uncertainty, nonlinearity, and complexity inherent in global economic systems [32]. In parallel, a re-evaluation of political economy is observed, seeking to transcend neoclassical and autocratic postulates to reintegrate social, institutional, ethical, and humanistic considerations into the economic analysis of digitisation [29, 33].

The profound current economic transformations are intrinsically linked to the rise of disruptive technologies and successive technological revolutions. These are not only reshaping entire sectors and automating complex processes, but they also raise fundamental questions about the future of employment, the distribution of wealth, and social sustainability. Fully understanding these implications requires an interdisciplinary approach that goes beyond economics, drawing on knowledge from various social sciences, as well as computer science and other disciplines. The urgency to develop mechanisms for a more equitable sharing of the prosperity generated by these technological innovations stands as a crucial challenge and a primary focus of research. Ultimately, the motivation for this exploration lies in the pressing need to adapt and enrich our economic understanding to address the challenges and seize the opportunities presented by the digital age, marked by these waves of technological change.

Within this framework of technology-driven transformation, it is useful to analyse economic and social dynamics through the lens of game theory. This discipline provides us with conceptual tools to understand the strategic interactions between different actors. Two fundamental concepts are Zero-Sum Games and Non-Zero-Sum Games.

In Zero-Sum Games, such as chess or poker (without rake), one participant's gain directly implies another's loss. The total value at stake remains constant. In contrast, most economic and social situations are NonZero-Sum Games. Here, actions can generate additional value (positive sum), such as a trade agreement beneficial to both parties, or destroy value (negative sum). The famous Prisoner's Dilemma illustrates how the individual pursuit of the best strategy can lead to a suboptimal outcome for everyone, demonstrating the complexity of nonzero-sum game.

Furthermore, we can distinguish between Competitive Games (Non-Cooperative), where actors seek to maximize their own benefit without binding agreements (such as competition between companies), and Collaborative Games (Cooperative), where coalitions can be formed to achieve common goals and distribute the gains (such as wage negotiations).

In strategic decision-making, players can employ Pure Strategies, choosing a specific action with certainty, or Mixed Strategies, assigning probabilities to different actions to make them unpredictable. A central concept for analysing the stability of these strategies is the Nash Equilibrium, where no player has an incentive to unilaterally change their strategy given those of the others. In the specific area of transport networks, the Wardrop Equilibrium describes a situation where no driver can improve their travel time by changing route.

Finally, the concept of Pareto Optimality helps us to evaluate the efficiency of an outcome. An outcome is Pareto optimal if it is not possible to improve one individual's situation without worsening another's. It is crucial to understand that efficiency (Pareto optimality) does not necessarily imply fairness in the distribution of the benefits generated by new technologies and technological revolutions.

In the context of digital transformation and disruptive technologies, understanding these concepts from game theory helps us to analyse how individual and collective decisions in an environment of constant technological change can lead to different outcomes in terms of wealth distribution, employment opportunities, and social well-being. The key will be to design mechanisms that foster nonzero-sum games with Pareto optimal and, as far as possible, equitable outcomes, to maximize the potential of these waves of technological innovation.

The aim of this study is to carry out a systematic review and a bibliometric analysis of quantum game theory in order to provide a comprehensive view of its evolution and applications. Through the systematic review, we seek to identify the main advances, methodological approaches and results obtained in the research of this emerging discipline. In addition, through bibliometric analysis, we aim to evaluate publication trends, collaboration networks between authors and institutions, and the areas of greatest impact within quantum game theory. This approach will allow us to position quantum game theory in the current context, identify knowledge gaps and suggest directions for future research in the area.

This paper is structured as follows: first, we outline the methodology employed, which includes a systematic review and bibliometric analysis of quantum game theory. In this section, we describe the process of selecting relevant studies, the criteria for inclusion, and the methods used for clustering and analysing the data. Next, we present the results, starting with the clustering of the literature, where we identify and categorize key themes and areas of research within the field. We then examine the evolution of quantum game theory, highlighting significant trends, milestones, and shifts in focus over time. Following this, we present a detailed discussion of the clusters, providing an in-depth analysis of the key topics within each cluster and their interconnections. Finally, we conclude the paper by summarizing the key findings, discussing the implications of the study, and suggesting avenues for future research in quantum game theory.

2 Methodology

A systematic review was carried out following PRISMA methodology [34] in which articles were selected from one of the most widely used databases in the scientific community: Scopus. The initial search equation was (“QUANTUM”) AND (“GAME THEORY”) OR ("QUANTUM GAME THEORY"). With the initial search equation, we found 5320 documents. In order to focus on the most relevant papers, a stronger restriction was necessary, so we limited the types of documents to articles, set as keywords "Game Theory", "Quantum Theory", "Quantum Game Theory", and determined English as the language, since most of the articles were written in that language. This new search was formulated as (ALL ("quantum") AND ALL ("game theory") OR ALL ("quantum game theory")) AND PUBYEAR > 1988 AND PUBYEAR < 2025 AND (LIMIT-TO (EXACTKEYWORD , "Game Theory") OR LIMIT-TO (EXACTKEYWORD , "Quantum Theory") OR LIMIT-TO (EXACTKEYWORD , "Quantum Game Theory")) AND (LIMIT-TO (DOCTYPE , "ar")) AND (LIMIT-TO (LANGUAGE , "English")) resulting in 957 articles (Fig. 1).

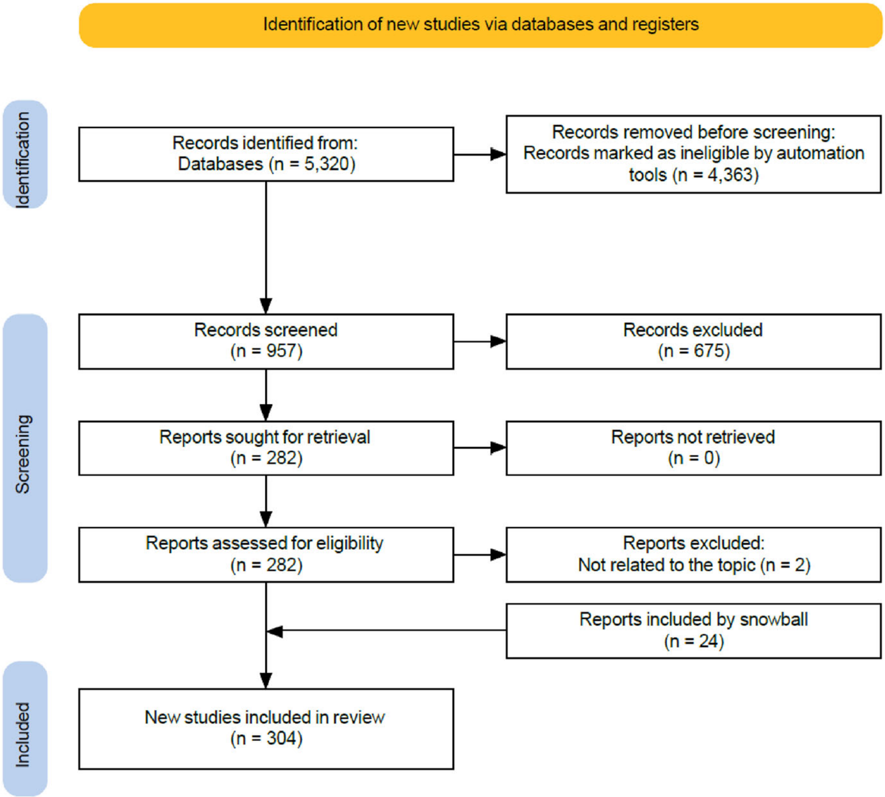


Fig. 1 Flow diagram of the PRISMA methodology. Adaptation from [35]

3 Results

3.1 Clustering

The dataset used comprised 1738 different keywords and was first imported to VOSviewer using the criterion of co-occurrence of all keywords and was limited to a minimum of nine occurrences, resulting in 29 keywords that were organized in four clusters of nine, eight, seven and five items, respectively, as is shown in Fig. 2. The clusters were named according to the keywords they had. Each node signifies a keyword, its size reflecting the frequency of occurrences. Lines between keywords represent their co-occurrences, forming clusters denoted by distinct colours.

3.2 Evolution

In Fig. 3, we can see the evolution over the years of the publication of articles on the topic under study. The figure shows a significant increase in the number of publications from 1989 to 2024, particularly from 2000 onwards. While the first years show a

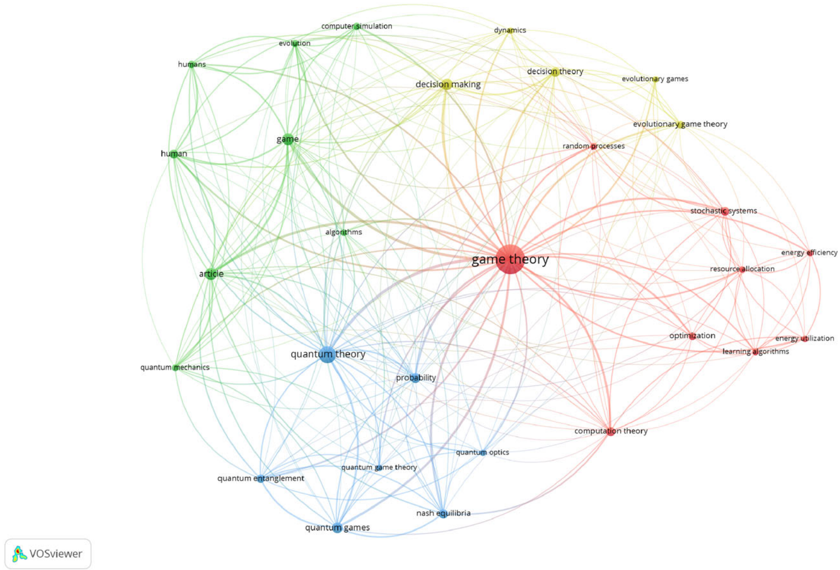


Fig. 2 Map of the clusters

steady decline in article production, interest in these fields has grown since 2002, with notable peaks in 2002, 2010, 2020, 2023, and 2024. Please note that in 2025, we have collected two articles by snowball, but as the year has not yet concluded, we have not included them in the graph to avoid any potential bias. The year 2020 stands out with the third-highest number of publications, likely driven by the COVID-19 pandemic and the growing interest in interdisciplinary research. Despite some declines in certain years, the overall trend suggests an increasing relevance and development in quantum theory and game theory, reflecting an expanding and dynamic field of study. Additionally, 2023 and 2024 shows a significant peak in the number of publications, reaching the highest point in the graph. This surge is likely driven by the ongoing developments in these fields, as well as the continued evolution of research across various interdisciplinary domains, further emphasizing the growing momentum and expanding interest in quantum theory and game theory.

The evolution of keywords in game theory and quantum theory over time can be seen in Fig. 4 and demonstrates the diversification and expansion of research in these fields. From the early years (2010–2014), keywords primarily focussed on traditional themes such as game theory and quantum theory, with an emphasis on theoretical bases like quantum mechanics and quantum entanglement. These concepts reflect the first applications of quantum theory and the exploration of its foundations. From 2015–2018, research began to expand into new areas, with key terms such as decision theory, evolutionary games, and random processes becoming more prominent. These fields focus on the dynamics of agent interactions and modelling complex and stochastic situations, becoming fundamental areas in research. There was also a growing interest in topics related to sustainability and efficiency, such as energy efficiency and resource allocation, reflecting practical applications of game theory. In recent

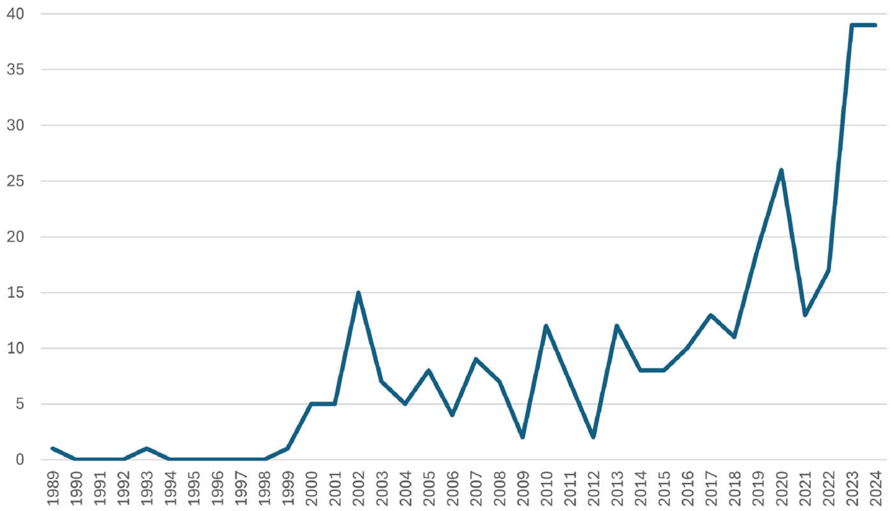


Fig. 3 Evolution of the publication of the papers selected

years (2019–2020), emerging areas have been consolidated, with an emphasis on optimization, learning algorithms, and energy utilization. This trend indicates a strong integration of game theory with applications in AI and sustainability, specifically in improving energy efficiency and resource optimization. Key terms like quantum theory and quantum games are increasingly linked to classic game theory topics, such as Nash equilibrium, and emerging areas like computation theory and quantum optics, reflecting a deeper integration of quantum theory in computer and system optimization contexts. In conclusion, the evolution of keywords in game theory and quantum theory highlights a clear trend towards integrating game theory with practical applications in areas like resource optimization and sustainability. This evolution has led to a more comprehensive application of game models in these fields, demonstrating how game theory has evolved towards interdisciplinary approaches to solve contemporary social and scientific problems.

Figure 5 shows that a few authors dominate the publications in our database. Wang H. leads with 7 publications, followed by Iqbat A., Abbott D., and Cheong KH. with 6 each. Li Y., Zhang H., Li H., Wang Y. Flitney AP., Li H., Sładkowski J., and Li C. have 5 articles each. This suggests that these authors have significantly contributed to the advancement of quantum theory and game theory.

Figure 6 shows the geographical distribution of articles on "Quantum and Game Theory." China leads with 110 publications, followed by the USA with 59, and Germany with 27. Australia and the UK contribute 22 and 21 articles, while Canada and Singapore each have 18. Japan, France, and Italy also contribute, with smaller numbers from Spain, Switzerland, and India. This highlights a strong concentration of research in both Asian and Western countries.

In Fig. 7 we see the word cloud of the most repeated words. Key concepts such as game theory and quantum theory appear most frequently, reflecting their importance in strategic analysis and quantum phenomena. Words such as 'game theory'

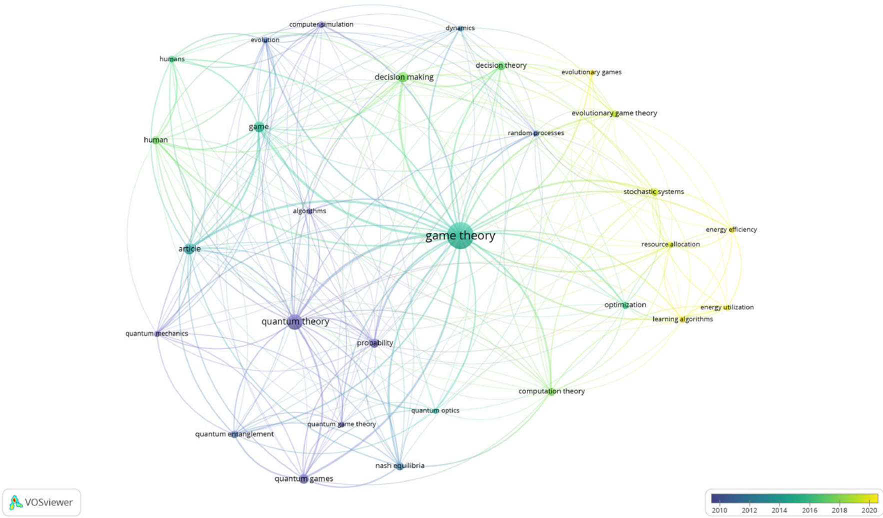


Fig. 4 Evolution of keywords

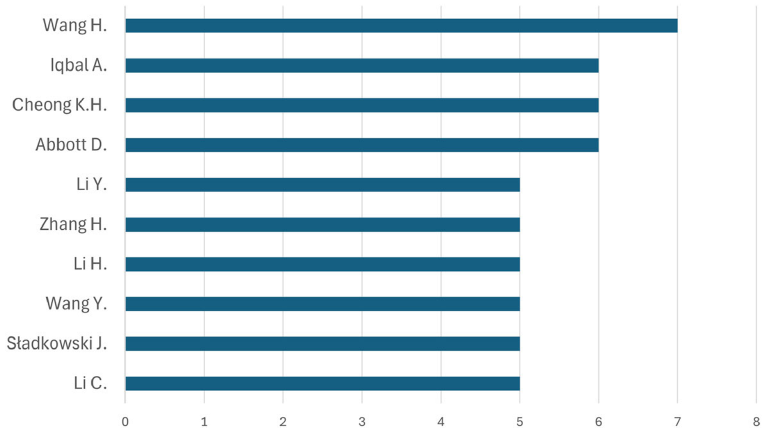


Fig. 5 Authors with most publications

and 'quantum theory' will be the most prominent, indicating their central role. Topics related to decision and probability, as well as models and algorithms, will also be emphasized, highlighting their relevance in research and simulation. Other notable concepts include cooperation in games and aspects of computation, providing a clear overview of the predominant approaches in strategy analysis and quantum theory.

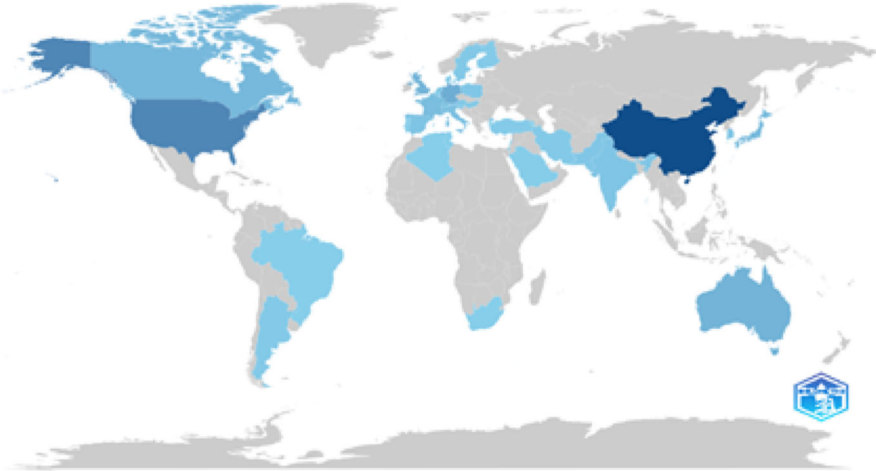


Fig. 6 Map of the geographical distribution

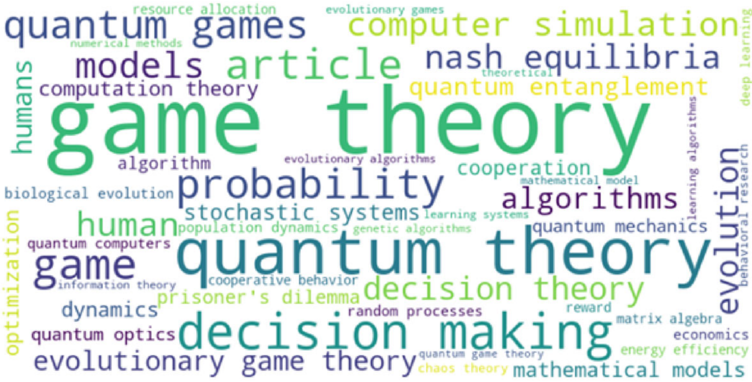


Fig. 7 Wordcloud of the most used words

4 Development of the identified clusters

The significance of quantum game theory lies in its ability to extend classical game theory into the quantum realm, offering new perspectives on strategic interactions in systems where quantum mechanics plays a role. This field has not only theoretical implications but also practical applications in areas such as cryptography, algorithm design, and complex systems modelling.

To further explore and harness the potential of quantum game theory, the following sections will develop clusters focussed on key areas of research. These clusters will delve into the optimized dynamics in evolutionary and stochastic game theory, as well as other emerging intersections within the field, providing a structured approach to advancing both the theoretical and practical aspects of quantum game theory.

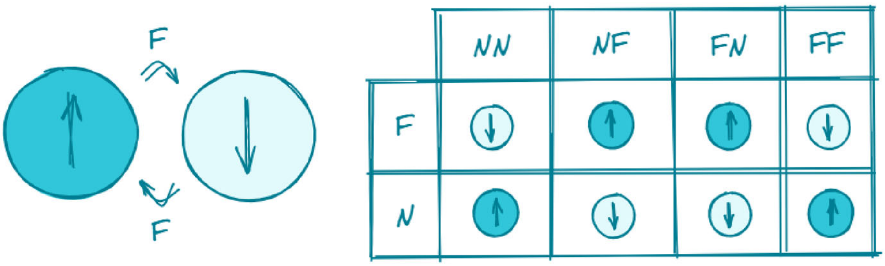


Fig. 8 On the left, coin heads up (↑) and tails down (↓), where F refers to the action of flipping. On the right, a table with all the possible game outcomes, where the different columns show A's possible strategies, and the rows show B's possible strategies

4.1 Quantum game theory: Nash equilibria and entanglement in computational systems

Historically, this field of study began with two articles that were developed and published almost simultaneously. The first article (published on 01/02/1999) by David A. Meyer [36] introduced the first scheme for quantizing a game (hereafter referred to as the Meyer scheme), the Penny Flip (PF) game. Later that year (published on 11/10/1999), Eisert, Wilkens, and Lewenstein introduced a quantization scheme (hereafter referred to as the EWL scheme) for a different game, the Prisoner's Dilemma (PD) [21]. The EWL scheme is arguably the most successful and impactful. Shortly thereafter (published on 07/08/2000), Luca Marinatto and Tullio Weber [37] proposed a third quantization scheme (hereafter referred to as the M&W scheme) for the Battle of Sexes (BOS) game. This third scheme, as the authors themselves admit, has much in common with the EWL scheme and is clearly inspired by it, although it has its own characteristics, as will be shown later.

All these schemes were conceived to offer better performance in competitive games. Without a doubt, the most successful of the three over the years has been the EWL scheme, as evidenced by its higher number of citations and as we will see in the development of this literature review.

The quantum coin game, described in Meyer's scheme, involves two players, A and B, who must decide whether or not to flip a coin. Initially, the coin is in the 'heads' position. Player A has access to quantum strategies, while player B can only use classical strategies. The goal is for player A to win if the coin ends in 'heads' and B to win if it ends in 'tails'. When A uses a quantum strategy, in particular the Hadamard operator, he will always win, regardless of the strategy of B. This happens because the Hadamard operator creates a quantum superposition that is not affected by the classical strategies of B. Thus, even if B applies the classical strategies of inversion or identity, the final result will always be 'heads', which guarantees the victory of A [36], as can be seen in Fig. 8.

Furthermore, the quantum advantage in this game is analysed, suggesting that in systems with only one qubit it is not possible to achieve a quantum advantage, as they can be described by classical models. However, by considering more than one qubit, the possibility of obtaining a real quantum advantage opens up. On the other hand,

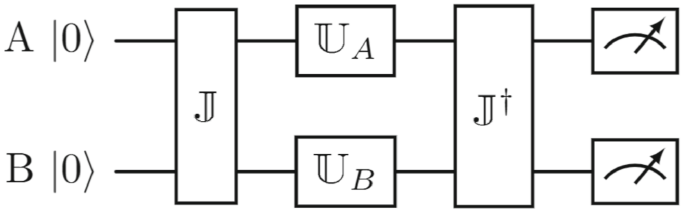
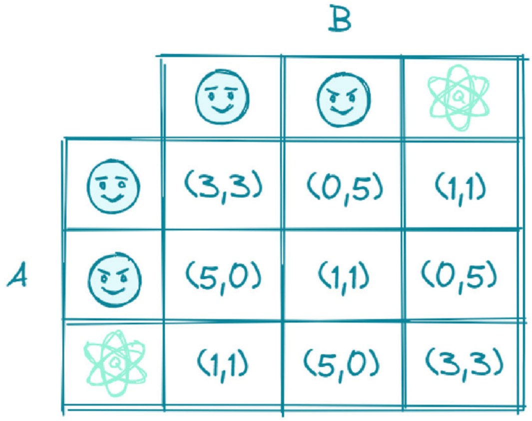


Fig. 9 Diagram of the EWL Scheme

Fig. 10 Payoff table for the Prisoner's Dilemma game with 2 players, A and B, adding the quantum strategy Q



decoherence, which is a phenomenon that degrades quantum states when interacting with the environment, can affect the actual implementation of the game. If B delays its strategy sufficiently, the game may revert to its classical behaviour, which represents a challenge in the practical implementation of quantum games [38]. Finally, although the strategies used by B (identity and Pauli σ_x) are called classical, they are actually quantum strategies because they can act on superposition states. It is also discussed that there are tools to simulate quantum games, which can be useful for educational and popularization purposes.

The EWL (Eisert, Wilkens and Lewenstein) [21] scheme was proposed to quantify the prisoner's dilemma using quantum tools. In the classical dilemma, players must decide whether to cooperate or betray, and although betrayal is a Nash equilibrium, cooperation is the optimal outcome for both players. In the quantum scheme, a judge prepares two qubits, and the players apply quantum operations to their respective qubits before the judge measures the outcome. Cooperation and betrayal are represented by specific quantum operators, and players can mix these operations, giving them access to a wider quantum strategy space than players with classical strategies (Fig. 9).

The use of quantum strategies allows players to choose from a wider range of options, giving them an advantage over players who only use classical strategies. This scheme ensures that, under certain conditions of quantum entanglement, the dilemma is resolved, as the game reaches a Nash equilibrium that coincides with the Pareto optimal, where both players can obtain the best possible outcome by cooperating (Fig. 10).

The concept of 'miracle move' refers to a quantum strategy that allows a player to obtain good results regardless of what the other player does, which generates debates about the validity of these strategies compared to classical ones. Furthermore, quantum entanglement plays a crucial role in the outcome of the game, since, according to the study by Eisert et al. [21], a certain threshold of entanglement is necessary to achieve a quantum advantage. However, it has been pointed out that the use of quantum strategies also generates controversy, especially when the available strategies are extended or when the entanglement conditions are modified. For example, in papers such as [39], it is argued that, when considering pure and mixed strategies, the Nash equilibrium may disappear in some situations, although it can be restored in cases with mixed strategies.

The EWL scheme proposes a quantum version of the prisoner's dilemma that solves the classical dilemma by using quantum strategies, but it also generates debates about its applicability and the influence of quantum entanglement on the outcomes of the game. Criticisms and extensions of this work have been addressed in articles such as [40, 41], which question certain aspects of the model and its relation to classical strategies. As for the practical implementation of the quantum scheme, it has been tested on quantum computers, such as nuclear magnetic resonance (NMR) and optical computers (as mentioned in [42, 43]). However, decoherence remains a challenge in real implementation, as it degrades quantum states when interacting with the environment, as discussed in [44].

The role of entanglement in the outcome of quantum games is crucial. In the original article by [21], its effect on the quantum advantage of miracle moves was studied, but it is also important for reaching new Nash-like equilibria or not. Several studies, such as [42], have addressed this issue, highlighting that for strategies belonging to general unitary operations, an entanglement threshold is found, beyond which the Nash equilibrium ceases to exist. This result suggests that, by controlling the degree of entanglement, a Nash equilibrium superior to the traditional one could be achieved for pure strategies of this set of general unitary operations.

In more general strategy spaces, some authors have delved into the existence of miracle moves, as proposed by [21]. The article [45] reiterates that a player in this strategy space has the ability to undo the action of the other player in the global state through local operations and then implement a betrayal strategy [39]. In addition, this paper explores the role of the degree of entanglement in quantum advantage, identifying new thresholds for different games: gamma $> \sqrt{1/3}$ for the Chicken Game, and gamma $= \sqrt{2/3}$ for the Stalemate Game. For the Deer Hunting Game, no quantum advantage was found, and for the Battle of the Sexes Game no clear threshold was identified. It underlines the difficulty of analytically studying quantum advantage in games with more players, where no quantum player can reach the desired final state, even knowing the strategies of the other players.

A more fundamental critique of the quantum game theory model and the EWL scheme is raised in [41], which argues that the EWL scheme neither solves the traditional game nor does it do so through quantum mechanics. It is argued that, since the payoff table in the quantum scheme does not match that of the traditional game, the proposed game is not a quantum version of the original, but a new game in which the traditional game is a subgame. Other authors, as in [46], have addressed this issue, arguing that the extension of the game is as valid as considering mixed strategies. Regarding the second criticism, it is argued that extension can be done classically by considering a new classical strategy Q and generating a new payoff table. Although, as acknowledged in [21], the quantum model is more efficient in terms of communication resources, since four real variables must be transmitted, and it is more complex to simulate situations with more players and strategies classically. However, for [41], efficiency is not relevant in game theory and a traditional model is considered valid.

As for the physical implementation of the EWL scheme in a quantum computer, remarkable examples have been achieved in nuclear magnetic resonance (NMR) computers [47] and in optical computers [48]. Other studies have also investigated how decoherence affects the performance of the [44] scheme. All previous implementations on physical hardware use quantum computers. However, implementing quantum games requires a delocalised architecture that enables players A and B to be in different locations. The paper [49] addresses this issue by implementing a delocalised circuit of the EWL scheme using an auxiliary qubit for the game of prisioner's dilemma. An application of this proposal for the magic square game can be find in [50].

The scheme proposed by Marinatto and Weber [51] to quantise the Battle of Sexes (BOS) game, as opposed to the Prisoner's Dilemma quantization. This scheme uses a quantum entangled state as a starting point, with the aim of obtaining more balanced results than in the traditional game. In this game, Alice and Bob must decide between two plans (one male and one female). If they both choose the same plan, they get a reward, although whoever chooses their preferred activity receives a higher reward. However, payoffs are asymmetric in Nash equilibria.

Marinatto and Weber's scheme uses a maximally entangled state and considers pure quantum strategies as the identity($\mathbb{I}$) and σ_x, with associated probabilities. This allows both players to receive the same payoff in the Nash equilibrium, eliminating the asymmetry of the traditional game.

However, this scheme has been criticized. Benjamin [52] points out the limitation of the strategy space, restricted to only $\mathbb{I}$ and σ_x, which could result in behaviour similar to that of the classical game if general unitary strategies were employed. In addition, a coordination problem persists, as players still depend on each other's choices to decide which strategy to use.

Marinatto and Weber respond to these criticisms by arguing that the restriction of strategies ensures that the game recovers classical behaviour when a non-interlocked state is used, such as $|00\rangle$. On the other hand, [53] highlights the dependence on the initial state, as certain initial states solve the coordination problem, making players choose the same strategy. The scheme has also been criticized for not correctly reflecting the paytable of the traditional game and for the insufficiency of the strategy space. To address this, Marinatto and Weber's extended proposal (eM&W) allows players to choose different initial states, not just limiting themselves to$|00\rangle$ [54]. In summary, Marinatto and Weber's scheme for quantifying the BOS game has advantages, but faces criticisms about the limitation of strategies and the dependence on the initial state.

Another equally important extension is the one made to consider games with strategies consisting of continuous variables. All that can be found on quantum game theory with continuous variables is applied to the problem of the Cournot Duopoly. For the quantization of the game, the scheme proposed by Li et al. [55] is practically identical to the EWL scheme [21] Fig. 9, though it presents some modifications. Now, the valid strategies are of the form $U_j = \exp(-ix_j P_j)$ with $P_j = i(a^\dagger - a)/\sqrt{2}$, where $x_j \in [0, \infty]$ and a and $a^\dagger$ are the creation and annihilation operators of the electromagnetic field. In other words, now the strategies correspond to displacement operators in the phase space. The main difference is that in this extension of the game, there is no fixed J operator that satisfies certain commutation relations with the "classical" strategies; instead, the J operator considered will determine whether the game is quantum or not. Thus, with $J = J^\dagger = \mathbb{I}$, we recover the results of the classical game. The general form of the J operator will be $J = \exp(-\gamma(a_1^\dagger a_2^\dagger - a_1 a_2))$, where as before, $\gamma \in [0, \infty]$ can be interpreted as the degree of entanglement. In the article, the new Nash equilibrium is found as a function of the strategies X_1 and x_2 of each player, as well as the parameter γ. It is found that when $\gamma \rightarrow \infty$, the new Nash equilibrium coincides with the Pareto optimum. Some extensions, such as [56], study the case of the previous quantum game when the distribution of information is asymmetric, with the second company knowing the production costs of both, while the first company only knows its own production costs. In this case, it is found that the effect of quantum entanglement is not clearly positive and can reduce the resulting payoffs for both players, even for the one with more information. Other mathematical models for describing duopolies, such as the Stackelberg duopoly, have also been quantized [57]. The fundamental difference between the Cournot model and the Stackelberg model is that in the latter, the game is sequential, as is the case with the Penny Flip game; one company first sets a production quantity q_1, and knowing this decision, the second company sets its production quantity q_2. In this case, the previous article uses the M&W scheme for quantization. Other works [58] focus on the different roles played by entanglement in the two models. We can also find articles comparing the performance of different schemes for the quantization of the Cournot duopoly [59]. On the one hand, they consider the scheme proposed in the previous article [57], based on the M&W scheme, and apply it to the Cournot model. On the other hand, they explain again the Li Du Massar scheme [55], based on the EWL scheme. Additionally, a new extension of the EWL scheme is introduced for the Cournot duopoly problem. This highlights the various valid ways to quantize a game, with the only condition being that the traditional game is contained as a particular case. In this way, the lack of a general formulation for quantum game theory is evident (Fig. 10).

Regarding the study of the Evolutionary Stable Strategies (ESS) concept in quantum game theory, we can highlight, for example [60], where the M&W scheme is employed. As with the Nash equilibrium in this scheme, a strong dependence on the initial state's entanglement is observed, finding that depending on how entangled the state is, some strategies will be ESS, while others will not. The same authors also considered the ESS idea in the case of the EWL quantization scheme [61]. As we have just seen above, classically in the Prisoner's Dilemma game, defecting is an ESS. In this article, they studied whether a population of mutants equipped with quantum strategies could invade a population of defectors. It is shown that if the mutants can access the set of strategies, the invasion can occur. The results of this article again highlight the importance of the initial state for the ESS in the M&W scheme. The same authors, in another article [22], again used the M&W scheme to study the ESS now for the RPS game. The results show how the entanglement of the initial state causes the Nash equilibrium associated with the mixed strategy consisting of each pure strategy with probability $1/3$ to also be an ESS, unlike in the classical case or similarly in the unentangled initial state. The potential of these results for evolutionary theory at the molecular level is also discussed. A similar approach is used in the paper [62] for the hawks-and-doves game, with a similar result showing strong dependency on initial quantum state.

In addition to the three schemes discussed in detail before, there are alternative schemes created ad hoc for specific games. Therefore, they are proposals that are difficult to generalize, although they can be useful in their particular use case. One notable example is the quantization of the Monty Hall game [63, 64]. In this quantization scheme, which deviates somewhat from the usual, qutrits are used instead of qubits. It bears a greater resemblance to the M&W scheme since quantization is performed by considering an entangled state as the starting point, a state on which no operation is performed at the end to invert the first, as happens in the EWL scheme. In this case, general unitary operations are considered, unlike in the M&W scheme. The results show that the outcomes of the quantum and classical versions are the same, except in the case where the player has access to quantum strategies, but the person running the game does not, in which case the player has a winning strategy. Another more innovative and counterintuitive proposal for quantizing the Monty Hall game can be found in [65]. Another game with more direct application that has sparked interest for being quantized is auctions. Noteworthy works here include [66–68]. In the first one [66], a quantization scheme is proposed where players can encode their bid in a quantum state so that a quantum algorithm can then search for the highest bid. A possible advantage lies in the ability to choose a quantum superposition state as a bidding strategy that involves several items. Another possibility is to introduce entangled states to encode bids involving several items in one package or to encode bids coordinated for multiple products with other players who may be partners. A point the authors highlight is that after the quantum search algorithm, losing bids disappear, thus maintaining privacy. An important issue emphasized is that players may be tempted not to follow the scheme's instructions to deceive the search algorithm and win with a lower bid. The precautions the auctioneer must take to prevent these failures from happening are also explained. In the second work [68], an alternative general scheme is introduced for the same purpose using a continuous state space, although it is clarified that the previous proposal is much easier to implement.

In the realm of quantum game theory, one of the most pressing and unresolved challenges is determining how players can autonomously reach the Nash equilibrium. This paves the way for the integration of a range of techniques pertinent to the field of AI, which have the potential to redefine the methods employed to identify and enhance equilibria. Variational methods, for instance, provide a systematic approach to approximate quantum states that minimize certain objective functions, which could be tailored to finding Nash equilibria in quantum games. Parametric quantum circuits, on the other hand, offer a flexible framework for exploring the vast landscape of quantum states that may correspond to an equilibrium, leveraging quantum gates and parameters that can be fine-tuned to approach the optimal strategy. The integration of these methods into the analysis of quantum games is particularly intriguing because, upon reviewing the extensive body of literature on quantum game theory, there is a notable lack of studies addressing the process by which players might actively discover the equilibrium in the absence of prior knowledge. This gap in the literature underscores the novelty and relevance of exploring how quantum-enhanced techniques could facilitate the discovery of Nash equilibria. In the future research, we aim to address this gap in the current state of the art by exploring how players in quantum games can autonomously discover the Nash equilibrium without prior knowledge of the equilibrium parameters.

Inspired by previous literature [69], we have identified some more realistic cases where quantum game theory could play a role in the future. These problems are related to traffic effects when transporting "something" from one point to another in a network or network. This "something" can be anything from vehicles in a road network, to data in a fibre optic network, to electricity or fuels. An example of this type of problem is the Braess paradox. The problem can be understood through the Braess paradox, which shows that adding a new road to a transportation network can, counter-intuitively, make traffic worse. This phenomenon reflects a situation in which players (drivers) who make selfish decisions end up with a worse outcome than if they considered the common good. Similar to the Prisoner's Dilemma, quantum game theory can offer new solutions to this type of congestion problems in transportation networks by applying quantum tools to manage and optimize decision-making in complex traffic systems.

In addition, quantum game theory can be extended to more general networks where players must choose among several alternative routes depending on congestion and traffic in each segment. Here, quantum strategies can help optimize user decisions in the network, improving efficiency and reducing congestion. Several works are advancing this direction. For example [70], proposes a solution to the Braess paradox based on quantum game theory. In particular, the multiplayer EWL scheme with three players and a three-parameter strategy set is used. An iterative method to adjust each parameter and reach Nash equilibrium is also presented. Interestingly, the best results, which coincide with the Pareto optimum, are not achieved with maximal entanglement, but with $\gamma = \pi/4$. Another notable work is [71], which develops an algorithm based on constructive interference of the desired end state and destructive interference of the less favourable end state. The originality of the scheme lies in the fact that it requires an initial state specifically intertwined with a parameter that the judge can modify to favour or hinder the outcome of certain final states, and where all players must implement a single strategy, without any degree of freedom. This scheme is applied to solve the spectrum allocation problem in cognitive radio systems. Although the problem addressed by this game is interesting, it cannot be considered a true game, since the players do not have different strategies to choose from. Similarly, [72] presents a situation where N packets wish to travel from one point to another in a network through the fastest channel. On congested channels, these packets can continue on the same channel or switch to the second best option. The payoff of the game is the total time, which is the sum of the path time and the travel time. The quantum game scheme used is the multiplayer EWL scheme. The results show that quantum game theory could offer solutions to traffic congestion problems in networks.

The relevance of applying quantum game theory to Braess paradox extends beyond previous problems, encompassing critical issues related to demographic challenges, including those faced by large cities but also by sparsely populated regions and areas with low population density. As global populations continue to grow and urbanization accelerates, the demand for efficient infrastructure and resource management becomes more pressing. Problems such as urban mobility [73], congestion management [74], and smart grid optimization [75] are increasingly critical in the context of both high-density urban centres and sparsely populated regions. On the other hand, efficient routing of vehicles [76] and energy distribution networks [77] are relevant to rural settings where the allocation of shared resources becomes a complex balancing act.

4.2 Optimized dynamics in evolutionary and stochastic game theory for decision-making

The application of optimisation methods in dynamic systems, essential for game theories and decision-making, is encompassed by the cluster "Optimized Dynamics in Evolutionary and Stochastic Game Theory for Decision Making". Evolutionary game theory, which analyses strategies and adaptive behaviours over time, is combined with stochastic game theory, focussing on gaming scenarios with elements of uncertainty and randomness. This concept's primary focus is on improving and optimizing decision-making. This integrated approach is particularly relevant for a cluster or research group dedicated to exploring these interdisciplinary fields, providing a holistic perspective that ranges from theoretical foundations to practical application in decision-making processes.

Game theory focuses on decision-making, examining how different parties make decisions based on their personal interests. It explores situations of competition, rivalry, or struggle, and has proven useful in social sciences, biology, and economics. It is particularly relevant for incomplete information games, where parties can choose their action plan based on rational knowledge but lack understanding of the others' decisions [78]. Nowadays, game theory defines a wide range of human-computer interactions and is already a science of logical decision-making, which assumes that players act rationally [79]. In game theory, a player's strategy usually considers what they have learned about the tactics of the other players. Probability is a significant subgroup of interactions in each of these domains [80]. Group decision-making is a complex procedure involving humans selecting an optimal alternative based on desired criteria, such as cost and benefit. However, uncertainty and complexity are common in real-world environments. To address these issues, domain experts evaluate alternatives, but uncertainty and subjectivity limit their ability to provide accurate evaluations [81]. With the competition of an adversary, dynamic game theory has attracted much interest as a potentially useful method for developing agent action plans in such a complicated scenario [82]. Large-Scale Group Decision-Making (LSGDM) has emerged as a key research topic in the field of decision-making, standing out for its unique advantages in solving complex decision-making problems due to the large number of participants (DMs). In this sense, [83] proposes a new optimal consensus mechanism, emphasizing fairness and rationality in consensus allocation, in order to improve the efficiency and cooperation of participants. In the future, we plan to extend this method to more complex situations, such as LSGDM with fuzzy information in decision-making, LSGDM in social networks, LSGDM with heterogeneous decision information, and LSGDM considering individual behaviour. Furthermore, it is suggested to apply this new method in various fields, such as decision-making in large-scale emergencies, COVID-19 pandemic, e-waste recycling, and environmental performance evaluation, among others.

Two significant assumptions of classical game theory are completely rational players and complete information. However, players' rationality is often bounded, especially when facing complex situations, which limits the real-world application of traditional game theory. Evolutionary game theory is an important breakthrough in game theory, emphasizing bounded rationality and dynamic equilibrium. It combines traditional game theory and dynamic evolution, adopting bounded rationality as an analysis framework, which is closer to the reality that decision-makers in the real world are not entirely rational. The evolutionary game theory holds that people's decision-making behaviours in a real-life situation reach dynamic equilibrium through continuous learning, imitation, and trial and error. It has two essential concepts: replicator dynamics and evolutionarily stable strategy (ESS) [84].

The techniques of quantum theory are employed merely for taking into account the hidden variables, such as emotions and biases of decision makers. The possibility of considering hidden variables is at the heart of the quantum-theory techniques that allow for their existence by modifying the rules of calculating the quantum probabilities. This is why the quantum techniques make it possible to characterize human decision making, incorporating in it the existence of such hidden variables as subconscious feelings and behavioural biases. The efficiency of quantum techniques for human decision-making is not because humans are quantum objects, but because these techniques are mathematically designed to accommodate the existence of hidden variables, which can be of a very different nature [85].

The concepts of optimization and game theory overlap. Put it differently, game theory can be considered an approximation of optimization [86]. Optimization is a decision-making process that consists of maximizing or minimizing a predefined function. Classical optimisation techniques, based on mathematical and probabilistic principles, have difficulties in providing adequate solutions to complex problems. Examples are gradient-based methods, which require the gradient of the objective function. These shortcomings have led to the introduction of new optimization techniques called 'metaheuristics' [87]. Metaheuristic optimization algorithms (MOAs) are widely used to solve real-world problems by finding optimal decision variables and solving engineering problems due to their high computational efficiency and complexity. MOAs consist of two main phases: exploration and exploitation, the angular balance being the key. If scanning is effective, random operators generate different regions of the search space, but the MOA may be stuck in the optimal locale. If exploitation is effective, the optimization method determines the most suitable solution in the search field. However, the MOA may still be trapped in the minimum local tramp, making the balance critical for convergence to the global optimum. Over the past decade, many new MOAs have been proposed to balance the exploitation and exploration phases [88].

Zhu et al. [89] Adressed the challenge of multirobot task assignment in cooperative robotics. It introduces a modified optimization method, which combines static optimization with evolutionary game theory to improve multirobot task assignment performance. The method uses an autoadaptive strategy based on evolutionary game theory and optimization by particulate enjection. It also provides a convergent parameter selection principle and a viability-based rule for handling multirobot task assignment restrictions. The method outperforms competitors in terms of solution quality and time computation time. The method's simulation results show a 126.41 s average computation time. Intermittent Brownian ratchets, in which diffusive particles exhibit unexpected drift when exposed to alternating periodic potentials, were the original idea behind Parrondo's paradox. It has since been used in a variety of disciplines in the physical and engineering sciences, such as diffusive and granular flow dynamics, information hermodynamics, chaos theory, switching problems and quantum phenomena. The paradox has also been widely used in the life sciences, such as ecology and evolutionary biology, social dynamics, and collaborative work [90].

Stochastic approximation theory also deals with computational procedures in which test results are error-prone. in which test results are error-prone. However, in this context, errors are defined as are understood as measurement inaccuracies. The problem is to find zeros and extreme values of real functions of one variable. The values of functions are to be determined by measurements subject to random errors. The objective is to construct a process that, under certain assumptions, stochastically converts to the zero value to be found. The question of efficiency is addressed by studying the speed of convergence [91]. The main difficulty associated with the stochastic minimax dynamic games is due to the presence of white noise and square integrable disturbances in both dynamics and observations. These games are more general than risk-sensitive stochastic control problems and deterministic minimax dynamic games. Risk-sensitive control problems are equivalent to minimax stochastic dynamic games, where white noise inputs and disturbances affect dynamics and observations through the same channel. These games result in robust controllers but cannot model physical phenomena where white noise affects dynamics and measurements through different channels. The presence of both white noise and stochastic disturbances is crucial for the mathematical value and generalization of the H1 rule to stochastic systems [92]. In this context [93] propose a framework that integrates quantum and classical approaches to decision-making, focussing on quantum stochastic walkers within classical networks. This approach investigates how behavioural choice probabilities emerge as the unique stationary distribution, shedding light on the interplay between unitary and irreversible dynamics, quantum coherence and cognitive realism. The applicability of the model is extended to large-scale surveys and games with incomplete information. The applicability of the model is extended to large-scale surveys and games with incomplete information. The paper analyses the theoretical connections between Bell's notion of non-locality and Harsanyi's theory of games with incomplete information. It proposes a complete formulation of quantum stochastic walks.

Bohl et al. [94] Analysed the application of game theory to cell and molecular biology, focussing on molecular scenarios involving RNA, genes, molecules, transposable elements, viruses and proteins. The strategies of genetic elements, their social behaviour and the application of Shapley's value concept to measure the contribution of enzymes are analysed. The article concludes that game theory can effectively analyse molecular scenarios, reveal unexpected results and highlight the important contribution of genes in metabolic pathways. It also suggests that game theory can be applied to macromolecules such as DNA, RNA and proteins.

Zhang et al. [95] Proposed a new approach to optimize subcarrier allocation in Filter-Based Multichannel Modulation (FBMC) systems, which leads to significant improvements in energy efficiency. The main technique used is evolutionary theory, which considers competition between subcarriers in FBMC systems. The method also analyses interference suppression in cognitive radio networks and resource allocation strategies. The method achieves optimal energy efficiency and system performance, while reducing interference and competition between subcarriers. Future research should focus on more complex scenarios and advanced techniques.

Cui et al. [96] Proposed a new method for multi-user computation on mobile devices using evolutionary gaming strategies for IoT devices. This approach can significantly reduce delay and energy consumption, making it a promising solution for IoT networks. The techniques used include Evolutionary Game Theory (EGT), dynamic replicator dynamics, reinforcement learning, genetic algorithms and Q-learning. The proposed game is effective in reducing computational costs and improving overall utility, with a stable and optimal strategy for multi-user computation.

The integration of evolutionary and stochastic game theory in decision optimization has proven to be a valuable tool for analysing and improving dynamic processes in various disciplines. Game theory, which studies decisions in competitive and uncertain situations, has been enriched by evolutionary and stochastic theories, adapting its models to more realistic situations where players may not always act rationally. The incorporation of quantum techniques has allowed for a better understanding of human decision making, considering hidden variables like emotions and cognitive biases. Optimization, viewed as a process of maximization or minimization of predefined functions, has evolved towards the use of metaheuristics, which are useful in solving complex and large-scale problems. These approaches have been applied in various fields, from cooperative robotics to molecular biology and IoT communication, demonstrating their effectiveness in task assignment, energy efficiency, and problem resolution in uncertain scenarios.

The future of research in these fields should focus on improving optimization techniques applied to complex systems, especially in adapting to rapid changes and integrating various uncertainty sources. The application of game theory in complex environments, such as communication networks and AI, presents an area for further exploration. The interaction between quantum game theory and advanced optimization methods, such as quantum metaheuristics, could open new possibilities for decision-making in uncertain and dynamic contexts. The study of multiagent systems and cooperation analysis is crucial for achieving sustainable and collaborative equilibriums. Lastly, the practical implementation of these theories in real systems, such as the Internet of Things (IoT), cooperative robotics, and biological systems, should advance towards more efficient and robust solutions, addressing the challenges inherent in these environments.

4.3 Simulated human evolution and cooperation in computational games

The utilization of computational simulations for studying and modelling human evolution, with a focus on the crucial role of cooperation in this process, is encompassed by the cluster "Simulated Human Evolution and Cooperation in Computational Games". The importance of cooperation within the context of computational games, a key aspect in the research of human evolution and social interactions, is highlighted by this approach. This comprehensive approach allows a deeper understanding of how cooperation has influenced and shaped human development.

The multidisciplinary topic of evolutionary game theory (EGT) combines game theory with evolutionary biology, emphasizing the dynamics of player tactics and the evolutionary mechanisms that drive change, such as natural selection and mutation. In biology, interactions can be characterized by stochastic processes or deterministic laws, and strategies are inherited algorithms that control individual behaviour. Evolutionary game theory is situated in the domain of nonlinear dynamics and stochastic processes, and this dynamic process may be seen as an iterative map or stochastic process. The idea of an evolutionary stable strategy (ESS), which stops mutant strategies from proliferating, is introduced by EGT. Furthermore, given symmetric information circumstances, classical games can develop into quantum games, as demonstrated by evolutionary quantum games. Nonetheless, entanglement may cause perturbations to the equilibrium of ESS [97, 98]. In this context [99], explores the interrelationships and evolution of strategic decisions in green construction sectors, focussing on the proposed three-part evolutionary model that simulates how actors adjust their strategies over time, considering factors like government subsidies and additional benefits of innovation in green building projects (GBTs). It also explores how actors within this ecosystem interact and how their decisions can lead to a high-level mutualistic innovation ecosystem, optimizing collaboration for sustainable construction. The study highlights the effects of government subsidies and the distribution of benefits and penalties on different actors' strategies. The study provides valuable theoretical and practical insights into integrating evolutionary game theory with innovation ecosystems.

There are many hostile situations in real life, where cooperation is not favoured and collective behaviour requires intervention to improve the efficiency of the system [100]. Cooperation between agents is a common phenomenon in biological, economic and social systems, despite the fact that competition and natural selection drive evolution. Scientific researchers from various fields study cooperative behaviours using evolutionary game theory as a framework [101]. The emergence of cooperation in multi-agent games has generated a great deal of interest. These games typically have NAll agents connected. Most multi-agent games rely on two-person outcomes in which tçhe agent competes with each of its N connected neighbours. Recent attempts have been made to incorporate the real effects of N persons in multi-person games where the interactions of N persons directly influence the outcomes [102]. The prisioner's dilemma is a model to explain cooperative behaviour through peer interactions. Most studies in experimental economics focussed on the Positive Group Games (PGG) for group interactions [103]. Individuals in (PGG) prefer cooperation if others also cooperate, contradicting the self-interest argument. However, later studies found that conditional cooperation can be explained by other preferences, such as inequity aversion or conformity. In the repeated PGG, conditional cooperators who observe that others take advantage of others will reduce their contributions, leading to a decline in cooperation [104]. Cooperative game theory suggests that a coalition should be formed in which each microgrid benefits from participating. A large coalition, a single coalition, maximizes the benefits to the players, but is practically impossible because it does not take into account the distance between microgrids. A similar approach, using the merge/split operation, provides stable partitions, but is NP-hard and has exponential computational complexity in the worst case [105]. Among the collective phenomena that can be examined within the theoretical framework of evolutionary games, the development of cooperation is probably the most intriguing. So-called social dilemmas are composed of a number of evolutionary games, the best known of which is the prisoner's dilemma game, in which it remains very difficult to understand how cooperation evolved. A social dilemma suggests that the success of each individual is not aligned with the welfare of the group, regardless of the particularities of the game [106].

Xu and Yu [107] Present a game theoretic resource allocation algorithm for cloud computing, aiming to improve fairness and optimize resource utilization. The algorithm balances efficiency and fairness, enhancing traditional methods. Experimental results show superior performance compared to Max–Min and Min–Max Algorithms. The algorithm requires fewer iterations, reducing computation time and emphasizing the importance of balancing efficiency and fairness in cloud resource allocation. This paper contributes to the advancement of resource allocation strategies in cloud computing.

Zhang and Huang [108] Introduce a configuration game model for optimizing supply chain management in the Simultaneous Configuration Space for Platform Products and Supply Chains (SCPPSC) problem. It analyses cooperative and non-cooperative supplier scenarios using a game-theoretic approach, highlighting their benefits for all parties involved. The paper introduces a global search algorithm to solve the SCPPSC game and evaluates its performance through numerical analysis. The results show that cooperative supplier scenarios improve supply chain management, customisation, cost reduction, price discounts and extended ordering intervals for suppliers.

Genetic algorithms and other evolutionary computation techniques can be easily parallelised, making it significantly easier to obtain competitive human results. Many results use parallel computing techniques, including quantum, electronic, analytical, optical and mechanical systems. The ability to compute in time, according to Moore's Law, has significantly facilitated genetic programming. The number and complexity of competing human outputs have slowly increased in recent decades [109].

From the development of this cluster, we can draw the following conclusions: cooperation is a central phenomenon in human evolution and modern social systems. Evolutionary game theory provides a suitable framework for modelling cooperative strategies, even in scenarios where individual interests are in conflict with collective welfare. The study of these behaviours in biological and social contexts allows for the development of more effective strategies to promote cooperation and innovation in areas such as sustainability and resource management. The application of game theory in practical areas such as sustainable construction, cloud computing and supply chain management means that evolutionary models can optimize cooperation between stakeholders. The results suggest that, through factors such as government subsidies, innovation incentives and cooperation between actors, a more efficient and sustainable ecosystem can be created. The ability to parallelise evolutionary algorithms, such as genetic algorithms, improves efficiency in information processing and optimizes the use of computational resources. This feature is especially relevant in the era of quantum computing and AI, where speed in obtaining solutions to complex problems is crucial.

4.4 Quantum probability and algorithmic models

In the context of quantum economics, the application of mathematical models [110], probability [111–114], algorithms [65, 111, 113], and quantum theory [111, 115–120] represents a groundbreaking approach to understanding economic systems. The synthesis of quantum probability and economic theory, underpinned by the interdisciplinary cluster "Quantum Probability and Algorithmic Models", opens new frontiers for analysing and predicting complex economic behaviours.

This approach leverages the probabilistic nature of quantum systems [121] to model uncertainties and fluctuations that are inherent in economic markets [122]. Traditional economic models often rely on classical probability theory, assuming deterministic outcomes or simple stochastic processes. However, quantum economics introduces a more sophisticated understanding of uncertainty, where economic agents and market behaviours can exist in superpositions of multiple states—analogous to quantum superposition in physics. The probabilistic phenomena in quantum mechanics offer new ways to conceptualize and model market volatility, investment decisions, and economic forecasts, which may be influenced by a variety of overlapping factors.

At the heart of this quantum approach is the use of quantum algorithms to process and analyse economic data in ways that classical algorithms cannot. Quantum computing's ability to handle vast datasets and perform parallel computations exponentially faster than classical computers is crucial in economic modelling, where large-scale simulations and optimizations are needed. These algorithms can solve complex problems such as portfolio optimization, risk assessment, and resource allocation in a more efficient and potentially more accurate manner than classical methods.

Furthermore, the intersection of quantum mechanics and algorithmic models provides a framework for understanding phenomena like market entanglement, where the actions of one economic agent can instantaneously affect others, resembling quantum entanglement. Quantum probability theories allow for more dynamic models of interdependent markets, incorporating non-local correlations that are absent from classical economic theories. This creates a richer and more holistic view of economic interactions, addressing the limitations of traditional models that fail to capture the complexity of modern financial systems.

The application of these quantum principles in economics is not merely theoretical but has practical implications. Quantum-inspired models could redefine how we approach economic policies, risk management, and financial technologies (FinTech). By uniting advanced quantum theory with cutting-edge algorithmic approaches, the field of quantum economics aims to transform economic analysis and decision-making, offering a novel lens through which to view the uncertainties and complexities of the global economy. Quantum economics also opens up new lines of research by reinterpreting classical concepts such as supply and demand curves, using tools such as Wigner pseudoprobailities. This allows us to model market behaviours that do not conform to traditional laws, such as Giffen goods, and explore the existence of optimal strategies in complex scenarios through adiabatic approximations and "Giffen strategies." These theoretical and methodological proposals broaden the range of possibilities for economic modelling and suggest that, as technologies advance, quantum economics could provide innovative answers to structural challenges that persist in global markets today [68].

A key concept that connects quantum probability to game theory is the non-factorizability of probabilities. In classical game theory, the probabilities of the outcomes of players' actions are factorizable, meaning that they can be expressed as the product of each player's individual probabilities. However, in quantum mechanics, entanglement can lead to non-factorizable probabilities, where the outcomes of measurements in an entangled system are correlated in a way that cannot be explained classically [112–114]. This non-factorizability can have a significant impact on the Nash equilibria (NEs) of a game [113, 115, 123–125]. NEs are sets of strategies in which no player can improve his outcome by unilaterally changing his strategy. In some quantum games, the non-factorizability of probabilities can lead to the emergence of new NEs that do not exist in the classical version of the game [113, 114]. A famous example is the Prisoner's Dilemma [111, 113, 126]. In the classical version of the game, the dominant strategy for both players is to "betray" the other, leading to a suboptimal outcome for both. However, in the quantum version of the game, with the use of entangled states and unfactorizable probabilities, a new NE can be found where both players "cooperate", leading to a better outcome for both. Quantum Games and EPR (Einstein–Podolsky–Rosen) experiments provide a platform to study and utilize unfactorizable probabilities in quantum games [113]. In these experiments, measurements on entangled particles show correlations that violate Bell inequalities, demonstrating the nonlocal nature of quantum mechanics. It is important to note that in order to ensure that the classical game is embedded within the quantum game, certain restrictions are imposed on the joint probabilities in EPR models. These restrictions ensure that when the probabilities are factorizable, the quantum game reduces to the classical game. This means that non-factorizability is an essential ingredient to obtain new outcomes in quantum games.

In the face of increasingly complex, uncertain, and interconnected economic systems, emerging interdisciplinary approaches are reshaping the analytical tools used to understand strategic decision-making and system dynamics. The integration of computational techniques for economic models [124, 125, 127–131], quantum contextuality [132, 133], and chaotic dynamics in quantum game theory [134, 135] offers a novel and powerful framework for exploring economic behaviour beyond the limitations of classical models. Computational methods—such as the calculation of minimum energy paths—allow for the simulation and prediction of dynamic transitions and structural changes in economic systems, revealing how economies navigate between equilibrium states under stress or innovation [131]. At the same time, the concept of quantum contextuality introduces a fundamentally different understanding of causality and dependence, where the outcome of economic decisions depends intrinsically on contextual variables, challenging the classical assumptions of separability and independence among agents [132]. This quantum lens provides a richer modelling language for describing market entanglement, inter-agent correlations, and the probabilistic nature of preferences and expectations [133]. Complementing these insights, the study of chaotic dynamics within quantum game theory highlights the interplay between randomness and order in strategic environments, where nonlinear interactions and feedback loops can generate emergent behaviour not captured by static or deterministic models [134]. Quantum game theory, with its inclusion of superposition, entanglement, and non-factorizable probabilities, offers a strategic space where new equilibria—sometimes Pareto-optimal—can emerge in situations traditionally constrained by classical dilemmas [113, 126]. Taken together, these three domains point towards a unifying paradigm in which economic systems are viewed as complex adaptive entities shaped by computation, context, and strategic uncertainty, thus opening new pathways for modelling, optimization, and policy design in the digital and quantum age.

5 Discussion

Traditionally framed within the logic of rationality and utility maximization, game theory is undergoing a profound transformation thanks to the incursion of concepts and tools from diverse fields.

However, this quantum incursion into game theory is not without its challenges. The threat of decoherence in practical implementation and the debate surrounding the interpretation of concepts such as the 'miracle move' remind us that translating quantum theory to the strategic sphere requires careful consideration of physical and conceptual limitations. Despite these caveats, the potential of quantum game theory to offer novel solutions to complex problems is undeniable. The capacity to model strategic interactions with a greater richness of possibilities, thanks to quantum states and unitary operations, opens new avenues for optimizing and understanding multi-agent systems. Quantum theory is expanding the boundaries of game theory, offering improvements in the outcomes of classical games and conceptual tools to address strategic challenges in an increasingly complex and interconnected world. Future research must focus on overcoming current limitations and fully exploring the potential of this fascinating fusion of two fundamental fields of science and decision-making.

Still, introducing quantum theory into this landscape represents a paradigm shift. Quantifying subjective and 'irrational' aspects of decision-making challenges traditional game theory's fundamental assumptions. While this approach is still in its early stages, its potential to offer a deeper understanding of human behaviour in strategic contexts is significant. Ultimately, the convergence of evolutionary and stochastic game theory with optimisation techniques and the emerging influence of quantum theory signal a promising direction for future research.

These integrated approaches are expected to enable the development of more precise and applicable decision-making models for a broader range of complex systems, from resource management to understanding social and biological behaviour. The key will be refining these methodologies and demonstrating their practical value in solving real-world problems. The capacity of evolutionary computation to parallelise algorithms introduces a dimension of computational efficiency crucial for addressing the complexity inherent in evolutionary models and real-world problems. This synergy between evolutionary modelling and computational power opens new avenues for research and the practical application of cooperative strategies in various fields. Cooperation is not merely an accident of evolution but a complex and multifaceted phenomenon that can be modelled, understood, and promoted by combining evolutionary game theory and computational simulation tools. Understanding the underlying mechanisms of cooperation has profound implications for addressing humanity's social, economic, and environmental challenges. The non-factorisation of probabilities in quantum game theory is not just a theoretical curiosity; it can redefine the outcomes of strategic interactions, as exemplified in the quantum resolution of the Prisoner's Dilemma. This could have profound implications for policy design and understanding cooperative behaviour in economic contexts. Ultimately, by merging the sophistication of quantum theory with the rigour of economic and algorithmic models, quantum economics does not simply seek to refine existing models but offers an entirely new lens through which to analyse the uncertainties and complexities of the global economy. While still in its initial stages, its potential to transform economic analysis, risk management, and financial technologies is undeniable, marking a paradigm shift in how we conceive and model the economic world. In consideration of the aforementioned factors, and in accordance with the provisions outlined in Sect. 4.1, these advances have the potential to play a pivotal role in addressing the demographic challenges that lie ahead. This is due to the fact that game theory addresses the interaction among individuals, and its quantum version possesses the capacity to enhance conventional performance.

6 Conclusions

In the current climate, where the productivity paradox coexists with growing concerns about inequality, quantum economics and quantum game theory are emerging as novel frameworks for addressing inherent uncertainty and complex strategic interactions.

In contrast to classical game theory, firmly anchored in probabilistic strategies within a Boolean logical framework, the application of quantum theory introduces an unprecedented dimension of complexity and potentiality. The pioneering schemes that have been analysed convincingly demonstrate that the incorporation of fundamental quantum principles, such as superposition and entanglement, not only radically transforms the strategic landscape but also, in specific paradigm-shifting scenarios like the quantum resolution of the Prisoner's Dilemma using the EWL scheme, even enables the overcoming of limitations imposed by classical equilibria, allowing for the achievement of Pareto-optimal and, therefore, more efficient outcomes for all participants.

Unlike the static approach based on complete rationality that characterizes classical game theory, the integration of evolutionary dynamics and the explicit consideration of stochastic uncertainty provide a considerably richer perspective closer to the reality of decision-making processes. Evolutionary game theory empowers us to understand how strategies adapt and evolve, more accurately reflecting the bounded rationality exhibited by real agents in their interactions. The subsequent incorporation of powerful optimisation tools within this dynamic framework provides the necessary mechanisms to identify and achieve superior outcomes in scenarios characterized by their complexity and constant change. In this regard, metaheuristics have proven their great utility in tackling problems where traditional analytical methods often encounter significant limitations, offering solutions that, while they may not be optimal in an absolute sense, are eminently practical and applicable.

Contrary to purely analytical approaches, computational simulation, solidly grounded in the principles of evolutionary game theory, presents an inherently dynamic and exploratory methodology for unraveling the intricate evolution of human cooperation. This approach allows us to transcend the rigidity of classical equilibria, offering the ability to observe directly how strategies modify and emerge in response to repeated interactions and the various evolutionary pressures acting on agents. Evolutionary game theory provides a robust conceptual framework for modelling cooperation, stripping it of the simplistic notion of a mere altruistic trait and revealing it as a strategy that can be evolutionarily advantageous under certain conditions. The exploration of diverse practical scenarios, ranging from sustainable ecological construction to the efficient allocation of cloud resources and the optimization of supply chain management, highlights the broad applicability of these models for improving collaboration and efficiency in complex systems operating in the real world both in big cities and in regions with low population density.

Finally, in contrast to traditional economics, which is based on classical probability and the assumption of complete rationality on the part of agents, quantum economics proposes a radical transformation in our fundamental understanding of economic systems. By importing the essential principles of quantum mechanics, such as superposition and entanglement, this new discipline offers innovative conceptual tools for modelling uncertainty and complex economic interdependencies in a fundamentally novel way. The promising ability of quantum algorithms to overcome the significant computational limitations faced by classical methods in economic analysis opens up a vast horizon of unprecedented possibilities for optimizing and predicting economic phenomena. The ability to model non-local correlations through market entanglement suggests achieving a much deeper and more accurate understanding of the intricate dynamics that govern global financial systems.

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Declarations

Conflict of interest The authors declare no Conflict of interest.

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