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Combination of the SecVtx 1.2fb^{-1} b-Tagging Scale Factors

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Summary

The winter 2007 combined results for the scale factors of the the SecVtx based taggers are presented. We find the following values for the loose, tight and ultra-tight versions of the algorithm:

Loose SecVtx SF	=	0.95	$\pm$	0.01(stat)	$\pm$	0.05(syst)
Tight SecVtx SF	=	0.95	$\pm$	0.01(stat)	$\pm$	0.04(syst)
Ultra-tight SecVtx SF	=	0.88	$\pm$	0.01(stat)	$\pm$	0.05(syst)

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1 Introduction

The SecVtx scale factors have been measured using two methods ("electron" [1] and "muon" [2]) and the full 1.2 fb^{-1} datasets ("gen6"). The uncertainties on these measurements are partially correlated.

One of the largest systematic uncertainties is due to the extrapolation in jet E_T from the samples in which the SF is measured (8 GeV electron and muon data), to that to which it is applied, for example b-jets in $t\bar{t}$ events. We combine the two estimates of the dependence of the SFs on jet E_T and extract a single uncertainty due to this effect for $t\bar{t}$ b-jets. A parameterization of the extrapolation error is provided, for readers who wish to convolute the E_T dependence error with a jet spectrum other than b-jets from $t\bar{t}$ events.

The SF measurements are combined using a best linear unbiased estimate technique (BLUE) [4], taking the approximate correlations into account. Results for the loose, tight and ultra-tight versions of the SecVtx tagger are presented.

2 Combined estimate of the E_T dependence systematic

To estimate the systematic uncertainty due to the jet E_T extrapolation, we start with the measurements of the SFs in bins of jet E_T made by the two methods. The weighted average SF in each jet E_T bin is calculated. Figures 1, 2 and 3, show the combined SF, where only the statistical errors are included. The jet E_Ts are corrected to Level 5.

Following [3], the combined SFs are fitted with two functions, one a constant, the other a linear function in jet E_T, using a χ^2 minimization. To obtain the E_T dependence systematic, the difference between these two functions is weighted with the jet E_T spectrum of b-jets from $t\bar{t}$ MC events (*ttop75*). However, we found that this procedure leads (with the results in this note) to a smaller systematic than just doing the error propagation of the linear fit (taking into account the correlation between parameters) and weight this function with the b-jet E_T spectrum. We adopt this last error. The absolute systematic uncertainties on the SF due to this effect are 0.020 ± 0.010, 0.028 ± 0.012, and 0.027 ± 0.012, for the loose, tight and ultra-tight SecVtx taggers respectively.

The functional form of the systematic (shown in Figures 1, 2 and 3) is tabulated in Table 1. These are provided in case the reader wishes to use the parameterizations to weight a different spectrum than b-jets from $t\bar{t}$ MC events. If the result differs significantly from the one presented here then the different errors on the SF would have to be recombined to account for this (or the systematic error would have to be increased). The data points of the different taggers are tabulated in Table 2.

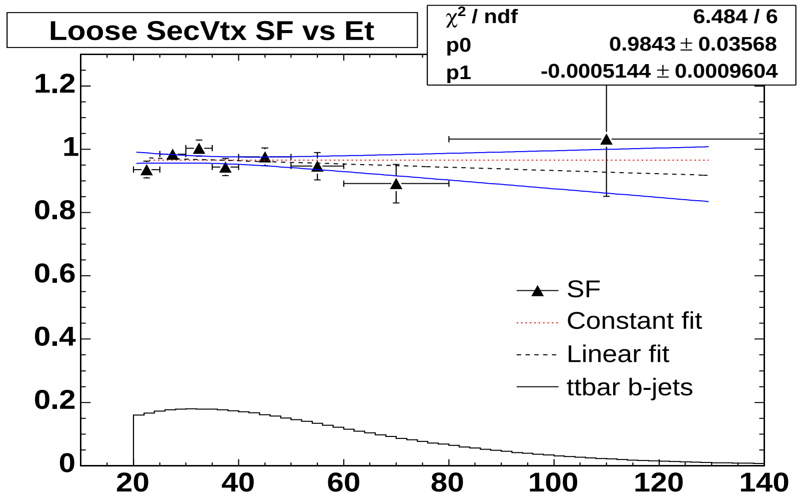

Figure 1: Average Scale Factor as a function of jet E_T for the loose SecVtx tagger. The b-jet spectrum (normalized) is also plotted. The fitted functions as well as the error band on the linear fit are shown. The data and the parameteriztion are tabulated in Tables 1 and 2.

$f^2_{(E_T)} = a_0 + 2a_1 E_T + a_2 E_T^2$			
Tagger	a_2	a_1	a_0
Loose	8.37×10^{-7}	-2.97×10^{-5}	0.0012
Tight	9.87×10^{-5}	-3.49×10^{-5}	0.0014
Ultra-tight	2.13×10^{-6}	-7.74×10^{-5}	0.0031

Table 1: Parameterization of the E_T error dependence for the different SecVtx taggers.

E_T bin	Bin Center	Loose	Tight	Ultra-tight
20-25	22.5	0.936±0.0263	0.927±0.0285	0.871±0.0478
25-30	27.5	0.985±0.0236	0.973±0.0255	0.906±0.0414
30-35	32.5	1.00±0.0256	1.010±0.0284	0.958±0.0439
35-40	37.5	0.944±0.0267	0.97±0.0304	0.952±0.0481
40-50	45	0.976±0.0279	0.984±0.0317	0.972±0.0497
50-60	55	0.947±0.043	0.921±0.0467	0.877±0.0706
60-80	70	0.891±0.0607	0.892±0.0666	0.783±0.0872
80-	110	1.030±0.1801	0.928±0.1870	0.912±0.2561

Table 2: Average SF for the different taggers.

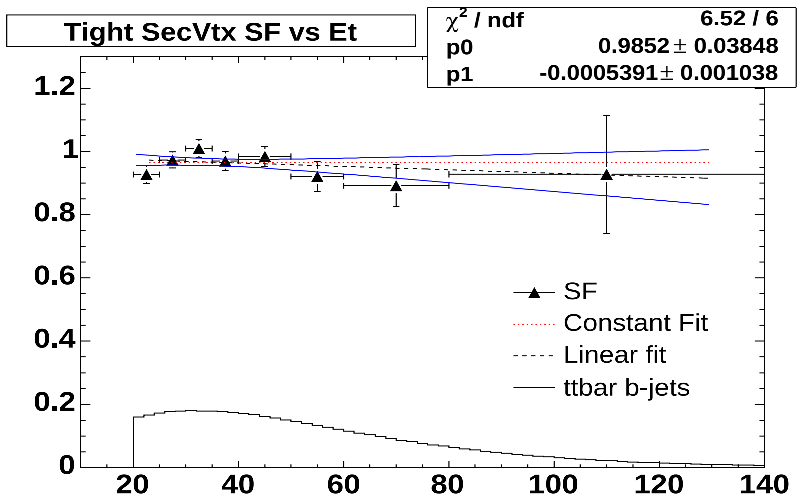

Figure 2: iAverage Scale Factor as a function of jet E_T for the tight SecVtx tagger. The b-jet spectrum (normalized) is also plotted. The fitted functions as well as the error band on the linear fit are shown.

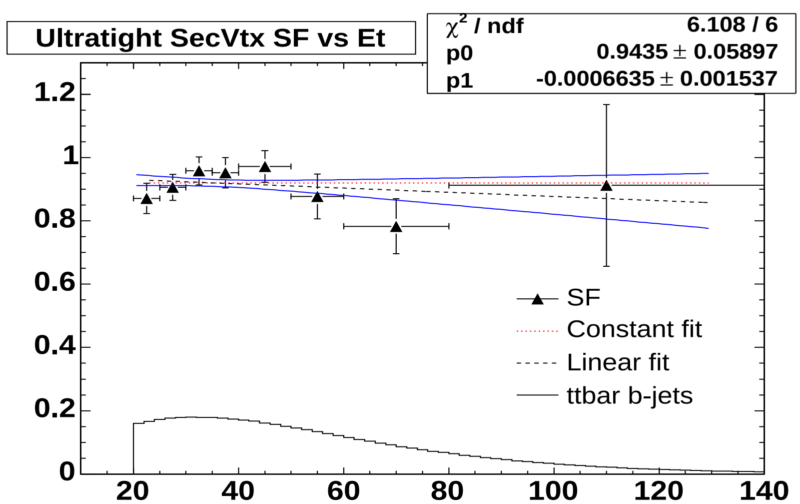

Figure 3: Scale Factor as a function of jet E_T for the ultra-tight SecVtx tagger. The b-jet spectrum (normalized) is also plotted. The fitted functions as well as the error band on the linear fit are shown.

3 Uncertainties and correlations

The combined SF results are based on the measurements reported in references [1] and [2]. While some systematics are unique to either measurement (due to the assumption that the b-jet tagging efficiency is the same with and without conversions, for example), others are highly correlated (that due to the SF E_T dependence, for example). We assume that that the error contributions are either uncorrelated or fully correlated between measurements as detailed in Table 3. The method-specific uncertainties are taken to be uncorrelated while uncertainties arising from the extrapolation from the jet sample in which the SF is measured to the one in which is applied (jet E_T, η, and semi-leptonic decay bias) are taken as fully correlated.

	Electron			Muon			Corr
	Loose	Tight	Ultra-tight	Loose	Tight	Ultra-tight	
SF value	0.954	0.977	0.854	0.944	0.932	0.908	
Stat err	0.016	0.018	0.027	0.015	0.016	0.026	0
Relative systematic errors (%)							
MC stat	1.3	1.5	2.5	-	-	-	0
Mistag Subtraction	3.4	1.5	0.5	-	-	-	0
Conversion bias	1.2	1.1	1.1	-	-	-	0
Charm SF	0.4	0.3	0.2	-	-	-	0
c/b ratio	1.5	1.4	3.5	-	-	-	0
Template Spread	-	-	-	1.5	1.5	1.5	0
Jet direction	-	-	-	0.3	0.7	4.4	0
b-Template un/tag bias	-	-	-	1.1	1.4	1.4	0
non-b Charm Fraction	-	-	-	0.2	0.2	0.2	0
Semi-leptonic decay model	-	-	-	0.8	0.8	0.8	0
Jet Et (absolute)	0.020	0.028	0.027	0.020	0.028	0.027	1
Jet η	3.4	1.5	1.2	3.4	1.5	1.2	1
Semi-leptonic bias	1.6	1.4	0.8	0.6	1.7	2.4	1

Table 3: Inputs to the combination. The assumed correlation between the electron and muon methods is shown.

4 Combination results

With the errors and correlations detailed in Table 3 an error matrix is constructed for each of the SecVtx algorithms. The matrices are listed in Table 4, together with the correlations. The measurements are combined following reference [4]. Using a best linear unbiased estimate technique (BLUE), the coefficients (α_i) which minimize the variance of the combined result are obtained ($SF_{comb} = \alpha_e SF_e + \alpha_\mu SF_\mu$).

	Loose		Tight		Ultra-tight	
	e	μ	e	μ	e	μ
e	0.0037	0.0018	0.0019	0.0009	0.0035	0.0013
μ	0.0018	0.0024	0.0009	0.0015	0.0013	0.0035
Corr	0.61		0.50		0.37	

Table 4: Error matrices for the SecVtx taggers.

The combination method provides a single uncertainty (per tagger). In order to estimate the statistical uncertainty on the combined values, the combinations are performed using only the (uncorrelated) statistical uncertainties. The variance of these combinations are taken to be the statistical components of the full combinations, while the systematic uncertainty is taken to be the quadratic difference between the total and statistical uncertainties. The results are listed in Table 5.

Measurement	Loose			Tight			Ultra-tight		
SF_e	0.954	±0.016	±0.063	0.977	±0.018	±0.051	0.854	±0.027	±0.057
SF_μ	0.944	±0.015	±0.051	0.932	±0.016	±0.048	0.908	±0.026	±0.068
SF_{comb}	0.946	±0.011	±0.047	0.949	±0.012	±0.035	0.880	±0.013	±0.047
Coefficients	$\alpha_e = 0.24$, $\alpha_\mu = 0.76$			$\alpha_e = 0.38$, $\alpha_\mu = 0.62$			$\alpha_e = 0.51$, $\alpha_\mu = 0.49$		

Table 5: Combined SF results. The systematic uncertainties quoted above for the electron and muon SFs correspond to the error matrices of Table 3 (and not the CDF notes [1] and [2]).

5 Cross check combination

The SecVtx scale factor results for the electron and muon p_T^{rel} techniques were combined in an alternative manner to cross check the results yielded from BLUE. The cross check follows the procedure utilized in the $700 pb^{-1}$ scale factor combination [3] and is based on the minimization of a generalized χ^2. BLUE performs a similar χ^2 minimization, so one should expect consistent results.

Each of the scale factor results has several sources of systematic error, listed in Table 3. In this section the jet E_T dependence systematic, listed in Table 6 was taken from reference [2]. Again, it is assumed that there are three common systematic errors between the two methods (jet E_T, η, and semi-leptonic decay bias), and that the effect of these common systematic errors are completely correlated between the two results. Under this assumption, the largest relative error between the two measurements was applied; where a relative error was not determined (for example, no η systematic error study was done in the context of the electron technique) the corresponding error from the other technique was applied.

Jet E_T dependence systematic			
SecVtx Tagger	Electron	Muon	Correlation
Tight	0.039	0.039	1
Loose	0.036	0.036	1
Ultra-tight	0.047	0.047	1

Table 6: Jet E_T dependence systematic for the different SecVtx taggers. Other systematics are identical to Table 3.

In the description of the method below, we use similar conventions as those employed in [3]. The total uncertainty on the $1.2 fb^{-1}$ electron (muon) scale factor result is defined to be $S^{e(\mu)}$. The correlation coefficient is defined then as

$$\rho_{tot} = \frac{\sum_{i=1}^{N} \rho_{e+\mu} y_i^e y_i^\mu}{S^e S^\mu} \tag{1}$$

where N is the total number of sources of systematic error among the two measurements ($N = 13$ in this case); each $\rho_{e+\mu}$ and the y_i for each measurement is listed in Table 6. The errors in the numerator and denominator of ρ_{tot} can be either the relative or raw values; care should be taken that the denominator and numerator are using the same convention.

The central value of the combined scale factor, $\langle Q \rangle$, is given by

$$\langle Q \rangle = w^e Q^e + w^\mu Q^\mu \tag{2}$$

where $Q^{e(\mu)}$ is the central value from the electron (muon) combination; the weights w^e and w^μ are given by

$$w^e = \frac{S^\mu (S^\mu - \rho_{tot} S^e)}{(S^e)^2 + (S^\mu)^2 - 2\rho_{tot} S^e S^\mu} \quad \text{and} \quad w^\mu = \frac{S^e (S^e - \rho_{tot} S^\mu)}{(S^e)^2 + (S^\mu)^2 - 2\rho_{tot} S^e S^\mu} \tag{3}$$

and $S^{e(\mu)}$ is the total error on the electron (muon) SF result. These w's are the weights with which each result is combined. The total uncertainty on $\langle Q \rangle$ is then given by

$$S_{\langle Q \rangle} = \sqrt{\frac{(S^e S^\mu)(1-\rho_{tot}^2)}{(S^e)^2 + (S^\mu)^2 - 2\rho_{tot} S^e S^\mu}} \tag{4}$$

As noted in [3] a drawback of this method is that there is no way to break the resulting total error on the combined measurement into its constituent statistical and systematic portions. An approximation of the total statistical error, $T_{\langle Q \rangle}$, takes into account the statistical errors from the individual results from each technique according to the weights defined above:

$$T_{\langle Q \rangle} = \sqrt{(w^e T^e)^2 + (w^\mu T^\mu)^2} \tag{5}$$

and then subtracts this from the total error on the combined measurement to get an approximate value for the tot systematic error, $Y_{\langle Q \rangle}$:

$$Y_{\langle Q \rangle} = \sqrt{S_{\langle Q \rangle}^2 - T_{\langle Q \rangle}^2} \tag{6}$$

The results of this combination procedure for the Tight SecVtx operating point are summarized in Table 7. One can see from the total correlation and the errors from the individual measurement techniques, the combined measurement is weighted towards the muon result in a similar way as the BLUE result. The final combined value is consistent with the BLUE result.

	Electron	Muon
Scale Factor	0.977 ± 0.051 ± 0.018	0.932 ± 0.047 ± 0.016
Correlation	0.674	
Weight	0.371	0.629
Combined Stat Error	0.012	
Combined Syst Error	0.046	
Combined SF	0.949 ± 0.047	
BLUE Correlation	0.68	
BLUE Combined SF	0.95 ± 0.01 ± 0.05	

Table 7: Summary of cross check calculation of combined electron+muon method results for Tight SecVtx.

This cross-check was also performed for the Loose and ultra-tight SecVtx operating points. Tables 8 and 9 summarize the results of the combination for these taggers. The consistency of these results with respect to those from BLUE is adequate.

	Electron	Muon
Scale Factor	0.954 ± 0.063 ± 0.016	0.944 ± 0.051 ± 0.015
Correlation	0.664	
Weight	0.200	0.800
Combined Stat Error	0.012	
Combined Syst Error	0.050	
Combined SF	0.946 ± 0.052	
BLUE Correlation	0.50	
BLUE Combined SF	0.95 ± 0.01 ± 0.04	

Table 8: Summary of cross check calculation of combined electron+muon method results for Loose SecVtx.

	Electron	Muon
Scale Factor	0.854 ± 0.060 ± 0.027	0.908 ± 0.063 ± 0.026
Correlation	0.386	
Weight	0.534	0.466
Combined Stat Error	0.019	
Combined Syst Error	0.053	
Combined SF	0.879 ± 0.056	
BLUE Correlation	0.53	
BLUE Combined SF	0.87 ± 0.01 ± 0.06	

Table 9: Summary of cross check calculation of combined electron+muon method results for utra-tight SecVtx.

6 Conclusions

The electron and muon method SecVtx efficiency scale factors have been combined for the loose, tight and ultra-tight SecVtx tagging algorithms. A best linear unbiased estimation technique was used for the combination and the correlation of the methods was taken into account (at least partially). An alternative method was used as a cross check and similar results were obtained. A functional form for the jet E_T error is provided for users that want to check the impact of this systematic on a different (from b-jets from $t\bar{t}$ events) E_T spectrum. The recommended values for the SFs for the different taggers are:

$$\begin{aligned}
\text{Loose SecVtx SF} &= 0.95 \pm 0.01(\text{stat}) \pm 0.05(\text{syst}) \\
\text{Tight SecVtx SF} &= 0.95 \pm 0.01(\text{stat}) \pm 0.04(\text{syst}) \\
\text{Ultra-tight SecVtx SF} &= 0.88 \pm 0.01(\text{stat}) \pm 0.05(\text{syst})
\end{aligned}$$

References

[1] S. Grinstein, J. Guimaraes da Costa, D. Sherman, CDF Note 8625.

[2] F. Garberson, J. Incandela, C. Neu, CDF Note 8640.

[3] F.Garberson, *et.al.*, CDF Note 8025.

[4] Lyons, Gibaut, Clifford, NIM A270 (1998) 110.