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Drag force of moving quark in STU background

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Abstract In this paper we consider a quark moving in $D=5$, $\mathcal{N}=2$ supergravity thermal plasma. By using the three charges non-extremal black hole solution (STU solution) we calculate the drag force on the quark and the diffusion constant from the AdS/CFT correspondence.

1 Introduction

The AdS/CFT correspondence, which is one of the interesting dualities in physics, was first suggested by Maldacena [1] and then developed by Witten and Gubser et al. [2, 3]. The most famous case of AdS/CFT correspondence is the duality between IIB string theory in $AdS_5 \times S^5$ space and $\mathcal{N}=4$ supersymmetric Yang–Mills gauge theory on the four-dimensional boundary of AdS_5 space [4–6]. One of the various applications of the AdS/CFT correspondence is in QCD. By using a string theory description one can attack the complicated problems of QCD. One of these complicated problems is moving charged particles in a medium. If one would like to consider a moving quark in plasma, there are some problems in the QCD description, while for instance the energy loss of a heavy quark moving in supersymmetric Yang–Mills thermal plasma has been investigated easily by the AdS/CFT correspondence [7–13]. This problem is also considered in the case of a quark–antiquark pair [7, 14–16] and the jet-quenching parameter has been calculated [17–27]. According to Maldacena's dictionary we have the following model: a moving quark in thermal plasma is considered such as a string in AdS space stretched from the D-brane to the black hole horizon. The end point of the string attached to the D-brane represents a quark. The drag force on the quark due to plasma is interpreted as a current of energy and momentum along the string. The energy and momentum is transferred from the string end point to the horizon.

The correspondence between $\mathcal{N}=2$ supergravity theory and string theory will be interesting in physics [28, 29]. Actually, solutions of $\mathcal{N}=2$ supergravity may be solutions of supergravity theory with more supersymmetry i.e. $\mathcal{N}=4$ and $\mathcal{N}=8$. The $\mathcal{N}=2$ supergravity theory in five dimensions can be obtained by compactifying eleven-dimensional supergravity in a three-fold Calabi–Yau case [30]. Also anti-de Sitter supergravity can be obtained by gauging the $U(1)$ subgroup of the $SU(2)$ group in $\mathcal{N}=2$ supersymmetric algebra.

Here we consider a moving quark in $D=5$, $\mathcal{N}=2$ supergravity thermal plasma and suppose that the quark is influenced by an external force F. It has the momentum P, mass M and velocity v; also the friction coefficient of the plasma is μ. We can write the equation of motion in the form of $\dot{p}=F-\mu p=F-\mu M v$. To obtain some information about the drag force, it is useful to consider two special cases. One, we assume that the momentum of the charged particle is constant. Hence by using $F=\mu p$ for a particle with mass M and non-relativistic momentum $p=Mv$ one can obtain $\mu M=\frac{F}{v}$. So, by measuring the velocity of a particle for a given force we can obtain μM. It shows that we cannot find μ independently. Second, we assume that an external force does not exist, so from the equation of motion one can find $p(t)=p(0)e^{-\mu t}$. In other words, by measuring the ratio $\frac{\dot{p}}{p}$ or $\frac{\dot{v}}{v}$ we can determine μ without any dependence to M. This leads us to obtaining the drag force for a moving quark in plasma [7, 8].

In order to calculate the drag force by using the AdS/CFT correspondence we will consider the three charges non-extremal black hole solution (STU model) [29, 31]. Already Behrndt et al. obtained the STU solutions as a special solutions of equation of motion for $D=5$, $\mathcal{N}=2$ gauge supergravity theory [29].

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This paper is organized as follows. In Sect. 2 we introduce the STU model and obtain canonical momentum densities for the non-extremal black hole background. Then in Sect. 3 we will obtain the drag force and the diffusion constant for various states of the string corresponding to the quark. Quasi-normal modes of strings are considered in Sect. 4, and finally in Sect. 5 we have the conclusion and discussion, and also we give some suggestions for future work.

2 STU model solution

STU model solution is given by the following background metric [31],

$$ds^2 = -\frac{f_k}{\mathcal{H}^{\frac{2}{3}}}dt^2 + \mathcal{H}^{\frac{1}{3}}\left(\frac{dr^2}{f_k} + r^2 d\Omega_{3,k}^2\right),$$
$$f_k = k - \frac{m_k}{r^2} + \frac{r^2}{L^2}\mathcal{H}, \tag{1}$$

where $\mathcal{H} = H_1 H_2 H_3$ and $H_i = 1 + \frac{q_i}{r^2}$ $(i = 1, 2, 3)$. Here we have three real scalar fields $X^i = \frac{\mathcal{H}^{\frac{1}{3}}}{H_i}$ which satisfy $X^1 X^2 X^3 = 1$. r is a radial coordinate along the black hole and $f_k(r) = 0$ shows that the horizon of the black hole is in $r = r_h$.

The three charges q_i are related to three independent angular momenta in ten dimensions and k indicates the curvature of space. $k = 1$ shows a three-dimensional sphere with unit radius, and $k = 0$ indicates three-dimensional flat real space. Using Kaluza–Klein dimensional reduction of a near-extremal spinning brane, we obtain the corresponding solution of this space [32]. Finally for $k = -1$ we have a metric of a pseudo sphere, i.e. the space is in hyperbolic form.

At first, if we considered $k = 1$ and supposed the three charges to be equal ($q_1 = q_2 = q_3 = q$); the results would be similar to Refs. [33, 34] and in this case $X^1 = X^2 = X^3 = 1$. In Ref. [34] we consider a moving quark through $\mathcal{N} = 2$ supergravity thermal plasma and we obtained the drag force. Also we found the interesting limit where our solutions in $\mathcal{N} = 2$ supergravity are corresponding to the solutions in the $\mathcal{N} = 4$ super Yang–Mills theory. In this paper we consider the STU background and will try to solve the general case for arbitrary values of k. From Ref. [29] we know that $d\Omega_{3,k}^2 = d\chi^2 + (\frac{\sin\sqrt{k}\chi}{\sqrt{k}})^2(d\theta^2 + \sin^2\theta\, d\phi^2)$, and we would like to assume that the motion in a manifold with metric $d\Omega_{3,k}^2$ is only on a transverse axis which we call x. Hence in this special case one can write $d\Omega_{3,k}^2 \to dx^2$.

Also we assume that only one charge exists, $q_1 = q$ ($q_2 = q_3 = 0$); then we can write

$$ds^2 = -\frac{f_k}{H^{\frac{2}{3}}}dt^2 + H^{\frac{1}{3}}\left(\frac{dr^2}{f_k} + r^2 dx^2\right),$$

$$f_k = k - \frac{m_k}{r^2} + \frac{r^2}{L^2}H, \tag{2}$$
$$H = 1 + \frac{q}{r^2}.$$

As we know, the dynamics of an open string is described by the Nambu–Goto action,

$$S = -T_0 \int d\tau\, d\sigma\, \sqrt{-g}, \tag{3}$$

where T_0 is the string tension and σ, τ are coordinates of the string world sheet. Also g_{ab} is the metric on the world sheet of the string, and $g = \det g_{ab}$.

Using the static gauge we set $\tau = t$, $\sigma = r$; then the string world sheet is described by $x(t, r)$, so one can obtain

$$-g = \frac{1}{H^{\frac{1}{3}}} - \frac{H^{\frac{2}{3}} r^2}{f_k L^2}\dot{x}^2 + \frac{f_k r^2}{L^2 H^{\frac{1}{3}}} x'^2, \tag{4}$$

and the Lagrangian density is $\mathcal{L} = -T_0\sqrt{-g}$. Therefore, the string equation of motion is the following:

$$\frac{\partial}{\partial r}\left(\frac{f_k r^2}{L^2 H^{\frac{1}{3}}\sqrt{-g}} x'\right) - \frac{H^{\frac{2}{3}} r^2}{f_k L^2}\frac{\partial}{\partial t}\left(\frac{\dot{x}}{\sqrt{-g}}\right) = 0. \tag{5}$$

But in order to obtain the total energy and momentum of the string and the drag force on it, we have to calculate the canonical momentum densities,

$$\pi_t^0 = -\frac{T_0}{\sqrt{-g}}\frac{(1 + \frac{f_k r^2}{L^2}x'^2)}{H^{\frac{1}{3}}},$$
$$\pi_x^0 = \frac{T_0}{\sqrt{-g}}\frac{H^{\frac{2}{3}} r^2}{f_k L^2}\dot{x}, \tag{6}$$
$$\pi_r^0 = -\frac{T_0}{\sqrt{-g}}\frac{H^{\frac{2}{3}} r^2}{f_k L^2}\dot{x}x',$$

and

$$\pi_t^1 = \frac{T_0}{\sqrt{-g}}\frac{f_k r^2}{L^2 H^{\frac{1}{3}}}\dot{x}x',$$
$$\pi_x^1 = -\frac{T_0}{\sqrt{-g}}\frac{f_k r^2}{L^2 H^{\frac{1}{3}}}x', \tag{7}$$
$$\pi_r^1 = \frac{T_0}{\sqrt{-g}}\left(-\frac{1}{H^{\frac{1}{3}}} + \frac{H^{\frac{2}{3}} r^2}{f_k L^2}\dot{x}^2\right).$$

Then we can find the drag force by $\dot{p} = \pi_x^1$. Also the total energy and momentum of the string are described by following relations:

$$E = -\int dr\, \pi_t^0,$$
$$p = \int dr\, \pi_x^0. \tag{8}$$

In the two next sections we use the above relations for some special cases and will obtain drag force, diffusion constant, total energy and momentum of the string.

3 Straight and curved string

In this section we will consider three different cases for a string and calculate the drag force. As we mentioned, a moving quark in $\mathcal{N}=2$ supergravity thermal plasma corresponds to the string which stretches from the D-brane to black hole, so the end point of the string on the D-brane represents the quark. It means that according to the Maldacena dictionary, a quark in CFT is equal to a string in AdS space. On the other hand, the existence of quark flavor causes one to have a D-brane, and by adding temperature to the plasma in CFT we have a black hole in AdS space.

At first we consider a static quark in a plasma which is equal to a static string stretched directly from $r=r_m$ on the D-brane to $r=r_h$ in black hole. It means that we deal with the simplest solution of (5), $x(t,r)=x_0$ where x_0 is a constant. In this case the total momentum of the string and drag force are zero and since $-g=H^{-\frac{1}{3}}$ we obtain the total energy of the string, by using the relations (6), (7) and (8),

$$E=T_0\left[\frac{3r(1+\frac{r^2}{q})^{\frac{1}{6}}}{4(1+\frac{q}{r^2})^{\frac{1}{6}}}2F_1\left(\frac{2}{3},\frac{1}{6},\frac{5}{3},-\frac{r^2}{q}\right)\right]_{r_h}^{r_m}, \tag{9}$$

where $2F_1(\frac{2}{3},\frac{1}{6},\frac{5}{3},-\frac{r^2}{q})$ is a hypergeometric function. It should be mentioned that in the limit of $q\to 0$ one can obtain $E=T_0(r_m-r_h)$, which agrees with the case of a moving quark in $\mathcal{N}=4$ SYM thermal plasma [7, 34].

In the second case the straight string is moving with constant velocity, $x(r,t)=x_0+vt$, which can be the solution of (5), so we have

$$-g=\frac{1}{H^{\frac{1}{3}}}-\frac{H^{\frac{2}{3}}r^2}{f_kL^2}v^2. \tag{10}$$

The canonical momentum densities are

$$\begin{aligned}\pi_t^0&=-\frac{T_0}{\sqrt{-g}}\frac{1}{H^{\frac{1}{3}}},\\ \pi_x^0&=\frac{T_0}{\sqrt{-g}}\frac{H^{\frac{2}{3}}r^2v}{f_kL^2},\\ \pi_r^1&=\frac{T_0}{\sqrt{-g}}\left(\frac{H^{\frac{2}{3}}r^2}{f_kL^2}v^2-\frac{1}{H^{\frac{1}{3}}}\right),\end{aligned} \tag{11}$$

and $\pi_r^0=\pi_t^1=\pi_x^1=0$. However, in that case we have imaginary $\sqrt{-g}$, so this is not a physical solution [7, 34]; therefore we consider the other case.

The most realistic and important case is a curved string moving with constant velocity v. So the relation

$$x(r,t)=x(r)+vt \tag{12}$$

satisfies (5) and leads us to the following expression:

$$-g=\frac{1}{H^{\frac{1}{3}}}-\frac{H^{\frac{2}{3}}r^2}{f_kL^2}v^2+\frac{f_kr^2}{L^2H^{\frac{1}{3}}}x'^2, \tag{13}$$

and therefore from (5) we have

$$\frac{\partial}{\partial r}\left(\frac{f_kr^2}{H^{\frac{1}{3}}L^2v\sqrt{-g}}x'\right)=0. \tag{14}$$

The solution of (14) can be obtained as

$$x'^2=\frac{L^2C^2v^2}{f_k^2r^4}\frac{f_kL^2H^{\frac{1}{3}}-H^{\frac{4}{3}}r^2v^2}{f_k-\frac{L^2C^2v^2H^{\frac{1}{3}}}{r^2}}, \tag{15}$$

where C is constant, and it is easy to find that

$$\begin{aligned}\pi_t^1&=\frac{T_0Cv^2}{L^2},\\ \pi_x^1&=-\frac{T_0Cv}{L^2}.\end{aligned} \tag{16}$$

Now the important problem is to calculate the constant C. It can be obtained by the condition $-g\geq 0$, otherwise we have imaginary action, energy and momentum. By using x'^2 in (13) in the case of $-g>0$, we find C as follows:

$$C=\pm\left(\frac{r_c}{L}\right)^2\left(1+\frac{q}{r_c^2}\right)^{\frac{1}{3}}, \tag{17}$$

where r_c is the root of the following equation,

$$r^4(1-v^2)\left(1+\frac{q}{r^2}\right)+kL^2r^2-m_kL^2=0. \tag{18}$$

r_c is the minimum radius, which gives a $-g$ equal to zero. In order to obtain π_t^1 and π_x^1 we use (16) and (17),

$$\begin{aligned}\pi_t^1&=T_0\frac{v^2}{L^4}r_c^2\left(1+\frac{q}{r_c^2}\right)^{\frac{1}{3}},\\ \pi_x^1&=-T_0\frac{v}{L^4}r_c^2\left(1+\frac{q}{r_c^2}\right)^{\frac{1}{3}},\end{aligned} \tag{19}$$

which leads us to the following equation:

$$\dot{p}=\pi_x^1=-\mu p=-\mu Mv. \tag{20}$$

In other words the drag force coefficient is given by

$$\mu M=\frac{T_0}{L^4}r_c^2\left(1+\frac{q}{r_c^2}\right)^{\frac{1}{3}}, \tag{21}$$

and we can also calculate the quark diffusion constant $D = \frac{T}{\mu M}$, where T is the Hawking temperature of the black hole. So we have

$$D = \frac{TL^4}{T_0} \frac{1}{r_c^2(1+\frac{q}{r_c^2})^{\frac{1}{3}}}. \tag{22}$$

Next we are going to consider a special condition; we take the $q \to 0$ limit in which there is a relation between $\mathcal{N}=2$ supergravity and $\mathcal{N}=4$ super Yang–Mills theory [34]. In this limit $H \longrightarrow 1$ and we have

$$r_c^2 = \frac{L}{2(1-v^2)}\left(-kL \pm \sqrt{k^2L^2 + 4m_k(1-v^2)}\right); \tag{23}$$

therefore,

$$C = \pm\frac{1}{2(1-v^2)}\left(-k \pm \sqrt{k^2 + \frac{4m_k(1-v^2)}{L^2}}\right), \tag{24}$$

and one can obtain x' as follows:

$$x' = \frac{vL^4}{r^4 + kL^2r^2 - m_kL^2} \frac{r_h^4}{r_c^4}. \tag{25}$$

By integrating over r and using (12) we find $x(r)$ to be proportional to $\tan^{-1} r$. In that case we must use a positive sign of the constant C in relation (17) or (24); it shows the energy current from the quark to the horizon. But the negative sign of C shows energy current from the horizon to the quark where the string moves and pulls the quark, so it is a non-physical situation and we leave this state and give a positive sign for the constant C.

The drag force in this limit can be given by

$$\pi_x^1 = -\frac{T_0 v}{2L^2(1-v^2)}\left(-k \pm \sqrt{k^2 + \frac{4m_k(1-v^2)}{L^2}}\right). \tag{26}$$

We can neglect the square terms of velocity for the heavy quark, therefore by increasing the speed, the drag force increases too. Then one can write the drag force coefficient as

$$\mu M = \frac{T_0}{2L^2}\left(-k \pm \sqrt{k^2 + \frac{4m_k}{L^2}}\right), \tag{27}$$

and the diffusion constant of the quark is

$$D = \frac{2TL^2}{T_0 m}\left(-k \pm \sqrt{k^2 + \frac{4m_k}{L^2}}\right)^{-1}. \tag{28}$$

Here is another interesting limit to add to $q \to 0$, which is $m_k \to 0$. In that case the critical radius is obtained by the following relation:

$$r_c^2 = -\frac{kL^2}{1-v^2}, \tag{29}$$

in that case we have two values for the constant C. The first is $C=0$, which yields zero drag force, and the second is

$$C = -\frac{k}{1-v^2}, \tag{30}$$

therefore we have the following components of energy and momentum density:

$$\pi_t^1 = -\frac{T_0 k v^2}{L^2(1-v^2)}, \qquad \pi_x^1 = \frac{T_0 k v}{L^2(1-v^2)}. \tag{31}$$

As before the constant C must be positive and on the other hand the critical radius r_c should be real; thus in (29) and (30) we must have $v^2 > 1$, so we cannot eliminate the square term of the velocity. Then we have $\mu M = \frac{T_0 k}{L^2(1-v^2)}$ and one can obtain

$$D = \frac{TL^2(v^2-1)}{T_0 k}. \tag{32}$$

We will explain the $q \to 0$ and $m_k \to 0$ limits in conclusion.

The physical description of the loss energy mechanism in terms of some microscopic parameters in $\mathcal{N}=2$ SYM field theory is a challenge. In the AdS dual description, energy and momentum current along the string flow from D-brane toward black hole horizon. Obviously that part of the string near the horizon has no interaction with the quarks. Therefore, the energy loss should not be regarded in the surrounding plasma as a result of scattering in thermal medium. The mentioned scattering would correspond to small fluctuations in the string world sheet. We will discuss such small fluctuations in the next section. In the classical string dynamics, there is nothing for relativistic velocities, $v \to 1$, so from (32) one can see that $D = 0$ for the relativistic limit.

4 Quasi-normal modes

In this section we consider a quark moving in the $\mathcal{N}=2$ supergravity thermal plasma without any external field. The goal of this section is to study the behavior of the string after a long time and with a low velocity. We consider the dynamics of such a system at a long time as a small perturbation in the static string which describes the rest of quark. Indeed quasi-normal modes are the classical perturbations with the non-zero scattering in a given gravitational background [35–40]. In that case we have only the out-going boundary conditions at the black hole horizon which capture the dissipative nature of the process. Therefore under such boundary conditions we have the quasi-normal modes of the string world sheet. Hence for low velocity we will neglect the $\dot{x}^2$ and the x'^2 in the expressions of $-g$, so (5) turns into

$$\frac{\partial}{\partial r}\left(\frac{f_k r^2}{L^2 H^{\frac{1}{6}}}x'\right)-\frac{H^{\frac{5}{6}}r^2}{f_k L^2}\ddot{x}=0. \tag{33}$$

Putting a solution in the form of $x(t,r)=x(r)e^{-\mu t}$ in (33) gives the following eigenvalue equation:

$$Ox=\mu^2 x, \tag{34}$$

where O is defined as

$$O=\frac{f_k L^2}{H^{\frac{5}{6}}r^2}\frac{d}{dr}\frac{f_k r^2}{L^2 H^{\frac{1}{6}}}\frac{d}{dr}. \tag{35}$$

In order to obtain $x(r)$, we expand it in terms of μ powers,

$$x(r)=x_0(r)+\mu x_1(r)+\mu^2 x_2(r)+\cdots, \tag{36}$$

then we can replace (36) in the eigenvalue equation (34) to find

$$\begin{aligned} &Ox_0(r)=0,\\ &Ox_1(r)=0,\\ &Ox_2(r)=x_0(r). \end{aligned} \tag{37}$$

Now, the boundary conditions can be applied. As we know, our solutions should satisfy the Neumann boundary condition, so $x'(r_m)=0$. Thus we select the $x_0=A$ for the first term in (36) where A is a constant. Then one can apply the Neumann boundary condition to (36) and obtain

$$x'(r_m)=\mu x_1'(r_m)+\mu^2 x_2'(r_m)=0. \tag{38}$$

Then by solving (37) in the $q\to 0$ and $m_k\to 0$ limits one can find

$$\begin{aligned} &x_1'(r)=\frac{-AL^4}{r^4+kL^2r^2},\\ &x_2'(r)=\frac{AL^4}{r^4+kL^2r^2}\left(r-\sqrt{kL^2}\tan^{-1}\frac{r}{\sqrt{kL^2}}\right). \end{aligned} \tag{39}$$

Now by inserting (39) into (38), we obtain the friction coefficient μ, and then write the diffusion constant as follows:

$$D=\frac{T}{M}\left[r_m-\sqrt{kL^2}\tan^{-1}\frac{r_m}{\sqrt{kL^2}}\right]. \tag{40}$$

In the other words, μ is the lowest eigenvalue of the operator O or the lowest quasi-normal mode of the string which leads us to the drag force and the diffusion constant of the quark.

Now we would like to obtain the total energy and momentum of the string. With respect to the exponential time-dependence of our solution one can rewrite the equation of motion as follows:

$$\frac{\partial}{\partial r}\left(\frac{f_k r^2}{L^2 H^{\frac{1}{6}}}x'\right)=-\mu\frac{H^{\frac{5}{6}}r^2}{f_k L^2}\dot{x}=\mu^2\frac{H^{\frac{5}{6}}r^2}{f_k L^2}x. \tag{41}$$

Hence by using the (6), (8) and (41), we find the total momentum of the string,

$$p=\frac{T_0}{\mu L^4}\frac{r_{\min}^4+(q+kL^2)r_{\min}^2-m_kL^2}{(1+\frac{q}{r_{\min}^2})^{\frac{1}{6}}}x'(r_{\min}), \tag{42}$$

where we used the Neumann boundary condition. In the $q\to 0$ limit, we have

$$p=\frac{T_0}{\mu L^4}\left(r_{\min}^4+kL^2r_{\min}^2-m_kL^2\right)x'(r_{\min}). \tag{43}$$

In order to calculate the total energy, we expand $(-g)^{-\frac{1}{2}}$ to the second order of the velocity and obtain

$$\begin{aligned} E=-\frac{T_0}{2}\Bigg[&\frac{3r_{\min}(1+\frac{r_{\min}^2}{q})^{\frac{1}{6}}}{2(1+\frac{q}{r_{\min}^2})^{\frac{1}{6}}}{}_2F_1\left(\frac{2}{3},\frac{1}{6},\frac{5}{3},\frac{-r_{\min}^2}{q}\right)\\ &+\frac{r_{\min}^4+(q+kL^2)r_{\min}^2-m_kL^2}{L^4(1+\frac{q}{r_{\min}^2})^{\frac{1}{6}}}\\ &\times x(r_{\min})x'(r_{\min})\Bigg], \end{aligned} \tag{44}$$

where we used the Neumann boundary condition too. For the $q\to 0$ limit we have

$$\begin{aligned} E=T_0\Bigg[&r_m-r_{\min}-\frac{1}{2L^4}\left(r_{\min}^4+kL^2r_{\min}^2-m_kL^2\right)\\ &\times x(r_{\min})x'(r_{\min})\Bigg], \end{aligned} \tag{45}$$

which is different only in the third term from Ref. [34]. We expect that relations (43) and (50) together with $x(r,t)=x(r)e^{-\mu t}$ lead us to the simple relation between the energy and momentum $E=T_0(r_m-r_{\min})+\frac{p^2}{2M_{\text{kin}}}$, so one can check validity of this relation easily. We can interpret the first term in the right hand side of above equation as M_{rest}.

5 Conclusion

In this paper, we considered a moving quark in the three charges non-extremal black hole background and obtained the drag force by using the AdS/CFT correspondence. However, we set $q_2=q_3=0$ and supposed that the only non-zero black hole charge is $q_1=q$. We obtained our results for general space with arbitrary k, and here we would like to represent solutions of different space.

First, we consider the flat space with $k=0$. In this case the roots of (18) are

$$r_c{}^2 = -\frac{q}{2} \pm \frac{\sqrt{q^2 + \frac{4m_0L^2}{1-v^2}}}{2}. \tag{46}$$

Here, there is an interesting limit: $\frac{4m_0L^2}{q^2(1-v^2)} \ll 1$. In this limit we have the physical solution for $v^2 < 1$ and one can neglect the square terms of velocity and find

$$D = \frac{TL^2}{T_0}\left(\frac{qL^2}{m_0^2}\right)^{\frac{1}{3}}. \tag{47}$$

It means that the drag force coefficient μM is proportional to $q^{-\frac{1}{3}}$, for small velocities, similar to (26), we find that the drag force is proportional to v.

Second, we consider the spherical space, i.e., $k=1$. In this configuration by setting the $H \to H^3$, $m_k \to \eta$, $\Lambda^2L^2 = 1$ and $q = \eta \sinh^2\beta$ we reduce our results to Ref. [34], where η is the non-extremality parameter, Λ is the cosmological constant and β is related to the electric charge of black hole.

We see that the $q \to 0$ limit is corresponding to the $\eta \to 0$ limit and one can interpret m_k as the non-extremality parameter. For this reason, to calculate (39) and (40) we take both limits, $q \to 0$ and $m_k \to 0$. In other words, we may be able to write $q \propto m_k$ (particularly for $k=1$).

Third, we consider the hyperbolic space corresponding to $k=-1$. This space is not interesting for us; indeed the only space we can give some physical relevance is the S^5 space, because from the AdS/CFT correspondence we know that a gauge theory on the boundary of AdS_5 space is corresponding to the string theory in $AdS_5 \times S^5$ space. Therefore, it seems that the $k=1$ solutions have more physical relevance in the present problem.

There are several interesting problems in the case of $q_1 \neq q_2 \neq q_3$, for example, one can obtain the drag force and the shear viscosity [31, 41–43] or the jet-quenching parameter [17–27]. In that case some authors have shown in two papers [17–19] that the string configuration that Liu et al. take to calculate the jet-quenching parameter is not the most energetically-favored configuration and once one uses the most energetically-favored configuration the result for the jet-quenching parameter becomes zero. This point of view may be checked in different backgrounds.

Also the effect of higher derivative corrections on the drag force, the diffusion constant [44–50] and the ratio of the shear viscosity η to the entropy density s [51–53] will be an interesting problem in future.

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