Alignment of the LHCb detector with Kalman filter fitted tracks

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Abstract. The LHCb detector, operating at the Large Hadron Collider at CERN, is a single arm spectrometer optimised for the detection of forward b and anti-b production for b physics studies. The reconstruction of vertices and tracks is done by silicon micro-strip and gaseous straw-tube based detectors. To obtain excellent momentum, mass and vertex resolutions, the detectors need to be aligned well within the hit resolution for a given detector. We present a general and easy to configure alignment framework which uses the closed from method of alignment with Kalman filter fitted tracks to determine the alignment parameters. This allows us to use the standard LHCb track model and fit, and correctly take complexities such as multiple scattering and energy loss corrections into account. With this framework it is possible to align any detector for any degree of freedom.

1. Introduction

The LHCb detector [1], see Fig. 1, operating at the Large Hadron Collider at CERN, is a single arm spectrometer optimised for the detection of forward b and anti-b production for b physics studies. The reconstruction of tracks and vertices is done by silicon micro-strip and gaseous straw-tube based detectors. The tracking system consists of a Vertex Locator (VELO), a large area silicon detector (TT), located upstream of the dipole magnet, and three tracking stations (T stations), located downstream of the magnet. Each T station consists of an Inner Tracker (IT) station and an Outer Tracker (OT) station. In addition, the reconstructed tracks in the tracking stations are extrapolated to the Muon stations, located downstream of the T stations, and used in the particle identification of muons.

Tracking performance studies on Monte Carlo data show that a momentum resolution of $3 - 4‰$ can be achieved for tracks that traverse the entire detector [2]. In order to achieve this momentum resolution, and a good mass resolution, with real data the positions of the detectors need to be determined with an accuracy well below the hit resolution for a given detector - 8μm, 60μm and 200μm for the VELO, IT and OT, respectively. For the Muon stations the requirements on the alignment are not so stringent, though major mis-alignments can lead to wrong transverse momenta estimates, likely charge dependent, in the L0 trigger.

Here we present an alignment framework that uses the global χ^2 or closed form method of alignment [3]. The novelty of this framework is that its track model and fit is based on the Kalman filter [4], which is also the default in the LHCb reconstruction [5]. This allows us to correctly take complexities such as the magnetic field, multiple scattering and energy loss corrections into account.

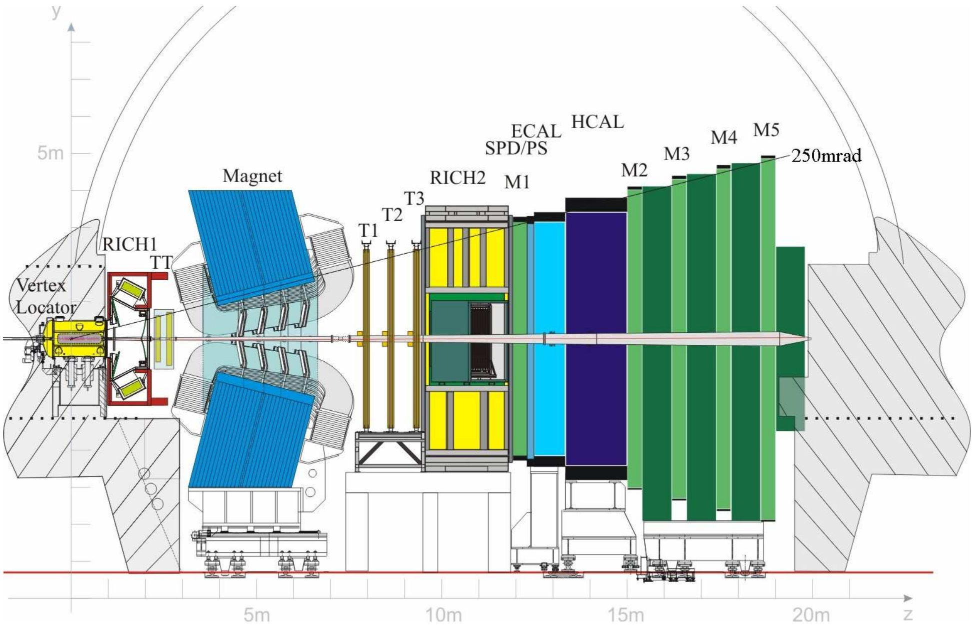

Figure 1. The LHCb detector.

The alignment framework is designed to be user friendly and easy to configure. Currently, the VELO, TT, IT, OT and Muon stations can be aligned for any relevant translations and rotations - at any granularity, *e.g.* module, layer, or station - with this framework.

2. Alignment Method

The method used to determine the alignment parameters is the minimisation of a total χ^2 given by

$$\chi^2 = \sum_t \chi_t^2,$$

where χ_t^2 is the track χ^2 for track t. The latter is given by

$$\chi^2 = \boldsymbol{r}_t^T V_t^{-1} \boldsymbol{r}_t,$$

where $\boldsymbol{r}$ is the residual and V^{-1} the measurement covariance matrix. The residual $\boldsymbol{r}$ is defined as

$$\boldsymbol{r} = \boldsymbol{m} - h\left(\boldsymbol{x}, \boldsymbol{\alpha}\right)$$

where $\boldsymbol{m}$ is the measurements and h is the measurement function which expresses the measurements in terms of the track, $\boldsymbol{x}$, and alignment parameters, $\boldsymbol{\alpha}$.

The total χ^2 needs to be minimised simultaneously with respect to $\boldsymbol{\alpha}$ and the track parameters of all tracks in a calibration sample. The alignment parameters are sometimes referred to as global parameters, since these are the same for all tracks. While the track parameters are sometimes referred to as local parameters, since these are independent, *i.e.* the tracks are uncorrelated. Exploiting the fact that the tracks are uncorrelated, the total χ^2 can be efficiently minimised by first minimising the total χ^2 with respect to the track parameters for an initial estimate of alignment parameters $\boldsymbol{\alpha}_0$, *i.e.* the tracks are fitted for $\boldsymbol{\alpha}_0$. Subsequently the corrections to the alignment parameters, $\Delta\boldsymbol{\alpha} = \boldsymbol{\alpha} - \boldsymbol{\alpha}_0$, that minimises the total χ^2 are calculated using the Newton-Raphson method. This leads to an expression of the form [3, 6]

$$\begin{aligned}\Delta\boldsymbol{\alpha} &= \left[\frac{d^2\chi^2}{d\boldsymbol{\alpha}^2}\right]^{-1}\left[\frac{d\chi^2}{d\boldsymbol{\alpha}}\right] \\ &= \left[\sum_t A^T V_t^{-1}\left(V_t - H_t C_t H_t^T\right) V_t^{-1} A\right]^{-1}\left[\sum_t \frac{\partial \boldsymbol{r}_t}{\partial \boldsymbol{\alpha}} V_t^{-1} \boldsymbol{r}_t\right],\end{aligned} \tag{1}$$

where H is the derivative of h with respect to $\boldsymbol{x}$, A is the derivative of $\boldsymbol{r}$ with respect to $\boldsymbol{\alpha}$, and C is the covariance matrix of $\boldsymbol{x}$. The matrix $\left(V - HCH^T\right)$ is the covariance matrix of the residuals. The off-diagonal elements of this matrix represent the correlations between the residuals, which enter through the matrix C. Taking these correlations into account the alignment parameters can be estimated with a single pass through the calibration data. (Note that if h is non-linear then more iterations are necessary.) If these correlations are ignored, then each detector element is, effectively, aligned independently of the others and more iterations are necessary to estimate the alignment parameters.

2.1. Global χ^2 alignment with Kalman filter fitted tracks

In a global χ^2 track model and fit $\boldsymbol{x}$ is a vector of N, generally five, parameters and C is an $N \times N$ matrix. This is not the case in a Kalman filter track model and fit. Here $\boldsymbol{x}$ has $5 \times N$ parameters, *i.e.*

$$\boldsymbol{x} = \begin{pmatrix} \boldsymbol{x}_1 & \boldsymbol{x}_2 & \boldsymbol{x}_3 & \ldots & \boldsymbol{x}_N \end{pmatrix},$$

where $\boldsymbol{x}_N$ is a state at hit N and has five parameters. The corresponding covariance matrix C is an $N \times N$ matrix of 5×5 matrices, *i.e.*

$$C = \begin{pmatrix} C_{11} & C_{12} & C_{13} & \ldots & C_{1N} \\ C_{21} & C_{22} & C_{23} & \ldots & C_{2N} \\ C_{31} & C_{32} & C_{33} & \ldots & C_{3N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ C_{N1} & C_{N3} & C_{N3} & \ldots & C_{NN} \end{pmatrix},$$

where C_{NN} is the 5×5 (smoothed) covariance matrix for state $\boldsymbol{x}_N$. In addition, in the global χ^2 track fit the correlation matrix C is always calculated, whereas in the Kalman filter track fit only the diagonal elements C_{ii} of C are calculated and the off-diagonal elements C_{ij} are either not calculated at all or only implicitly. Consequently, the correlations between the residuals, *i.e.* the off diagonal elements of $\left(V - HCH^T\right)$, are ignored.

Recently a method was proposed to determine the global correlation matrix C in the Kalman filter fit [6]. It can be shown that the correlation between state $\boldsymbol{x}_i$ and state $\boldsymbol{x}_j$ in the Kalman filter fit is given by (using the notation in [4])

$$C^n_{i-1,j} = A_{i-1} C^n_{i,j} \text{ for } i \leq j, \tag{2}$$

where A is the smoother gain matrix. In addition, the procedure described in [6] to calculate the correlations can be used to include vertex constraints without refitting the tracks.

Using Eq. 2 the correlations between states can be calculated and the Kalman filter fit can be used in the closed form method of alignment. This enables us to use the standard LHCb track fit and model, and reconstruction tools. In addition, all the complexities of the track model, *e.g.* multiple scattering, energy loss corrections and magnetic field, are correctly taken into account. Moreover, the estimated alignment parameters are consistent with the track model and fit used in the reconstruction and analysis.

3. Software and Procedure

The alignment software is developed within the LHCb C++ software framework GAUDI [7]. This framework is also used by the reconstruction and analysis software. Consequently, the alignment has access to the same information and tools as the reconstruction and analysis software. These include, amongst others, access to the detector elements and their conditions, tracks and vertices, and reconstruction, track selection and monitoring tools.

3.1. Alignment Framework

The main components of the alignment framework are shown in Fig. 2. It has the following features:

- easy and simple configuration via Python;
- uses the standard reconstruction algorithms and tools;
- can align any detector for any degree of freedom (*i.e.* translations and rotations);
- implements methods to constrain (a combination of) alignment parameters with external information, such as survey constraints.

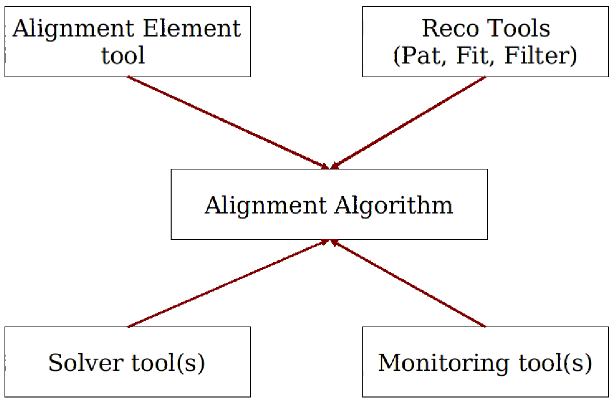

Figure 2. The main components of the LHCb alignment framework

At the core of the alignment framework is a single generic C++ algorithm. The inputs to the algorithm are the detector elements to be aligned and the tracks to be used for alignment. The former is provided by a tool that gets the detector elements and their conditions, specified by the user, and converts them to generic objects called alignables. The latter is provided by the reconstruction tools and algorithms. The algorithm determines which hits belong to which alignables and gets the corresponding residuals $\boldsymbol{r}$ and measurement covariance matrices V. Subsequently, the global covariance matrix C is calculated. This information is then used to calculate the first and second derivative in Eq. 1. After all the tracks in a calibration sample have been processed, the derivatives are passed on to a solver tool to determine $\Delta\boldsymbol{\alpha}$.

There exist also various reconstruction and alignment monitoring tools to monitor track quality, residuals and pulls, alignment parameters, *etc.* during the alignment procedure.

3.2. Procedure

A general alignment procedure consists of a number of phases executed in the following order: reconstruction, process, solve, update. In the reconstruction phase the pattern recognition, track fit and (optionally) track selection algorithms are executed. (In track selection, tracks are selected based on track quality criteria, *e.g.* the number of hits on a track, the track χ^2, *etc.*) In the process phase, the selected tracks are processed, and the first and second derivatives are calculated, *i.e.* the matrix and vector are filled. The corrections to the alignment parameters are then determined in the solving phase. This procedure is repeated until a convergence criterion is met, at which point the alignment parameters are updated in the update phase.

A useful convergence criterion is the change in the χ^2 given by

$$\Delta\chi^2 = \left(\frac{d\chi^2}{d\alpha}\right)^T \Delta\alpha = -\Delta\alpha^T \mathrm{Cov}(\alpha)^{-1}\Delta\alpha.$$

For $\Delta\chi^2$ values less than one the changes in the alignment parameters are statistically insignificant and, therefore, the problem has converged.

During the whole procedure the number of used hits in a detector element, number of tracks, the average track χ^2 and the alignment parameters are monitored. These quantities indicate whether the alignment procedure is stable and correct. The flow of the number of tracks and hits, the average track χ^2 and alignment parameters are indicators for the stability of the procedure.

4. Configuration via Python

GAUDI is modular framework in which the algorithms and the tools they use are configured via Python. It allows the user to choose and configure a sequence of algorithms. For a typical alignment run the user would first run the reconstruction and subsequently the alignment. To configure the alignment algorithms the user needs to specify the detector elements and the degrees of freedom to align for, the type of constraint to use, the location of the tracks, the solving method to use and the number of iterations.

Since the alignment software is developed within the GAUDI framework, the user has direct access to the reconstruction algorithms. They can be easily configured for specific reconstruction tasks or conditions. This includes reconstruction with or without magnetic field, with or without drift time information, or reconstruction of cosmics. They have also been tuned and optimised for various detector conditions. In addition, there exists already a set of predefined configurations for these algorithms. Therefore, the user needs only to specify a configuration in his or her alignment job or, in the worst case, write or modify a configuration. The latter can be the case, *e.g.* if the user wants to use a different tuning of the pattern recognition or track fit.

The alignment framework can align any detector whose hits are on a track, *i.e.* for which a measurement function, h, exists in the LHCb track model. Currently measurement functions exist for the VELO, TT, IT, OT and the Muon stations. In principle the framework can align any detector, as long as there is a corresponding measurement function for it. It is possible to align all the detectors simultaneously with respect to each other in an alignment run. Currently, the degrees of freedom that can be aligned for are translations along and rotations about a detector element's axes, *i.e.* the parameters that are stored in the detector conditions database.

The detector elements and degrees of freedom to align for are specified with a Python list of strings, e.g.

```
elements = ['T1X1UCSide:T/OT/T1/(X1|U)/Q(0|2):TxTzRz', ...].
```

The format of the string is as follows (note that the colon, ":", is a separator):

- *Name*: This is the name that will be assigned to the alignable object. If a name is not specified, then a default name will be determined from the detector element. If the user wants to group these elements into one single alignable, *i.e.* an ensemble of detector elements, he can either specify a name or use the keyword *Group*.
- *Perl Regular Expression*: This is used to match against the name of a detector element. All true matches are either grouped together to form a single alignable object, if a name is specified or the keyword *Group* is used, or a unique alignable object is created for each match.

- *Degrees of Freedom*: Currently the degrees of freedom that can be aligned for are translations in x, y and z, and rotations about x, y and z. In the alignment framework the translations are denoted as Tx, Ty and Tz, and the rotations are denoted as Rx, Ry and Rz.

Thus, the example above shows that the user wants to align an ensemble of OT quarters in the first two layers in the first station on the C-Side, which is called *T1X1UCSide*, for translations in x and z, and a rotation about z.

In some cases the second derivative in Eq. 1, which is a matrix, can be singular. This is for example the case in the absence of external constraints, which introduces global translations, rotations and shearings that the detector is insensitive to. To constrain these so-called weak modes the following methods are available in the alignment framework:

- Explicitly remove detector elements or constrain them with Lagrange multipliers. The syntax to do is similar to the example given above.
- Fix degrees of freedom by cutting on the eigenvalue for a degree of freedom.
- χ^2 penalty terms to constrain some detector elements to their nominal or survey coordinates.

5. Results

Here we present a selection of results, from two alignment studies on Monte Carlo data [8, 9] and an alignment study on cosmic data, obtained with the alignment framework. The two Monte Carlo studies focused on the effects of mis-alignments on physics quantities and whether these quantities can be recovered. In the third study [10] the OT was aligned using cosmic ray data taken during the commissioning of the LHCb detector in September 2008.

For a more detailed overview of first alignment results for the VELO, IT and OT, obtained with beam dump and cosmic ray events during the commissioning of the LHCb detector in August-September 2008, see [11]. This also includes results obtained with the MILLIPEDE framework [3].

5.1. Effects of mis-alignments in the IT and OT on the di-muon reconstruction

In this study the IT and OT layers were each mis-aligned for a translation in x and a rotation about z. The effect of these mis-alignments on the di-muon reconstruction were then studied with 50000 magnet-off beam gas events at a beam energy of 450 GeV.

The effect of the mis-alignments on the invariant di-muon mass resolution as a function of the momentum is shown in Fig. 3. Also shown in Fig. 3 is the di-muon mass resolution after aligning the IT and OT layers simultaneously for a total of 64 degrees of freedom. The IT and OT were aligned locally, *i.e.* there were no external constraints. To constrain the global translations and shearings the first two and last two layers of the IT and OT were fixed.

Both the mis-aligned and aligned scenarios should be compared to the di-muon mass resolution obtained with the default geometry. As can be seen the default di-muon mass resolution is fully recovered after alignment.

5.2. Effects of mis-alignments in the Velo on the reconstruction of $B_s \to \mu^+\mu^-$

In this study the VELO modules and sensors were mis-aligned. The effect of these mis-alignments on the distance of closest approach between a μ^+ and μ^- from the decay $B_s \to \mu^+\mu^-$ is shown in Fig. 4. Also shown in Fig. 4 is the result after alignment. The VELO modules and sensors were aligned for all relevant degrees of freedom using 2000 events (magnet-on data) and vertex constraints. As can be seen the default result is recovered after alignment.

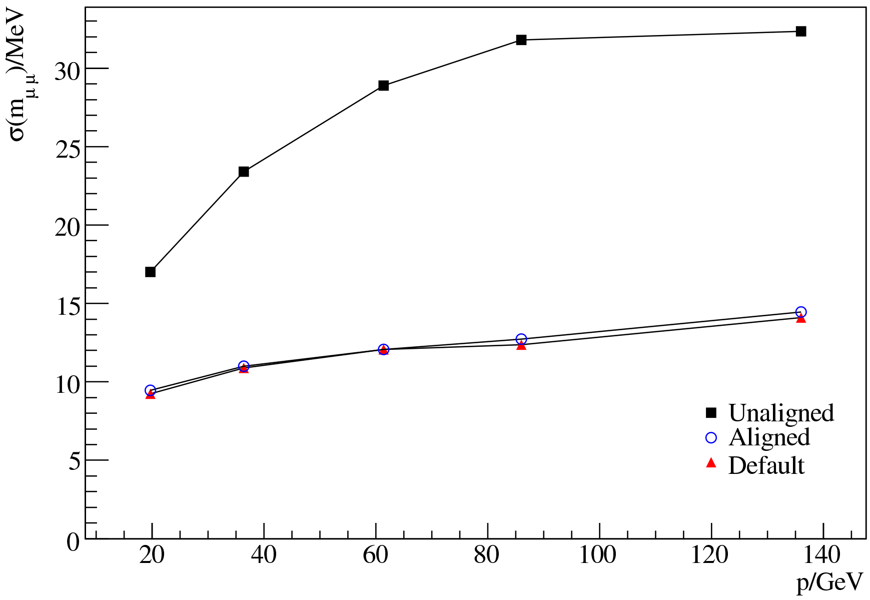

Figure 3. The di-muon mass resolution as function of momentum for default, mis-aligned and aligned scenarios.

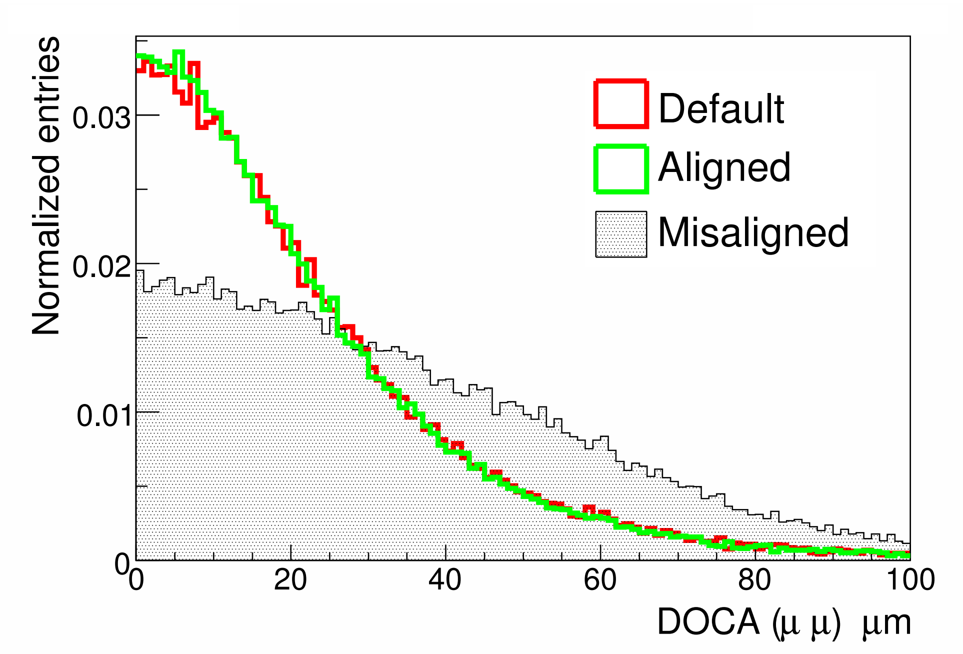

Figure 4. The $\mu^+\mu^-$ distance of closest approach for default, mis-aligned and aligned cases.

5.3. Alignment of the OT with cosmic ray data

In this study the OT C-Frames - an OT C-Frame consists of two layers with different stereo angles to measure x and y - were aligned using cosmic ray data that were acquired in September 2008. A total of 20000 tracks were used to align the OT C-Frames for translations in x (Tx) and z (Tz) and a rotation about the z (Rz) axis. To constrain global translations and shearings the first and second-to-last C-Frames were fixed to their survey positions. The tracks were fitted without drift-time information. In this case the hit resolution of the OT is 1.44 mm.

The difference between preliminary survey measurements and the alignment parameters obtained with the framework are shown in Fig. 5. The error bars in Fig. 5 reflect the uncertainty in the survey. The errors on the alignment parameters are negligible. A few microns compared to 500 μm of the survey. On the x-axis are the nominal z coordinates of the C-Frames. The alignment parameters are in good agreement with the survey measurements.

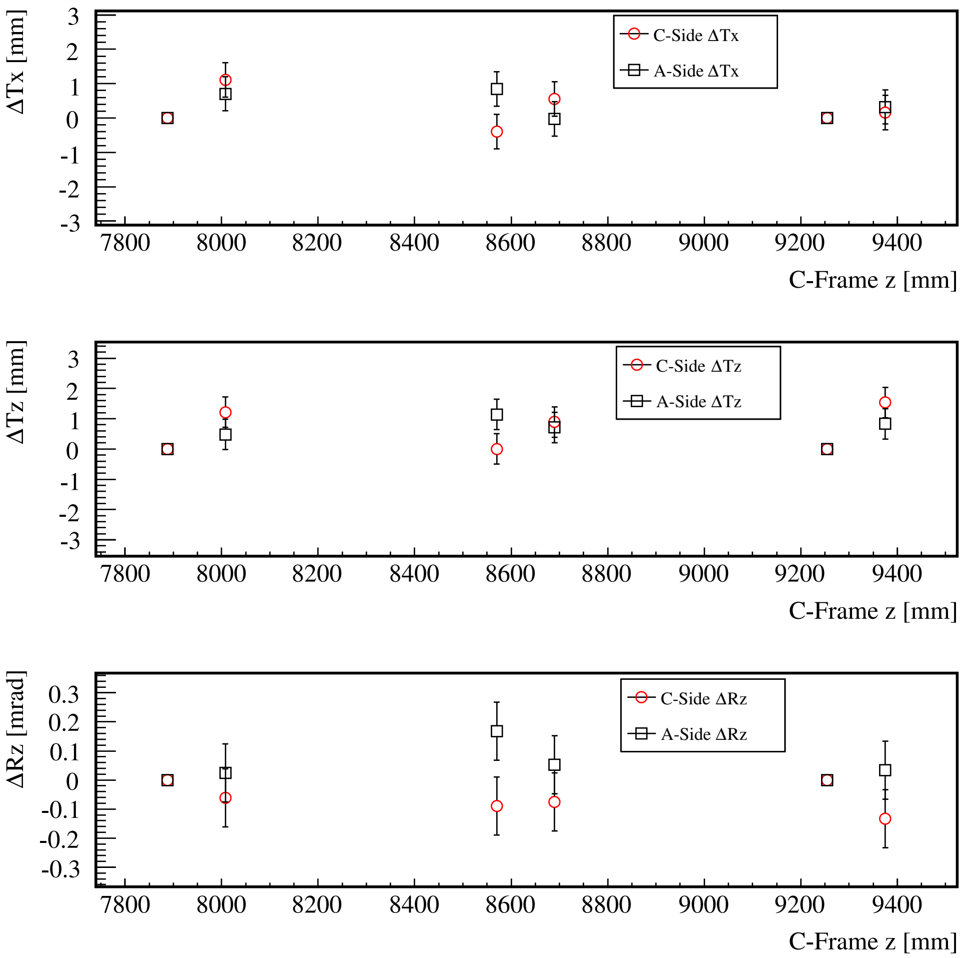

Figure 5. Result of the OT C-Frames alignment with cosmic rays: the difference between the survey and alignment parameters are shown. The error bars reflect the uncertainty in the survey. The points without error bars are the C-Frames that were fixed to their survey positions.

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6. Conclusion

We have presented an alignment framework that uses the closed form method of alignment with Kalman filter fitted tracks [6]. This allows us to use the standard LHCb track model and fit, and to determine the alignment parameters with a single pass through the calibration data. In addition, complexities of the track model such as multiple scattering and energy loss corrections are correctly taken into account. Moreover, the obtained alignment parameters are consistent with the track model and fit used in the reconstruction.

At the core of the framework is a generic C++ algorithm with which any detector element can be aligned for any degree of freedom and is easy to configure. An alignment job can be configured using standard reconstruction algorithms and configurations. The elements and the degrees of freedom to align for, and similarly for constraints, are specified with a simple Python lists of strings.

We have shown that the alignment framework works with tracks from beam events or cosmic ray events, and under different settings and or conditions, *e.g.* magnet off or on, with or without drift time information, *etc.*

We have also presented a selection of results showing that physics quantities can be recovered using this framework. In addition, we have presented alignment results for the OT C-Frames using cosmic ray events acquired during the LHCb detector commissioning in September 2008.

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