NUCLEAR STRUCTURE AND NUCLEAR β-DECAY

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1. INTRODUCTION

For a long time the nuclear β-decays had an isolated position among nuclear processes. Although the ft-values were of great importance for assigning spin and parities to nuclear levels, β-decays otherwise contributed rather little to the understanding of the general features of nuclear structure. The Alaga selection rules were one of a few examples of connections between β-decay and more general properties of nuclei.

This poor situation started to change when the V-A interaction was established as the predominant component in weak interactions and, during the last years, further progress has been made in relating nuclear β-decays to the rest of nuclear physics. The study of isobaric analogue states and the discovery that the isospin is a good quantum number even in heavy nuclei have initiated a better understanding of the Fermi decays. The old problem of strong hindrance for Gamow-Teller decays in nuclei with a neutron excess has now been related, at least in a qualitative way, to more general components in nuclear interactions. Also, the general relationship between the vector part of the β-current and the electromagnetic current (C.V.C.) together with the isobaric symmetry will allow us to deduce more general properties for some of forbidden β-moments.

2. ALLOWED DECAYS

The allowed decays have a large spread in ft-values. The general trends can, however, be related to simple effects of nuclear structure. The most conspicuous feature of both the Fermi and the Gamow-Teller decays is the strong hindrance in nuclei with neutron excess. For the Fermi transitions this is due to the fact that all decays in such nuclei ($N > 2$) are isospin forbidden.

2a. Fermi decays

The Fermi transitions provide a direct test of the isobaric symmetry since the transition operator is given by

$$T_{\pm} = T_x \pm iT_y = \sum_k t_x(k) \pm it_y(k) \ . \qquad (1)$$

This structure of the operator is a direct consequence of the relationship between the vector β-current and the electromagnetic current (the conserved vector current hypothesis), and thus the calculation of Fermi matrix elements implies no specific assumptions regarding the internal nuclear properties. The matrix element vanishes except for transitions between members of an isobaric multiplet for which we have

$$M_F = \langle T, M_T \pm 1 | T_\pm | T\ M_T \rangle = ((T \pm M_T + 1)(T \mp M_T))^{1/2}. \quad (2)$$

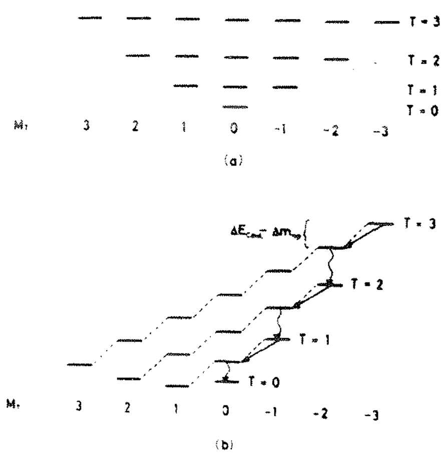

Fig. 1. Isospin multiplets: a) without and b) with Coulomb displacement. In b) isobaric analogue states are connected with dotted lines. The superallowed Fermi transitions, connecting states of the same isospin, are only possible in nuclei with Z > N.

The strong Fermi decay must then be found in that part of the periodic table where transitions between isobaric analogue states are energetically possible. The mutual position of isobaric analogue states in neighbouring nuclei (see Fig. 1) is determined by the large Coulomb energies, and this restricts the Fermi transitions to nuclei with $Z \gtrsim N$, as shown in Fig. 1.

The mixed Fermi and G.T. decays $n \rightarrow p$ and $H^3 \rightarrow H_e^3$ do not follow this rule because the Coulomb energy vanishes or is smaller than the neutron-proton mass difference.

The most precisely measured Fermi decays[1)] are the $0^+ \rightarrow 0^+$ ($T_i = T_f = 1$) transitions for which the matrix element is $M_F = \sqrt{2}$. It has been observed that the experimental ft-values for these decays are extremely constant and thus provide evidence for the isobaric purity of the corresponding states. The very small deviations ($\lesssim 1\%$) actually observed are not understood in detail, but must be due to 1) deviations from complete isobaric symmetry, 2) second forbidden corrections to the Fermi matrix element and the weak radial dependence of the Fermi operator, arising from the radial variations of the electron wave functions inside a finite nucleus. Recent discussions of the two types of effects are given in references 2 and 3.

The isospin forbidden transitions ($\Delta T=1$) are hindered by factors of the order of 10^4 or more as compared to the $\Delta T=0$ transitions. In Table 1 we have collected the known $0^+ \rightarrow 0^+$ transitions between states of different isospin ($\Delta T=1$). Matrix elements for such transitions have also been extracted from mixed Fermi and G.T. decays[8,9,13,14)] and are as small as the ones quoted in Table 1; in fact, the largest isospin forbidden matrix elements are found in the decays of

TABLE 1

$0^+ \rightarrow 0^+$ decays between states with different isospin

	T_i	T_f	ft(sec)	$\|M\|$
$Ga^{64} \rightarrow Zn^{64}$	1	2	$4\ 10^6$	$4\ 10^{-2}$
$Ge^{66} \rightarrow Ga^{66}$	1	2	$6\ 10^6$	$3\ 10^{-2}$
$Ga^{66} \rightarrow Zn^{66}$	2	3	$8\ 10^7$	$9\ 10^{-3}$
$Eu^{156} \rightarrow Gd^{156}$	15	14	$5.8\ 10^9$	$1.0\ 10^{-3}$
$Eu^{156} \rightarrow Gd^{156}$ (1.05)	15	14	$1.5\ 10^{10}$	$0.64\ 10^{-3}$
$Lu^{170} \rightarrow Yb^{170}$	14	15	$5.7\ 10^9$	$1.1\ 10^{-3}$
$Np^{234} \rightarrow U^{234}$	24	25	$1.8\ 10^8$	$5.9\ 10^{-3}$
$Np^{234} \rightarrow U^{234}$ (0.81)	24	25	$1.5\ 10^9$	$2.0\ 10^{-3}$
$Np^{234} \rightarrow U^{234}$ (1.04)	24	25	$1.4\ 10^9$	$2.1\ 10^{-3}$

The experimental values are taken from references 4, 5,6 and 7. The matrix elements in the last column are obtained from the ft-value, using the relation $|M| = \sqrt{6200/ft}$. If higher order corrections to the Fermi matrix element are small, as they seem to be in most cases, the last column gives an approximate value for the Fermi matrix element.

Ga^{64} and Ge^{66}.

The observed small transition strength for the $\Delta T=1$ decays arises from isobaric spin impurities and higher order corrections to the Fermi matrix element. In most cases the largest contribution seems to come from isobaric spin impurities.

For illustration and discussion of isospin forbidden transitions the best example is provided by the decay of Ca^{49} to Sc^{49} (see Fig. 2). These nuclei have a single particle outside the double closed shells of Ca^{48} and we therefore expect the ground states and some of the low lying states to be single-particle states. This has also been confirmed by experiment. The ground state of Ca^{49} is a $p_{3/2}$ state and the ground state of Sc^{49} an $f_{7/2}$ state.

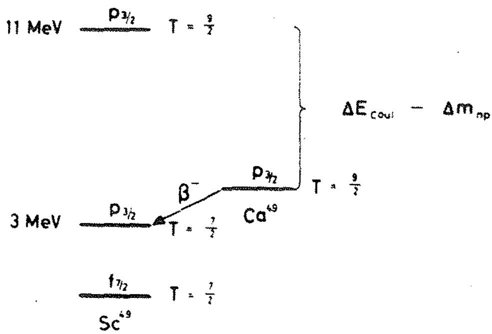

Fig. 2. Single-particle states in Ca^{49} and Sc^{49}.

The Ca^{48} core has the isospin $(T,M_T) = (4.4)$ and Ca^{49} then has $(T,M_T) = (\frac{9}{2}, \frac{9}{2})$. The $f_{7/2}$ ground state in Sc^{49} has $(T,M_T) = (\frac{7}{2}, \frac{7}{2})$, but the $p_{3/2}$ state in Sc^{49} can have both the isospin $(T,M_T) = (\frac{9}{2}, \frac{7}{2})$ and $(T,M_T) = (\frac{7}{2}, \frac{7}{2})$. From experiment[10,11)] two $p_{3/2}$ states are known in Sc^{49}, viz. a $T = \frac{7}{2}$ state at 3 MeV and a $T = \frac{9}{2}$ state at 11 MeV. The high-lying $T = \frac{9}{2}$ state is the isobaric analogue of the Ca^{49} ground state.

The possibility of having two $p_{3/2}$ states with different isospin is due to the configuration arising from the charge exchange between the $p_{3/2}$ proton and one of the $f_{7/2}$ neutrons. The two components building the $p_{3/2}$ states are shown in Figs. 3a and 3b. The neutron hole and the proton in the state ν_i are coupled to the spin 0^+ and so the 7 neutrons and the proton in the $f_{7/2}$ orbit are supposed to have the same structure as the closed $f_{7/2}$ neutron shell in Ca^{48} and to have the same total isospin T=4.

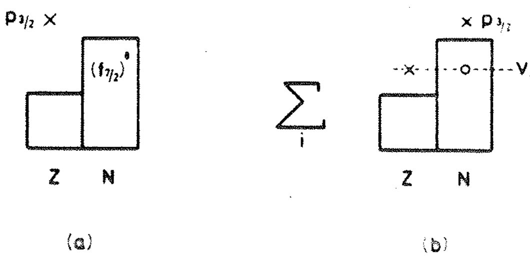

Fig. 3. In Sc^{49} there are two $p_{3/2}$ configurations a) and b). Eigenstates of isospin are combinations of a) and b).

The two configurations 3a and 3b taken separately have no definite isospin. To obtain the eigenstates of the isospin, we must take linear combinations of the two configurations. These combinations are given by Clebsch-Gordan coefficients in isospin and can in a shorthand notation be written:

$$\mathrm{Sc}^{49}\left[\begin{array}{l} |p_{\frac{3}{2}},(T,M_T)=(\frac{7}{2},\frac{7}{2})\rangle = \sqrt{\frac{8}{9}}\ \overset{p}{p}_{\frac{3}{2}}\left(\overset{n}{f}_{\frac{7}{2}}{}^{8}\right)_{0^+} - \sqrt{\frac{1}{9}}\ \overset{n}{p}_{\frac{3}{2}}\left(\overset{p}{f}_{\frac{7}{2}}\ \overset{n}{f}_{\frac{7}{2}}{}^{7}\right)_{0^+} \\ |p_{\frac{3}{2}},(T,M_T)=(\frac{9}{2},\frac{7}{2})\rangle = \sqrt{\frac{1}{9}}\ \overset{p}{p}_{\frac{3}{2}}\left(\overset{n}{f}_{\frac{7}{2}}{}^{8}\right)_{0^+} + \sqrt{\frac{8}{9}}\ \overset{n}{p}_{\frac{3}{2}}\left(\overset{p}{f}_{\frac{7}{2}}\ \overset{n}{f}_{\frac{7}{2}}{}^{7}\right)_{0^+} \end{array}\right.$$

$$\mathrm{Ca}^{49}\quad |p_{\frac{3}{2}},(T,M_T) = (\frac{9}{2},\frac{9}{2})\rangle = \overset{n}{p}_{\frac{3}{2}}\left(\overset{n}{f}_{\frac{7}{2}}{}^{8}\right)_{0^+} \quad . \qquad (3)$$

For the Fermi matrix elements we then have

$$M_F = \langle \mathrm{Sc}^{49}\ p_{\frac{3}{2}},\ T=\frac{7}{2}\ |T_-\ |\ \mathrm{Ca}^{49},\ p_{\frac{3}{2}}\ \ T=\frac{9}{2}\rangle = 0$$

$$M_F = \langle \mathrm{Sc}^{49}\ p_{\frac{3}{2}},\ T=\frac{9}{2}\ |T_-\ |\ \mathrm{Ca}^{49},\ p_{\frac{3}{2}}\ \ T=\frac{9}{2}\rangle = 3$$

So we see that, if the isospin is strictly conserved, then the Fermi matrix element to the low-lying $p_{3/2}$ state will vanish.

However, the Coulomb force violates the isobaric symmetry and we expect the low-lying $p_{3/2}$ state to contain a component with $T=\frac{9}{2}$.

This component will be small and can be calculated in perturbation theory. Thus for the low-lying $p_{3/2}$ state we write

$$\psi(p_{3/2}) = \psi(T=\frac{7}{2}) - \frac{\langle T=\frac{9}{2}\,|\,V_{\mathrm{Coul.}}\,|\,T=\frac{7}{2}\rangle}{\Delta E}\ \psi(T=\frac{9}{2}) \ ,$$

where (4)

$$\Delta E = E(T = \tfrac{9}{2}) - E(T = \tfrac{7}{2}) = 8 \text{ MeV} .$$

The Fermi matrix element is then given by

$$M_F = -3 \frac{\langle T = \tfrac{9}{2} \,|\, V_{Coul.} \,|\, T = \tfrac{7}{2} \rangle}{\Delta E} . \tag{5}$$

From the wave functions given above we obtain for the Coulomb matrix element

$$\langle T = \tfrac{9}{2} \,|V_{Coul.}\,| T = \tfrac{7}{2} \rangle = \tag{6}$$

$$\frac{\sqrt{8}}{9} \left\{ \langle p_{3/2} |V_{Coul.}| p_{3/2} \rangle - \langle f_{7/2} |V_{Coul.}| f_{7/2} \rangle \right\}$$

Using a Woods-Saxon potential and assuming the Coulomb potential to be of the form

$$V_{Coul.} = \frac{Ze^2}{R} \left(\frac{3}{2} - \frac{1}{2} \left(\frac{r}{R} \right)^2 \right) \tag{7}$$

we can calculate the Coulomb energy for a proton in a $p_{3/2}$ orbit and in an $f_{7/2}$ orbit. We obtain

$$\langle f_{7/2} |V_{Coul.}| f_{7/2} \rangle - \langle p_{3/2} |V_{Coul.}| p_{3/2} \rangle \simeq 100 \text{ keV.}$$

This gives the Fermi matrix element

$$|M_F| = 1.2 \; 10^{-2} . \tag{8}$$

The value of the Coulomb matrix element calculated above depends on the parameters chosen for the Woods-Saxon potential. However, this number can also be taken from experiment. The study of isobaric analogue states in the Ca-region gives Coulomb energies for a proton in a $p_{3/2}$ and in an $f_{7/2}$ orbit. From the position of isobaric analogue states in Ca^{47} and Sc^{47} one has found[12)]

$$\langle f_{7/2}|V_{Coul.}|f_{7/2}\rangle - \langle p_{3/2}|V_{Coul.}|p_{3/2}\rangle = 108 \text{ keV} ,$$

which is close to the value used above for the A=49 system.

A calculation of higher order corrections to the Fermi matrix element shows that they are small in the present case.

So far the measurements have given us the ft-value[15)] only, which is determined by the G.T. matrix element alone since the Fermi matrix element is small. To obtain M_F, one must measure a β-circular polarized γ-correlation which is also possible (the half-life of Ca^{49} is 8 min. and the subsequent γ-ray has an energy of 3 MeV).

The Fermi matrix element calculated above is of the same order of magnitude as the Fermi matrix element found experimentally for Ga and Ge, but an order of magnitude larger than those for heavy nuclei. This is also to be expected. In Ga, Ge, and Ca the neutron excess is small so that neutrons and protons fill the same orbits and the Coulomb matrix element is determined by the difference between some single-particle Coulomb energies. In heavy nuclei the neutron excess is large, neutrons and protons fill very different orbits, and the components in the wave functions which can contribute to the Coulomb matrix element are small. For a further discussion of isospin forbidden Fermi transitions, see references 16, 17, and 18.

2b. Gamow-Teller decays

We first consider the superallowed Gamow-Teller transitions. The best examples for comparison of calculation and experiment are provided by mirror transitions between nuclei with a single particle or a single hole outside the closed shells with N=Z. The experimental evidence on these decays is collected in Table 2. The G.T. matrix elements in column 4 were calculated from the ft-value, assuming $M_F = 1$, as expected for transitions between isobaric analogue states with $T = \frac{1}{2}$. We further used $\frac{g_A}{g_V} = -1.18 \pm 0.02$ and took g_V from the O^{14} decay. Using $\frac{g_A}{g_V} = -1.23$, as obtained from the new half-life of the neutron, the numbers in column 4 will be reduced by 8%. In column 5 we give

$$M^2_{GT\ s.p.} = \left(\frac{j+1}{j}\right)^{\pm 1} \quad \text{for} \quad j_i = j_f = j = \ell \pm \frac{1}{2} \quad . \tag{9}$$

The experimental values of M^2_{GT} are in qualitative agreement with the single-particle values but are systematically smaller, except for A=3. The largest correction to the G.T. matrix element comes from the weak magnetism. The interference term between the G.T. matrix element and the matrix element of the weak magnetism term reduces the total matrix element M^2 thus reducing the discrepancy between the experimental and the calculated matrix elements in Table 2. However, the corrections are too small to give any substantial change. The relative reductions are found to be -2% (O^{15}), -1% (F^{17}), -4% (Ca^{39}) and -1% (Sc^{41}) and vanishing values for H^3 and n.

TABLE 2

Comparison of calculated and experimental G.T. matrix elements for single-particle states in mirror nuclei

Nuclei	j	ft(sec)	M^2_{GT} exp	M^2_{GT} s.p.
n → p	$s_{1/2}$	1212± 40	3	3
H^3 → He^3	$s_{1/2}^{-1}$	1060±100	3.4 ±0.3	3
O^{15} → N^{15}	$p_{1/2}^{-1}$	4470± 30	0.28±0.02	1/3
F^{17} → O^{17}	$d_{5/2}$	2370± 50	1.17±0.1	7/5
Ca^{39} → K^{39}	$d_{3/2}^{-1}$	4330±150	0.32±0.05	3/5
Sc^{41} → Ca^{41}	$f_{7/2}$	2780±100	0.90±0.1	9/7

The experimental data are taken from:

O^{15} and F^{17}: Nuclear Data Sheets
Ca^{39}: W.L. Talbert, Phys. Rev. 119 (1960) 272
Sc^{41}: D.H. Youngblood et al., Nucl.Phys. 65 (1965) 602.

The ft-value for H^3 is based on the discussion given by J.N. Bahcall, Nuclear Physics 75 (1966) 10.

The largest discrepancies between experimental and calculated matrix elements are found for A=39 and A=41. This may indicate the presence of more complicated components in nuclear states. Also, recent stripping and pick-up experiments[19] show imported deviations from the closed shell structure for Ca^{40}. For the A = 15 and A = 17 systems the reduction of the experimental matrix elements is somewhat smaller and might be explained by minor deviations from the single-particle picture of the states. However, such deviations should also affect the magnetic dipole moments. The G.T. moments are related to the isovector part of the spin contribution to the magnetic dipole moments by the isobaric symmetry

$$\frac{\langle jm;\ T=\frac{1}{2},\ M_T \mid \sum_i \sigma_z(i)\ t_z(i) \mid jm;\ T=\frac{1}{2}\ \ M_T \rangle}{\langle jm;\ T=\frac{1}{2}, M_T=\frac{1}{2} \mid \sum_i \sigma_z\ (i)\ t_+(i) \mid jm;\ T=\frac{1}{2}\ \ M_T=-\frac{1}{2} \rangle} = -\frac{1}{\sqrt{2}}\ \frac{\langle \frac{1}{2}\ M_T\ 1\ 0 \mid \frac{1}{2}\ M_T \rangle}{\langle \frac{1}{2} - \frac{1}{2}\ 1\ 1 \mid \frac{1}{2}\ \frac{1}{2} \rangle} = -M_T \quad . \tag{10}$$

The deviations from isobaric symmetry for these nuclei are expected to be extremely small[2] and of little importance to the present discussion. Let us take, for example, the A=17 system. The observation that the M_{GT}^2 is reduced by 16% from the single-particle value would imply a reduction of 8% in the isovector spin contribution to the magnetic moments of these nuclei. This would shift the magnetic moments by 0.2

magnetons. However, the magnetic moments both of O^{17} and of F^{17} are extremely close to the single-particle values. For O^{17}, $\mu_{obs} = -1.89$, $\mu_{sp} = -1.91$, and for F^{17} $\mu_{obs} = 4.72$ and $\mu_{sp} = 4.79$. So the comparison with the magnetic moments points to the existence of interaction terms either in the M1 or in the G.T. moments.

For nuclei with $N > Z$ we encounter dramatic hindrance effects for the G.T. transitions. The hindrance factors due to the neutron excess are of the order of 10–100.

The only example of a G.T. transition of a single particle outside double-closed shells in a nucleus with $N > Z$ is the decay $Ca^{49}(p_{3/2}) \rightarrow Sc^{49}(p_{3/2}, 3\text{MeV})$ (see Fig. 2). We have already discussed the Fermi matrix element for this decay. From the ft-value (log ft = 5.1) the G.T. transition is seen to be hindered by a factor of about 30 as compared to the single-particle value. As a further example of hindrance due to the neutron excess we may mention that the so-called unhindered G.T. decays in the deformed nuclei, when corrected for pairing, still are too slow by about an order of magnitude as compared to the values obtained from the Nilsson wave functions[20].

The correlation effects which give rise to the quenching of the G.T.-matrix elements have been[21] compared with the correlation effects which remove the Fermi strength from the single-particle transitions with $T_f = T_i - 1$. The Fermi strength is concentrated on the isobaric analogue state of the parent. In a similar manner, the interactions tend to concentrate the G.T. strength on a high-lying collective state. For the Fermi strength the concentration on the isobaric analogue state is a direct consequence of the isospin invariance, but for the G.T. strength the correlations are not related to any simple symmetry and must be deduced from definite components in the nuclear force. The force relevant to G.T. decays must be of the form

$$V(1,2) = \kappa\, \vec{\tau}_1 \cdot \vec{\tau}_2 \;\vec{\sigma}_1 \cdot \vec{\sigma}_2 \quad . \tag{11}$$

Calculations with such a force were first done by Fujita and Ikeda[21] and later by Halbleib and Sorensen[22]. Let us consider a transition between spin-orbit partners, for example,

$$\overset{n}{d}_{3/2} \;\overset{\beta^-}{\rightarrow}\; \overset{p}{d}_{5/2} \quad .$$

The force quoted above will then, in the final state, admix components of the form

$$\sum_{(\epsilon_F)_p \leq \epsilon(\nu_i) \leq (\epsilon_F)_n} a_i \left[\overset{n}{d}_{3/2} \left(\nu_i(p)\; \nu_i(n)^{-1} \right)_{1^+} \right]_{\frac{5}{2}} \tag{12}$$

where the summation runs over all those orbits ν_i which are occupied by neutrons but not by protons. These components give a contribution to the G.T. matrix element and are responsible for the quenching of the matrix element. We also note that these components only contribute in second order to the magnetic dipole moments. So the reduction of the G.T. moments are not followed by a similar reduction of the magnetic moments.

3. FORBIDDEN DECAYS

In forbidden β-decays we encounter difficulties of two kinds. Firstly, there are several matrix elements so that the relevant part of nuclear structure is not so uniquely determined as for the Fermi and G.T. transitions. Secondly, we do not know precisely if the β-operators we use are correct or if there are major corrections from induced interactions, exchange currents or other unknown effects (see the contributions of Eman and Tadić to this conference). However, the vector part of beta interaction is better known than the axial vector part, if we assume the validity of the conserved vector current hypothesis. This hypothesis also implies an intimate relationship between the matrix elements of the β-vector current and the corresponding β-matrix elements. For example, the observation that the E1 transition strength is concentrated on the giant photoresonance allows the conclusion that the strength of the β-matrix element $\int \vec{r}$ is concentrated in a state which is the isobaric analogue state of the giant resonance of the parent, and that we must expect $\int \vec{r}$ to be small for transitions between low-lying states.

Many relations have been deduced between forbidden β-matrix elements. However, only one obtained from the C.V.C. hypothesis is reliable. It was first given by Fujita[23] and also by Eichler[24] and was written in the form

$$< \vec{\alpha} > = -(W_0 \mp 2.5\, m_e \pm 2.4\, \frac{\alpha Z}{2R}) < i\vec{r} > \qquad (13)$$

for first forbidden decays.

We will consider this relation in more detail and first indicate how it can be deduced[30]. The vector part of the β-current corresponds to the electromagnetic current, in fact it is supposed to be the same current, except for the isospin structure. Thus we can connect the β- and the electromagnetic 4-vectors by the relation

$$(j_V^{\beta})_\mu = - \frac{g_V}{e} [T_-, (j^{el})_\mu] \quad . \qquad (14)$$

The electromagnetic current fulfils the continuity equation or the Siegert theorem, which can be written

$$\partial_\mu \; j_\mu^{el} = \vec{\nabla} \; \vec{j}^{el} + \frac{1}{c} \; \frac{\partial \rho^{el}}{\partial t} = 0 \qquad (15)$$

From (14) and (15) we obtain

$$- \; \vec{\nabla} \; \vec{j}_V^{\beta} + \frac{1}{c} \frac{\partial \rho_V^{\beta}}{\partial t} = - \frac{i}{\hbar c} [\rho_V^{\beta} , H^{el}] \qquad (16)$$

We have inserted

$$\frac{\partial T_-}{\partial t} = \frac{i}{\hbar} [H, T_-] = \frac{i}{\hbar} [H^{el}, T_-]$$

and used

$$[H^{el}, \rho^{el}] = 0 .$$

In Eq. (16) we insert

$$H^{el} = \sum_i \left(\frac{1}{2} - t_z(i)\right) e\, \phi_{Coul.}(\vec{r}) \qquad (17)$$

and

$$\rho_V^\beta = g_V \sum_i t_-(i)\ \delta(\vec{r}-\vec{r}_i) \qquad (18)$$

We then obtain

$$\vec{\nabla} \vec{j}_V^\beta = - \frac{1}{c} \frac{\partial \rho_V^\beta}{\partial t} + \frac{i g_V}{\hbar c} \sum_i t_-(i)\ e\,\phi_{Coul.}\ \delta(\vec{r}-\vec{r}_i) \ . \qquad (19)$$

For simplicity, we consider only the first forbidden matrix element of the vector current. This is usually denoted by $\int \vec{\alpha}$ or $< \vec{\alpha} >$. We use the notation

$$M(j_V, \kappa = 0, \lambda = 1) = \int \vec{j}_V^\beta \cdot \vec{\Phi}_{o\,1\mu}\, d\tau$$

where Φ denotes the vector spherical harmonics (see Edmonds, p. 81). Using (19) we obtain

$$\int \vec{j}_V^\beta\ \vec{\Phi}_{0\,1\mu}\, d\tau = \frac{1}{\sqrt{3}} \int \vec{j}_V^\beta\ \vec{\nabla}\, (r\, Y_{1\mu})\, d\tau$$

$$= - \frac{1}{\sqrt{3}} \int (\vec{\nabla}\, \vec{j}_V^\beta)\, r\, Y_{1\mu}\ d\tau$$

$$= - \frac{i\, g_V}{\hbar c} \frac{1}{\sqrt{3}} \sum_i t_-(i)\, (e\, \phi_{Coul.} + \Delta E)\, r_i\, Y_{1\mu}(\theta_i, \phi_i) \ , \qquad (20)$$

where $\Delta E > 0$ is the total transition energy.

We can now proceed in two different ways.

1) We can insert the average value of the Coulomb potential $2.4 \frac{\alpha Z}{2 R}$ and obtain (13) the relation of Fujita or
2) we can use an explicit expression for the Coulomb potential and evaluate the matrix element with suitable wave functions.

Both methods are approximate. The first method corresponds to setting the nondiagonal matrix elements of the Coulomb potential equal to zero (see below). The second method depends on the reliability of the wave functions used.

The relation between the matrix elements of $\vec{j}_V^\beta$ and ρ_V^β can also be deduced in a very convincing and illustrative manner by relating the β-matrix elements to the corresponding matrix elements for an E1 γ-decay (see Fig. 4). We consider a β^--decay between the states i and f. The initial state has isospin T_0 and the final state then has isospin T_0-1 in a nucleus with $N > Z$. The isobaric analogue state of the initial state is denoted by i′.

From Eq. (13) we see that the vector β-operators O^β and the electromagnetic operators O^{el} of the same space and spin dependence are related by

$$O_V^\beta = -\frac{g_V}{e} [T_-, O^{el}] \quad . \tag{21}$$

We can then rewrite a β-matrix element

$$< f(T_o-1,T_o-1)|O_\gamma^\beta|i(T_o,T_o) >$$

$$= -\frac{g_V}{e} < f(T_o-1,T_o-1) \; |[T_-,O^{el}]| \; i(T_o,T_o) > \qquad (22)$$

$$= \frac{g_V}{e} \sqrt{2T_o} < f(T_o-1,T_o-1) \; | \; O^{el} \; | \; i'(T_o,T_o-1) > \quad ,$$

where we used

$$T_- \; | \; i(T_o,T_o) > \; = \; \sqrt{2T_o} \; | \; i'(T_o,T_o-1) >$$

and

$$T_+ \; | \; f(T_o-1,T_o-1) > \; = 0 \; . \qquad (23)$$

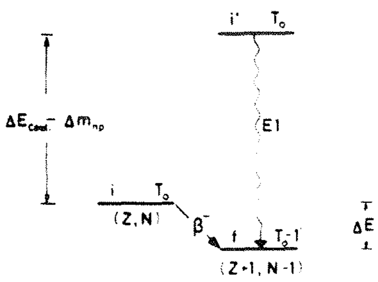

Fig. 4. The vector matrix elements for the first forbidden transition i → f are proportional to the γ-decay matrix element for the E1 transition i′ → f.

So a matrix element for the β-transition shown in Fig. 4 can be translated into a γ-matrix element for the E1 γ-decay i′→f. Using (22) and the continuity equation (15) for the electromagnetic current, we then obtain

$$\frac{\langle f\, \vec{j}_V^{\beta}\, \vec{\Phi}_{o\,1\mu} |\, i \rangle}{\langle f | i\, \rho_V^{\beta}\;\; r\, Y_{1\mu} |\, i} = \frac{\langle f\, |\, \vec{j}^{el}\; \vec{\Phi}_{o\,1\mu} |\, i' \rangle}{\langle f\, |\, i\, \rho^{el}\;\, r\, Y_{1\mu}\, |\, i' \rangle}$$

$$= - \frac{1}{\sqrt{3}}\; \frac{\langle f\; (\vec{\nabla}\, \vec{j}^{el})\;\, r\, Y_{1\mu}\, |\, i' \rangle}{\langle f\, |\, i\, \rho^{el}\;\, r\, Y_{1\mu} |\, i' \rangle}$$

$$= - \frac{1}{\sqrt{3}}\; \frac{1}{\hbar\, c}\;\, E_{\gamma}\;, \qquad (24)$$

where $E_{\gamma} = E_{i'} - E_f$ is the energy of the γ-ray. The relation thus deduced is identical to the Fujita relation (13), since $E_{\gamma} = \Delta E_{Coul.} + \Delta E$. This relation is often claimed to be accurate to better than 10% [25]. However, larger deviations might be found especially when the β-matrix elements are small[26,27]. The crucial point in the derivation is the assumption that the states i and i′ are exact isobaric analogue states. The deviations from formula (13) are connected with the nondiagonal effects of the Coulomb force in distorting the state i′ as compared to i. The mere fact that the state i′ in many cases is particle unstable, whereas i is not, shows a deviation from the exact isobaric symmetry.

The matrix element of the current can also be calculated directly without using C.V.C. Let us consider a transition between single-particle states with wave functions calculated from a potential of the Woods-Saxon type. In such a potential we must use a different depth for neutrons and protons to reproduce the correct binding energies of the single-particle states. This is usually done by including the symmetry potential in the single-particle Hamiltonian

$$H_{s.p.} = H_{kin.} + V_0 + (\frac{1}{2} - t_z) e \phi_{Coul.} + \frac{V_1}{A} \vec{t} \cdot \vec{T} \quad ,$$

where $\vec{t}$ is the isospin of the particle and $\vec{T}$ the isospin of the core. The necessity of the symmetry potential is well known also from the optical model calculations and it has especially been investigated in recent years since the discovery of isobaric analogue states in heavy nuclei and the more intensive study of the isospin symmetry.

To calculate $< \vec{\alpha} >$ directly, we must calculate the matrix element of

$$t_-(i) \; (\vec{v}_i \, \vec{\Phi}_{o1\mu})$$

$$= t_-(i) \; (\vec{v}_i \; \vec{\nabla} \; r_i \, Y_{1\mu}) \qquad (25)$$

$$= t_-(i) \frac{d}{dt} (r_i \, Y_{1\mu})$$

$$= \frac{d}{dt} (t_-(i) \; r_i \, Y_{1\mu}) - r_i \, Y_{1\mu} \frac{d \; t_-(i)}{dt}$$

$$= \frac{d}{dt} (t_-(i) \; r_i \, Y_{1\mu}) - i \; r_i \, Y_{1\mu} \; \cdot$$

$$\cdot \; (t \div e \, \phi_{Coul.} + \frac{V_1}{A} \; \{ t_z(i) T_- - T_z \; t_- \} \;)$$

where we have evaluated the time derivative of $t_{\div}(i)$

with our single-particle Hamiltonian $\frac{d\ t_-(i)}{dt} =$ $= \frac{i}{\hbar} [H_{s.p.}, t_-(i)]$. The first two terms correspond directly to the expression obtained from the C.V.C. relation (19), whereas the last term arises from the symmetry potential and is new. Because of this last term the direct calculation gives a different result than the calculation based on C.V.C.

$$\langle \vec{\alpha} \rangle_{C.V.C.} \neq \langle \vec{\alpha} \rangle_{directly} \quad . \tag{26}$$

We encounter this difference in all nuclei with a neutron excess since a neutron and a proton interact in a different way with such nuclei, and we can therefore not allow the symmetry potential to break the continuity equation. So a more correct way to calculate $\langle \vec{\alpha} \rangle$ is to use the C.V.C. relation. The contribution from the symmetry potential in the direct calculation can be as large as the contribution from the Coulomb potential[28)], and the results of the two calculations can therefore differ by a factor of two.

In the direct calculation we neglect the important exchange effects depending on the coordinates of the nucleons in the closed shells. In using a single-particle model Hamiltonian we should therefore also use some β-operators which depend on the model and take the exchange currents into account. Such operators have not been constructed and are also not strictly necessary for the vector part of the β-interaction since we have the C.V.C. relation to transform operators depending on the β-current density into operators depending on the β-charge density. We then assume, just for the electromagnetic moments, that exchange currents modify the current but leave the charge distribution practically unchanged.

By utilizing the C.V.C. relation and comparing two different methods for calculating $< \vec{\alpha} >$, we have seen that exchange currents are important for the vector β-current. For the axial vector interaction we have less powerful tools to obtain information, but similar effects can of course be expected.

In conclusion, I should like to point out that although we have little knowledge about the effective β-operators, the calculations made so far definitely reproduce trends in the experimental data. For example, the calculated and the experimental ft-values for first forbidden transitions between single-particle states differ by less than a factor of two[28]. At present, only three such transitions are known, but in the deformed nuclei we have also states which might be characterized as single-particle states, although the cores are not closed shells, and for these nuclei the agreement is reasonable[29].

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