



PAPER

Dissipation process in eternal black holes

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E-mail: escargoviolin@infoseek.jp**Keywords:** eternal black hole, dissipation process, thermo field dynamics, AdS/CFT correspondence**Abstract**

We consider the effect of the double trace deformation on the eternal black hole. On the boundary CFTs, the deformation can be considered the dissipation in the thermofield dynamics framework. In this framework, the entanglement operator describes the dissipation effect in boundary CFT. Corresponding to CFTs, the wormhole in spacetime and the defect in code subspace are formed as dissipative structures. These dissipative structures realize efficient processing through hierarchical information in the gravitational system. Further, the Fisher information metric renders the Lyapunov functional, which gives a criterion for the stability of the eternal black hole.

1. Introduction

Spacetime geometry is considered to be deeply related to quantum entanglement. ER=EPR conjecture attempts to solve the black hole information paradox by creating a wormhole connecting far-apart spacetime pieces [1]. Wormholes are solutions to the Einstein equations that work as tunnels in spacetime. Einstein-Rosen bridge first gives geometric interpretation as an object connecting two copies of the black hole geometry. Traversable wormholes are considered for human short-cut travel through wormholes [2, 3]. The study of traversable wormholes is examined by general relativity. The stress energy tensor on the wormhole's throat violates the null energy condition (NEC) from Einstein equations. In other words, wormholes are supported by negative-energy matter, i.e., exotic matter. NEC violation is a generic feature of all wormholes, whether they are time-dependent or static. From this fact, the quantum effect plays a major role in their foundation [3–5].

EPR paradox is a gedankenexperiment proposed by Einstein, Podolsky, and Rosen. This paradox results from a nonlocal property of quantum mechanics: two distant quantum objects do not always behave independently, even if they are extremely far apart from each other. Namely, the locality is not established in quantum mechanics. These two objects are called entangled. The reason is that quantum entanglement exists only when the state of a composite system cannot be expressed as a product of the quantum states of its individual subsystems. The ER=EPR conjecture asserts that there exists an Einstein-Rosen bridge connecting black holes (ER), which is equivalent to the quantum entanglement (EPR) of quantum particles [6].

An eternal black hole establishes the situation verifying this conjecture. In the context of AdS/CFT correspondence, the eternal black hole with two asymptotically AdS provides the two-boundary CFTs. The description of the eternal black hole state is based on thermo field dynamics (TFD). The above picture is agreed with the doubled multi-scale entanglement renormalization ansatz (MERA) network [7]. A doubled MERA network is constructed by gluing two copies of the standard MERA together at infrared points by a bridge state [8]. The continuous version of MERA (cMERA) gives the surface-state correspondence [9].

The deformed theory gives the coupling of two CFTs, leading to the NEC violation for the wormholes [10, 11]. This deformed theory has a dissipative character, which has already been pointed out [12]. The dissipation has universality phenomena from a micro to a macro perspective, such as non-equilibrium physics or decoherence [13]. For quantum dissipation in the scalar field, the dissipation promotes a gapped momentum space and reduces the particle energy [14]. The framework of TFD is adequate for detailing quantum dissipation [15, 16].

When we turn our attention to an exotic matter violating NEC, we find a squeezed quantum state as a candidate, which realizes matter with negative energy density. Gravitational interaction leads to a squeezed matter vacuum, further supporting the wormhole structure, accompanied by negative energy density [3, 17]. The framework of TFD is also adequate for detailing squeezed quantum states. This study addresses that the dissipation process relates directly to double trace deformation in an eternal black hole.

2. Eternal black hole

Bañados, Teitelboim, and Zanelli (BTZ) discovered that (2+1)-dimensional gravity has a black-hole solution [18–20]. The BTZ black hole is asymptotically anti-de Sitter rather than asymptotically flat and has no curvature singularity at the origin. At the same time, it has an event horizon and an inner horizon when rotating. Further, it has thermodynamic properties as those of a (3 + 1)-dimensional black hole. The BTZ black hole in Schwarzschild coordinates is given as follows:

$$ds^2 = -\left(-M + \frac{r^2}{\ell^2} + \frac{J^2}{4r^2}\right)dt^2 + \frac{1}{\left(-M + \frac{r^2}{\ell^2} + \frac{J^2}{4r^2}\right)}dr^2 + r^2\left(d\phi - \frac{J}{2r^2}dt\right)^2 \quad (1)$$

This metric is the vacuum field equations of (2 + 1)-dimensional general relativity with a cosmological constant $-1/\ell^2$. The parameters M and J are the standard ADM mass and angular momentum, respectively. (2 + 1)-dimensional Einstein-Maxwell action is given as follows:

$$S = \frac{1}{16\pi G} \int d^3x \sqrt{-g} \left(R + \frac{2}{\ell^2} - 4\pi G F_{\mu\nu} F^{\mu\nu} \right) \text{ with } F_{tr} = \frac{Q}{r}, \quad (2)$$

From this action, we have the following charged BTZ black hole solution:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2d\theta^2, \quad (3)$$

where $f(r) = \frac{r^2}{\ell^2} - 8GM - 8\pi GQ^2 \ln\left(\frac{r}{\omega}\right)$. ω is constant in this expression. It is also possible to consider the nonlinear Born-Infeld electrodynamics [21, 22].

Further modifications of the BTZ black hole solutions are found by allowing scale-dependent coupling [23–28], by connection with string theory or holography [29–33], and by introducing gravitational Chern-Simons term [34].

We consider the eternal BTZ black hole, whose metric is given as follows:

$$ds^2 = -\frac{r^2 - r_h^2}{\ell^2}dt^2 + \frac{\ell^2}{r^2 - r_h^2}dr^2 + r^2d\phi^2. \quad (4)$$

The inverse temperature of the BTZ black hole $\frac{2\pi\ell^2}{r_h}$ is determined by its horizon radius r_h . Here and below, we will set AdS radius $\ell = 1$. Figure 1 is the Penrose diagram of an eternal black hole.

An eternal black hole has a holographic description, which is dual to two copies of CFT in an entangled state: The system is described by two identical uncoupled CFT on the disconnected boundary: $\mathcal{H} = \mathcal{H}_L \otimes \mathcal{H}_R$. This entangled state $|0(\beta)\rangle \in \mathcal{H}$ is described by thermofield double state [35].

$$\begin{aligned} |0(\beta)\rangle &= \exp \sum_k \tanh^{-1} \left(-\frac{\beta\omega_k}{2} \right) (\mathcal{O}_k^{(R)\dagger} \mathcal{O}_k^{(L)\dagger} - \mathcal{O}_k^{(L)} \mathcal{O}_k^{(R)}) |0\rangle \\ &= \mathcal{N} \sum_n \exp \left(-\frac{\beta E_n}{2} \right) |E_n\rangle_L \otimes |E_n\rangle_R, \end{aligned} \quad (5)$$

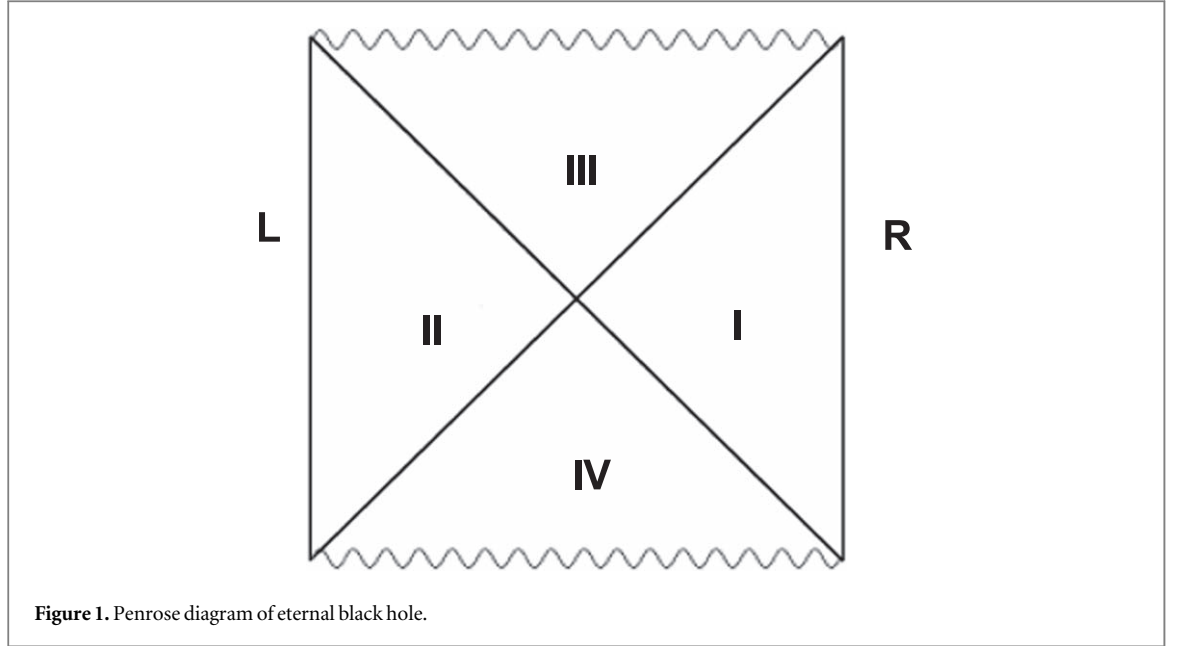
where β is the inverse temperature of the black hole and $|0\rangle = |0\rangle_R \otimes |0\rangle_L$. In the above equation, $\mathcal{O}_k^{(R)}$ and $\mathcal{O}_k^{(L)}$ are annihilation operators at right CFT and left CFT, respectively, i.e., $\mathcal{O}_k^{(R)}|0\rangle_R = \mathcal{O}_k^{(L)}|0\rangle_L = 0$. The energy level is discrete, and corresponding energy states are $|E_n\rangle_L, |E_n\rangle_R$.

The state $|0(\beta)\rangle$ represents a single black hole on thermal equilibrium. $|0(\beta)\rangle$ is invariant under H , which corresponds to the bulk Killing symmetry and the difference of Hamiltonians of the two CFTs.

$$H = H_R - H_L. \quad (6)$$

3. Dissipation from double trace deformation

Double trace deformation considered by Gao, Jefferis, Wall (GJW) can be understood from the dissipative viewpoint [11, 12]. The main characteristic of the GJW construction is a non-local interaction between the two



boundaries, introduced through a small deformation of the original CFT Hamiltonian. From the BDHM (Banks, Douglas, Horowitz, Martinec) prescription [12, 36, 37], a field near the boundary of the AdS black hole can be a linear combination of the creation and annihilation operators. Especially, we choose the same operators shown in equation (5):

$$\begin{aligned} A_R &= \frac{\mathcal{O}^{(R)\dagger} - \mathcal{O}^{(R)}}{2i}, & A'_R &= \frac{\mathcal{O}^{(R)\dagger} + \mathcal{O}^{(R)}}{2i} \\ B_L &= \frac{\mathcal{O}^{(L)\dagger} + \mathcal{O}^{(L)}}{2}, & B'_L &= \frac{\mathcal{O}^{(L)\dagger} - \mathcal{O}^{(L)}}{2}. \end{aligned} \quad (7)$$

Then we have the following relation:

$$A_R B_L + A'_R B'_L = i\frac{\gamma}{2} (\mathcal{O}^{(L)} \mathcal{O}^{(R)} - \mathcal{O}^{(R)\dagger} \mathcal{O}^{(L)\dagger}). \quad (8)$$

We build a modified Hamiltonian by summing up these terms:

$$\delta H_b = i\frac{\gamma}{2} \sum_{k \in \mathcal{K}} (\mathcal{O}_k^{(L)} \mathcal{O}_k^{(R)} - \mathcal{O}_k^{(R)\dagger} \mathcal{O}_k^{(L)\dagger}), \quad (9)$$

where the finite sum of the momentum is denoted by the set \mathcal{K} .

3.1. Time evolution of the boundary CFT

We outline the dissipative model of the conformal field. The Lagrangian of a dissipative model of the conformal field is given by

$$L = \int d^d x \left\{ \partial_\mu \phi \partial^\mu \psi + \frac{\gamma}{2} (\phi \partial_t \psi - \psi \partial_t \phi) \right\}, \quad (10)$$

where γ is the damping coefficient. We have finite a propagation range from this Lagrangian: $\phi, \psi \sim e^{-\gamma t/2}$ [14]. The second term leads to damping and corresponds to the modified Hamiltonian of equation (9).

From this viewpoint, double trace deformation can be understood as quantum dissipation [15, 16]. The time evolution of the state $|0(\beta)\rangle$ is given by

$$\begin{aligned}
|0_{\text{bdy}}(t)\rangle &= \exp(-i\delta H_b t)|0(\beta)\rangle \\
&= \exp\sum_k \mathcal{D}(t)(\mathcal{O}_k^{(R)\dagger}\mathcal{O}_k^{(L)\dagger} - \mathcal{O}_k^{(L)}\mathcal{O}_k^{(R)})|0\rangle \\
&= \exp\left(-\frac{\mathcal{S}_R}{2}\right)\exp\left(\sum_k \mathcal{O}_k^{(R)\dagger}\mathcal{O}_k^{(L)\dagger}\right)|0\rangle \\
&= \exp\left(-\frac{\mathcal{S}_L}{2}\right)\exp\left(\sum_k \mathcal{O}_k^{(R)\dagger}\mathcal{O}_k^{(L)\dagger}\right)|0\rangle,
\end{aligned} \tag{11}$$

where

$$\mathcal{S}_R = -\left(\sum_k \mathcal{O}_k^{(R)\dagger}\mathcal{O}_k^{(R)}\text{lnsinh}^2(\mathcal{D}(t)) - \mathcal{O}_k^{(R)}\mathcal{O}_k^{(R)\dagger}\text{lncosh}^2(\mathcal{D}(t))\right). \tag{12}$$

\mathcal{S}_L is expressed by replacing $\mathcal{O}_k^{(R)}$ with $\mathcal{O}_k^{(L)}$. \mathcal{S}_R and \mathcal{S}_L are the entropy operators for the dissipative system [15]. By the modified Hamiltonian δH_b , a portion of $\mathcal{D}(t)$ is time-dependent:

$$\mathcal{D}(t) = \begin{cases} \tanh^{-1}[e^{(-\frac{\beta\omega}{2})}] + \frac{\gamma t}{2}: k \in \mathcal{K}, \\ \tanh^{-1}[e^{(-\frac{\beta\omega}{2})}]: \text{otherwise.} \end{cases} \tag{13}$$

As a result, CFT calculates the entropy which is realized in spacetime:

$$\langle 0_{\text{bdy}}(t)|\mathcal{S}_R|0_{\text{bdy}}(t)\rangle = \frac{\sigma}{4G_3} + S_{\text{bulk}}, \tag{14}$$

where G_3 is the three-dimensional Newton constant, and σ is the minimal surface area in the holographic geometry [38]. Concretely, a minimal area surface is placed at the wormhole's throat. In addition, S_{bulk} is the bulk entanglement entropy across the throat [39] derived from the dissipation process.

3.2. Time evolution of the gravitational system

We consider the gravitational system's evolution corresponding to the boundary CFT. The state corresponding to the gravitational system can be understood from the following: The vacuum state of the eternal black hole is given by the Kruskal patch. A similar situation can be seen among the Minkowski vacuum and Rindler thermofield double state [40, 41]. By selecting the bulk mode $H_{\mathbf{k}}^{(\pm)}$, the global bulk field operator living in the entire eternal black hole is given as follows [11]:

$$\phi(t, x, z) = \int d\mathbf{k}(H_{\mathbf{k}}^{(+)}b_{\mathbf{k}}^{(+)} + H_{\mathbf{k}}^{(-)}b_{\mathbf{k}}^{(-)\dagger} + h.c.). \tag{15}$$

The deformation does not constrain these modes' asymptotic behavior at order γ . $b_{\mathbf{k}}^{(\pm)}$ is the annihilation operator which defines the vacuum $|0(\kappa)\rangle$. Because the bulk-free field two-point function in the BTZ background and that in the Kruskal vacuum $|0(\kappa)\rangle$ are same as up to normalization, $|0(\kappa)\rangle$ can be recognized bulk state corresponding boundary CFT [11, 42].

Annihilation operators in the regions (I) and (II) are given as

$$\begin{aligned}
a_{\mathbf{k}}^{(I)} &= b_{\mathbf{k}}^{(+)}\cosh\Phi_\omega + b_{\mathbf{k}}^{(-)\dagger}\sinh\Phi_\omega \\
a_{\mathbf{k}}^{(II)} &= b_{\mathbf{k}}^{(-)}\cosh\Phi_\omega + b_{\mathbf{k}}^{(+)\dagger}\sinh\Phi_\omega,
\end{aligned} \tag{16}$$

and $H_{\mathbf{k}}^{(\pm)}$ is related to the Killing mode $F_{\mathbf{k}}^{(I)}$ and $F_{\mathbf{k}}^{(II)}$:

$$\begin{aligned}
H_{\mathbf{k}}^{(+)} &= F_{\mathbf{k}}^{(I)}\cosh\Phi_\omega + F_{\mathbf{k}}^{(II)}\sinh\Phi_\omega \\
H_{\mathbf{k}}^{(-)} &= F_{\mathbf{k}}^{(II)}\cosh\Phi_\omega + F_{\mathbf{k}}^{(I)}\sinh\Phi_\omega.
\end{aligned} \tag{17}$$

In the above expression, $\tanh\Phi_\omega = \exp(-\beta\omega/2)$. Bulk field operator in equation (15) can be expressed via Killing modes:

$$\phi(t, x, z) = \int d\mathbf{k}(F_{\mathbf{k}}^{(I)}a_{\mathbf{k}}^{(I)} + F_{\mathbf{k}}^{(II)}a_{\mathbf{k}}^{(II)\dagger} + h.c.). \tag{18}$$

Using the operator defined in equation (5), we have

$$\begin{aligned}
\langle 0(\beta)|\mathcal{O}_{\mathbf{k}}^{(R)\dagger}\mathcal{O}_{\mathbf{k}'}^{(R)}|0(\beta)\rangle &= \langle 0(\kappa)|a_{\mathbf{k}}^{(I)\dagger}a_{\mathbf{k}'}^{(I)}|0(\kappa)\rangle \\
= \langle 0(\beta)|\mathcal{O}_{\mathbf{k}}^{(L)\dagger}\mathcal{O}_{\mathbf{k}'}^{(L)}|0(\beta)\rangle &= \langle 0(\kappa)|a_{\mathbf{k}}^{(II)\dagger}a_{\mathbf{k}'}^{(II)}|0(\kappa)\rangle \\
&= \frac{1}{e^{\beta\omega} - 1}\delta(\mathbf{k} - \mathbf{k}').
\end{aligned} \tag{19}$$

equation (19) shows that operators $\mathcal{O}_{\mathbf{k}}^{(R)}$ and $\mathcal{O}_{\mathbf{k}}^{(L)}$ are gauge theory modes rescaled by a two-point Green function. From this rescaling, the identification among $\mathcal{O}_{\mathbf{k}}^{(R)}$, $\mathcal{O}_{\mathbf{k}}^{(L)}$, and $a_{\mathbf{k}}^{(1)}$, $a_{\mathbf{k}}^{(1)\dagger}$ is possible [43].

The states dual to a convex closed surface are related by the surface/state correspondence.

$$|\Phi(\Sigma_1)\rangle = U(s_1, s_2)|\Phi(\Sigma_2)\rangle, \quad (20)$$

where a smooth deformation preserving convexity connects two surfaces, and $U(s_1, s_2)$ is a unitary transformation [44]. From this correspondence, the state $|0(\beta)\rangle$ of the boundary CFT and the state $|0(\kappa)\rangle$ have the following relation:

$$|0(\beta)\rangle = U|0(\kappa)\rangle, \quad (21)$$

where U is the unitary transformation. The bulk state has the following expression:

$$\begin{aligned} |0(\kappa)\rangle &= e^{(-i\sum_{\mathbf{k}} i\Phi_{\omega}(a_{\mathbf{k}}^{(1)\dagger} a_{\mathbf{k}}^{(1)\dagger} - a_{\mathbf{k}}^{(1)} a_{\mathbf{k}}^{(1)})} |0\rangle \\ &= \frac{1}{Z} \prod_{\mathbf{k}} \left\{ \sum_{n_{\mathbf{k}}}^{\infty} \exp(-n_{\mathbf{k}} \omega\beta/2) |n_{\mathbf{k}}\rangle^{(L)} |n_{\mathbf{k}}\rangle^{(R)} \right\}. \end{aligned} \quad (22)$$

$|0(\kappa)\rangle$ is not invariant by the generator $\mathcal{G} = \sum_{\mathbf{k}} (i\Phi_{\omega}(a_{\mathbf{k}}^{(1)\dagger} a_{\mathbf{k}}^{(1)\dagger} - a_{\mathbf{k}}^{(1)} a_{\mathbf{k}}^{(1)}))$ given in equation (22). $|0(\kappa)\rangle$ has a configuration where the pair $a_{\mathbf{k}}^{(1)} a_{\mathbf{k}}^{(1)\dagger}$ is condensed. This state varies by the change in the pair's distribution. The pair is Nambu-Goldstone (NG) mode corresponding to the symmetry \mathcal{G} . From this fact, the pair $a_{\mathbf{k}}^{(1)} a_{\mathbf{k}}^{(1)\dagger}$ has zero energy. $|0(\kappa)\rangle$ possesses the entanglement entropy between different momentum distributions as shown in equation (22).

Without the dissipation, the change $|0(\beta)\rangle \rightarrow |0(\beta + \Delta\beta)\rangle$ in boundary CFT can be transitioned among thermal state. This transition corresponds to the transition $|0(\kappa)\rangle \rightarrow |0(\kappa + \Delta\kappa)\rangle$ in the gravitational system and results from the change of spacetime structure, including the variation of the horizon radius.

In general, mapping among boundary CFT's states and bulk's multiparticle states can be complicated [45]. For establishing the thermal state, generators have different properties. From equation (5), the generator of the boundary CFT is given as $G = \sum_{\mathbf{k}} \tanh^{-1}(\frac{-\beta\omega_{\mathbf{k}}}{2})(\mathcal{O}_{\mathbf{k}}^{(R)\dagger} \mathcal{O}_{\mathbf{k}}^{(L)\dagger} - \mathcal{O}_{\mathbf{k}}^{(L)} \mathcal{O}_{\mathbf{k}}^{(R)})$, where the summation is taken over discrete momentum. On the other hand, the generator of the gravitational system is given as $\mathcal{G} = \sum_{\mathbf{k}} \Phi_{\omega}(a_{\mathbf{k}}^{(1)\dagger} a_{\mathbf{k}}^{(1)\dagger} - a_{\mathbf{k}}^{(1)} a_{\mathbf{k}}^{(1)})$, where summation over \mathbf{k} symbolizes the integral. The gravitational system needs NG mode made with continuous momentum for thermal transition. Equation (5) and equation (22) show that the discrete sum of momentum in the boundary CFT corresponds to the infinite integral interval in momentum for the gravitational system when considering the thermal state. Then, the finite sum in boundary CFT corresponds to the finite integral in momentum for the gravitational system.

From this consideration, the boundary CFTs' deformation shown in equation (11) corresponds to dissipation to the gravitational system as follows:

$$\begin{aligned} |0_{\text{Grav}}(t)\rangle &= \exp(-i\delta H_g t) |0(\kappa)\rangle = e^{(-i\delta H_g t)} e^{(-i\sum_{\mathbf{k}} i\Phi_{\omega}(a_{\mathbf{k}}^{(1)\dagger} a_{\mathbf{k}}^{(1)\dagger} - a_{\mathbf{k}}^{(1)} a_{\mathbf{k}}^{(1)})} |0\rangle \\ &= \exp \sum_{\mathbf{k}} (\mathcal{D}_G(t)(a_{\mathbf{k}}^{(1)\dagger} a_{\mathbf{k}}^{(1)\dagger} - a_{\mathbf{k}}^{(1)} a_{\mathbf{k}}^{(1)})) |0\rangle \\ &= \exp(-i\mathcal{G}_D(t)) |0\rangle. \end{aligned} \quad (23)$$

In the above expression, we define the modified Hamiltonian δH_g from the identification in equation (19):

$$\delta H_g = i\frac{\gamma}{2} \sum_{\mathbf{k} \in \hat{\mathcal{K}}} (a_{\mathbf{k}}^{(1)\dagger} a_{\mathbf{k}}^{(1)\dagger} - a_{\mathbf{k}}^{(1)} a_{\mathbf{k}}^{(1)}), \quad (24)$$

and

$$\mathcal{D}_G(t) = \begin{cases} \frac{\gamma t}{2} + \Phi_{\omega}: & \mathbf{k} \in \hat{\mathcal{K}}, \\ \Phi_{\omega}: & \text{otherwise.} \end{cases} \quad (25)$$

In the above expression, $\hat{\mathcal{K}}$ is obtained by interpolating the value in \mathcal{K} with a continuous value.

We concentrate on the deformation effect on the state of the gravitational system. The transition is caused by the generator $\mathcal{G}_D(t)$ given in equation (23) when the dissipation is applied. The fact that $\mathcal{G}_D(t)$ does not annihilate the state $|0_{\text{Grav}}(t)\rangle$:

$$\mathcal{G}_D(t) |0_{\text{Grav}}(t)\rangle \neq 0 \quad (26)$$

shows that $|0_{\text{Grav}}(t)\rangle$ is not invariant by the generator $\mathcal{G}_D(t)$. From the generator $\mathcal{G}_D(t)$, the quasi-particle operator is given by the Bogoliubov matrix $B_{\mathbf{k}}$:

$$\begin{pmatrix} \alpha_{\mathbf{k}}^{(I)} \\ \alpha_{\mathbf{k}}^{(I)\dagger} \end{pmatrix} = B_{\mathbf{k}} \begin{pmatrix} a_{\mathbf{k}}^{(I)} \\ a_{\mathbf{k}}^{(I)\dagger} \end{pmatrix},$$

$$B_{\mathbf{k}} = \begin{pmatrix} \cosh(\mathcal{D}_G(t)) & -\sinh(\mathcal{D}_G(t)) \\ -\sinh(\mathcal{D}_G(t)) & \cosh(\mathcal{D}_G(t)) \end{pmatrix}. \quad (27)$$

Because the quasi-particle operators are not affected by the dissipation, the following equation is satisfied:

$$\begin{pmatrix} \frac{d}{dt} + i\omega \\ \frac{d}{dt} + i\omega \end{pmatrix} \alpha_{\mathbf{k}}^{(I)} = 0,$$

$$\begin{pmatrix} \frac{d}{dt} + i\omega \\ \frac{d}{dt} + i\omega \end{pmatrix} \alpha_{\mathbf{k}}^{(I)\dagger} = 0. \quad (28)$$

Together with the relation (27) and (28), the following equations are derived:

$$\begin{pmatrix} \frac{d}{dt} + i\omega + B_{\mathbf{k}}^{-1} \frac{d}{dt} B_{\mathbf{k}} \\ \frac{d}{dt} + i\omega + B_{\mathbf{k}}^{-1} \frac{d}{dt} B_{\mathbf{k}} \end{pmatrix} a_{\mathbf{k}}^{(I)} = 0,$$

$$\begin{pmatrix} \frac{d}{dt} + i\omega + B_{\mathbf{k}}^{-1} \frac{d}{dt} B_{\mathbf{k}} \\ \frac{d}{dt} + i\omega + B_{\mathbf{k}}^{-1} \frac{d}{dt} B_{\mathbf{k}} \end{pmatrix} a_{\mathbf{k}}^{(I)\dagger} = 0. \quad (29)$$

$B_{\mathbf{k}}^{-1} \frac{d}{dt} B_{\mathbf{k}}$ ($\mathbf{k} \in \hat{\mathcal{K}}$) only has a time dependence in the above equations. Because the time-dependency of the Bogoliubov matrix modifies the equation for $a_{\mathbf{k}}^{(I)}$ -quantum and $a_{\mathbf{k}}^{(I)\dagger}$ -quantum, the dissipative nature is associated with the $a_{\mathbf{k}}^{(I)}$, $a_{\mathbf{k}}^{(I)\dagger}$ -quanta. From this equation, $a_{\mathbf{k}}^{(I)}$, $a_{\mathbf{k}}^{(I)\dagger} \sim e^{-i\omega t} e^{-t \frac{d}{dt} \mathcal{D}_G(t)}$ ($\mathbf{k} \in \hat{\mathcal{K}}$). These quanta are time-decaying exponentially from dissipation. The evolution of $|0_{\text{Grav}}(t)\rangle$ is obtained by differentiating equation (23):

$$\frac{\partial}{\partial t} |0_{\text{Grav}}(t)\rangle = \frac{\gamma}{2} \sum_{\mathbf{k} \in \hat{\mathcal{K}}} (a_{\mathbf{k}}^{(I)\dagger} a_{\mathbf{k}}^{(I)\dagger} - a_{\mathbf{k}}^{(I)} a_{\mathbf{k}}^{(I)}) |0_{\text{Grav}}(t)\rangle. \quad (30)$$

The above equation has the following solution:

$$|0_{\text{Grav}}(t)\rangle = \exp\left(\frac{\gamma}{2} \sum_{\mathbf{k} \in \hat{\mathcal{K}}} \{a_{\mathbf{k}}^{(I)\dagger} a_{\mathbf{k}}^{(I)\dagger} - a_{\mathbf{k}}^{(I)} a_{\mathbf{k}}^{(I)}\}\right) |0(\kappa)\rangle \quad (31)$$

This equation shows that the time evolution of the vacuum in a non-equilibrium state is realized by the condensation of the time decaying NG mode $a_{\mathbf{k}}^{(I)} a_{\mathbf{k}}^{(I)}$ ($\mathbf{k} \in \hat{\mathcal{K}}$) into the thermal vacuum.

If the deformed Hamiltonian is stopped to operate, the system returns to a thermal state, and the energy of the excitation becomes zero superficially. During the above process, energy by the deformation spreads over the spacetime and is lost at the end. Because the system of quantum fields has an infinite number of degrees of freedom, the system absorbs finite changes in energy without changing the thermal state. However, applying larger deformation leads to the instability of spacetime.

Next, we turn our attention to the characteristic phenomena on the wormhole's throat. The degrees of freedom of annihilation and creation operators are generated from separately manipulating the quantum field in regions (I) and (I I). The degrees of freedom reduce to half in the neighborhood of the midpoint of the throat. The vanishing energy of $a_{\mathbf{k}}^{(I)} a_{\mathbf{k}}^{(I)}$ shows that $a_{\mathbf{k}}^{(I)}$ behaves like $a_{\mathbf{k}}^{(I)}$ with the opposite sign of energy. This fact leads to the situation that $\sum_{\mathbf{k}} (\mathcal{D}_G(t)) (a_{\mathbf{k}}^{(I)\dagger} a_{\mathbf{k}}^{(I)\dagger} - a_{\mathbf{k}}^{(I)} a_{\mathbf{k}}^{(I)})$ is equivalent to $\sum_{\omega, k} (\mathcal{D}_G(t)) (a_{\mathbf{k}}^{(I)\dagger} a_{-\mathbf{k}}^{(I)\dagger} - a_{\mathbf{k}}^{(I)} a_{-\mathbf{k}}^{(I)})$ around the throat. ($-\mathbf{k}$) in $a_{-\mathbf{k}}$ denotes the opposite sign of energy. This effect has been discussed in [46].

As a result, \mathcal{G}_D causes a time-dependent single-mode squeezed state around the throat. Concretely, the damping characteristic of the pair $a_{\mathbf{k}}^{(I)} a_{-\mathbf{k}}^{(I)}$ ($\mathbf{k} \in \hat{\mathcal{K}}$) induces a time-dependent single-mode squeezed state. In this state, the fluctuation increases the expectation value of negative energy density. Accordingly, the single-mode squeezed state violates NEC. The violation of NEC widens the area of the throat. However, an astronaut is not allowed to enter the wormhole because TFD is a fine-tuned state [11].

From the holographic viewpoint, the widened area of the throat increases the entanglement entropy. The following is a specific explanation of entropy generation. The generator corresponding to the single-mode squeezed state at the throat has the same expression as time-dependent quantum quenches. The information metric is considered in connection with cMERA [47]. In parallel with this argument, we consider the gravitational system. $|0_{\text{Grav}}(t)\rangle$ can be expressed as follows:

$$|0_{\text{Grav}}(t)\rangle = e^{-\mathcal{S}_G/2} \exp\left\{\sum_{\mathbf{k}} a_{\mathbf{k}}^{(I)\dagger} a_{\mathbf{k}}^{(I)\dagger}\right\} |0\rangle, \quad (32)$$

where

$$\mathcal{S}_G = -\sum_{\mathbf{k}} a_{\mathbf{k}}^{(1)\dagger} a_{\mathbf{k}}^{(1)} \log \sinh^2 \mathcal{D}_G(t) - a_{\mathbf{k}}^{(1)} a_{\mathbf{k}}^{(1)\dagger} \log \cosh^2 \mathcal{D}_G(t) = -\sum_{\mathbf{k}} \mathcal{S}_{G\mathbf{k}}. \quad (33)$$

\mathcal{S}_G corresponds to the entropy operators of the boundary \mathcal{S}_R and \mathcal{S}_L , which are defined in equation (12). The expectation value of \mathcal{S}_G is equivalent to equation (14). From equation (32), we have the following equation.

$$\frac{\partial}{\partial t} |0_{Grav}(t)\rangle = -\frac{1}{2} \frac{\partial \mathcal{S}_G}{\partial t} |0_{Grav}(t)\rangle. \quad (34)$$

This equation shows the entropy variations give time evolution.

Σ_u is the one-parameter family of codimension-two closed surface, where u is the scale where $u = -\infty$ corresponds to IR and $u = 0$ corresponds to UV. Fisher information metric measuring the distance between the two different quantum states is given by follows [44, 47].

$$G_{uu} du^2 = 1 - |\langle \Phi(\Sigma_u) | \Phi(\Sigma_{u+du}) \rangle|. \quad (35)$$

In our situation, quantum distance is given as follows:

$$\begin{aligned} G_{uu} du^2 &= 1 - |\langle 0_{Grav}(t, u) | 0_{Grav}(t, u + du) \rangle| \\ &= 1 - |\langle 0_{Grav}(t, u) | e^{-\mathcal{S}_G/2} \exp\left\{ \sum_{\mathbf{k}} a_{\mathbf{k}}^{(1)\dagger} a_{\mathbf{k}}^{(1)\dagger} \right\} | 0 \rangle|. \end{aligned} \quad (36)$$

equation (32) shows that $|0_{Grav}(t)\rangle$ is determined by the $a_{\mathbf{k}}^{(1)}$ variables, which projects the system (I) with the elimination of the $a_{\mathbf{k}}^{(1)}$ variables. This fact suggests that the quantum distance among single-mode and two-mode squeezed states is different from that among two-mode squeezed states. Then, we can consider the time variation of the metric.

$$\begin{aligned} \partial_t G_{uu} du^2 &= \frac{\langle 0_{Grav}(t, u) | \frac{\partial \mathcal{S}_G}{\partial t} | 0_{Grav}(t, u + du) \rangle \langle 0_{Grav}(t, u + du) | 0_{Grav}(t, u) \rangle}{|\langle 0_{Grav}(t, u) | 0_{Grav}(t, u + du) \rangle|} \\ &+ \frac{\langle 0_{Grav}(t, u) | 0_{Grav}(t, u + du) \rangle \langle 0_{Grav}(t, u + du) | \frac{\partial \mathcal{S}_G}{\partial t} | 0_{Grav}(t, u) \rangle}{|\langle 0_{Grav}(t, u) | 0_{Grav}(t, u + du) \rangle|}. \end{aligned} \quad (37)$$

This equation shows that the time derivative of the Fisher information relates to the entropy generation rate. On the other hand, without dissipation, $\frac{\partial \mathcal{S}_G}{\partial t} = 0$ leads to $\partial_t G_{uu} = 0$. This state is an equilibrium state.

Let A and B be the locations at equal displacements $\frac{du}{2}$ from the center of the throat M in the regions (I) and (I I) directions.

$$\partial_t G_{uu} du^2 = -\frac{\partial}{\partial t} |\langle 0_{Grav}(t, u_A) | 0_{Grav}(t, u_M) \rangle| = -\frac{\partial}{\partial t} |\langle 0_{Grav}(t, u_B) | 0_{Grav}(t, u_M) \rangle|. \quad (38)$$

This equation shows that the $\partial_t G_{uu}$ corresponds to the rate of change in information from the NEC violation by a single-mode squeezed state at the throat. The wormhole's throat functions as an interface between (I) and (I I), which is a trigger for generating the correlation between quanta $a_{\mathbf{k}}^{(1)}$ and $a_{\mathbf{k}}^{(1)}$. This correlation leads to the generation of bulk entanglement entropy S_{bulk} . At the same time, equation (29) and (31) show that condensed quanta $a_{\mathbf{k}}^{(1)}$, $a_{\mathbf{k}}^{(1)}$ ($\mathbf{k} \in \hat{\mathcal{K}}$) decay with time, which decreases the correlation and also contributes to S_{bulk} .

3.3. Information processing

The previous sections describe transitions among the thermal states and the dissipative process. These descriptions suggest that boundary CFTs function as a gravitational system controller, which implements the information processing between boundary CFT and the bulk gravitational system.

We detail the information processing between boundary CFTs and the bulk gravitational system in the following. Two Fock spaces are inequivalent if the state vectors in one Fock space are not a superposition of the state vector of another Fock space [48]. Because the same observed particles describe the system on the dissipation process, the equivalent Fock space is used, and efficient information processing is realized.

The code subspace is made from the vacuum $|0_{Grav}(t)\rangle$, which belongs to the Fock space [49]. NG mode is a communication device among the particles in the condensate for information processing, which can propagate over spacetime and enables robust functioning code. Further, information transmission based on the squeezed NG mode reduces the number of steps of the information process by using the generation of quantum entanglement [48].

The eternal black hole provides an open system. If the deformation is applied, the system approaches to non-equilibrium state. The deformation brings about an outstanding feature in CFT's information processing. The main point is that the deformation brings about hierarchy in information. Deformation differentiates NG

Table 1. Hierarchy of information.

Underlayer	Upper layer	Dissipative structure	Function
Time decay	Single-mode squeezed	Wormhole	Information production
			Information transmission
	Two-mode squeezed	Defect	Information dissipation
No time decay	Single-mode squeezed	Wormhole	Information transmission
			Information transmission
	Two-mode squeezed	Defect	Information transmission

modes into those that decay with time and those that do not. This differentiation creates the underlayer of the information, which is the base structure of information. Further, the underlayer of the information is subdivided from functionality. One function is to dissipate the information, and the other function is to increase the fluctuation of the information.

Specifically, it can be explained as follows: degrees of freedom become a half around the wormhole's throat. This characteristic provides different types of squeezing at the throat and outside the throat. This functionality provides the upper layer of the information, which plays a crucial role in information processing. This differentiation of functionality creates a hierarchy of information shown as table 1.

Single-mode squeezed NG mode with time-decay constructs a wormhole, which dynamically changes the topology of spacetime to stabilize the non-equilibrium state. The wormhole's throat functions as an interface between (I) and (I I) and correlates them. This correlation leads to the generation of bulk entanglement entropy S_{bulk} . In this process, $\mathcal{S}_{R/L}$ in CFTs can calculate the generated entropy. Controller CFT gives the entropy \mathcal{S}_G for the gravitational system by interpolating the $\mathcal{S}_{R/L}$. Calculating the expectation value of $\mathcal{S}_{R/L}$ at the CFT gives the expectation value of \mathcal{S}_G , which lightens the burden of CFTs' processing power.

In this context, the wormhole is called a dissipative structure. A non-equilibrium dissipative system is a system in which dynamic stability is maintained by taking in information from the external environment and dissipating it. This system needs a dissipative structure, which is a self-organized macroscopic structure for maintaining dynamic stability [13]. The function of increasing fluctuation of information and dissipating information as an interface establishes a wormhole's self-organization.

Another realized dissipative structure is the defect formed in the code subspace $|0_{Grav}(t)\rangle$ by the time-decay NG mode shown in equation (31). The defect acquires information from the time-decaying NG mode and changes the distribution of pair-condensate in the thermal vacuum.

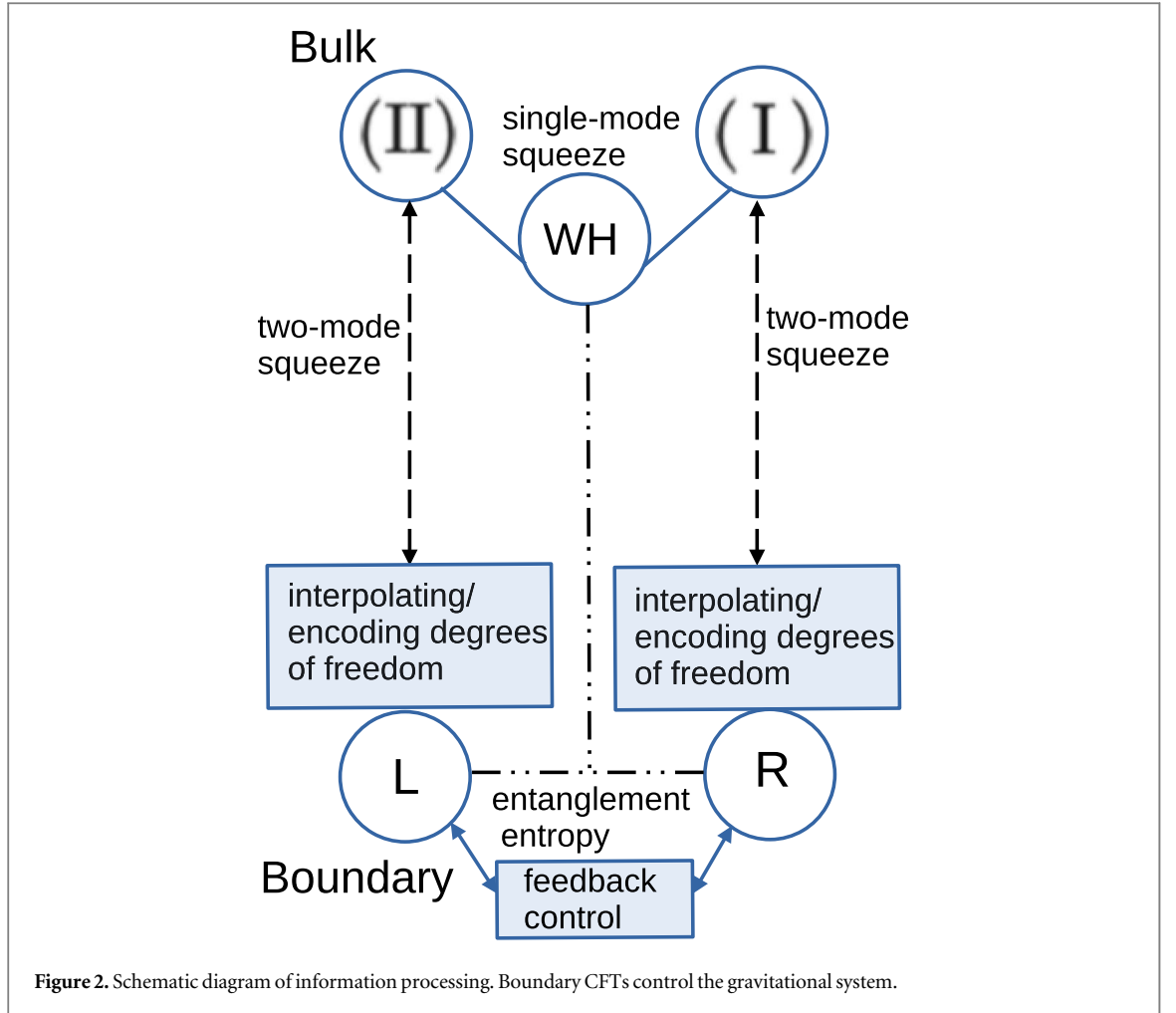
In the following, we explain the uplink and downlink of the information shown in figure 2.

Uplink: By equation (21), boundary CFT's vacuum is converted to the vacuum of the gravitational system. Deformation on the boundary CFT is expressed in modified Hamiltonian δH_b , shown in equation (9). When the deformation δH_b is applied to the boundary CFTs, CFTs as controllers cause the dissipative process to the gravitating system by operating δH_g , shown in equation (24). The identification in equation (19) is used to convert the discrete sum of CFT operators to the continuous sum of operators in the gravitational system. This manipulation leads to interpolating the degrees of freedom of δH_b .

Instead of using the identification here, entanglement manipulation can achieve interpolating. During the thermal state, the information of NG mode is entanglement concentrated for storing ebits or distilled to a maximally entangled state. This stored information can be restored for generating the necessary degrees of freedom. Classical communication among boundary CFTs for this manipulation can be neglected when storing a large number of NG modes for entanglement concentration. On the other hand, a maximally entangled state needs classical communication for manipulation. In the processing, we do not find the generation of entanglement entropy [50, 51]. This operation allows the CFTs to realize interpolating but imposes a heavy burden on the CFTs.

The added deformation creates a time-decay NG mode, which causes a change in pair-condensate by forming a defect in the code subspace. The two-mode squeezed NG mode communicates this change in the distribution of pair-condensate. The two-mode squeezed NG mode changes to the single-mode squeezed state on the throat. Further, its time decay nature widens the throat. This effect contributes to entropy generation and allows the throat to interface between the two regions (I) and (I I). This interface allows a single-mode squeezed NG mode to transmit information between the two regions. Regions (I) and (I I) can share the information through the information transmission. Then, this interface helps to correlate quanta between them, which leads to the generation of S_{Bulk} .

Downlink: A single-mode squeezed NG mode will change to a two-mode squeeze in the bulk after exchanging the information. This variation causes a change in the distribution of pair-condensate in the code subspace. This change in the distribution of pair-condensate is transmitted by the two-mode squeezed NG mode.



If CFTs can take over the entanglement manipulation, CFTs store this information for generating degrees of freedom. CFTs detect NG modes and entanglement concentrate or distill to a maximally entangled state. From this processing, bulk degrees of freedom are encoded into the boundary CFT. Boundary CFTs can keep the spacetime stable if they have the feedback function which force-quit the deformation when detecting the instability.

4. Stability of black hole

A black hole under the deformation is in a non-equilibrium state. In the following, we will explain whether it is stable or not.

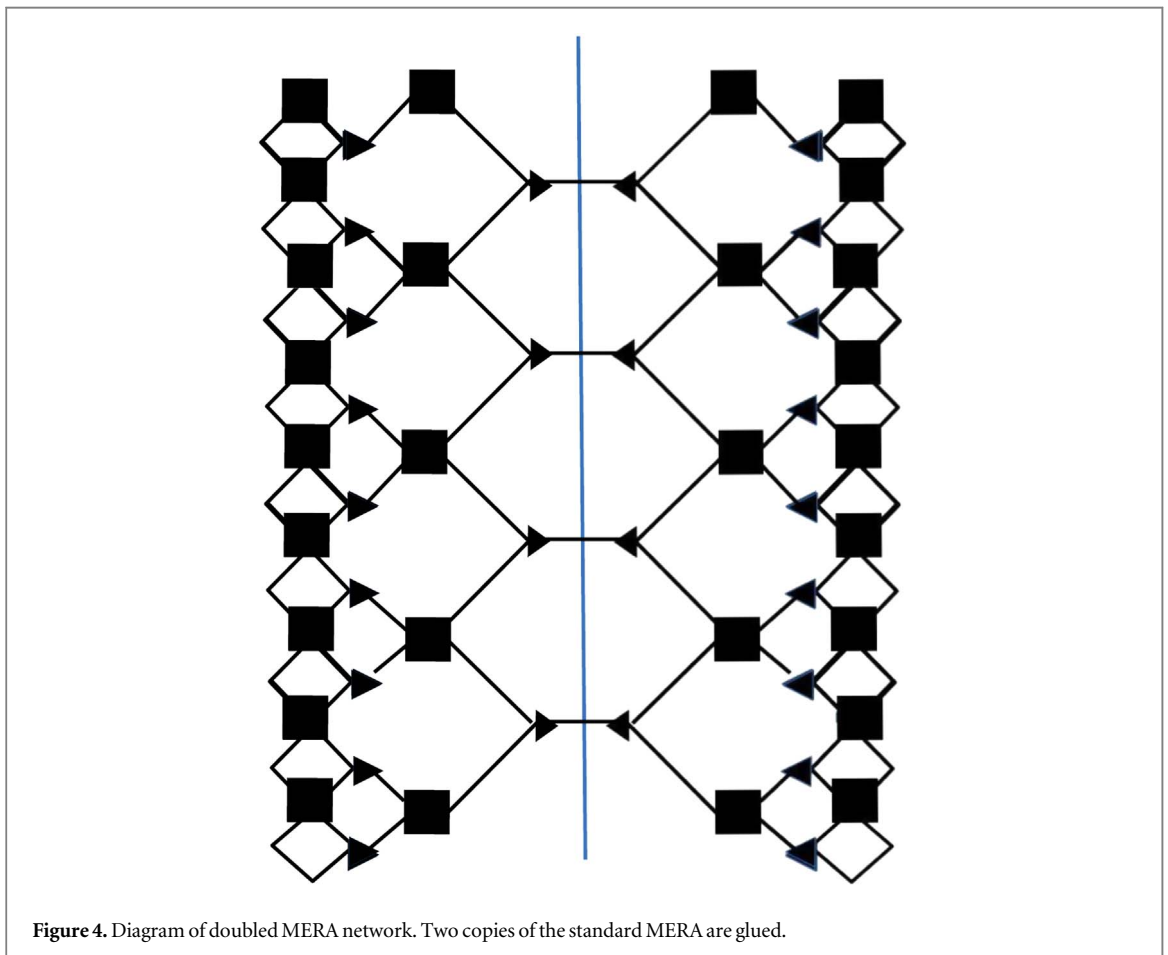
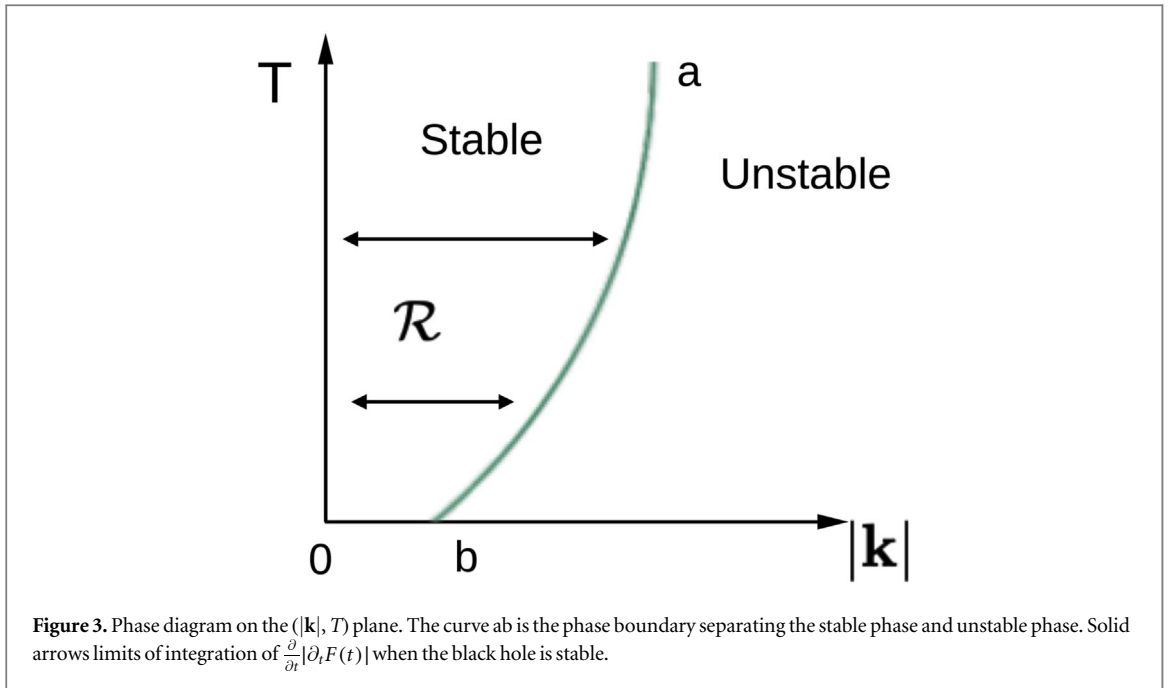
The time derivative of the quantity $F(t) = \langle 0_{Grav}(t, u_A) | 0_{Grav}(t, u_B) \rangle$, where A and B be the locations at equal displacements $\frac{du}{2}$ from the center of the throat, gives the expectation value of entropy generation rate:

$$\frac{\partial}{\partial t} F(t) = \frac{\partial}{\partial t} \langle 0_{Grav}(t, u_A) | 0_{Grav}(t, u_B) \rangle = \langle 0_{Grav}(t, u_A) | \frac{\partial \mathcal{S}_G}{\partial t} | 0_{Grav}(t, u_B) \rangle \quad (39)$$

As shown in equation (38), the information metric between A and B is given as follows:

$$\partial_t G_{uu} du^2 = \frac{(\partial_t F(t)) F(t)^* + F(t) (\partial_t F(t))^*}{|F(t)|} \quad (40)$$

$|\partial_t F(t)|$ can be regarded as Lyapunov functional, which gives a criterion for the stability of the eternal black hole. This is because $|\partial_t F(t)|$ satisfies $|\partial_t F(t)| > 0$ in non-equilibrium processes and $|\partial_t F(t)| = 0$ in equilibrium states. In a stable non-equilibrium state, $|\partial_t F(t)| > \varepsilon > 0$. According to the criterion, the non-equilibrium state is stable when $\frac{\partial}{\partial t} |\partial_t F(t)| < 0$. Because $|\partial_t F(t)|$ represents the rate of change of the non-equilibrium state, stability requires that the rate of change be decelerated. The information at the wormhole's throat describes the non-equilibrium state of the eternal black hole. Methods for analyzing stability from information geometry have already been developed [52].



In the following, we summarize the eternal black hole's stability analysis from $|\partial_t F(t)|$. figure 3 indicates limits of integration that ensure the stability of a black hole. Let \mathcal{R} be the region bounded by the curve ab , momentum $|k| = 0$, and temperature $T = 0$. Under constant temperature, the black hole is stable when the interval of integration of $\frac{\partial}{\partial t}|\partial_t F(t)|$ is a finite interval where both limits belong to \mathcal{R} , as shown by the solid arrows. Figure 3 shows that the interval of integration that gives stability becomes large at higher temperatures.

Usually, the Glansdorf-Prigogine criterion is used for non-equilibrium stability. This criterion utilizes quadratic variations of the system's entropy $-\delta^2 S/2$ as a Lyapunov functional. The quadratic variation of entropy that is locally in equilibrium is negative. This fact can be understood from positive heat capacity: $\frac{1}{2}\delta^2 S = -\frac{C_V(\delta T)^2}{2T^2}$, where C_V is heat capacity.

This functional gives the criterion for instability: the excess entropy production $\frac{d}{dt}\frac{\delta^2 S}{2}$ gives the criterion for instability. The non-equilibrium state is stable if the $\frac{d}{dt}\frac{\delta^2 S}{2} > 0$. The condition $\frac{d}{dt}\frac{\delta^2 S}{2} < 0$ is the necessary condition of the instability, but it is not a sufficient condition [13]. $\frac{d}{dt}\frac{\delta^2 S}{2} > 0$ indicates that the absolute value of the quadratic variations of entropy decrease, i.e., the fluctuations of entropy decrease. On the other hand, our discussion shows that the Lyapunov functional $|\partial_t F|$ corresponds to the rate of entropy change and that stability is achieved when the rate is decelerated. Our argument and the Glansdorf-Prigogine criterion include the same context, i.e., both criteria are consistent in that the fluctuations of entropy are decreasing. As a result, there is a possibility of a thermodynamic trade-off between $\frac{\partial}{\partial t}|\partial_t F(t)|$ and excess entropy production $\frac{d}{dt}\frac{\delta^2 S}{2}$ [52].

As the deformation becomes large, the damping characteristic becomes strong. This effect leads to the widening of the wormhole's throat and a decrease in correlation. As a result, instability in the TFD is induced, which is similar to the effect of shockwaves [53, 54].

Holographic entanglement entropy's increase can be understood from the viewpoint of doubled MERA network shown in figure 4. Dissipation accompanies the time-decay of $a_{\mathbf{k}}^{(1)}$ and $a_{\mathbf{k}}^{(1,1)}$. Boundary CFT detects the time-decay modes made of quantum $a_{\mathbf{k}}^{(1)}$ and $a_{\mathbf{k}}^{(1,1)}$. This detection shows a decrease in the physical degrees of freedom of the corresponding operator on the boundary CFTs. Then, the doubled MERA network's layer number reduces, and the holographic entanglement entropy increases.

5. Conclusion

We consider the effect of the double trace deformation on the eternal black hole. On the boundary CFTs, the deformation can be considered the dissipation in the thermofield dynamics framework. In this framework, the entanglement operator describes the dissipation effect in boundary CFT. CFTs can become the controller with a small processing load and implement a non-equilibrium state in the gravitational system. Further, CFTs utilize NG mode to detect condensation configuration and encode bulk degrees of freedom.

Corresponding to CFTs, the wormhole in spacetime and the defect in code subspace are formed as dissipative structures. These dissipative structures realize efficient processing through hierarchical information in the gravitational system. In this processing, NG mode functions as a communication device over spacetime.

Further, the Fisher information metric renders the Lyapunov functional, which gives a criterion for the stability of the eternal black hole. Instability is caused as the deformation becomes large. Strong damping characteristic leads to the widening of the wormhole's throat and the decrease in correlation. As a result, instability in the TFD is induced, which is similar to the effect of shockwaves [53, 54]. Since these factors that create instability are dimension-independent, similar instabilities exist for higher-dimensional black holes.

Charged black holes have rich spacetime structure when the model parameters are varied, where spacetime properties determine the condition of phase transition. Further, the presence of charge introduces a correction term to the entropy. As a result, we have a more complex phase diagram for an eternal charged black hole [22, 55, 56].

It is known that topological phase transitions can be detected from the c -function [57]. In our situation, although NEC has been broken in three-dimensional spacetime, the information metric detects the instability and plays a similar role as the c -function.

Data availability statement

No new data were created or analysed in this study.

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