Empirical residual *n-p* interactions in odd-odd, even-even, and odd-A nuclei

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Introduction

On the basis of similarity between the pairing of two nucleons with opposite projections of angular momentum and the quasi particle formed by two electrons with equal and opposite momenta [1-3], the theory of superconductivity was introduced in the nuclear regime. This utilization of the BCS approximation into the nuclei, lead to an anomaly, as the introduction of a proton and a neutron in the even-even core altered the measured mass values lesser than the mass of the added nucleons. This energy difference was attributed to the pairing interaction between the odd neutron and odd proton, which is referred to as the residual *n-p* interaction energy.

In our present work, we have tried to parameterize this residual interaction for four different species of nuclei using the latest B.E. values from AME-2020 [4] and the fourth-order expansion of the Taylor series for the concerned nuclear mass for the first time. The parameters have been obtained for even-even, even-odd, odd-even and odd-odd nuclei falling in the mass range between $50 < A < 253$.

Methodology

We begin with the evaluation of the semi-empirical residual n-p interaction energies. This requires the expansion of the Taylor series mass formula to fourth order of its derivative. The explicit form of these formulae has been presented in our recent work [5]. The formulation uses recently updated AME-2020 for the calculation of the semi-empirical values of the interaction energy. Mass of each nucleon encounters different correction term. Even-even mass being the core nuclei, odd-neutron and odd-proton experience just the neutron and proton pairing energies, respectively. But, the anomaly of mass in the odd-odd nuclei is, that the measured mass is lesser than the one expected on addition of the two nucleons. This residual energy has been predicted using several theoretical models including the traditional model [6], the Vogel et al. model [7] and the Madland Model [8]. The residual interaction energy formulation proposed by these approaches are shown in Table 1.

Table 1: General forms of residual interaction.

Model	$\overline{\boldsymbol{\delta}}$
Traditional [6]	d/A
Vogel et al. [7]	d/A
Madland et al.[8]	$h/(B_s A^{2/3})$

After the semi-empirical evaluation, we re-determine the parameters which appear in the theoretical formulations, using the minimization of the chi-square deviation. A significant amount of deviation was observed in the obtained parameters for different species of the nuclei. Thus, our parameters are classified on the basis of the species of the nuclei which include the even-even, even-odd, odd-even and odd-odd nuclei. For the purpose of estimation of the residual n-p interaction energy values, the generalized parameters for the entire nuclear regime are calculated as well.

Results and discussion

In this work, we have successfully derived the semi-empirical expressions for the residual n-p interactions. Making use of the formulations, we tabulate residual n-p interaction energy values for nuclei ranging between $50 < A < 253$. The values generated are used for parameter re-determination for the theoretically proposed expressions. For instance, for the entire nuclear regime, irrespective of the species of nuclei, general parameters have been fit using the traditional model. The expression for which is listed below,

$$\bar{\delta} = {}^{d}/_{A}$$

Here, the evaluated parameter for the general nuclear range is calculated to be $d = 32.51\ MeVs$ and 'A' is the mass number of the nucleus. The fitting for the same can be seen in Figure 1. Using the traditional model, we have calculated the parameters for different species of nuclei. For example, the 'd' parameter for even-even is 34.59 MeV and for odd-odd is 31.99 MeV. Since the two values appear to vary appreciably.

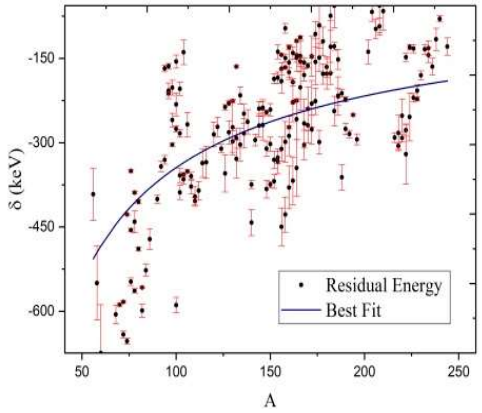

Figure 1: Variation of residual interaction with mass number irrespective of nuclear species.

A similar approach has been used to obtain the parameters for the Vogel et al. model and the Madland model. The subsequent results can be found in the paper for the nuclei raging between $50 < A < 253$. We have also optimized the parameters for these models separately for different nuclear species and the values of these parameter and corresponding procedure is discussed somewhere else [5].

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