Perspectives on Theory of Weak Interactions

Benjamin W. Lee*

Institute for Theoretical Physics
State University of New York at Stony Brook
Stony Brook, New York 11790

and

National Accelerator Laboratory†
Batavia, Illinois 60510

1. HISTORICAL INTRODUCTION

Perhaps the most significant development in weak-interaction theory in the last two years, both from the viewpoints of theory and of possible impact on future experiments, has been in the construction of renormalizable models of weak interactions based on the notion of spontaneously broken gauge symmetry. The basic strategy of this construction appears first in Weinberg's[1] paper published in 1967 and also in Salam's,[2] published in 1968. In these papers, weak interactions and electromagnetic interactions are unified in a Yang-Mills gauge theory with the intermediate vector bosons $W^{\pm}$ and the photon as gauge bosons. This idea by itself was not new, having previously been discussed by Schwinger,[3] Glashow,[4] Salam and Ward,[5] and others. What was new in the Weinberg-Salam strategy was to attribute the observed dissimilarities between weak and electromagnetic interactions to a spontaneous breakdown of gauge summetry (which is known as the Higgs mechanism).

This mechanism was studied by Higgs,[6] Kibble,[7] Guralnik, Hagen, and others[8] since 1964. The Higgs mechanism takes place in a gauge theory in which the stable vacuum is not invariant under gauge transformations. In the absence of gauge bosons, noninvariance of the vacuum under a continuous symmetry of the Lagrangian implies the existence of massless scalar bosons, by the Goldstone theorem.[9-13] In a gauge theory, these would-be Goldstone bosons combine with would-be massless gauge bosons (with two transverse polarizations) to produce a set of massive vector bosons (with three polarizations). Suppose that the gauge group in question has n generators. A gauge theory based on this symmetry group contains n gauge bosons. Suppose further that

*Work supported in part by National Science Foundation Grant No. GP-32998X.
†Operated by Universities Research Association Inc. under contract with the United States Atomic Energy Commission.

spontaneous breakdown of symmetry leaves the physical vacuum invariant under a subgroup of dimension $m < n$. Then the m gauge bosons of this subgroup remain massless. The other n-m gauge bosons become massive. This theorem was first stated and proved by Kibble.[7]

Let me explain very briefly how this mechanism works to generate the observed differences between weak and electromagnetic interactions in models of the sort we are considering. We set up a gauge-invariant Lagrangian which unifies weak and electromagnetic interactions. This requires at least two charged gauge vector bosons $W^{\pm}$ that mediate weak interactions, and the photon as gauge bosons, so the gauge group must be nonabelian with at least three generators. We arrange the dynamics of scalar fields (Higgs scalar) in the Lagrangian in such a way that the vacuum is invariant only under the U(1) gauge transformation associated with electric charge conservation. In this way we endow all gauge bosons but the photon with finite masses. In the original model of Weinberg and Salam, the gauge group used to unify electromagnetic and weak interactions was $SU(2) \times U(1)$. In such a theory, one has the photon, two massive charged vector boson $W^{\pm}$, and a massive neutral vector boson Z.

The main points of Weinberg's and Salam's papers are twofold: The first is the unification of electromagnetic and weak interactions. In the particular model they discussed, there is a relation among G_F, e and m_W, the mass of the W meson:

$$\frac{G_F}{\sqrt{2}} = \frac{e^2}{8m_W^2 \sin^2 \theta_W} \quad \text{or} \quad m_W = (37.2 \text{ GeV})/\sin \theta_W, \tag{1}$$

where θ_W is a parameter of the theory. The number 37.2 GeV was also derived by T. D. Lee[14] from a similar, but less specific consideration. The second point is the suggestion, stressed by these authors, that the theory of this kind may well be renormalizable because the equations of motion are identical to those of an unbroken gauge theory. Nothing much had been done on the second point, and the whole subject rested dormant until 1971.

In the meantime there were two developments which were necessary for the resurgence of interest in these ideas in 1971. The first is the quantization of Yang-Mills theory. The first serious effort at construction of quantum theory of Yang-Mills fields goes back to Feynman[15] who reported on his work at a meeting in Poland in 1962. Since then, the subject had attracted a number of eminent physicists including deWitt,[16] Popov and Faddeev,[17,18,19] Mandelstam,[20] Fradkin and Tyutin,[21] and Veltman.[22] By 1968, thanks especially to the work of Faddeev and Popov, Feynman rules for Yang-Mills fields were well understood. The second development was the study of renormalization of the σ model of Schwinger,[3] and Gell-Mann and Levy.[23] The σ-model is the simplest, semi-realistic field theory model which exhibits spontaneous breakdown of symmetry. We learned from this study that the model is renormalizable even when the symmetry of the Lagrangian is spontaneously broken, and in fact the same renormalization counterterms remove the divergences of the theory whether the vacuum is invariant under the summetry of the Lagrangian or not.[24,25]

At the Amsterdam conference last year, a young Dutch physicist, G. 't Hooft, not yet out of graduate school, presented a paper which would change our way of thinking in gauge field theory in a most profound way.[26,27] In addition to rediscovering the Higgs mechanism and the Weinberg-Salam theory by himself, he

presented a formulation of spontaneously broken gauge theories which is manifestly renormalizable, i.e., all Feynman graphs are finite except for a small number of primitively divergent vertices. The formulation takes advantage of the gauge freedom afforded in such a theory. In this formulation Green's functions are defined in a big Hilbert space which contains, in addition to physical states, unphysical ones which possess indefinite metric. 't Hooft gave a convincing argument that the S-matrix is nevertheless unitary in such a theory, unphysical states decoupling from physical ones on the mass shell. The rest of this review deals with the developments since the Summer of 1971.

In concluding this section, let me emphasize a few points in order to place this enterprise in perspective. The unification of weak interactions and electromagnetism is esthetically pleasing. In this sense the present attempt is superior to other attempts at making weak interactions finite. The second point is that renormalizability is a desirable (but not an essential) feature of a theory. If a theory is nor renormalizable, one requires additional prescriptions to specify a complete theory. What is necessary in a logically consistent theory of weak interactions is that higher order corrections are finite and unambiguously predictable, and that they are small enough up to some moderate energies to protect the experimentally well-established phenomenology based on lowest order theory. Ensuring renormalizability is one possible way, and the only way I know, of arranging this in the framework of local field theory.

2. PHENOMENOLOGICAL IMPLICATIONS

Before discussing various theoretical ramifications, I think it worthwhile to discuss certain physical conditions that a renormalizable theory of weak and electromagnetic interactions must satisfy, and explore their phenomenological implications. For this purpose, let us accept the validity of quantum electrodynamics and the premise that the β- and μ- decays are mediated by charged vector bosons $W^{\pm}$.

Let us consider the process $\nu + \bar{\nu} \to W^+ + W^-$. In lowest order, this process receives a contribution from electron exchange in the t-channel (see Fig. 1) and in fact this is the only diagram for this process in the conven-

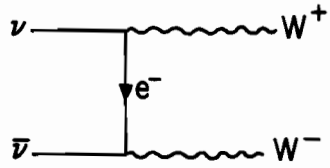


Fig. 1

tional phenomenology of weak interactions. One finds that this amplitude grows like s for large s:

$$F(\nu + \bar{\nu} \to W^+ + W^-) \sim s\, e^{i\phi} \sin\theta + O(s^{-1}), \qquad (2)$$

where θ and φ are the polar and azimuthal angles of the W^+ in the center-of-mass system. The most violent growth at high energy occurs in the J = 1 state with W^+ and W^- polarized longitudinally.[28] This linear growth with $(\text{energy})^2$ of the amplitude for $\nu + \bar{\nu} \to W^+ + W^-$ is responsible for the quadratic divergence in "conventional" theory of the amplitude for the elastic process $\nu + \bar{\nu} \to \nu + \bar{\nu}$, whose imaginary part is proportional to the absolute square of the former.

Therefore, in a renormalizable theory where no divergence can be tolerated in a four-fermion coupling, the linear growth of Eq. (1) must be suppressed. In renormalizable theories, amplitudes for (fermion) + (antifermion) → two bosons typically behave as $1/s$ as $s \to \infty$. There are essentially two possibilities of suppressing this behavior by using renormalizable interactions. They correspond to adding single-particle poles in the s- and u- channels to cancel the leading term Eq. (2). Let us discuss them in turn.

The first possibility is to add a pole term in the s-channel. We need a boson of spin 1 which couples to the neutrino-antineutrino pair (it cannot be the photon). See Fig. 2. In order that the cancellation of the leading term

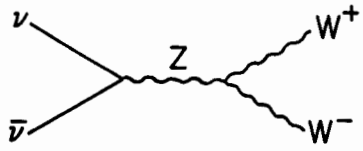


Fig. 2

takes place for all helicities of W^+ and W^-, the coupling of the neutral heavy vector boson Z to W^+ and W^- must be precisely as in the Yang-Mills gauge theory. Weinberg's original model embodies these features.

The second possibility is to add a pole term in the u-channel. This calls for the existence of a lepton of the opposite electric charge and the same lepton number as the electron. See Fig. 3. The model advanced by Georgi

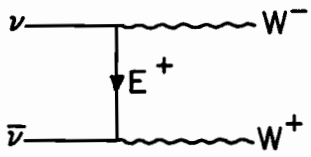


Fig. 3

and Glashow[29] achieves the asymptotic vanishing of the amplitude $\nu + \bar{\nu} \to W^+ W^-$ by the cancellation of the e^- and E^+ (heavy electron) exchange diagrams.

A renormalizable model of weak interactions must, therefore, contain one or both of the following features: neutral current, and/or heavy leptons. Let us consider the experimental situation with regards these two possibilities:

1. Neutral Current: For purely leptonic processes, e.g., $\bar{\nu}_e + e \to \bar{\nu}_e + e$, $\nu_\mu + e \to \nu_\mu + e$, the upper bounds presently available are only moderately restrictive[30,31,32] (see Table I). The situation as regards neutral current effects is somewhat more restrictive in the case of strangeness-conserving semileptonic processes, e.g., ν + nucleon $\to \nu$ + hadrons;[33] in fact, the upper bounds have recently diminished sufficiently to make serious trouble for certain models which feature neutral currents.[34-37] See Table II. Most decisive are $\Delta S \neq 0$, $\Delta Q = 0$ semileptonic processes mediated by neutral current such as $K_L \to \mu + \bar{\mu}$, $K^+ \to \pi^+ + \ell + \bar{\ell}$ and the $K_1 - K_2$ mass difference. The upper bounds are so restrictive (see Tables III and IV) that one takes it as a principle of model building to banish $\Delta S \neq 0$ neutral currents altogether, using, for example, the device of Glashow, Iliopoulos and Maiani.[38] More on this later. In this review, we shall not reject any model on the grounds that it disagrees with present data on $\Delta S = 0$ neutral currents.

2. Heavy leptons: One must assume that they are sufficiently massive to have so far escaped detection. The heavy leptons that interest us here carry either the electron or muon number, so they can be produced in reactions initiated by the usual neutrinos, electrons, and muons and, of cours, they can be produced in pairs in other reactions. Various experimental consequences of the existence of heavy leptons have been discussed recently by Perl,[39] Bjorken, and Llewellyn-Smith.[40] Production processes and decay modes of E^+, E^o (heavy electrons) and M^+, M^o (heavy muons) are listed in Table IV.

3. RENORMALIZABILITY

Let us turn to the question of the renormalizability of spontaneously broken gquge theories. After all, you recall, it was the renormalizability which was directly responsible for the revival of interest in these theories.

For simplicity let us consider a system of an O(3) triplet of gauge bosons and a triplet of scalars. For the moment, let us ignore fermions. The Lagrangian of the system is

$$L = -\frac{1}{4}(\partial_\mu \underset{\sim}{A}_\nu - \partial_\nu \underset{\sim}{A}_\mu + g\underset{\sim}{A}_\mu \times \underset{\sim}{A}_\nu)^2$$

$$+ \tfrac{1}{2}(\partial_\nu \underset{\sim}{\phi} + g\underset{\sim}{A}_\mu \times \underset{\sim}{\phi})^2 - V(\phi) \qquad (3)$$

where the "potential" V is an O(3)-invariant quartic polynomial of the scalar fields $\underset{\sim}{\phi}$. The vacuum expectation values of $\underset{\sim}{\phi}$ are determined to lowest order by minimizing the potential energy. In order to induce a spontaneous breakdown of the O(3) symmetry, the potential V must be so chosen that the absolute minimum occurs at some nonzero value of $\underset{\sim}{\phi}$. One can always choose the third axis to coincide with the direction of this vector, $\langle\underset{\sim}{\phi}\rangle_0 = \underset{\sim}{v} = v\hat{e}_3$. When we translate the scalar fields $\underset{\sim}{\phi}$ by their vacuum expectation values $\underset{\sim}{S} = \underset{\sim}{\phi} - \underset{\sim}{v}$, and express the Lagrangian in terms of $\underset{\sim}{A}_\mu$ and $\underset{\sim}{S}$, the bilinear terms of the Lagrangian can be written as

$$L_0 = \sum_{i=1}^{2}\left[-\tfrac{1}{4}(\partial_\mu A_\nu^{\,i} - \partial_\nu A_\mu^{\,i})^2 + \frac{(gv)^2}{2}\,(A_\mu^{\,i})^2\right] - \tfrac{1}{4}(\partial_\mu A_\nu^{\,3} - \partial_\nu A_\mu^{\,3})^2$$

$$+ \tfrac{1}{2}\sum_{i=1}^{2}(\partial_\mu S^i)^2 + gv(A_\mu^{\,2}\partial^\mu\phi^1 - A_\mu^{\,1}\partial^\mu\phi^2)$$

$$+ \tfrac{1}{2}\left[(\partial_\mu S^3)^2 - \mu^2\,(S^3)^2\right], \qquad (4)$$

where μ^2 is a positive number determined from V.

The "free" Lagrangian (4) is singular, in the sense that the matrix which defines this bilinear form is not invertible.[18,19] This is a typical situation one encounters in a gauge-invariant theory. A way of quantizing a system of this type, discussed by Popov and Faddeev[17,18] and perfected by 't Hooft,[26] is to add a gauge-variant term to the Lagrangian. A suitable choice for the gauge-variant term, which "defines the gauge" is [41, 42]

$$L_0' = \frac{\xi}{2}\left[(\partial^\mu A_\mu^{\,1} + \frac{gv}{\xi}\phi^2)^2 + (\partial^\mu A_\mu^{\,2} - \frac{gv}{\xi}\phi^1)^2\right] + \frac{1}{2\eta}(\partial^\mu A_\mu^{\,3})^2, \qquad (5)$$

where ξ and η are real paramers. This device is known as Fermi's trick in quantum electrodynamics. When the above gauge-defining term is added to Eq. (4), the resulting "free" Lagrangian is no longer singular and can be quantized in the usual way. The propagators for various fields are

$$\begin{aligned}
A_\mu^{\,1,2}&: \quad -i\left[g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2 - \frac{1}{\xi}m^2}(1 - \frac{1}{\xi})\right]\frac{1}{k^2-m^2}\ ,\\
A_\mu^{\,3}&: \quad -i\left[g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\ (1-\eta)\right]\frac{1}{k^2}\ ,\\
\phi^{1,2}&: \quad i\,\frac{1}{k^2 - \frac{1}{\xi}m^2}\ ,\\
\phi^{3}&: \quad i\,\frac{1}{k^2-\mu^2}\ ,
\end{aligned} \qquad (6)$$

where $m = gv$. The vector propagator here is the same as that of the ξ- limiting process.[43,44,45]

In nonabelian gauge theories, the S-matrix becomes unitary only when a suitable "gauge compensating term" is added to $L + L_0'$. This was the important discovery of Feynman, and the gauge compensating term can be viewed as internal loops in Feynman graphs generated by a complex scalar triplet which obey Fermi

statistics and interact with $\underset{\sim}{A}_\mu$ and $\underset{\sim}{S}$. We shall not write down this expression explicitly, but suffice it to say that its structure is such that the vertices implied by Eq. (3) and this term, together with the propagators of Eq. (6) make the theory renormalizable, by the usual power-counting argument.

Formal arguments and explicit calculations show that, while the Green's functions of the theory depend on the parameters ξ and η, the S-matrix does not. The particle spectrum of the theory is most easily deduced by letting $\xi,\eta \to 0$. In this limit the propagator for $A^{1,2}$ becomes the canonical one for a massive vector boson, and the would-be Goldstone bosons $\phi^{1,2}$ simply disappear from the physical spectrum. The limit $\xi \to 0$ is referred to in the literature as the U-gauge formulation, since in this formulation the unitarity of the theory is manifest. The choice $\xi = \eta = 1$ corresponds to the Feynman gauge in electrodynamics. This gauge is the one used by 't Hooft[27] in his discussion, and turns out to be a very convenient one for practical computations.

The formal arguments referred to above are based on the gauge invariance of the Lagrangian [Eq. (3)]. These arguments would be rigorous but for the divergences in Feynman integrals. Thus it is crucial to demonstrate that it is possible to remove the divergences from the theory in such a way that the formal argument for the gauge independence of the S-matrix is still correct after renormalization. A demonstration that the S-matrix is both renormalizable and unitary was given by Zinn-Justin and Lee[46,47] in the "R-gauge" ($\xi \to \infty, \eta \to 0$) where the renormalizability of the theory is manifest, but the Green's functions are not unitary in general because of the $k^2 = 0$ poles in the propagators. 't Hooft's original argument was expounded by him and Veltman and constitutes an alternative proof.[48,49] There are also very informative discussions of the renormalizability by Salam and Strathdee,[50] and by J. C. Taylor[50b].

The demonstration proceeds first by showing that the symmetric theory (i.e., without spontaneous breakdown) can be renormalized in such a way that renormalized Green's functions satisfy Ward-Takahashi identities which are the precise mathematical statement of gauge invariance of the theory. This was done first, and independently, by Slavnov[51]. Secondly, it is shown that the same renormalization counter terms as in the symmetric theory render finite the spontaneously broken gauge theory (which is obtained from the former by varying the coefficients of subdominant terms of the potential V), and the resulting finite Green's functions satisfy Ward identities appropriate to spontaneously broken gauge invariance. Thirdly, it is shown that the Ward identities imply that the spurious singularities at $k^2 = 0$ in the $A_\mu^{1,2}$ - and $\phi^{1,2}$ - propagators cancel in the S-matrix, thereby insuring the unitarity of the S-matrix.

In the proof of renormalizability and in practical computations, it is essential to regulate Feynman integrals in a gauge-invariant way. A most ingenious and convenient regularization which preserves Ward identities was devised by 't Hooft and Veltman.[48] Their method consists in continuing Feynman integrals in the number of space-time dimensions.[52,53,54] The divergence of the Feynman integral now appears as singularities of the dimensionally continued amplitude at $n = 4$, and the method is in some sense reminiscent of the analytic renormalization of Speer.[55] The essential advantage of this method is the economy in not requiring auxiliary fields and the deeper understanding it affords on anomalies in Ward identities.

The above discussions fail in the presence of fermion fields if there are Adler-Bell-Jackiw anomalies,[56,57] as pointed by Veltman, Bouchiat, Ilioupoulos and Meyer,[58] and Gross and Jackiw.[59] These anomalies are present, in general, when there are

fermions in the model,and destroy gauge invariance of the second kind which is needed to make the theory renormalizable and unitary. One way of understanding their origin is to observe that any theory must be regulated when perturbation calculations are performed. The anomalies of the axial vector current are a consequance of the absence of a chirally invariant regulation procedure for fermion loops. More specifically in the dimensional regulation of 't Hooft and Veltman,[48] they are a consequence of the fact that the Dirac matrix γ_5 and the tenser $\varepsilon_{\alpha\beta\rho\sigma}$ are unique to four-dimensional space-time and do not allow unique extensions to arbitrary dimensions.[54] Since renormalizability is desirable, the absence of anomalies may place an important constraint on model building. In order to eliminate the Adler anomaly from a model, the fermion fields must be so arranged that the anomalous contributions of various fermion loops cancel, between leptons and hadrons, for example. On the other hand it may be well to bear in mind that physically observable effects of anomalies in weak interactions occur at a fantastically high order such as $G_F\alpha^2$, and that the anomaly can be eliminated from the theory by postulating heavy fermions with appropriate couplings to gauge bosons, which are massive enough not to influence low energy phenomenology substantially. The anomalies that might arise among the strongly interacting vector gluon and weak-gauge bosons are much more serious in their observable effects, and should not be tolerated in realistic models.

Georgi and Glashow[60] have discussed a necessary and sufficient condition for anomaly-free gauge theories. In its most general form, the condition is that the quantity

$$C_{ijk} \equiv \mathrm{Tr}\,[\gamma_5(F_i,F_j)F_k] \tag{7}$$

vanishes identically for all i,j, and k, where F_i is the matrix which specifies the couplings of gauge bosons to spinor fermion fields through the interaction

Lagrangian $A_\mu{}^i\bar{\psi}F_i\gamma^\mu\psi$. The above condition guarantees that all the triangle-graph anomalies are absent. It follows from the work of Bardeen[61] and Wess and Zumino[62] that if the triangular anomalies are absent, then all other anomalies are absent. In the foregoing discussion it is tacitly assumed that the numerical values of anomalies are not modified by higher order corrections so that their absence in lowest order suffices to make a theory anomaly-free. While experience in electrodynamics renders support to this assumption, an explicit demonstration in the context of nonabelian gauge theoreis is desirable. I am happy to learn that Bardeen has completed such a proof (see W. A. Bardeen's contribution to the parallel session).

4. MODEL BUILDING

A. Leptons

Theoretical possibilities on model building are enormously varied, if one is allowed to freely invent intermediate vector bosons, Higgs scalar particles, new heavy leptons, charmed quarks, etc., all sufficiently massive to have eluded detection so far.

The unification of electromagnetism and weak interactions requires that we treat the charged vector bosons $W^{\pm}$ and the photon on an equal footing as gauge bosons so that any scheme of this sort must contain either O(3) or SU(2) × U(1) as a subgroup. We shall discuss "economical" models based on the minimum groups as they apply to leptons.

The principles of model building have spelled out by Weinberg, and more recently by Bjorken and Llewellyn-Smith. It is worth reproducing the recipe here [see Table V]:

1. Choose a gauge group.
2. Choose the representation of the Higgs fields and their charge assignments.
3. Choose the representations of the spin ½ chiral fermions.
4. Couple the gauge fields invariantly to Higgs fields and fermions.
5. Couple the Higgs fields invariantly and renormalizably to themselves.
6. Choose these couplings so that the potential of the Higgs fields is a minimum when neutral Higgs fields have nonvanishing vacuum expectation values.
7. Couple the Higgs fields invariantly to fermions.
8. Rewrite the Lagrangian in terms of the translated fields $S = \phi - \langle\phi\rangle_0$, and quantize:

(a) Some intermediate bosons acquire masses:

$$\tfrac{1}{2}(\partial_\mu \phi + gW_\mu \phi)^2 \to \tfrac{1}{2} g^2 W_\mu^2 \langle\phi\rangle^2.$$

(b) Some fermions acquire masses:

$$\bar{\psi}_R \psi_L \phi \to \text{h.c.} \to \langle\phi\rangle \bar{\psi}\psi.$$

(c) At least one vector boson is massless because electric charge conservation is unbroken.

(d) Some of the scalar fields become redundant; they turn into longitudinal components of massive vector bosons.

The original model of Weinberg and Salam[1,2] is based on the SU(2) × U(1) scheme; the symmetries act on a left-handed SU(2) doublet

$$L = \tfrac{1}{2}(1-\gamma_5)\begin{pmatrix} \nu \\ \ell^- \end{pmatrix} \tag{8}$$

with the leptonic hypercharge Y = 1 and a right-handed singlet

$$R = \tfrac{1}{2}(1+\gamma_5)\ell^- \tag{9}$$

with the leptonic hypercharge Y = -2. The electric charge is given by

$$Q = T_3 + Y/2 \tag{10}$$

We need four gauge bosons, two charged and two neutral. In addition, we need a complex scalar doublet to break the symmetry spontaneously down to the U(1) of electric charge. This is achieved by letting the neutral component of the Higgs doublet develop a vacuum expectation value. The nonvanishing lepton masses are also due to this mechanism. The physical photon, for example, is a linear combination of the hypercharge gauge boson (Y_μ) and the neutral isospin gauge boson (W_μ^3):

$$A_\mu = \frac{1}{\sqrt{g^2+g'^2}} \; (g' W_\mu^{\,3} + g Y_\mu) \qquad (11)$$

where g and g' are the isospin and hypercharge gauge coupling constants. The Weinberg mixing angle θ_W of Eq. (1) is defined as $\tan \theta_W = g'/g$.

The model of Georgi and Glashow[29] is based on O(3); the charged intermediate vector bosons $W^{\pm}$ and the photon form a triplet of gauge bosons. Leptons are placed in triplets and singlets:

$$\begin{pmatrix} e^- \\ \nu \sin\beta + E^0 \cos\beta \\ E^+ \end{pmatrix}_L , \quad \begin{pmatrix} e^- \\ E^0 \\ E^+ \end{pmatrix}_R , \qquad (12)$$

$$(-E^0 \sin\beta + \nu \cos\beta)_L ,$$

and similarly for the muon and its relatives (ν_μ, M^0, M^+). A triplet of Higgs scalar mesons provides spontaneous breakdown of symmetry. In this scheme, the universality of the electron and muon (and hadrons) in their couplings to the $W^{\pm}$ is extremely artifical since the mixing angles β have to be the same for electron and muon by accident. Nevertheless the model is a very interesting one in not having any neutral current other than the electromagnetic current. In this scheme the mass of the $W^{\pm}$ is

$$m^W = (52.8 \text{ GeV}/c^2) \sin\beta .$$

The main features of the Weinberg-Salam and Salam and George-Glashow models are summarized in Table VI.

A number of variations is possible on the Weinberg-Salam scheme so that the neutral current does not contain the neutrino term $\bar{\nu}\gamma^\mu(1-\gamma_5)\nu$.

In one model discussed by Lee,[64] and Prentki and Zumino,[65] the left-handed lepton (electron or muon) and its neutrino are placed in a triplet instead of a doublet. In this model the heavy neutral vector boson couples only to charged fermions. In the second model discussed by Prentki and Zumino,[65] the neutral component of the left-handed electron doublet is not ν_L, but $(\nu+E^o)_L/\sqrt{2}$, $(\nu-E^o)_L/\sqrt{2}$ and E^+ forming another doublet. In this scheme the neutral current contains the term $E^o\gamma^\mu(1-\gamma_5)\nu$ + h.c., but not the diagonal neutrino term.

The above examples serve to illustrate the general strategy in constructing models of leptons of this type. The left-handed lepton (e_L or μ_L) and its neutrino are placed in a multiplet of SU(2), the right-handed component to another multiplet, by inventing heavy leptons as they are needed. If the multiplets chosen are such that $Q = T_3$, a neutral vector boson is not needed and the unification can be achieved in an O(3) framework. Otherwise we need an SU(2) × U(1) scheme. There are many variations to this basic theme. For example, (e,ν_e) and (μ,ν_μ) need not belong to multiplets of the same dimension provided that one can arrange the μ-e universality in weak interactions. The physical leptons need not be eigenstates of T_3 or Y - the possibilities are myriad.

There may be certain advantages in considering not so economical schemes. The main impetus for such an enterprise comes from the esthetic desire for unifying the electron and muon in a single multiplet, and from the possibility of understanding thereby the muon electron mass ratio. The works of Weinberg[66] and also Freund,[67] are typical of this class of theories, and I shall outline Weinberg's SU(3) × SU(3) scheme in the briefest terms. The four component leptons $(e^-,\nu,\mu^+)_L$ and $(e^-,\nu,\mu^+)_R$ form the fundamental representations (3,1) and (1,3) of SU(3) × SU(3). The spontaneous breakdown scheme is so concocted, in terms of a very large number of Higgs scalar fields, that only the SU(2) × U(1) gauge bosons play important roles in generating the observed phenomenology of electromagnetic interactions,

the other gauge bosons being much more massive. A consequance of embedding the SU(2) × U(1) symmetry in a much bigger one is that the two coupling constants g and g' are no longer independent, but their ratio must be fixed. Another consequence of the model is that the electron muon mass ratio is in principle calculable but in practice it will depend on a number of inaccessible parameters. More on mass differences later.

A remark on the Weinberg model in parting: One might suppose that one can suppress the effects of neutral current by increasing the mass of the neutral vector boson Z. Let us recall that in the Weinberg model,

$$m_Z^2 = (g^2+g'^2)v^2$$

$$m_W^2 = g^2v^2$$

So one can push m_Z to infinity by letting $g' \to \infty$. However such a limit does not attenuate the neutral current effects since the coupling of Z to the neutral current is proportional to $\sqrt{g^2 + g'^2}$. What one must do to suppress the neutral current effects is to postulate a large number of Higgs scalar multiplets, whose netural members develop vacuum expectation values. Let ϕ_i be a multiplet with the SU(2) quantum number I_i, and the leptonic hypercharge Y_i. The neutral member, which acquires the vacuum expectation value v_i has $I_3 = -Y_i/2$. The masses of Z and W are now given by

$$m_Z^2 = (g^2+g'^2) \sum_i v_i^2 \, |Y_i|^2/4$$

$$m_W^2 = g^2 \tfrac{1}{2} \sum_i v_i^2 \, (I_i^2 - Y_i^2/4 + I_i)$$

Thus one can arrange $m_Z >> m_W$ either by having a few multiplets with $I_i \simeq |Y_i|/2$ and I_i very large, or by having a very large number of multiplets (or both). In any case, the prospect of having such a large number of scalars ($\simeq$ 100, if you wand $m_Z \simeq 10\ m_W$) is unappetizing and the model so constructed is unattractive, even if the masses of these scalars are large enough, so as to be compatible with presently available experimental data.

Recently Achiman[67b] proposed a scheme in which the SU(3) group is taken to be the symmetry of electromagnetic and weak interactions and the leptons $\bar{\nu}_\mu$, $\bar{\mu}$, ν_e and e are placed in an octet with 3 additional heavy leptons and quarks p, n_c and λ_c in a triplet. The scheme is interesting, but suffers from $\Delta S = 1$, neutral current effects, to be discussed below.

B. Hadrons

We shall discuss how hadrons may fit into these schemes. In building models of hadrons, it is important to bear in mind that explicit breaking of the gauge symmetry would destroy the renormalizability. Thus all interactions - strong, weak and electromagnetic - must respect the gauge symmetry which unifies weak and electromagnetic interactions.[63] Any observable departure from this symmetry must arise from the Higgs mechanism.[68]

In this scheme then, exact and approximate symmetries of hadrons must be understood as follows: they are the symmetries of the Lagrangian when all leptons and the weak and electromagnetic gauge bosons are neglected, and when the Higgs scalar fields are replaced by their vacuum expectation values.[68]

There are several constraints one can impose on hadronic models.

They are, for example:

1. the β- and μ- decay universality,
2. nonvanishing Cabibbo angle,
3. absence of $\Delta S = 1$ neutral currents.

Additional constraints which derive from considerations of higher-order corrections will be discussed separately. In most models, the Cabibbo angle is incorporated in the scheme by arranging $n_L^c = n_L \cos\theta + \lambda_L \sin\theta$ and p_L to belong to the same multiplet, and arranging n and λ to be eigenstates of the mass matrix. In a spontaneously broken gauge scheme, the mass matrix M is given by

$$M = M_0 + \langle\phi^i\rangle_0 \Gamma_i, \tag{13}$$

where $\bar{\psi} M_0 \psi$ is the gauge-invariant mass term and $\bar{\psi}\Gamma_i \Psi \phi^i$ is the gauge-invariant coupling of the Higgs scalars ϕ^i to fundamental fermions.

As for the absence of $\Delta S = 1$ neutral currents, models without massive neutral vector bosons present no problem in lowest order. For other models it is necessary to arrange the matters so that

$$\langle n|T_o|\lambda\rangle = \langle n|[T_+, T_-]|\lambda\rangle = \langle n|Y|\lambda\rangle = 0, \tag{14}$$

where T_+, T_-, T_0 and Y are representations of generators of the leptonic (i.e., weak interaction) SU(2) × U(1). A way of achieving this is to borrow the construction of Glashow, Iliopoulos and Maiani[38] (GIM), who arrange

$$\langle n|T_+T_-|\lambda\rangle = \langle n|T_-T_+|\lambda\rangle = \langle n|Y|\lambda\rangle = 0 \tag{15}$$

by including a fourth quark p' which couples to $\lambda_L^c = (\lambda_L \cos\theta - n_L \sin\theta)$, such that there is a permutation symmetry of the interaction under the exchange

$$p \leftrightarrow p'; \; n_c \leftrightarrow \lambda_c,$$

$$n_c = n \cos\theta + \lambda \sin\theta, \; \lambda_c = \lambda \cos\theta - n \sin\theta$$

except for the mass terms. Then in the absence of fermion masses all neutral current effects (both intrinsic and induced - we shall discuss the latter later) occur in the combination

$$n_c^+ n_c + \lambda_c^+ \lambda_c = n^+ n + \lambda^+ \lambda.$$

For a pictorial representation of the suppression mechanism, see the figure in Table VII.

In order to include quarks in either the Weinberg-Salam SU(2) × U(1) or Georgi-Glashow O(3) model, they must be integrally charged. For this reason, Lipkin[69,70] advocates the marriage of these models with the Han-Nambu quarks.[71]

A possible scheme based on SU(2) × U(1) is to form two left-handed quark doublets

$$N_{1L} = \begin{pmatrix} p^+ \\ n_c^o \end{pmatrix}_L \qquad N_{2L} = \begin{pmatrix} p'^+ \\ \lambda_c^o \end{pmatrix}_L$$

and place the four right-handed quarks in singlets. The mass terms for quarks can be constructed from the couplings of right-handed and left-handed quarks to the doublet Higgs mesons. Thus the masses of n and λ, for instance, are generated by

the invariant coupling

$$\left(\frac{m_n}{v}\right)\bar{n}_R(\phi\cdot N_{1L}\cos\theta - \phi\cdot N_{2L}\sin\theta)$$

$$+\left(\frac{m_\lambda}{v}\right)\bar{\lambda}_R(\phi\cdot N_{1L}\sin\theta + \phi\cdot N_{2L}\cos\theta) + h.c$$

$$\rightarrow m_n\,\bar{n}\,n + m_\lambda\,\bar{\lambda}\lambda$$

as $\langle\phi\rangle_0 = \begin{pmatrix} 0 \\ \\ v \end{pmatrix}$.

Similarly the mimimum scheme based on O(3) requires 5 quarks. Alternatively an eight-quark version of the Georgi-Glashow model can be constructed which incorporates the GIM construction.

Models of hadrons constructed along this strategy may be classified in two catagories, depending on whether hadronic symmetries such as SU(2) and SU(3) are incorporated "naturally" or "artificially". To explain this concept, let me first recall a simple theorem: (This theorem is a corollary of the fact that spontaneously broken gauge theory requires the same renormalization counterterms as the unbroken counterpart[47]). A (spontaneously broken) gauge theory is renormalizable in the strict sense if it contains all possible terms of dimension 4 or less which are gauge invariant. If this condition is satisfied, the Lagrangian contains all the necessary counter-terms for renormalization (this is the meaning or renormalizability in the strict sense[72,73]). An artificial model of hadronic symmetry is a model which exhibits the hadronic symmetry in question in lowest order only if we take a subset of these terms or constrain the coefficients of gauge-invariant terms in a specified way. In such a model, the symmetry is lost in general in

higher orders because the terms excluded in lowest order have to be supplied as renormalization counter terms in higher orders. A natural model is a model in which the hadronic symmetry in question holds in the presence of all possible gauge-invariant terms. In all the models discussed above the hadronic SU(2) and SU(3) symmetries are artificial in the sense discussed here. In the model we shall discuss presently the approximate SU(3) symmetry of hadrons appears naturally.

Recently Bars, Halpern and Yoshimura[74,75] proposed a new model which combines leptons and hadrons in a grand scheme based on $U(3) \times U(3) \times SU(2) \times U(1)$. The first two factors refer to the usual hadronic $U_L(3) \times U_R(3)$; the last two to the Weinberg-Salam $SU(2) \times U(1)$. The scheme contains altogether 22 gauge bosons. I shall describe the model in its barest form which may not do justice to the original paper. A salient feature of this scheme is to assign all quarks to singlets of $SU(2) \times U(1)$, and to postulate 2 sets of mesons which transform like (3,1) under $U_L(3) \times U_R(3)$ and like ($\frac{1}{2}$) with $Y = -1/3$ under $SU(2) \times U(1)$. The couplings of quarks to the photon and weak bosons are through the intermediary of the $U_L(3)$ gauge bosons (i.e., $\rho + A1$) in a manner reminiscent of (but not identical to) the field algebra scheme. The couplings of weak bosons to hadronic vector bosons are induced by the vacuum expectation values of the mesons which have both hadronic and leptonic indices. Two sets of these mesons are necessary to suppress the $\Delta S = 1$ neutral current. The authors have promised to discuss the dynamics of the complicated Higgs scalar system in a future publication. In any case, I think the model is extremely interesting in its originality and in that the approximate SU(3) symmetry arises in this scheme naturally in the sense discussed earlier. It is well worth one's while to study various ramifications of this general scheme and physical constraints on these kinds of models imposed by experiment.

5. PHYSICAL CONSTRAINTS ON MODELS AND HIGHER ORDER EFFECTS

Models based on spontaneously broken gauge symmetry contain additional interactions arising from exchange of the Higgs scalar particles and/or neutral vector bosons. Further, higher-order corrections are finite and therefore should be taken seriously. We shall investigate what constraints are imposed on models by exchange of Higgs scalars and higher order effects. J. Primack summarized various existing calculations on higher-order effects at one of the parallel sessions. His summary is included in these Proceedings.

Let me summarize very briefly various higher order calculations performed so far, leaving a more complete and detailed discussion to Primack's contribution. There were initially several papers which demonstrated that the physical S-matrix elements were finite in the U-gauge formulation. Weinberg[63] showed that quartic and quadratic divergences in many processes were absent when graphs of the same order in perturbation expansion were taken together. Pursuing further, Appelquist and Quinn[76] demonstrated cancellation of logarithmic divergences in a simplified model. The papers of S. Y. Lee[77] and Rajasekaran[78] show much the same thing for processes of physical interest such as $\mu \to e + \nu_\mu + \nu_e$. Then there are a large number of papers dealing with weak correction to the anomalous magnetic moment of the muon:

1. Jackiw and Weinberg[79]: Weinberg model, in the U-formalism.
2. Bars and Yoshimura[80]: Weinberg model, in the U-formalism .
3. Altarelli, Cabibbo and Maiani[81]: Weinberg model, using the Drell-Hearn sum rule.
4. Bardeen, Gastmans and Lautrup[82]: Weinberg model, in the U-formalism, using the 't Hooft-Veltman dimensional regulation.
5. Primack and Quinn[83]: Georgi-Glashow model, in the U-formalism.

6. Fujikawa, Lee and Sanda[84]: Weinberg, Georgi-Glashow, and Lee-Prentki-Zumino models, in renormalizable gauges.

In the last paper, the gauge independence of the results is explicitly demonstrated. Fukuda and Sasaki[85], and Kummer and Lane[86] have also contributed to this subject. Appelquist, Primack and Quinn[87] have computed the radiative correction to the μ-decay in the Weinberg model using the dispersion technique. Bollini, Giambiagi and Sirlin[88] have also computed the radiative corrections to the μ-decay in the U-formalism. High order calculations for processes not included in this summary will be discussed below.

To begin with, it is worthwhile to discuss the general order of magnitude of higher-order effects on dimensional and other general grounds. In higher-order processes, where large momentum contributions are more important than the low momentum contributions, there is no difference between the contributions from the intermediate vector boson propagators and the photon propagator, so the T-matrix for weak processes will have the expansion

$$T \sim \frac{G_F}{\sqrt{2}} \left[a_0 + a_1 \left(\frac{\alpha}{2\pi}\right) + a_2 \left(\frac{\alpha}{2\pi}\right)^2 + \ldots \right]$$

or

$$\sim \frac{G_F}{\sqrt{2}} \left[b_0 + b_1 (G_F M^2) + b_2 (G_F M^2)^2 + \ldots \right]$$

where M is the largest mass scale in the theory (usually m_W or m_Z, the mass of the neutral boson), depending on whether α is bigger than $G_F M^2$ or not. It is generally true that higher-order effects are bigger in models with large M^2; if M is much larger than, say 50 - 100 GeV, then at least some higher-order effects

become intolerably large. On the other hand higher-order effects, as a rule, are not suppressed by making M small; unless a specific cancellation mechanism is operative, second-order effects are of order $G_F\alpha$, and so forth, if $G_F M^2 \lesssim \alpha$. A related question in this connection is whether there are parity or strangeness violations of order α in these theories. The question does make sense, since, for example, radiative correction to strong processes due to the Z meson can in principle be of this order if high momentum components contribute significantly, and is parity violating. Weinberg's preliminary result (private communication) indicates that there are no such violations of order α, at least in a certain class of models.

The second remark we wish to make is that in making estimates of higher-order effects for semileptonic and hadronic processes, we shall ignore strong-interaction effects completely, despite the warning of Ken Wilson, arguing that relevant hadronic matrix elements are governed by the operator product expansion for short distances which does not seem to be affected by strong interactions. Thus the results obtained ignoring strong interactions may be regarded as asymptotically valid in the parameter $(m/m_W)^2$, where $1/m$ is the characteristic expansion parameter in the operator product expansion (m = quark mass?).

In models in which the Higgs scalar couples to $(\bar{\lambda}n)$, processes such as $K^+ \to \pi^+ + e + \bar{e}$, or $K_L \to \mu + \bar{\mu}$ can occur already in lowest order (see Table VIII,1). The 5-quark version of the Georgi-Glashow model has this feature: thus a very stringent lower bound ($m_\phi > 10$ GeV) can be placed on the mass of the scalar particles in this model. On the other hand, in an 8-quark version, the $\bar{\lambda}n\phi$ coupling is altogether forbidden and the constraint on m_ϕ is eliminated.[89]

The anomalous magnetic moments of the electron and muon are known experimentally to good accuracy and provide several useful constraints on models. The weak contributions should fall within the bounds

$$-3 \times 10^{-7} \leq \left(\frac{g_\mu - 2}{2}\right)_{\text{weak}} \leq 9 \times 10^{-7}$$

allowing for a discrepancy of two standard deviations. In the Weinberg-Salam model, the weak correction to the muon g-2 is of order $G_F m_\mu^2 \sim 10^{-8}$ and does not provide any useful constraints. In the Georgi-Glashow model, both the M^o and ϕ-exchange diagrams are important (see Table VIII 4): the former is of order $G_F m_\mu m(M^o)$, the latter $G_F[m_\mu m(M^o)]^2/m_\phi^2$. Unfortunately the two contributions are opposite in sign, so no firm conclusion can be drawn about $m(M^+)$ or m_ϕ from experiment. However, if we disregard the possibility of cancellation and note that $m(M^o) \gtrsim m(K)$, we obtain the bound $m_W \gtrsim 18$ GeV/c^2 in this model. (For the electron g-2, the contirubition of the ϕ-exchange is negligible, so one obtains the bound $m_W \gtrsim 10$ GeV/c^2 with more certainty.[90])

More useful constraints are available from neutral K-decays (see Table VIII 2,3). Even in those models where there is no neutral current in lowest order, there are in general higher order induced effects such as $K_L \to \mu\bar{\mu}$ and $K^o \leftrightarrow \bar{K}^o$. Unless the GIM construction (Table VII) is used to cancel the p exchange by the p' exchange in the fundamental process $n\bar{\lambda} \to W^+W^-$, the effective intereaction for $K_L \to \mu\bar{\mu}$ is typically of the form [76]

$$\mathcal{L} = \frac{3G\alpha}{\sqrt{2}\pi} \sin\theta_c \cos\theta_c \left[\bar{\lambda}\gamma_\alpha\left(\frac{1-\gamma_5}{2}\right)n\right]\left[\bar{\mu}\gamma^\alpha\gamma^5\mu\right]$$

independently of m_W. This gives the branching ratio

$$\Gamma(K_L \to \bar{\mu}\mu)/\Gamma(K_L \to \text{all}) \simeq 3 \times 10^{-4} ,$$

which is clearly inconsistent with experiment. For those models which incorporate the GIM mechanism, the corresponding expression for the effective interaction is typically of order $G_F\alpha(\Delta m^2/M_W^2)$, where Δm^2 is the difference between the squared masses of "charmed" and "uncharmed" quarks, and it is possible to imagine that the suppression factor $\Delta m^2/M_W^2$ is small enough to be within experimental upper bounds.[77]

6. OTHER MATTERS

1. Induced $\Delta S = 0$ Neutral Current Effects.[91,92] In models where the neutral current is absent, or is present but does not contain the neutrino term $\bar{\nu}\gamma^{\mu}(1-\gamma_5)\nu$, processes such as $\nu_\mu + e \to \nu_\mu + e$ and $\nu + p \to \nu + p$ occurs in higher order. In conventional theory such processes occur via the intervention of weak and electromagnetic interaction, and as second-order weak processes. In a gauge theory there is no intrinsic difference between these two mechanisms and gauge-invariant results are obtained only if the two effects, which are formally of the same order, are taken into account. The magnitude of these amplitudes is precisely of the order of $G_F\alpha$: for example, in the Georgi-Glashow model, the sum of Feynman diagrams shown in Fig. 4 gives

$$T(\nu_\mu + e \to \nu_\mu + e) = i \frac{3G_F\alpha}{2\pi\sqrt{2}} \left[\bar{\nu}\gamma^{\mu}(1-\gamma_5)\nu\right]\left[\bar{e}\gamma_\mu \left(\log \frac{m(M^+)}{m_\mu} + \gamma_5\right) e\right]$$

2. Very High Energy Weak Processes.[93-96] In renormalizable gauge theories, the unitarity limits of partial wave amplitudes are reached typically at energy $\sqrt{s} \sim m_W \exp(1/\alpha)$. Thus lower-order amplitudes are presumably trustworthy up to, say, $\sqrt{s} \sim 10^3$ GeV.

In theories without intrinsic neutral currents, the Pomeranchuk limit

$$\lim_{s\to\infty} \left[\sigma_{\text{total}}(\nu_\mu e) - \sigma_{\text{total}}(\bar{\nu}_\mu e)\right] = 0$$

is reached very early, at $s \gtrsim$ (a few $m_W)^2$, so that the dispersion relation

$$\lim_{s\to 0} \frac{T(\nu_\mu + e \to \nu_\mu + e)}{s} = \frac{1}{\pi}\int_0^\infty \frac{ds}{s}\left[\sigma_{\text{tot}}(\nu_\mu e) - \sigma_{\text{tot}}(\bar{\nu}_\mu e)\right] + \text{the t-channel pole},$$

is superconvergent. In these models, weak interactions always remain weak, never exceeding the strength of electromagnetism even at ultra high energies.

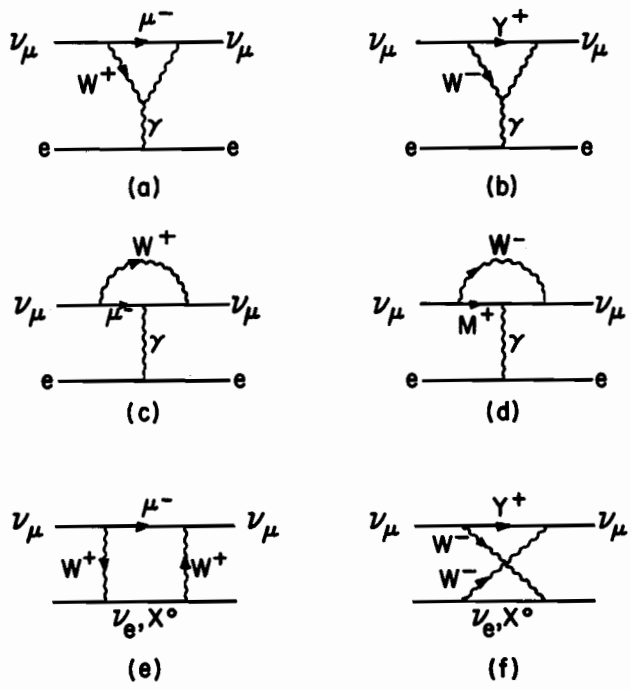


Fig. 4. Diagrams for the process $\nu_\mu + e \rightarrow \nu_\mu + e$ in the Georgi-Glashow model.

3. Electromagnetic and Weak Masses. This topic has been pursued very actively by Weinberg,[97] and Georgi and Glashow[98] in recent months. The idea that electromagnetism is responsible for intramultiplet mass differences of hadrons, or that the mass of the electron is due entirely to its interaction with the electromagnetic field is an old one. The recent study is to examine under what circumstances these quantities are finite and computable in spontaneously broken gauge theories. We shall borrow heavily the terminology and concepts of these authors in this discussion.

A mass difference or a mass is computable if it does not receive contributions from renormalization counterterms. Thus an intramultiplet mass difference is computable if and only if the symmetry is a natural one in the sense defined earlier. The electron mass is computable only if it is zero in the zeroth order (i.e., in the zero-loop approximation) when all gauge invariant terms of dimension 4 or lower are included in the Lagrangian, and the little group that leaves the vacuum invariant does not imply a vanishing electron mass.

A zeroth order mass relation is a relation valid in the zero-loop approximation in the presence of all possible renormalizable (i.e., dim ≤ 4) gauge-invariant terms. From the simple theorem quoted in Section IV B it follows that departures from zeroth order mass relations are computable. Recalling that [Eq. (13)] the zeroth order mass matrix M is of the form $M = M_0 + \langle\phi^i\rangle_0 \Gamma_i$, we see that there are three classes of zeroth order mass relations:

(a). Relations that follow from the invariance under the little group of the vacuum (i.e., the subgroup that leaves the vacuum invariant). These relations are of no interest, being exact in all orders.

(b). Relations that follow from the representation contents of scalar fields. An example is $m(e) + m(E^+) = 2m(E^o)\cos\beta$ in the Georgi-Glashow model, which follows from the fact that the mass matrix M is a combination of $\Delta I = 0, 1$ matrices.

(c). Relations that follow from the renormalizable dynamics of the potential of scalar fields and do not follow from group theoretic considerations.

The class (b) relation is especially emphasized by Weinberg as a reasonable basis for understanding electromagnetic and weak masses. We have yet to invent a model in which one can derive relations such as $m(e) = \alpha m(\mu)$. The foregoing discussion lays a foundation, hopefully, for such an invention.

I must emphasize here, though, that there is another class of relations which are not of the type discussed above but are interesting nevertheless. These are the relations among masses and coupling constants which hold in lowest order in the presence of all renormalization counter terms. Examples of this type are

$$m_p - m_n = f_\pi(g_{pp\pi^o} - g_{nn\pi^o}) + \text{finite correction}$$

which is true[99,100] in a model which combines the σ-model and the Weinberg-Salam lepton model, and the relation

$$\frac{m_W^2}{m_Z^2} = \cos^2\theta_W + \text{finite correction of } O(\alpha)$$

which hold in the Weinberg-Salam model. In fact, any relationship which is true in lowest order in the presence of all gauge invariant counter terms is also true in higher orders, with a finite, computable correction.

4. Radiative Correction as Source of Spontaneous Breakdown. This is an idea due to S. Coleman and E. Weinberg and has not yet been published. For a more detailed discussion, I refer you to the discussion of J. D. Bjorken in the parallel session. In the usual discussion of the Higgs phenomenon, the instability of the normal vacuum is caused by the displaced minimum of the potential of scalar fields in lowest order. In the approach of Coleman, the instability is caused not by the lowest order potential, but by the higher order correction to it. This idea is full of promises: for example, in an abelian realization of this idea, they show that

$$m_\phi^2/m_W^2 = 3\alpha/2\pi + O(\alpha^2).$$

More extensive exploration of this idea is clearly called for.

5. CP Violation. There are at least two ways of incorporating CP violation without doing violence to gauge invariance. The first is to make the Yukiwa couplings of Higgs mesons to fermions CP violating;[101] in order to do this, one needs in general more than one multiplet of the Higgs scalars. The second way, which has not been discussed in this context, is to have the Higgs mechanism violate CP simultaneously.

7. CONCLUDING REMARKS

The pioneering work of Weinberg, Salam, and 't Hooft led to various unified, renormalizable schemes for weak and electromagnetic interactions. I consider this class of theories to have passed an initial battery of tests on renormalizability. As emphasized before, what is really necessary for a logically consistent theory is that lowest order corrections to the phenomenological theory are finite, unambiguous and small enough not to disturb agreement with experiment. In this, the scheme succeeds admirably.

On the other hand, we have not succeeded in constructing a "natural" model of hadrons and leptons. This is a task that lies ahead of us. There are many models that have been discussed in the literature (and many more in notebooks). None of them may turn out to correspond exactly to the real world, but it may be that general features shared by some of these models, or specific features of one or another of them may survive.

The development of the last year brought a mild disappointment to some of us. I hoped, at the beginning, that the constraint of gauge invariance and renormalizability might shed some light on the origin of the Cabibbo angle, the size of CP violation, the structure of the hadronic SU(3) × SU(3) breaking terms, etc. Now this possibility seems unlikely. In the models discussed so far, these things can be put in, and you get out only what you put in. Perhaps in a more satisfactory model, these things will come out from a more reasonable, as-yet undiscovered dynamical principle.

Aside from the esthetic attractiveness, the merit of this theory is that its general phenomenological implications are testable in the near future. Discoveries of heavy leptons, or neutral current effects which fit in any one of

possible models will be a great relief, and triumph, for the enthusiast.

Our difficulty in producing an attractive model makes me wonder if we do know what we think we know about weak interactions. This is meant to be a plea to our experimental colleagues to reexamine the so-called "well-established" facts on all aspects of weak interactions.[102]

Finally, this review cannot be complete without my acknowledgement to those who have given me freely the benefit of their time and wisdom. Bill Bardeen, J. D. Bjorken, Joel Primack, Valya Zakharov and Bruno Zumino have given excellent survey talks in the parallel session. B. J. Bjorken, Bram Pais, Joel Primack, Sam Treiman and especially Steve Weinberg have been available to me for encouragement, enlightenment, and criticism.

APPENDIX TO SECTION 2

The Higgs scalar meson plays a role in making a spontaneously broken gauge theory finite. An example is given in Primack's discussion in which he shows that the Higgs scalar contribution is necessary to remove logarithmic divergences from one loop diagrams of fermion-fermion scattering. H. Quinn[103] has given a very interesting discussion as to the role of the Higgs meson in making two-loop contributions to fermion-antifermion scattering finite. The following is a brief summary of her result.

In order for the amplitude for $\nu\bar{e} \to \nu\bar{e}$ to be finite, the amplitude for $\nu\bar{e} \to W^+W^-W^+$ must grow at most like s^{-1} as $s \to \infty$. In a massive Yang-Mills theory, Vainshtein and Khriplovich[104] have shown that this condition is not met for the production of three longitudinally polarized vector bosons. When the extra term arising from the Higgs meson exchange (see Figure 5) is added to the above it cancels the leading term that grows like const $s^{-\frac{1}{2}}$ as $s \to \infty$, leaving an amplitude of order m_W/s, which is sufficiently convergent.

This is an explicit demonstration of the role of the Higgsian scalar in making the S-matrix finite from the viewpoint of the S-matrix theory, and repudiates the view that the Higgsian scalar is an artifact peculiar to operator field theory.

I am indebted to J. D. Bjorken, J. Primack and H. Quinn for teaching me about this argument.

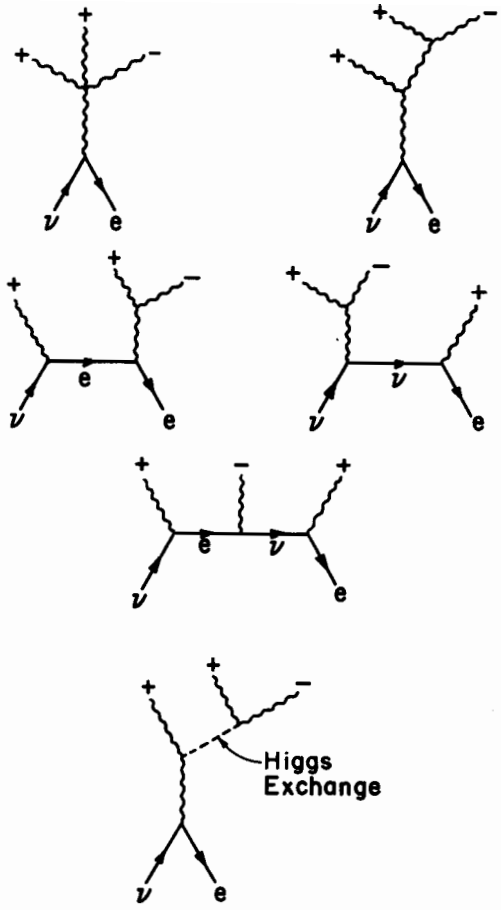


Fig. 5. Diagrams for the process $\nu\bar{e} \to W^+W^-W^+$. The bottom contribution, involving the exchange of a Higgs boson, is essential to obtain a sufficiently convergent amplitude as $s \to \infty$.

APPENDIX TO SECTION 3

I wish to expand my remarks on the Bars, Halpern, Yoshimura model. We shall ignore the anomaly problem altogether and use the notation appropriate to $[U_L(3) \times U_R(S)]_{\text{hadronic}} \times [SU(2) \times U(1)]_{\text{leptonic}}$. The mesons which transform like (3.1) with respect to the hadronic symmetry and $(\frac{1}{2}, -\frac{1}{3})$ with respect to the leptonic may be written as

$$M_a^{\ \alpha} = \begin{bmatrix} J^0 & J^+ \\ J_1^- & K^0 \\ J_2^- & L^0 \end{bmatrix}, \qquad a = 1,2,3, \quad \alpha = 1,2.$$

We need two such multiplets; we denote the second set by primed symbols.

Suppose we arrange the dynamics of the scalar complex so that M and M' develop vacuum expectation values.

$$\langle M \rangle = \begin{bmatrix} 1 & 0 \\ 0 & \cos\theta \\ 0 & \sin\theta \end{bmatrix} v$$

$$\langle M' \rangle = \begin{bmatrix} 0 & 0 \\ 0 & -\sin\theta \\ 0 & \cos\theta \end{bmatrix} v$$

Let V_μ^+ and W_μ be, respectively, the 3×3 gauge bosons corresponding to $[SU_L(3)]_{\text{hadronic}}$ and the 2×2 bosons for $[U(2)]_{\text{leptonic}}$:

$$V_\mu^+ = \begin{bmatrix} ---, & \frac{\rho_\mu^+ + (A1)_\mu}{\sqrt{2}}, & \frac{K^{*+} + (KA)_\mu}{\sqrt{2}} \\ ---, & ---, & --- \\ ---, & ---, & --- \end{bmatrix}$$

$$W_\mu = \begin{bmatrix} ---, & W \\ W_\mu^+, & --- \end{bmatrix}$$

The coupling between V_μ and W_μ is generated by

$$gf\ \mathrm{Tr}(V_\mu^+ M W^\mu M^\dagger + V_\mu^+ M' W^\mu M'^\dagger)$$

$$\to gg'\ \mathrm{Tr}(V_\mu^+ \langle M\rangle_0 W^\mu \langle M^\dagger\rangle_0 + V_\mu^+ \langle M'\rangle_0 W^\mu \langle M'^\dagger\rangle_0)$$

which will include the term

$$gfv^2\ W_\mu^+ \left(\cos\theta \frac{\rho_\mu^+ + (A1)_\mu}{\sqrt{2}} + \sin\theta \frac{K^{*+} + (KA)_\mu}{\sqrt{2}}\right)$$

but not the coupling of $K^0 + \bar{K}^0$ to the neutral members of W_μ, i.e., the photon and Z. In the above f and g are the gauge coupling constants for $[SU_L(3)]_{\text{hadronic}}$ and $[SU(2)]_{\text{leptonic}}$.

The authors argue that, in the approximation in which the meson complex M is replaced by its vacuum expectation value, the scheme is essentially identical to the field algebra of T. D. Lee, Weinberg and Zumino.

In this scheme the induced neutral current effects such as $K_L \to 2\mu$ and $K^0 \leftrightarrow \bar{K}^0$ are expected to be of order of

$$G_F(G_F m_\rho^2) \lesssim G_F\, \alpha \left(\frac{m_\rho}{m_W}\right)^2 \approx G_F\, \alpha \times 10^{-4}$$

and safe.

I am indebted to Dr. I. Bars for very stimulating discussions on his model.

REFERENCES

[1] S. Weinberg, Phys. Rev. Letters 19, 1264 (1967).

[2] A. Salam, Elementary Particle Theory, N. Svartholm, Editor (Almquist and Forlag A. B., Stockholm, 1968).

[3] J. Schwinger, Ann. Phys. (N.Y.) 2, 407 (1957).

[4] S. L. Glashow, Nucl. Phys. 22, 579 (1961).

[5] A. Salam and J. Ward, Phys. Letters 13, 168 (1964).

[6] P. Higgs, Phys. Letters 12, 132 (1964).

[7] T. W. B. Kibble, Phys. Rev. 155, 1554 (1967).

[8] G. Guralnik, C. R. Hagen, and T. W. B. Kibble, Phys. Rev. Letters 13, 585 (1964); F. Englert and R. Brout, Phys. Rev. Letters 13, 321 (1964).

[9] J. Goldstone, Nuovo Cimento 19, 154 (1961).

[10] J. Goldstone, A. Salam, and S. Weinberg, Phys. Rev. 127, 965 (1962).

[11] S. Bludman and A. Klein, Phys. Rev. 131, 2363 (1963).

[12] G. Jona-Lasinio, Nuovo Cimento 34, 1790 (1964).

[13] An excellent survey in this subject is given in T. W. B. Kibble in Proceedings of the 1967 International Conference on Particles and Fields, C. R. Hagen et al., Editor (Interscience, New York, 1967); C. R. Hagen and T. W. B. Kibble in Advances in Particle Physics, Vol. II, R. L. Cool and R. E. Marshak, Editors (Interscience, New York, 1968).

[14] T. D. Lee, Phys. Rev. Letters 26, 801 (1971).

[15] R. P. Feyman, Acta Phys. Polon. 26, 697 (1963).

[16] B. deWitt, Phys. Rev. 162, 1195, 1239 (1967).

[17] L. D. Faddeev and V. N. Popov, Phys. Letters 25B, 29 (1967).

[18] V. N. Popov and L. D. Faddeev, Kiev ITP Report ITP 67-36; translation, NAL-THY-57, May 1972.

[19] L. D. Faddeev, Theoretical and Mathematical Physics 1, 1 (1970).

[20] S. Mandelstam, Phys. Rev. 175, 1580 (1968).

[21] E. S. Fradkin and I. V. Tyutin, Phys. Letters 30B, 562 (1969); Phys. Rev. D2, 2841 (1970).

[22] M. Veltman, Nucl. Phys. B21, 288 (1970).

[23] M. Gell-Mann and M. Lévy, Nuovo Cimento 16, 705 (1960).

[24] B. W. Lee, Nucl. Phys. B9, 649 (1969); J.-L. Gervais and B. W. Lee, Nucl. Phys. B12, 627 (1969).

[25] K. Symanzik, Lettere al Nuovo Cimento 2, 10 (1969); Commun. Math. Phys. 16, 48 (1970).

[26] G. 't Hooft, Nucl. Phys. B33, 173 (1971).

[27] G. 't Hooft, Nucl. Phys. B35, 167 (1971).

[28] M. Gell-Mann, M. Goldberger, N. Kroll, and F. E. Low, Phys. Rev. 179, 1518 (1969); S. Weinberg, Phys. Rev. Letters 27, 1688 (1971).

[29] H. Georgi and S. L. Glashow, Phys. Rev. Letters 28, 1494 (1972).

[30] G. 't Hooft, Phys. Letters 37B, 195 (1971).

[31]H. H. Chen and B. W. Lee, Phys. Rev. D5, 1874 (1972).

[32]H. S. Gurr, F. Reines, and H. W. Sobel, Phys. Rev. Letters 28, 1406 (1972).

[33]S. Weinberg, Phys. Rev. D5, 1412 (1972).

[34]W. Lee, Phys. Letters 40B, 423 (1972).

[35]B. W. Lee, Phys. Letters 40B, 420 (1970).

[36]A. Pais and S. B. Treiman, Neutral Current Effects in a Class of Gauge Field Theories, COO-3502-25 (to be published in Phys. Rev. D).

[37]E. A. Paschos and L. Wolfenstein, Tests for Neutral Currents in Neutrino Reactions, National Accelerator Laboratory Report NAL-THY-69, July 1972.

[38]S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D2, 1285 (1970).

[39]M. L. Perl, Search for Heavy Leptons and Anomalous Lepton Behavior, SLAC Report SLAC-PUB-1062, July 1972.

[40]J. D. Bjorken and C. H. Llewellyn Smith, Spontaneously Broken Gauge Theories of Weak Interactions and Heavy Leptons, SLAC Preprint, August 1972.

[41]K. Fujikawa, B. W. Lee, and A. I. Sanda, Generalized Renormalizable Gauge Formulation of Spontaneously Broken Gauge Theories, (to be published in Phys. Rev. D).

[42]Y.-P. Yao, On Quantization and Gauge Freedom in a Theory with Spontaneously Broken Symmetry, Michigan preprint HE 72-16.

[43]T. D. Lee and C. N. Yang, Phys. Rev. 128, 885 (1962); T. D. Lee, Phys. Rev. 128, 899 (1962).

[44]K. Fujikawa, The ξ-Limiting Process in Spontaneously Broken Gauge Theories, Chicago preprint, June 1972. In this paper, a nonlinear gauge condition is employed, to make the Feynman rules as close as possible to those of ref. 43.

[45]J. L. Gervais and A. Neveu, Feynman Rules for Massive Gauge Fields with Dual Diagram Topology, LPTHE 72/13, March 1972. The gauge condition used here is very similar to that of reference 44.

[46]B. W. Lee, Phys. Rev. D5, 823 (1972).

[47]B. W. Lee and J. Zinn-Justin, Phys. Rev. D5, 3121, 3137, 3155 (1972).

[48]G. 't Hooft and M. Veltman, Nucl. Phys. B44, 189 (1972).

[49]G. 't Hooft and M. Veltman, Combinatorics of Gauge Fields, Utrecht preprint, July 1972.

[50]A. Salam and J. Strathdee, Nuovo Cimento, to be published.

[50b]J. C. Taylor, Nucl. Phys. B33, 436 (1971).

[51]A. Slavnov, Generalized Ward Identities for Gauge Fields, Kiev ITP-71-83E (1971).

[52]C. G. Bollini, J.-J. Giambiagi, and A. Gonzales Dominguez, Nuovo Cimento 31, 550 (1964). See also G. Cicuta, Analytic Continuation in the Dimension of Space-Time, and reference cited therein.

[53]W. A. Bardeen, R. Gastmans, and B. Lautrup, Static Quantities in Weinberg's Model of Weak and Electromagnetic Interactions, CERN TH. 1485.

[54]W. A. Bardeen, (to be published).

[55]E. R. Speer, Generalized Feynman Amplitudes, (Princeton University Press, Princeton, 1969).

[56] J. S. Bell and R. Jackiw, Nuovo Cimento 51, 47 (1969).

[57] S. L. Adler, Phys. Rev. 177, 2426 (1969). For a review see S. L. Adler, Perturbation Theory Anomalies in Lectures on Elementary Particles and Quantum Field Theory, S. Deser, M. Grisaru, and H. Pendelton, Editors (The Massachusetts Institute of Technology Press, Cambridge, Massachusetts, 1970).
R. Jackiw, Field Theoretic Investigations in Current Algebra in Lectures on Current Algebra and Its Applications, (Princeton University Press, Princeton, New Jersey).

[58] C. Bouchiat, J. Iliopoulos, and P. Meyer, Phys. Letters 38B, 519 (1972).

[59] D. J. Gross and R. Jackiw, Phys. Rev. D6, 477 (1972).

[60] H. Georgi and S. L. Glashow, Phys. Rev. D6, 429 (1972).

[61] W. A. Bardeen, Phys. Rev. 184, 1848 (1969).

[62] J. Wess and B. Zumino, Phys. Letters 37B, 95 (1971).

[63] S. Weinberg, Phys. Rev. Letters 27, 1688 (1971).

[64] B. W. Lee, Model of Weak and Electromagnetic Interactions, National Accelerator Laboratory Report NAL-THY-51, April 1972, (to be published in Phys. Rev. D).

[65] J. Prentki and B. Zumino, CERN TH 1504 (1972), (to be published in Nuclear Physics B).

[66] S. Weinberg, Phys. Rev. D5, 1412 (1972).

[67a] P. G. O. Freund, Nuclear Physics B41, 35 (1972).

[67b] Y. Achiman, A Model of Weak and Electromagnetic Interactions with No New Quarks, Heidelberg preprint.

[68]The hadronic SU(3) × SU(3) symmetry breaking is discussed from this viewpoint by J. Schecter and Y. Ueda, D5, 2846 (1972); W. F. Palmer, Phys. Rev. D., to be published, and contribution to this conference.

[69]H. Lipkin, unpublished memorandum.

[70]H. Georgi and S. L. Glashow, Gauge Theory of Weak and Electromagnetic Interactions with Han-Nambu Quarks, Harvard preprint, 1972.

[71]M. Han and Y. Nambu, Phys. Rev. 139, 1006 (1965).

[72]K. Symanzik, in Fundamental Interactions at High Energies, A. Perlmutter, Editor (Gordon and Breach, 1970).

[73]S. Coleman, Renormalization and Symmetry: A Review for Non-Specialists, lectures delivered at the 1971 International Summer School of Physics, Ettore Majorana, (to be published).

[74]I. Bars, M. B. Halpern, and M. Yoshimura, Gauge Theory of Strong, Weak and Electromagnetic Interactions, Berkeley preprint, LBL-990, 1972.

[75]K. Bardakci and M. B. Halpern, Phys. Rev. D6, 696 (1972).

[76]T. W. Appelquist and H. R. Quinn, Phys. Letters 39B, 229 (1972).

[77]S. Y. Lee, UCSD preprints, April 1972.

[78]G. Rajasekaran, Tata Institute preprint, March 1972.

[79]R. Jackiw and S. Weinberg, Phys. Rev. D5, 2396 (1972).

[80]I. Bars and M. Yoshimura, Phys. Rev. D6, 374 (1972).

[81]G. Altarelli, N. Cabibbo and L. Maiani, Physics Letters 40B, (1972).

[82]B. W. Bardeen, R. Gastmans and B. Lautrup, CERN preprint, May 1972.

[83]J. Primack and H. Quinn, Harvard preprint, May 1972.

[84]K. Fujikawa, B. Lee and A. Sanda, reference 41.

[85]R. Fukuda and R. Sasaki, University of Tokyo preprint, July 1972.

[86]W. Kummer and K. Lane, contribution to this conference.

[87]T. W. Appelquist, J. R. Primack and H. R. Quinn, Harvard preprint, May 1972.

[88]C. G. Bollini, J.-J. Giambiagi and A. Sirlin, contribution to this conference.

[89]B. W. Lee, J. Primack, and S. B. Treiman, Physical Constraints on Gauge Models of Weak Interactions, National Accelerator Laboratory Report NAL-THY-74, August 1972.

[90]J. Primack and H. Quinn, Muon G-2 and Other Constraints on a Model of Weak and Electromagnetic Interactions without Neutral Currents, (to be published in Phys. Rev. D).

[91]K. Fujikawa, B. W. Lee, A. I. Sanda, and S. B. Treiman, Induced Neutral Current Effects in Unified Models of Weak and Electromagnetic Interactions, National Accelerator Laboratory Report NAL-THY-65, June 1972.

[92]C. Bouchiat, J. Iliopoulos and Ph. Meyer, Elastic Neutrino Scattering in Renormalizable Models of Weak and Electromagnetic Interactions without Neutral Currents , Orsay preprint, August 1972.

[93]I. Ya. Pomeranchuk, Yad. Fiz. 11, 852 (1970).

[94]M. Gell-Mann, unpublished lecture.

[95]A. D. Dolgov, L. B. Okun, V. I. Zakharov, Nuclear Physics B37, 493 (1972).

[96]T. Appelquist and J. D. Bjorken, Phys. Rev. D4, 3726 (1972).

[97]S. Weinberg, Phys. Rev. Letters 29, 388 (1972).

[98]H. Georgi and S. L. Glashow, Spontaneously Broken Gauge Symmetry and Elementary Particle Masses, (to be published in Phys. Rev. D).

[99]T. Hagiwara and B. W. Lee, to be published.

[100]See, however, Fayyazuddin and Riazuddin, Proton-Neutron Mass Difference in a Unified Theory of Weak and Electromagnetic Interactions, Trieste IC/72/75: A Love and G. G. Ross, as quoted therein.

[101]R. N. Mohapatra, Unified Theory of Weak and Electromagnetic Interactions with CP Violation, Maryland preprint.

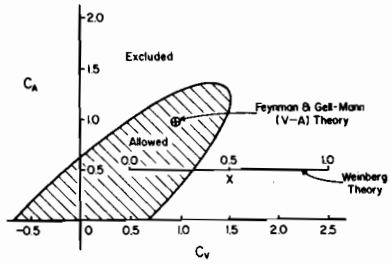
[102]See, for instance, A. Pais, Phys. Rev. D5, 1170 (1972).

[103]H. R. Quinn, unpublished notes.

[104]A. I. Vainshtein and I. B. Khriplovich, Soviet Journal of Nuclear Physics 13, 111 (1971); E. B. Bogomdny, L. B. Okun and V. I. Zakharov, Soviet Journal of Nuclear Physics, to be published.

Table I

Status of Leptonic Neutral Current Effects

	Process		Weinberg Theory
1.	$\bar{\nu}_e + e \to \bar{\nu}_e + e$	$\sigma_{exp}/\sigma_{V-A} < 1.9$ (Gurr, Reines, and Sobel[a])	$\frac{G_F}{\sqrt{2}} \bar{\nu}\gamma^\alpha(1-\gamma_5)\nu\bar{e}\gamma_\alpha(c_V - c_A\gamma_5)e$ 
2.	$\nu_\mu + e \to \nu_\mu + e$	$\sigma_{exp} \leq .7 \times 10^{-41}(E_\nu/\text{GeV})\text{cm}^2$ (CERN Group, this conference[b])	$\sin^2\theta_W = x < 0.6$
3.	$\bar{\nu}_\mu + e \to \bar{\nu}_\mu + e$	$\sigma_{exp} \leq 1.1 \times 10^{-41}(E_\nu/\text{GeV})\text{cm}^2$ (CERN Group, this conference[b])	

a. H. Gurr, F. Reines and H. W. Sobel, Phys. Rev. Letters 28, 1406 (1972)

b. CERN Group, as reported at this Conference.

Table II

Status of $\Delta S = 0$ Neutral Current Effects (Hadronic)

	Process	Experiment	Weinberg Theory
1.	$\nu+p\to\nu+n+\pi^+$	$R_1 = \frac{\sigma(\nu+p\to\nu+n+\pi^+)}{\sigma(\nu+p\to\mu+p+\pi^+)}$ = 0.08 ± 0.04 (Cundy et al.[a])	$R_1 \simeq \frac{1}{9}$ (Weinberg[b]) $R_1 \geq \frac{1}{9} \times 0.4$ (Albright et al.[c])
2.	$\nu+p\to\nu+p+\pi^0$ $\nu+n\to\nu+n+\pi^0$	$R_2 = \frac{\sigma(\nu p\to\nu p\pi^0) + \sigma(\nu n\to\nu n\pi^0)}{2(\nu n\to\mu p\pi^0)}$ ≤ 0.14 (W. Lee[d])	$R_2 \geq 0.4$ (B. Lee[e]: Δ dominance, static model) ≥ 0.4 (Paschos and Wolfenstein[f]: Δ dominance) ≥ 0.19 (Albright et al.[c]: incoherence of the I=1/2 and 3/2 final states)
3.	$\nu+p\to\nu+p$	$R_3 = \frac{\sigma(\nu+p\to\nu+p)}{\sigma(\nu+n\to\mu+p)}$ = 0.12 ± 0.06 (Cundy et al.[a])	$0.15 \leq R_1 \leq 0.25$ (Pais and Treiman[g])

a. D. C. Cundy et al., Phys. Letters 31B, 478 (1970).

b. S. Weinberg, Phys. Rev. D5, 1874 (1972).

c. C. Albright, B. W. Lee and E. Paschos, "Bounds on Neutral Current Interactions in Weak Pion Production", to be published.

d. W. Lee, Phys. Letters 40B, 423 (1972).

e. B. Lee, Phys. Letters 40B, 420 (1972).

f. E. Paschos and L. Wolfenstein, "Tests for Neutral Currents in Neutrino Reactions", to be published.

g. A. Pais and S. B. Treiman, "Neutral Current Effects in a Class of Gauge Field Theories", to be published.

Table IIIa

Status of $\Delta S \neq 0$ Neutral Current Effects

Processes	Upper Bounds (B.R.) (90% C.L.)	Sources[a]
$K^+ \to \pi^+ e^+ e^-$	$< 4 \times 10^{-7}$	Cline et al. (1968)
$K^+ \to \pi^+ \mu^+ \mu^-$	$< 2.4 \times 10^{-6}$	Bisi et al. (1967)
$K^+ \to \pi^+ \pi^o e^+ e^-$	$< 8 \times 10^{-6}$	Cline (1965)
$K_L^o \to \mu^\pm e^\mp$	$< 1.9 \times 10^{-9}$	Clark et al. (1971)
$K_L^o \to e^+ e^-$	$< 1.9 \times 10^{-9}$	Clark et al. (1971)
$K_L^o \to \mu^+ \mu^-$	$< 1.9 \times 10^{-9}$	Clark et al. (1971)
$K_s^o \to \mu^+ \mu^-$	$< 7.3 \times 10^{-6}$	Hyams et al. (1969)
$K^+ \to \pi^+ \nu\bar{\nu}$	$< 1.4 \times 10^{-6}$	Klems et al. (1971)
	$< 7.5 \times 10^{-7}$	Cable et al. (this Conference)
$K^+ \to \mu^+ \nu\bar{\nu}\nu$	$< 7 \times 10^{-6}$	Cable et al. (this Conference)

a. Pre-conference data are taken from J. H. Klems, R. H. Hildebrand, and R. Stiening, Phys. Rev. D4, 66 (1971).

Table IIIb

$\Delta S = 1$ Neutral and Double Charged Current Effects

Type	Decay Mode	Ref.	Limit
Neutral	$\left[\frac{\Gamma(K^+ \to \pi^+ e^+ e^-)}{\Gamma(K^+ \to \pi^o e^+ \nu_e)}\right]^{\frac{1}{2}}$	(a)	$< 3.0 \times 10^{-3}$
	$\left[\frac{\Gamma(K^+ \to \pi^+ \nu\bar{\nu})}{\Gamma(K^+ \to \pi^o e^+ \nu_e)}\right]^{\frac{1}{2}}$	(b)	$< 5.0 \times 10^{-3}$
	$\left[\frac{\Gamma(K_L^o \to \mu^+ \mu^-)}{\Gamma(K^+ \to \mu^+ \nu_\mu)}\right]^{\frac{1}{2}}$	(b)	$< 2.6 \times 10^{-5}$
Doubly-charged	$\left[\frac{\Gamma(K^+ \to \pi^- e^+ \mu^+)}{\Gamma(K^+ \to \pi^o \nu_\mu \mu^+)}\right]^{\frac{1}{2}}$	(c)	$< 9.4 \times 10^{-4}$
	$\left[\frac{\sigma(\mu^- + Cu \to e^+ + Co)}{\sigma(\mu^- + Cu \to \nu_\mu + Ni)}\right]^{\frac{1}{2}}$	(d)	$< 1.6 \times 10^{-4}$

a. D. B. Clarke, et al., Bull. Am. Phys. Soc. 17, 493 (1972).

b. Review of Particle Properties, Phys. Letters 39B, 1 (1972)

c. E. W. Beier, D. A. Pouchholtz, A. K. Mann, and S. H. Parker, "Search for Doubly Charged Weak Currents Through $K^+ \to \pi^- e^+ \mu^+$", to be published.

d. D. A. Buryman, et al., Phys. Rev. Letters 28, 1469 (1972).

This compilation is taken from c.

Table IV

Heavy Leptons (E^+, E^o; M^+, M^o)

Decay Modes (assume $m(E^+) > m(E^o)$):

$E^+ \rightarrow E^o e^+ \nu_e$

$E^o \mu^+ \nu_\mu$

E^o + hadrons

$\rightarrow \nu_e \mu^+ \nu_\mu$

$\nu_e e^+ \nu_e$

ν_e + hadrons

$E^o \rightarrow e^- \mu^+ \nu_\mu$ (good signature)

$e^- e^+ \nu_e$

e^- + hadrons

$E^o \rightarrow \nu\gamma$

Production Mechanisms:

$e^+ + e^- \rightarrow E^+ + E^-$

$\rightarrow$ hadrons + γ

$\downarrow \rightarrow E^+ E^-$

$e^- + e^+ \rightarrow E^o + \bar{\nu}_e$

$\nu_\mu + N \rightarrow M^+ + \text{hadrons}$

$\mu + N \rightarrow M^o + \text{hadrons}$

$\gamma + N \rightarrow M^+ + M^- + \text{hadrons}$

Table V. Recipe.

1. Choose a gauge group G SU(2).
2. Choose Higgs scalar fields ϕ.
3. Construct an invariant, renormalizable $V(\phi)$.

   $$\partial V(\phi)/\partial\phi\big|_{\phi=v} = 0.$$

   The little group of the vacuum = $U_Q(1)$.
4. Choose chiral spinor fields ψ_L, ψ_R.
5. Form Yukawa couplings $\bar{\psi}_L \phi \psi_R$ + H.c. .
6. Couple gauge bosons to Higgs ϕ, ψ_L, and ψ_R.
7. Quantize.

   A. $\frac{1}{2}(\partial_\mu \phi + gV_\mu \phi)^2 \rightarrow \frac{1}{2}(gv)^2 V_\mu{}^2$.

   B. $\gamma \psi_L \phi \psi_R$ + h.c. $\rightarrow \gamma v \bar{\psi}\psi$.

   C. Some of Higgs scalars $\rightarrow$ Longitudinal components of massive vectors.

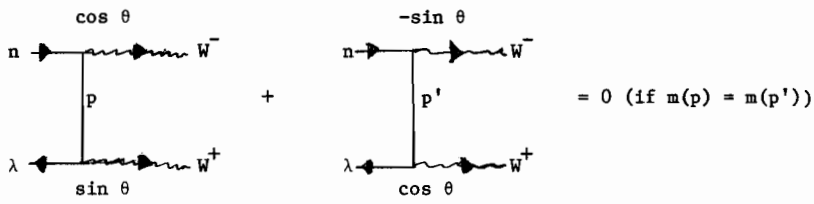
Table VI

Models of Leptons

	Weinberg-Salam	Georgi-Glashow
Group	U(2)	O(3)
Gauge Bosons	W^+, W^-, Z, γ	W^+, W^-, γ
Leptons	$\begin{pmatrix} e \\ \nu \end{pmatrix}_L , e_R$	$\begin{pmatrix} e^- \\ \nu \sin\beta + E^0 \cos\beta \\ E^+ \end{pmatrix}_L \begin{pmatrix} e^- \\ E^0 \\ E^+ \end{pmatrix}_R ,$ $(\nu \cos\beta - E^0 \sin\beta)_L .$
Higgs Scalar	$\begin{pmatrix} \phi^0 \\ \phi^+ \end{pmatrix} \rightarrow \begin{pmatrix} v \\ 0 \end{pmatrix}$	$\begin{pmatrix} \phi^+ \\ \phi^0 \\ \phi^- \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix}$
Electric Charge	$Q = T_3 + Y/2$	$Q = T_3$
W Mass	> 32.7 GeV/c^2	< 52.8 GeV/c^2

Table VII

GIM Construction



$= 0$ (if $m(p) = m(p')$)

$$\begin{bmatrix} p \\ n\cos\theta+\lambda\sin\theta \end{bmatrix} \leftrightarrow \begin{bmatrix} p' \\ -n\sin\theta+\lambda\cos\theta \end{bmatrix}$$

Weinberg Model

$$L_1 = \begin{pmatrix} p \\ n\cos\theta+\lambda\sin\theta \end{pmatrix}_L , \quad L_2 = \begin{pmatrix} p' \\ -n\sin\theta+\lambda\cos\theta \end{pmatrix}_L$$

$$\frac{m_n}{v}\ \bar{n}_R\,[\cos\theta(\phi\cdot L_1) - \sin\theta(\phi\cdot L_2)] + \text{h.c.} \rightarrow m_n\bar{n}n$$

Table VIII

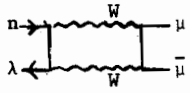
Physical Constraints

1. $K^+ \to \pi^+ + \ell + \bar{\ell}$

$$\sim \frac{G_F}{\sqrt{2}} \sin\theta \left(\frac{m_\lambda + m_n}{m_\lambda - m_n} \right) \frac{m(M^+) m(K)}{m_\phi^2}$$

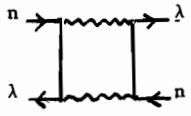
(if present)

2. $K_L \to \mu\bar{\mu}$



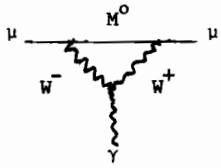
$\sim G_F \alpha$ without GIM,

3. $K^o \leftrightarrow \bar{K}^o$



$\sim G_F \alpha \dfrac{\Delta m^2}{m_W^2}$ with GIM.

4. Anomalous Magnetic Moment



$\sim G_F m_\mu m(M^o)$.

(cf. $G_F m_\mu^2$ for other diagrams)

DISCUSSION

K. A. Ter-Martirosyan (ITEP, Moscow): You have described many different models of weak interactions, but nature can clearly choose only one. Are there any tests to distinguish between these different models?

B. W. Lee: Of course no single experiment can prove any particular theory. But the search for (a) neutral currents and (b) heavy leptons should be crucial in proving or disproving these theories.