

# ESME at 18: Realistic Numerical Modeling of Synchrotron Motion

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## 1 Prolegomena

The program ESME has been developed intermittently during eighteen years for accurate concrete modeling of synchrotron beam dynamics in phase-energy coordinates of particles and particle distributions. That is, its domain is longitudinal phase space motion, and the relevant system constituents are rf cavities, longitudinal coupling impedances, feedback loops, beam space charge, and the lattice parameters governing the beam circulation period and its momentum dependence. Both rf experts and beginners have at some times found it efficient to use ESME instead of creating particular code for a model. Many features have been added in response to the needs of these users but always with an effort to do so in the most generally applicable practical form. Nonetheless, most of the code has been written by the author for his own needs rather than as a package for general use; many of the consequent questions of validity, generality, and documentation have been answered during efforts to make the code useful to others.

A simple problem should be simple to represent. In this respect the development of ESME has been a qualified success. A general feature, like setting the lattice parameters or describing the program output desired, is invoked by a single character command in the standard input stream; the number of these commands has increased rather little, and the new commands can frequently be ignored in routine use. A command normally reads parameters that immediately follow it and, sometimes, data tables from auxiliary files. Unfortunately, the number of parameters has grown substantially for many of the commands. The saving grace is that almost all parameters have useful default values that retain the behavior of the older, simpler versions.

For a casual user, documentation is often crucial. The user's guide for ESME[1] contains tables of all the command parameters and reasonably full discussion of most features; however, it is probably inadequate for a majority of first-time users as a sole source of information. The version available through the web page [www-ap.fnl.gov/ESME/](http://www-ap.fnl.gov/ESME/) is updated as errors are found or user requests for clarification are pursued. The internal documentation in the source code is quite informative for those sufficiently mo-

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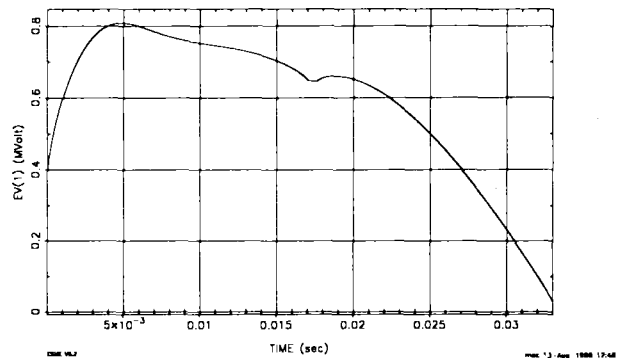


Figure 1: RF amplitude to maintain 0.06 eVs bucket area in Fermilab booster

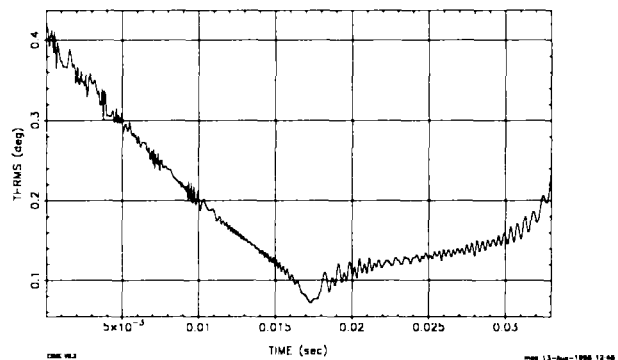


Figure 2: RMS width of 0.02 eVs bunch during booster cycle; ordinate scale is  $\pm 180^\circ$  for full circumference.

tivated to open the black box. One of the easiest ways to get started is to try some of the demo data sets or borrow one from an application like the one of interest.

## 2 Illustrative Examples

Considering that the user's guide is incomplete at fifty-five pages, the space-efficient introduction to features is by example. The first is intended to demonstrate how very simple a useful calculation can be by giving the complete input data; the others aim for suggestive variety.

### 2.1 Fixed bucket area, sine ramp

Problem: Find the voltage curve to maintain constant 0.6 eVs bucket area during acceleration on a sinusoidal magnet ramp and find the width of a 0.02 eVs bunch. The required data are

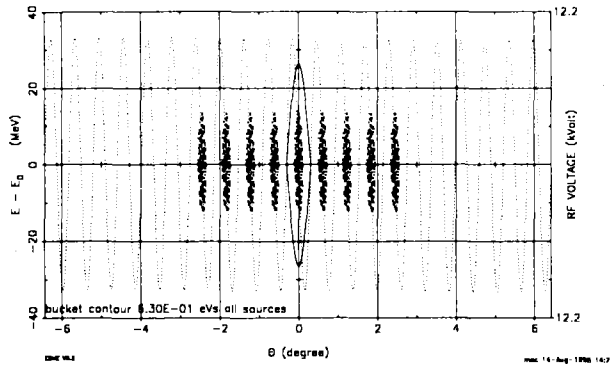


Figure 3: Nine bunch train at 53 MHz

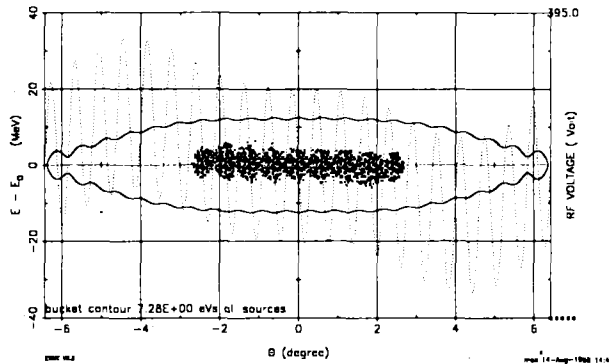


Figure 4: Bunch train fully debunched at 2.5 MHz, 53 MHz still present

```
R Similar to the Fermilab Booster
$RING Req=75.47 gammat=5.446 W0i=200.
W0f=8000. tf=.0333 kurveB=3 ncav=8 $
A V-curve from const. bucket area
$RF h=84 Vi=.4 Vkon=F holdBA=T isync=1 $
P 0.02 eVs elliptical distribution
$POPL8 kind=14 Sbnch=.02 npoint=2000 $
T Track 33 ms with turn-by-turn history
$CYCLE ttrack=0.033 histry=T $
H History of voltage & rms bunch width
$HISTORY nplt=1,101,1,6 $
Q Stop here; nothing more to do
```

One can almost read a data set like that; the orbit radius is 75 m, the transition gamma is 5.446, the injection energy is 200 MeV, the final energy 8 GeV, the ramp lasts for 33 ms, and, less obviously, a B-curve of type 3 is sinusoidal. The results of this little exercise are not trivial; the calculated voltage program is shown in Fig. 1 and the rms bunch width in Fig. 2.

## 2.2 Adiabatic multi-bunch coalescing

A typical hadron collider problem: One wants to collide high-intensity bunches at high energy that won't fit into single buckets somewhere in the injector chain. A train of nine high frequency ( $h=588$ ) bunches are adiabatically debunched after acceleration into a matched  $h=28$  bucket and the ensemble is then rotated  $90^\circ$  and recaptured at  $h=588$ . Four rf systems are used:  $h=588$ ,  $h=1176$ ,  $h=28$ , and  $h=56$ .

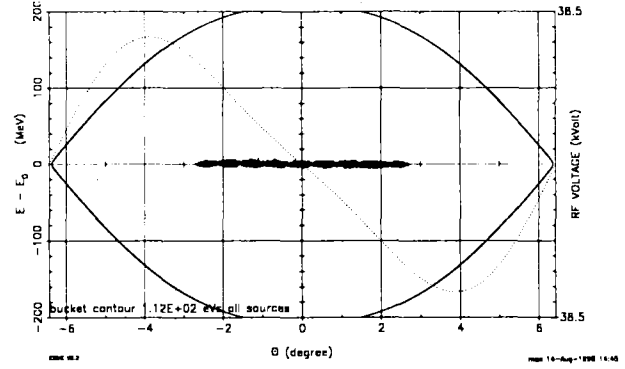


Figure 5: Large 2.5 MHz bucket to rotate train

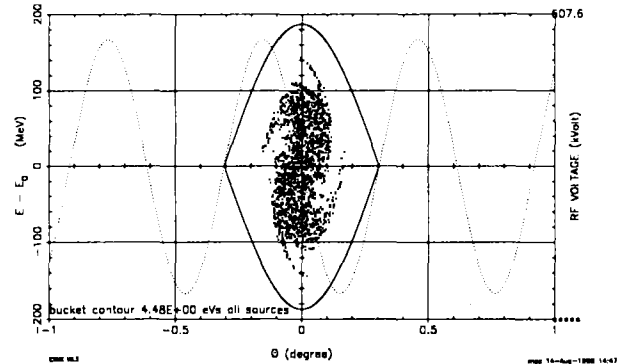


Figure 6: Train after recapture at 53 MHz showing filamentation caused by bucket mis-match to train boundary

The  $h=1176$  is a second harmonic for linearizing the phase rotation and the  $h=56$  system shapes the  $h=28$  bucket to better match the  $h=588$  bunch train. Some of the steps are illustrated in Figs. 3 – 6. The energy spread in Fig. 4 is only about  $\pm 4$  MeV; a more complete modeling would include some broadband  $Z_{||}$  to take account of limitations posed by microwave instability.

## 2.3 Negative mass instability (NMI)

It is arguable whether there is *need* to simulate negative mass instability because the theory for the threshold appears quite adequate,[2] but there are some reasons to see if one can, e.g., to calculate practical effect above threshold or to check the numerical machinery at very high frequency. Numerically it is a difficult problem because it appears necessary to calculate to at least millimeter scale. Macroparticle modeling done many years ago already shows the qualitative effect but is not very compelling because, with only 2000 macroparticles,[3] numerical noise exceeds the natural Schottky spectrum over most of the relevant frequency range, say 1 – 100 GHz. With faster computers and cheap memory it is reasonable to raise the macroparticle number  $10^7$  or so for short calculations, and it is also possible to start with a low noise distribution which makes the starting condition as quiet or quieter than a physical beam. Space does not permit a systematic develop-

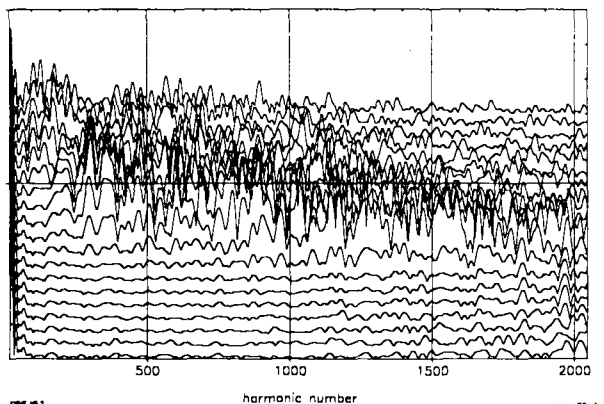


Figure 7: Fourier spectra of bunched beam in the Fermilab Main Ring over the range 1.1–100.8 GHz for  $6 \cdot 10^{10}$  protons per 0.08 eVs bunch from 420  $\mu$ s before to 1.26 ms after transition at 84  $\mu$ s intervals

ment; some comments could be of interest:

1. To increase the valid frequency band of a calculation by a factor  $f$  using ordinary (pseudo-) random numbers, the number of macro-particles must be increased by a factor  $f^3$ . This is a binning argument.
2. For certain quasi-random numbers, *e. g.* Sobol sequences, the numerical noise is  $\propto n^{-1}$ , so that initial distributions of  $10^6 - 10^7$  are as quiet as beams of  $10^{12}$  or so. From binning considerations it follows that a factor of  $f$  in bandwidth requires a factor  $f^2$  in macroparticles.
3. Normal mixing in phase motion eventually randomizes low noise distributions; the risetime for NMI is usually fast by comparison. Phase flow, however, does not produce the local clumps that always occur to some degree in ordinary random distributions.

By modeling a beam above negative mass threshold in successive trials of greater bandwidth, one finds that the amount of emittance dilution is not so sensitive to bandwidth. Following the development in ref. [2] one finds that for the Fermilab Main Ring the frequency with the shortest rise time is about 77 GHz. In simulations where the upper frequency cut-off is less than this value, the instability shows up at the highest frequency but rapidly quenches to much lower frequencies. The risetime goes inversely as the top frequency, but, after initial onset, disruption of the bunch is nearly independent of it. Therefore, for the practical purpose of including beam degradation by NMI into some system model, it is apparently not necessary to make the bandwidth of the modeling completely realistic.

In Fig. 7 is shown the fourier spectrum over the interval 1.1–109 GHz for bunches of 0.08 eVs with  $6 \cdot 10^{10}$  protons in the Main Ring ( $\gamma_T = 18.75$ ) at 84  $\mu$ s intervals from 420  $\mu$ s before transition to 1.26 ms

after on a linear ramp with  $\dot{\gamma} = 128 \text{ s}^{-1}$ . One sees high frequency structure appearing before transition which is clearly not simple mixing randomization. There is space charge driven emittance growth on a considerably slower time scale than the NMI. The instability becomes obvious to the eye by 168  $\mu$ s after transition. A faithful modeling of the negative mass instability requires attention to detail beyond what has been described; however, a practical estimate of the beam disruption is not an unreasonably extravagant undertaking.

### 3 So?

It is not so hard to develop a model for some particular process, but it is painful to do it over and over. The advantage in using a veteran code is that, because few problems are fundamentally new, most or all one needs is already there. The negatives are uncertainty about the correctness of someone else's code and the effort to understand exactly the meaning of the input parameters. For any serious work, both negatives must be addressed by test cases which can be checked against another code or, probably better yet, against experience. The history of ESME is the accrual of features that experience has shown useful into a conceptually simple structure which reflects concrete features of accelerator systems as faithfully as is practical. Despite years of elaboration, new facilities continue to be in demand. Most can be accommodated using existing hooks in the code for user subroutines and a provision for history plotting of user-defined quantities. The code is regarded as mature (legacy?), but some new material has been introduced into a test version in the last year, including expanded time domain wakefield capabilities and arbitrary particle species. The test code works for electrons but does not include synchrotron radiation. The emergence of the test version as standard distribution depends on interest of willing beta-testers, because the author is not currently engaged in related activities.

### Acknowledgement

I gratefully acknowledge the efforts of Jean-François Ostiguy in recent enhancements[4] in color graphics, memory management, and source management.

### References

- [1] J. MacLachlan, FERMILAB-TM-1856 (8/93)
- [2] W. Hardt, "Gamma-transition-jump scheme of the CPS", HEACC74. Stanford (1974)
- [3] W. W. Lee and L. C. Teng, HEACC71, Geneva (1971), p327
- [4] J. MacLachlan and J-F Ostiguy, USPAC97, Vancouver (1997)