The Progress of Black Holes: Principles & Physical Detection Technology

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Abstract. Humans have been trying to explain black holes since the 19th century, using theoretical understanding and observations of the universe. Incandescent body thermal radiation has been the focus of extensive theoretical and experimental research for over a century. This paper will discuss the concept of a black hole and use information retrieval to generate a panoramic image of it. The content illustrates black holes from three different perspectives: description of black holes using mathematical methods and data, models, and detection. A black hole is a controversial object, and no one knows for sure what it is. Indeed, multiple numerical simulations have been carried out to investigate the critical behaviors of black holes with various geometrical configurations in arbitrary dimensions. We will sum up the black hole knowledge so far and discuss the recent progress in terms of black hole exploration, i.e., shed light for future black holes model construction and detection.

Keywords: Black hole, Schwarzschild black hole, Kerr black hole, mutual motion, accretion disk X-ray detection, Einstein’s Field equation, stellar black hole, supermassive black hole

1. Introduction

The Black hole is a particular type of celestial body that possesses enormous mass within a little volume (gigantic gravitational force for which even light couldn’t escape within the event horizon). The level of technology falls short in the direct detection of black holes. Yet, "edge information" emitted by friction against the acceleration of the black hole's gravity before the object was sucked into the event horizon and the movement of the surrounding celestial bodies comprehensively inferred the characteristics and some basic information. About one year after Einstein published the gravitational field equation, Schwarzschild used it to generate an exact solution: As the radius of a star shrinks to a critical value known as the Schwarzschild radius, no signal can escape from the star's surface. The wavelength of radiation at the object's surface (event horizon) would be stretched to infinity, and the frequency tends to be zero [1].

For a long time in the past, a black hole has been regarded as a very controversial object. It is only a celestial body speculated by scientist Schwarzschild according to Einstein's theory of relativity. Some even doubt its existence. However, in 2019, scientists finally got a real picture of the black hole, which provided direct "visual" evidence of the existence of the black hole. The first glance at the black hole makes it possible to verify Einstein's general relativity in the strong gravitational field and carefully study the matter accretion and relativistic jet near the black hole. In 2021, astrophysicist Dan Wilkins from Stanford University observed X-rays in a black hole. This result is the first time human beings have detected light from behind a black hole, entirely in line with the prediction of light in Einstein's general theory of relativity. With the progress of various astrophysical observation equipment and the improvement of computer ability, more and more data about black holes are being collected. Information about the black hole with higher accuracy makes our current research on black holes more convenient and persuasive. By studying various strange phenomena of black holes, we can understand a lot of unknown knowledge about the universe and physics and even use these findings to develop and improve scientific and technological products. For example, black hole/qubit correspondence can help us better understand how the information will be disrupted and reorganized in quantum entanglement with the help of black holes.

Since the gravity of black holes and other physical properties of the gas in the accretion disk caused by black holes are tremendous (e.g., Ultra-high temperature, high pressure, super magnetic field, etc.). These extreme conditions assist scientists in verifying some basic theories in physics. Furthermore, violent explosions and high-energy phenomena in the universe are associated with black holes. The direct study of black holes would help scientists understand the causes of these phenomena, e.g., gamma-ray bursts. Moreover, after studying the black hole, it is expected to infer the issues associated with the formation and evolution of galaxies.

2. Black holes descriptions

Before describe the black holes, the Einstein's Field Equation is introduced [2]:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu} \Lambda = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{1}$$

The left side of the equation refers to the curvature of the universe's spacetime, and the right-hand side of Einstein's field equation corresponds to concepts of mass and energy. Ricci curvature tensor R is simply an indication of curvature in space. The metric tensor (second-rank tensor) is used to measure distances, areas, and angles in a metric space. Cosmological constant (proportional to the metric tensor) plays an indispensable role in large cosmic scales (negligible on galactic scales). It exists in the equation to account for the existence of a static universe where the density of matter isn't zero. The stress-energy momentum tensor is a physical concept that generalizes the stress tensor of Newtonian physics and describes the density and flux of energy and momentum in spacetime. Matter, radiation, and non-gravitational force fields all have this property. In the Einstein field equations of general relativity, this density and flux of energy and momentum are the sources of the gravitational field, just as mass density is in Newtonian gravity [3].

2.1. Schwarzschild Radius

As a matter of a particular mass is compressed within its Schwarzschild Radius, there will be no known type of force that can prevent the matter from collapsing itself into a black hole under its gravity. Since the centripetal force is provided by the gravitational force F_C=F_G. Substituting

$$m\frac{v^2}{r} = \frac{GMm}{r^2} \tag{2}$$

one derives

$$v = \sqrt{\frac{GM}{r}} \tag{3}$$

According to Newton's law of gravitation and Newton's second law of motion, one obtains:

$$g = \frac{GM}{R^2} \tag{4}$$

Substituting

$$E_p = mgh \tag{5}$$

Into

$$g = \frac{GM}{R^2} \tag{6}$$

we know

$$E_g = \frac{GMmh}{R^2} \tag{7}$$

on the surface of a celestial body R=h. Hence,

$$E_g = \frac{GMm}{R} \tag{8}$$

The kinetic energy $E_k = \frac{1}{2}mv^2 \geq E_g = \frac{GMmh}{R^2}$(matter escape the gravity of celestial bodies and fly into interstellar space). Take the critical value E_K=E_g, one obtains

$$\frac{1}{2}mv^2 = \frac{GMm}{R} \tag{9}$$

Hence Schwarzschild Radius is derived:

$$R = \frac{2MG}{v^2} \tag{10}$$

When the escape velocity is less than or equal to the speed of light, it can't escape the celestial body's gravity and enter interstellar space. As the escape velocity is equal to the speed of light, the object will be in a ladder orbit. As the escape velocity is less than the speed of light, it tends to fall towards the celestial body. The distance from the ladder orbit to the center of the black hole and the radius of the Event horizon equals the Schwarzschild Radius in a numerical perspective.

2.2. Black hole thermodynamics

Black holes are troublesome because they represent the problematic collision with quantum theory and classical theory. On the edge of the black hole, there is no longer possible to assume quantum effects can be ignored on a classical macroscale. Black holes are formed by the gravitational collapse (Radius of the celestial body reached the Schwarzschild Radius In 1975, Stephen Hawking combined the quantum field theory and general relativity and proposed that black holes emit radiation.

Vacuum is not empty due to the phenomenon like the Casimir effect. Virtual particle and anti-particle spontaneously appear only to immediately annihilate and leave the time-average energy the same. As the creation and annihilation of particles occur nearby the event horizon. A particle and anti-particle pair is generated as the anti-particle gets sucked into the black hole, and the particle escapes off to infinity. Hence, the anti-particle decreases the mass of the black hole. If this situation tends to repeat, the mass of the black hole would appear to be in a consistent reduction while radiating particles to the interstellar space. Such radiation is a product of the extreme radius of curvature of space-time around the black hole. The curvature creates an indeterminacy in particle number and energy density, mostly for photons and neutrinos. Steven Hawking concludes that black holes are black body radiators. The temperature of this black body radiation is defined by the following:

$$T_{BH} = \frac{hc^3}{8\pi GMK_B} \tag{11}$$

Within the formula, h is the Planck constant; c represents the speed of light; G is the gravitational constant; M is the numerical mass of the black hole, and KB is the Stephan-Boltzmann constant. The temperature of the black hole TBH is inversely proportional to its mass. As the mass of the black hole decrease, the temperature would increase. According to Einstein's famous equation of energy E=Mc2, the specific heat of a black hole can be derived. Substituting:

$$E = Mc^2 \tag{12}$$

Into

$$T_{BH} = \frac{hc^3}{8\pi GM\ \ _B} \tag{13}$$

One derives

$$T_{BH} = \frac{hc^5}{8\pi GEK_B} \tag{14}$$

Taking derivative, we obtain:

$$C_v = -\frac{hc^5}{8\pi GM^2K_B} \tag{15}$$

Noticeably, the specific heat C_v result is negative (As an energy being added to the black hole, its temperature decreases). As a black hole gets smaller in size, its temperature increases and radiates larger amounts of energy. Stephan Hawking shows that if a black hole becomes hot enough to emit massed particles like electrons. As the black hole gets smaller, it will eventually release the remaining contents in a short and burst radiation.

3. Black holes model

Black hole models can be classified according to mass, charge, rotation, and other characteristics. Black holes of different masses origin in various ways. Black holes with different charges and angular momentum need to be described by different models. In terms of mass, there are four types of black holes: stellar black hole, supermassive black hole, intermediate black hole, and miniature black hole (primordial blackhole). Besides, by classifying the blackholes by their mass, rotation, and charge, there will be three kinds of black holes: Schwarzschild black hole, Kerr black hole, and Charged black hole.

3.1. Stellar black hole

The stellar black hole is formed by dead stars, as schematically shown in Fig. 1. When the star stops burning, the star's surface will collapse under the action of gravity with various substances of the star. All stars will go to the end of gravitational collapse when their energy is exhausted, which is inevitable. Different collapse masses will leave different kinds of celestial bodies. When the mass of the collapsed part is below the Tolman Oppenheimer-Volkoff (TOV) limit [4], the dead star will eventually form a white dwarf or neutron star. Suppose the collapsed part exceeds the TOV limit. In that case, the stellar material will not stop squeezing until the final volume becomes 0, and a stellar material keeps squeezing until the final volume becomes 0 and a stellar black hole is formed (see from Fig.1). When a massive star collapses, it releases so much energy that it heats the surrounding matter to hundreds of millions of degrees. Such a high-temperature substance will produce x-rays and be observed by humans.

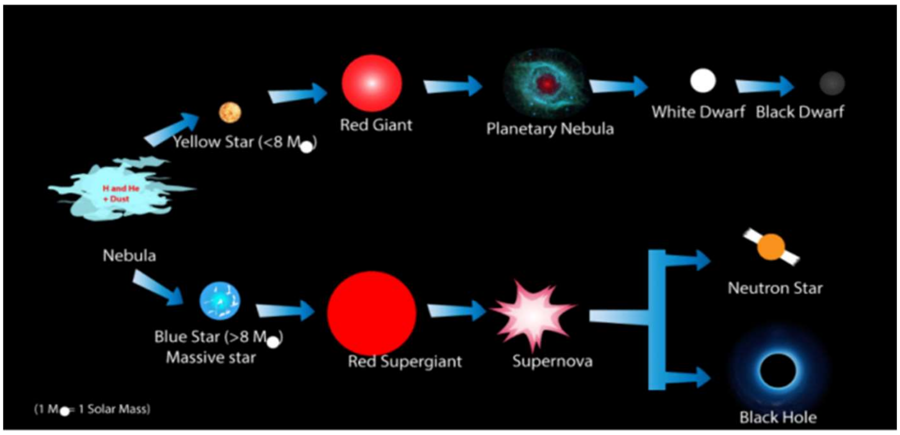

Figure 1. Steps of a star turn to a stellar black hole [5].

3.2. Supermassive black hole

The mass of a Supermassive Black Hole is generally one million to one billion times that of a Stellar Black Hole. Since supermassive black holes are often too far away from the earth, very few confirmed supermassive black holes exist. At present, most detections of supermassive black holes are based on indirect evidence. Among them, quasars are considered a typical celestial body that proves the existence of supermassive black holes [6].

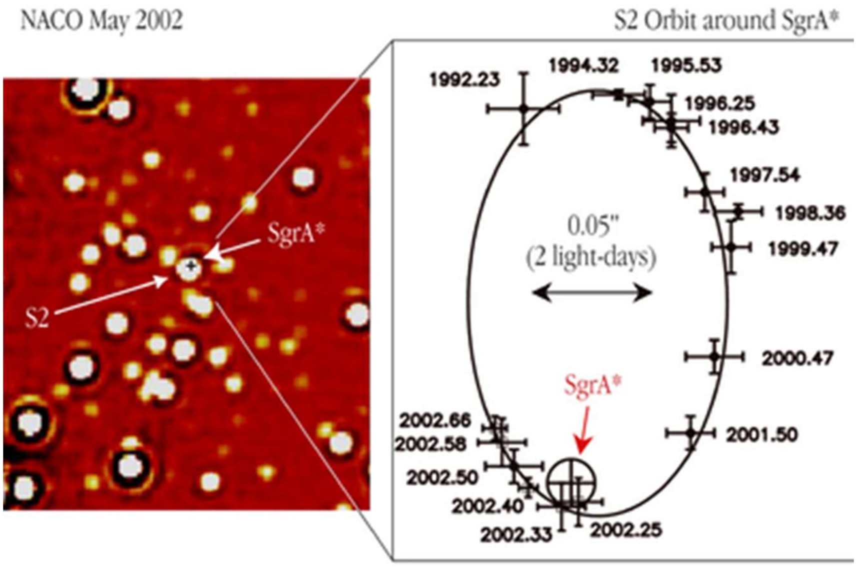

Figure 2. The orbital motion of the star S2 around the center of the Milky way shows evidence of a supermassive black hole [7].

Quasars can produce huge radiant energy in a very small range. The strength of this energy is very inconsistent with the volume of the quasar. The only way to generate such powerful energy is through black holes to convert gravity into the light. The phenomenon of quasars provides evidence for the existence of supermassive black holes [6].

In addition, supermassive black holes have been proved to exist in the centers of many galaxies (including the Milky Way, see from Fig. 2). By observing the speed of the celestial bodies in the center of the galaxy, scientists have found that the gravity that can provide such high speeds to these celestial bodies in a small area is likely to come from massive black holes. So far, the cause of the supermassive black hole is not clear. Some conjectures believe that the merger of multiple black holes produces supermassive black holes.

3.3. Schwarzschild black hole

As shown in Fig. 3, it is revealed that there are some differences between the 4 different types of black hole models. Besides, we will have some general descriptions of it before looking at the specific information. Firstly, singularity is one of the important points that we will mention in every type of black hole. Singularity is a point that we can understand as something we haven't understood, something with no correct description. It is a point with no volume, however, with infinite mass. Some experts think that things can travel into another universe by going through singularity, which means the singularity can be a canal or a door that connects our universe with other different universes.

Before someone goes into the singularity, it is revealed that the person has to go through the "Horizon", and for the Schwarzschild black hole, there is only one horizon. For others, there may be 2 horizons: the inner event horizon and the outer event horizon.

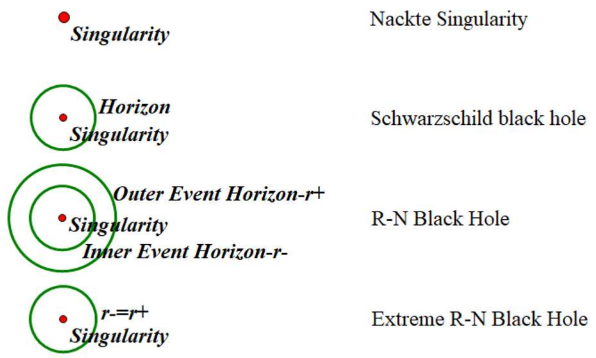

Figure 3. Different Types of Black Holes in terms of horizons.

Schwarzschild's black hole, which is the most understandable black hole model, was described by Schwarzschild. Additionally, this type of black hole has the following characters:

With mass

Without electric charge

Without spin

In this kind of black hole, there is one event horizon. The deduction of the horizon will be put in the part of black hole descriptions. The result of the deduction can reveal the horizon of the black hole, which is

$$r = \frac{2GM}{c^2} \tag{16}$$

This equation was called the Schwarzschild radius. If the distance between an object and the black hole is smaller than the Schwarzschild radius, that object will have no possibility to escape. Through the theory, everything can become a black hole if the mass and radius reach a certain level (see Fig. 4). For example, if the sun's radius is equal to or smaller than 3 km and the mass constant, it will become a black hole. At that time, anything that has a distance smaller than 3 km between the sun cannot escape. When a celestial body shrinks into the horizon's radius, nothing can escape, and everything will fall into the center of the black hole. For any black holes or objects with huge mass, the time and space will bend, and the bent space can affect the object's movement reversely.

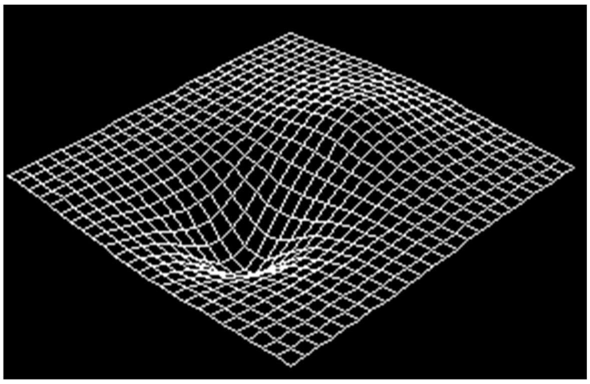

Figure 4. The bending space, this figure shows how the accumulation of mass can influence the space. [8]

The horizon is a kind of spherical surface that meets the condition of $r=2GM/c^2$ (as illustrated in Fig. 5). At this distance, if one clock falls close to it, the observer sees that the clock is constantly approaching a stationary state, the indicator will get slower and slower. The surface of infinite redshift can explain the condition. The surface of infinite redshift is at the same position where the Schwarzschild black hole's horizon is.

We can find specific information by looking at the equation of time slowing down in the gravitational field and the equation of gravitational redshift.

$$\Delta t = \frac{\Delta\tau}{exp\left(-\frac{GM}{Rc^2}\right)}, \nu = \nu_1 exp\left(-\frac{GM}{Rc^2}\right) \tag{17}$$

and there are
$$\lim_{r\to\frac{2GM}{c^2}} \Delta t \to \infty,\ \lim_{r\to\frac{2GM}{c^2}} \nu \to 0 \tag{18}$$

It shows that if one light source satisfies $r\to\frac{2GM}{c^2}$, the frequency of the light that emitted from the source will become smaller and smaller, the length of the light will become longer and longer, which means there will be an infinite redshift if the light source is very close to the surface of it. Actually, this is the important character of the Schwarzschild black hole.

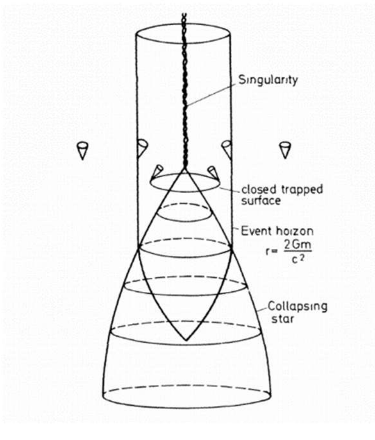

Figure 5. Spherical collapse shows the structure of a black hole generally [9].

3.4. Kerr black hole

Kerr's black hole is a black hole model with angular momentum but no electric charge, which is closer to the real situation than Schwarzschild's black hole. The black hole consists of a ring singularity, an inner event horizon, and an outer event horizon. Under the natural unit system, which means Planck constant, speed of light and Boltzmann constant will be set to dimensionless number 1. We can get the line segment of it:

$$ds^2 = -\left(1 - \frac{2Mr}{\rho^2}\right)dt^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 - \frac{4Mrasin^2\theta}{\rho^2}dtd + \left[(r^2 + a^2)sin^2\theta + \frac{2Mra^2sin^4\theta}{\rho^2}\right]d\varphi^2 \quad (18)$$

Thus, there are the 2 positions of two horizons:

$$r+= M + \sqrt{M^2 - a^2}, r-= M - \sqrt{M^2 - a^2} \quad (19)$$

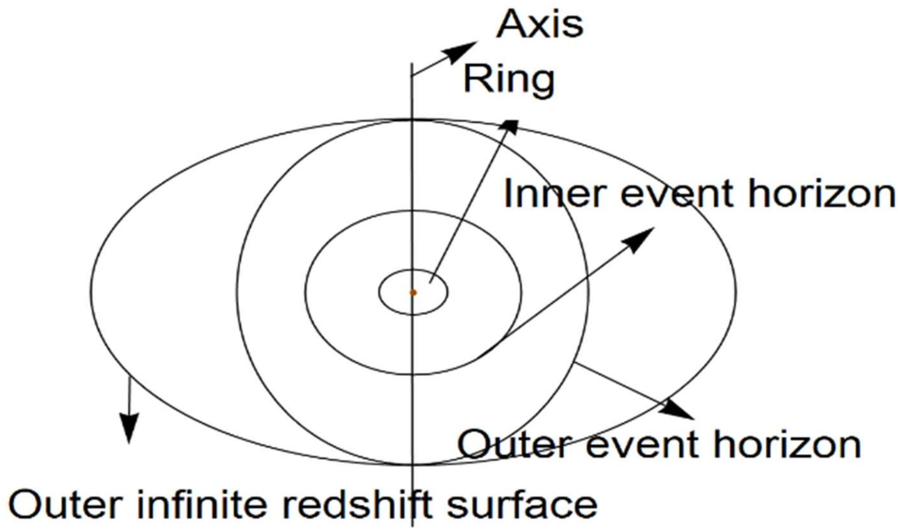

Figure 6. The structure of The Kerr black hole, event horizons are not coinciding with the infinite redshift surface.

In the Kerr black hole, as depicted in Fig. 6, singularity becomes a ring called ring singularity. It is because time is stretched into a line and connected end to end to form a ring. Between the Outer infinite redshift surface and outer event, the horizon is called the outer ergosphere. Between the Inner event horizon and the Ring singularity is the inner ergosphere. However, the outer and inner ergosphere area is not a unidirectional membrane region, which means objects can escape even if they get into the region. The unidirectional membrane region is the area between the outer horizon and the inner horizon. When an object gets into the region, it has to go through and fall into the inner ergosphere region.

In extreme conditions, which $M^2 = a^2$, the outer event horizon would be combined into one horizon. This kind of black hole is called the extreme Kerr black hole. The differences types between Kerr black holes and other black holes are indicated in Fig. 7 continually, if we add the momentum of the black hole till $M^2 < a^2$, the nackte ring singularity would be produced, which will be similar to the *R-N* black hole, which we are going to talk about later. In this condition, the naked ring singularity will affect the surrounding space and time.

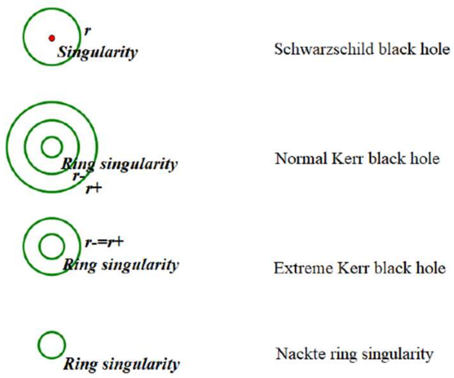

Figure 7. The comparison of 4 different situations of black holes, including normal Kerr black hole and extreme Kerr black hole.

3.5. Charged black hole

It is illustrated that this kind of black hole also has 2 horizons, an inner event horizon, and an outer event horizon. If something falls into the charged black hole, it may be able to get out from the white hole by going through its singular point, which means things may be able to travel to another universe by crossing the charged black hole.

This type of black hole can be described mainly by using two models: Reissner–Nordström black hole (R-N black hole) and Kerr–Newman black hole. The first kind of black hole has an electric charge but no momentum. The second kind of black hole has both momentum and electric charge. An R-N black hole is a static, charged black hole, as exhibited in Fig. 8. Its line segment can be deducted from Einstein's equation(Under the natural unit system, which means Planck constant, speed of light, and Boltzmann constant will be set to dimensionless number 1).

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 sin^2\theta d\varphi^2 \quad (20)$$

By looking at this equation, we can know that when r=0, there is a singularity because there is no solution. At this point, the curvature is infinite, mass, density is also infinite.

Besides, there are two solutions of the equation at where $r^2 + Q^2 - 2Mr = 0$(Under natural unit system)

$$r+= M + \sqrt{M^2 - Q^2}, r-= M - \sqrt{M^2 - Q^2} \quad (21)$$

Here, Q is the amount of charge, r is the position of the horizon. Thus, we can conclude that there are two horizons.

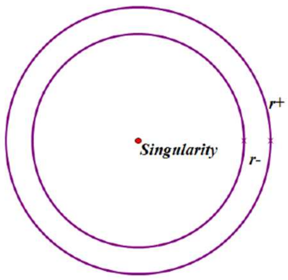

Figure 8. The structure of the R-N black hole

As a result, it is shown that the black hole contains 2 event horizons, 2 infinite redshift surfaces, which coincide with the 2 horizons, and one singularity. However, there are some problems here: when $M^2 = Q^2$, there can be only one horizon. In this condition, if we add the charges to a normal R-N black hole, the two horizons would coincide together and become a single horizon. It is suggested that if we add more charges until it satisfies the condition, the singularity which was at position r=0 would be exposed. Finally, a Naked singularity would be produced. The problem was, nowadays, nobody really understands the character of singularity. There are too many unknown questions for people to solve. It is also not sure if the singularity can really make objects travel through universes or if objects can really reach the singularity. All humans did was just use theoretical knowledge to explain them, although there seems not as much data as we need.

The line segment of the Kerr-Newman black hole is

$$ds^2 = \left[(r^2 + a^2)sin^2\theta + \frac{(2Mr - Q^2)a^2 sin^4\theta}{\rho^2}\right]d\varphi^2 - \left(1 - \frac{2Mr - Q^2}{\rho^2}\right)dt^2 + \rho^2 d\theta^2 - \frac{2(2Mr - Q^2)sin^2\theta \times a}{\rho^2}d\varphi dt \quad (22)$$

The positions where the ring singularity are

$$r+= M + \sqrt{M^2 - a^2 - Q^2}, r-= M - \sqrt{M^2 - a^2 - Q^2} \quad (23)$$

The difference between the Kerr black hole and the Kerr-Newman black hole is that the Kerr-Newman

The black hole's ring singularity does not contact its infinite redshift surface.

4. Black holes detection

4.1. mutual motion

In a binary system, two stars will form mutual motion under the action of mutual gravity, as shown in Fig. 9, because the huge gravity of the black hole itself will affect the surrounding celestial bodies. In many galaxies, the stars in the galaxy seem to be rotating around a blank area, i.e., scientists have carried out observation and calculation. Since 1995, astronomers have observed and recorded 90 stars in the central region of the Milky Way. From the records, it can be seen that these stars are moving in a circle around a position. After 20 years, a star named S2 completed a detour [10]. Finally, according to the data of the star and its orbit, scientists calculated that the target mass it surrounds is 4.3 million times the mass of the sun, and the radius is about 0.002 light-years. Therefore, for a non-luminous celestial body, it is basically determined to be a large black hole.

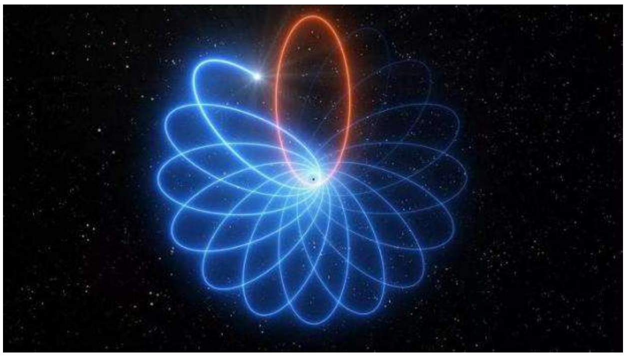

Figure 9. Southern Observatory in Chile revealed for the first time that a star moves around the supermassive black hole in the center of the Milky way, as predicted by Einstein's general theory of relativity [11].

4.2. Accretion disk X-ray detection method
An accretion disk is a structure composed of dispersed matter rotating around a central body. It is a gas disk surrounding a black hole or neutron star. The friction in the disk causes the gas to spiral down gradually and be sucked into a black hole or star. The black hole is a kind of star with huge gravity. All objects within its Schwarzschild radius, including electromagnetic waves, cannot escape. In addition, he also has gravity on the celestial body outside the event horizon of the black hole. When the speed of the celestial body is not enough, it will enter the event horizon under the action of gravity. When the matter is sucked into a black hole, a high-speed rotating gas accretion disk will be formed around the event horizon under the conservation of angular momentum.

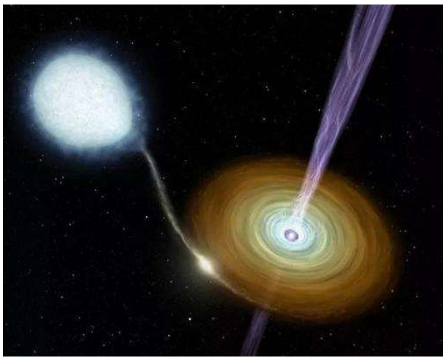

Figure 10. The schematically shown absorption of matter by a black hole, where it will be adsorbed to the center along a spiral. The orange-yellow area in the figure, is the accretion disk, and the purple area above and below is the jet [12].

It is precisely because of the high-speed rotation of the gas in the accretion disk (the closer it is to the event horizon, the higher the speed of rotation), the high-speed friction between the gases will produce a lot of heat energy [13], because the higher the temperature of the gas, the smaller the electromagnetic wavelength of the corresponding thermal radiation. Therefore, we can observe the x-rays emitted from the high-speed gas in the accretion disk.

4.3. Hawking radiation
Some scholars have detected strong radio radiation, but no radio source can be found. It is speculated that the radio source is likely to be a black hole. According to quantum physics, virtual particle pairs, a positive particle and a negative particle can be produced in energy. Usually, pairs of particles created in energy will annihilate each other almost instantaneously and will not be observed. But the discussion of black holes is somewhat complicated. Particle pairs can be created inside and outside the black hole and at the edge of the event horizon of the black hole. Because the gravity of the black hole is so strong that even light cannot escape, the pair of particles created in the event horizon of the black hole cannot escape the black hole and will annihilate in the black hole. The pair of particles created outside the black hole will not be sucked into the black hole because it is far enough away from the black hole but will soon annihilate. The particle pair created at the edge of the event horizon of the black hole may not annihilate as long as it has enough energy [14]. Particles that enter the event horizon of the black hole after creation cannot escape from the black hole, while particles that do not enter the event horizon of the black hole after creation may be far away from the black hole because of the loss of annihilation objects. The particles far away from the black hole are the black hole radiation (Hawking radiation) we observed.

4.4. Gravitational lens phenomenon
Gravity can turn light. A strong gravitational object attracts the light around it, making it focus, just like a convex lens. Galaxies usually cause gravitational lensing, magnifying objects in the background sky. In astronomical observation, the gravitational lens effect will have a great impact on the observation

results [15]. Because the gravity of the black hole itself is great, the light will deflect in its gravitational field and converge. Finally, it looks like a lens for imaging. Therefore, we can detect black holes by observing this phenomenon.

5. Conclusion

In summary, this paper gives a general introduction to different black hole models, as well as the currently feasible methods for detecting black holes. In the first part, we used Einstein's gravitational field equation to describe the black hole and discussed the physical meaning of each term in the equation. We clarified the physical meaning of the Schwarzschild radius and discussed the thermodynamics of black holes. In the second part, we classified the black hole model by mass, charge, and rotation. In this section, we describe the origins and properties of different types of black holes. In the third part, we introduced four methods for detecting black holes. The principles and advantages of the four methods are revealed in part three.

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