ing the separatrix due to quantum effects is less than 5×10^{-3}. The ratio of power losses in copper to beam power is kept greater than 1,5.

Fig. 4 shows some possible arrangements of experimental areas. The hall A is designed for handling electron beams, while gamma beams are produced in halls like B or C. The latter called "fishbone hall" seems attractive from the point of view of building. It is detailed in Fig. 5. Electron travel the central part and can strike a target at six possible locations, each of them being equivalent for the optical properties (apart from a slight enlargement of the beam due to radiation effects); the beam transport system between two neighbouring possible locations is achromatic and afocal.

Preliminary estimates lead to the following tentative budget (see Table IV).

This machine could be built in 5-6 years with 800 men-years. A model of magnet and a model of cavity are ordered and will be delivered at the end of 1965.

DISCUSSION

SCHAFFER: I should like to give a comment concerning the generation of r.f. power for extended energy electron synchrotrons. At DESY, we are specifying a new klystron amplifier in order to increase the energy of the synchrotron up to 7.5 or 8 GeV. This klystron will have a peak power of 500 kW and an average power of 250 kW at a frequency of 500 Mc/s. We hope that the new tube will also be useful for any future planning of electron synchrotrons or electron-positron storage rings with extended energy and intensity.

COMPUTER STUDIES OF CAPTURE OF LARGE INJECTED BEAMS INTO AN ELECTRON SYNCHROTRON

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(Presented by D. J. Thompson)

1. INTRODUCTION

In an Electron Synchrotron, one normally uses an accelerating structure with a high shunt impedance, so as to attain the highest possible energy. If one then attempts to accelerate an intense beam, (to a lower energy) the voltage induced on the r.f. cavities by the circulating current is out of phase with, and may be larger than the accelerating voltage. The system is said to be heavily beam-loaded. This leads to two basic problems in the dynamics of the system; the stability of the beam and the feasibility of trapping. The former has been analysed by Robinson (1), who derived a quartic equation for the complex resonant frequencies of the beam-cavity system, and hence, a stability criterion. The more complex transient problem at injection is the subject of this paper. The calculations have been carried out for the parameters appropriate to NINA; that is, we are considering a 4 GeV machine, 50 c/s repetition rate, injecting up to 500 mA for one turn, and attempting to trap 272 mA, this being equivalent to 10 μA mean current or 1.2×10^{12} electrons per pulse. There are five accelerating cavities, each having an unloaded shunt impedance of 16.9 MΩ and a coupling factor of 4.8 the waveguide feed, which can be considered to be mat-

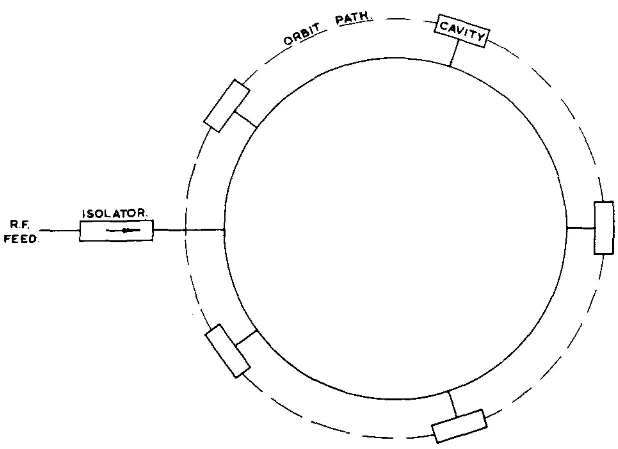

Fig. 1 - The r.f. cavity system.

ched at the power source, as it is fed through an isolator. The injector frequency is seven times the synchrotron r.f. frequency. In considering the trapping process, the mathematical complexities rule out an analytical solution, and one is forced to approximations and to running many particular cases on a computer to gain an insight into the problem.

At DESY, Passow (2) has used an analogue computer for this purpose, which gives the results in a simple pictorial form, but only six bunches (equivalent to seven in NINA) were used. We have used a large digital computer to follow up to 420 bunches, which are provided with a realistic statistical energy spread.

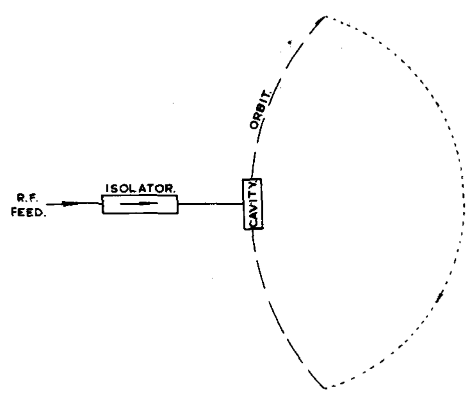

Fig. 2 - Single cavity approximation.

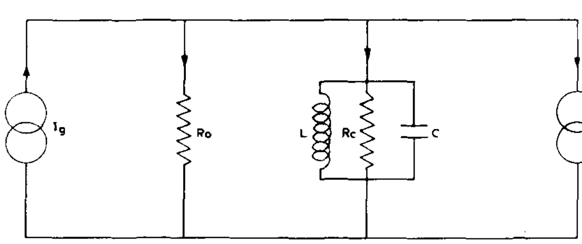

Fig. 3 - Equivalent circuit.

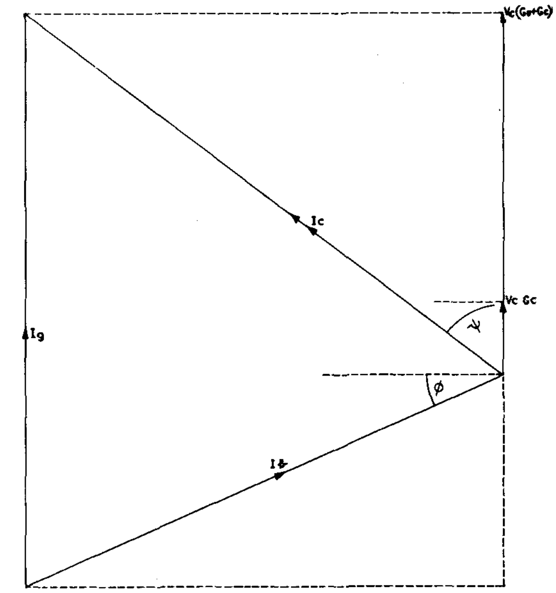

Fig. 4 - Vector diagram.

2. APPROXIMATIONS

In the synchrotron r.f. system, power is fed through an isolator to a waveguide ring (Fig. 1). The five cavities are attached to waveguide branches, coupled to the ring by tee-junctions. If the ring is correctly tuned and if each cavity presents the same impedance to the waveguide, then the voltage and reflection coefficients at each cavity are the same, and electrically the system can be represented to a very good approximation by a single cavity fed directly from the isolator. With beam loading this requires the assumption that the motion of the beam in each of the five segments of the orbit is the same, making the beam-induced voltage the same in each cavity. Though this is not strictly true, it seems that, taking into account the strong coupling between cavities through the waveguide, the cavity voltages and currents will be sufficiently alike to justify the single cavity approximation (Fig. 2). In the synchrotron the r.f. frequency is 300 times the orbital frequency and $1/7^{th}$ the injector frequency. This meams that 2,100 bunches each containing over 10^8 electrons are injected. The electrons in any one bunch have energy spread, which is a function of the injector. The first approximation is to consider each bunch as one massive particle of a certain energy, but to apply an energy distribution to the bunches. For mathematical simplicity we approximate the energy distribution to a cosine function. Next as there are 420 bunches in a segment of the machine between two cavities, and as we are approximating to one cavity, we consider only 420 bunches. This is the

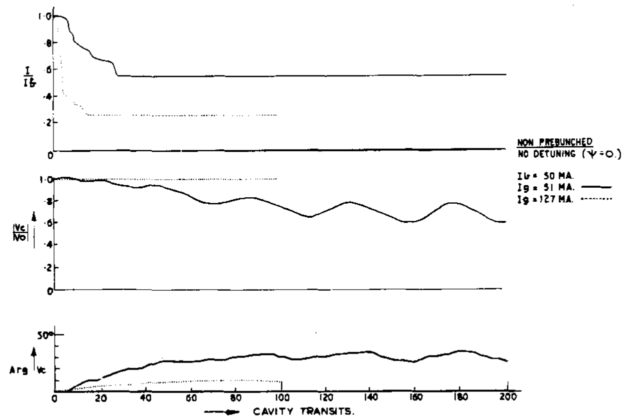

Fig. 5 - No detuning, 50 mA injected current.

basis of the calculation, but to cut down the computer time, the calculation is usually done with 60 bunches, taking one per r.f. cycle but successive bunches taken at successive phases from the injector. All particularly interesting runs are repeated with 420 bunches, and so far, this has shown only difference in detail with hardly any difference in final captured current. The other approximation is that the gap in the circulating beam, due to the inflector turn-off time, has been ignored and instead the ring considered to be filled with bunches containing correspondingly fewer electrons. There will be an additional 1.36 Mc/s component of cavity voltage, but as this is not a resonant frequency of the r.f. system, any effects are expected to be small. A few cases showing good trapping are being repeated with this gap inclûded. Betatron oscillations are assumed to be quite independent of synchrotron oscillations.

3. THE CALCULATION

The source can be considered as a constant current generator shunted by the waveguide impedance (Fig. 3). This source impedance is in parallel with the cavity shunt impedance, giving the cavity a loaded Q value Q_L. It has a resonant frequency ω_o and the generator frequency is ω. We define $\alpha = \omega_o/2\,Q_L$ and $\delta\,\omega = \omega - \omega_o$. The cavity voltage is V_c (the factor $e^{j\omega t}$ is implied throughout). The electron bunches are considered as δ-functions of current, of amplitude I_o, which, in passing through the cavity, induce on it a voltage V. At time $t = 0$ the voltage V_o, which is given by the applied generator current I_g, has a phase angle of zero. If I_g remains constant at injection, V_c, the cavity voltage is given by V_o minus the beam-induced voltage V. If however I_g is altered at injection by a phase and amplitude jump from I_{g1} to I_{g2}, then V_o is given by I_{g2}, and V_c is V_o plus the time-decaying contribution from $(I_{g1} - {}_{g2})$ minus the beam-induced voltage.

A bunch of electrons enters the cavity at time t_1. The response of the cavity voltage to this impulse is $V = K\exp(-\alpha - j\,\delta\,\omega)\ (t - t_1)$ where $K = I_o/C$ (C being the capacitance in the equivalent circuit). Then $V_c = V_o - V$. At time t_2 another bunch enters the cavity so now

$$V_c = V_o - [K\exp(-\alpha - j\delta\omega)(t_2 - t_1) + \\ +K]\exp(-\alpha - j\delta\omega)(t - t_2)$$

The process is reparated until all the bunches have traversed the cavity once. From the orbit parameters, new t values can be calculated for when the electron bunches enter the cavity for the second time. The energy deviation from the synchronous energy is also calculated. This value and the phase of entry is printed out for selected particle bunches for every cavity transit, the *cavity voltage, reflection coefficient and the beam* induced voltage are also printed out, when all the bunches have passed through, or would have passed through, the cavity. If the energy of any. bunch exceeds the maximum permissible energy deviation, then it is eliminated from further part in the calculation. This fact is printed out, and final captured current is obtained by counting the eliminated bunches and subtracting from the total injected. In some cases the bunches "cross" and $t_n - t_{(n-1)}$ becomes negative. This means the cavity voltage is calculated at $t = t_{(n-1)}$ and is effective in calculations at t_n. Since the increments of voltage are very small, this has negligible effect on the calculations. A re-ordering of the bunches to correct this defect would extend the time required for the calculation by a very large factor.

4. THE VECTOR DIAGRAM

When interpreting the results, and when deciding on values of parameters to put in the

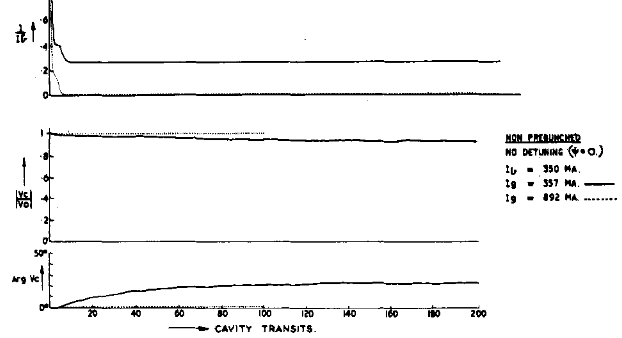

Fig. 6 - No detuning, 350 mA injected current.

calculation, use is made of the equivalent circuit (Fig. 3) *and the vector diagram shown in* Fig. 4. In Fgs. 3 and 4 I_1 is the 408 Mc/s component of the beam current. For tight bunching and unity transit time factor this would have a peak value of twice I_b, the meam circulating current.

5. RESULTS

The following values of parameters have been used throughout:

Mean injection energy = 40 MeV.

Energy spread at injection = ± 1/2% (non-prebunched) or ± 1% (prebunched).

Nominal energy gain per cavity transit = 18.5 kV.

Cavity loaded Q = 6340.

Cavity shunt admittance = 5.91×10^{-8} ohm^{-1}.

Source shunt admittance = 2.76×10^{-7} ohm^{-1}.

Transit time factor = 0.75.

Allowed energy deviation before hitting the walls = ± 2%.

The following quantities are treated as variables:

δ = phase of injection of first bunch, usually put at — 79° for non prebunched beams.

I_b = magnitude of injected current, usually put at 500 mA for non-prebunched and 272 mA for prebunched beams.

I_g = generator current.

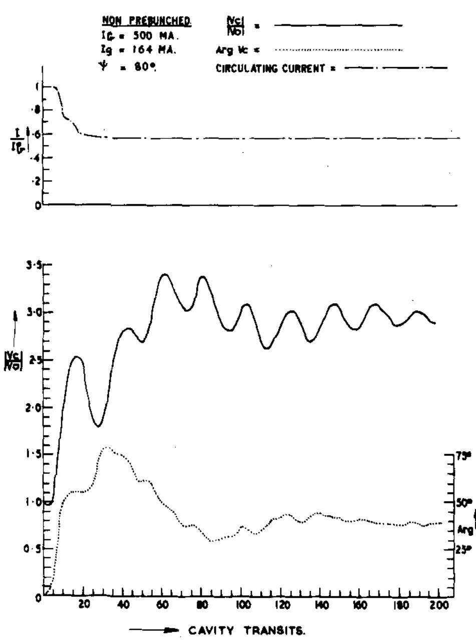

Fig. 7 - 500 mA injected current with cavity detuned.

ψ = cavity detuning angle, defined by $\tan\psi = 2\,Q_L\,\delta\omega/\omega$.

A phase and amplitude change in generator current at injection can also be included.

V_c, the amplitude of the cavity voltage, is plotted against the number of transits and is normalised to V_o. Also plotted are the phase of V_c, and the circulating beam current.

For the simple case of a beam injected into the high-Q cavity with no special precautions, if the generator current is sufficiently large to support the desired beam current (from the equivalent circuit, we must have $I_g = I_1 + I_c$ where I_c is determined by the required value of accelerating voltage), then the initial voltage before the beam is bunched and trapped is large, and beam is lost by excessive amplitude of synchrotron oscillations. If the voltage is reduced then the loading effect is too great, the voltage falls and not only are particles lost from the phase-stable region, but instability can result. For small injected currents, 4 out of 7 injected bunches *may be retained, but for larger currents,* not more than 2 out of 7. Examples are shown in Figs. 5 and 6.

A major improvement may be obtained by detuning the cavities. I_g can then be much greater, and when the beam current is injected, the vector difference of I_g and I_b can lead to a cavity voltage which, though it must vary considerably in phase, is always of about the right amplitude. The computations show that quite large currents can be trapped by this means, one of the best runs to date being shown in Fig. 7, when 4 out of 7 bunches were completely captured, with maximum injected current.

The problem can be simplified and currents increased by prebunching the injected beam at the cavity frequency. It is hoped to do this on NINA, bunching at the input to the injector,

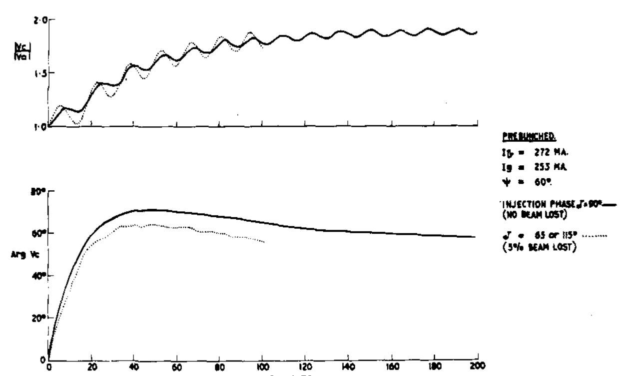

Fig. 8 - Prebunched beam.

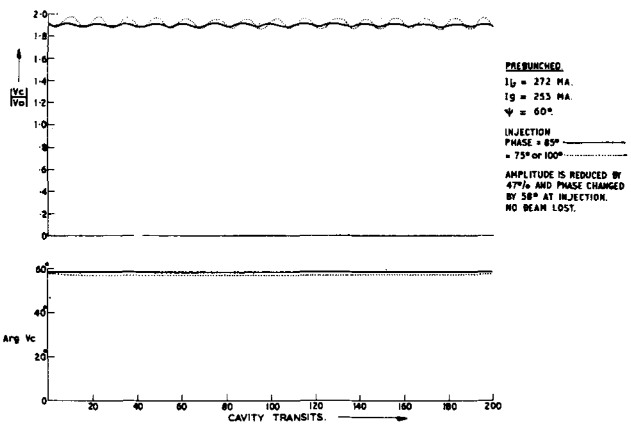

Fig. 9 - Prebunched beam with phase and amplitude jump.

and phase-locking the buncher to the synchrotron r.f. computations have been done in wich the whole of a prebunched beam was trapped. Examples are shown in Fig. 8, which shows also the reduction in oscillation amplitude achieved by correct phasing of injection.

With or without prebunching and even with cavity detuning, phase and amplitude changes in cavity voltage must be produced by the injected beam, and this must tend to cause loss of particles. If the phase and amplitude of the generator current can be changed at the moment of injection (suggested also by Schaffer (3)) it would .be possible to further reduce voltage variations and hence particle loss. This is shown clearly in Fig. 9 which is again for a prebunched beam equivalent to the previous case, but with a phase and amplitude jump in addition. For the correct injection phase, there is hardly any transient or oscillation.

For the non-prebunched beam, some bunches must always be lost, and as this takes time, (as does the bunching process for the captured particles), voltage fluctuations must always occur. However, computations have confirmed that combined with detuning, a phase and amplitude jump can lead to very good capture, typically just under 5 out of 7 bunches for a 500 mA injected beam. The margin of improvement is only slight over the best case without a jump at injection, but conditions are much less critical. Also it is hoped that with a suitable injection jump a detuning angle less than 82° (used in all the above cases) may be possible.

6. CONCLUSIONS

In NINA, it is planned to accelerate intense currents to 4 GeV and smaller currents to more than 5 GeV.

The latter requirement means that a high impedance accelerating structure is desirable. For the former, the cavities can be loaded, possibly in a time varying manner, but an alternative and possibly more elegant and convenient solution is to apply suitable modulation to the r.f. generator and suitably adjust the cavity tuning. The computer programme described above is sufficiently comprehensive and versatile to indicate the feasibility of such action, with the results described. The most basic conclusions are that a pre-bunched beam is very well worth while achieving, and that very great detuning of the synchrotron r.f. at injection appears to be necessary for optimum capture, and the repercussions of this on the r.f. structure must therefore be investigated thoroughly.

The most important feature not taken into account is that there are not one but five cavities, linked by a resonant waveguide ring. This must affect to some degree the response of the system to beam-loading.

The one-cavity approximation will be less good for the cases where the cavity voltage, and hence the synchrotron frequency, is high. This applies to the case of a prebunched beam, but here the fast transients are of small amplitude and we consider their effects are probably too small to seriously affect machine performance for the synchrotron frequencies which occur with our parameters.

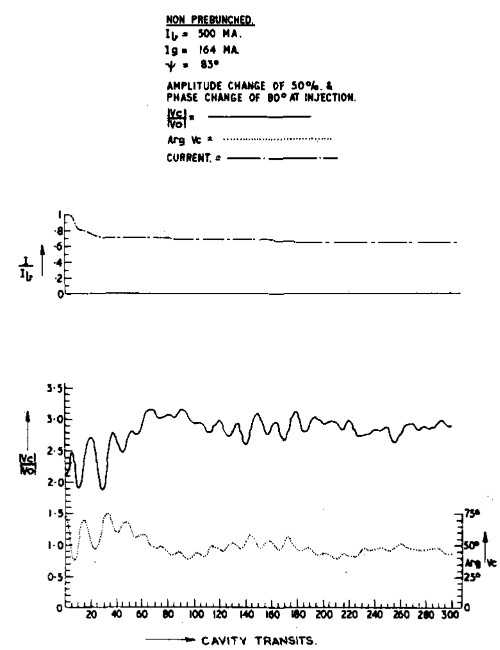

Fig. 10 - Non-prebunched beam with phase and amplitude jump.

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MEASUREMENT OF THE SYNCHROTRON RADIATION IN THE X-RAY REGION

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(Presented by R. Haensel)

Synchrotron radiation has two different aspects, one for the electron machine designers, the other for the experimentalists. For the former group the radiation limits the available energy because it represents a loss which increases as the fourth power of the energy. The strong irradiation of the walls of the vacuum chambers causes serious vacuum problems (1). For the experimentalists the synchrotron radiation is the strongest source in the extreme ultraviolet and soft X-ray region which has a continuous spectral distribution, high polarization and an absolutely evaluable intensity. The know-ledge of the characteristics of this radiation is important as well for application of this radiation (2-5) and the interpretation of certain astrophysical observations (6) as for machine designing.

Experimental investigations of the spectrum, the angular distribution and the polarization were up to now mainly carried out in the visible and in the extreme ultraviolet at accelerators with electron energies up to 1.2 GeV (7-16).

The theory of the synchrotron radiation has been developed by several authors (17-25). In the following we are referring to Schwinger (17). For any possible angle ψ, which is defined as the azimuth angle of observation, measured relative to the orbital plane, one gets for the radiation intensity I at an electron energy E_e, and a frequency of the emitted photon ω

$$\frac{\partial^2 I(\omega_1 \psi_1 E_e)}{\partial\omega\,\partial\psi} = \frac{3}{4\pi^2}\frac{e^2}{R}\left(\frac{\omega}{\omega_c}\right)^2 \gamma^2 (\lambda + \gamma^2 \psi^2)^2 .$$

$$\cdot\left[K^2_{213}(\xi) + \frac{\gamma^2\psi^2}{1+\gamma^2\psi^2} K^2_{113}(\xi)\right]$$

where

$$\xi = \frac{1}{2}\frac{\omega}{\omega_0}(1+\gamma^2\psi^2)^{3/2},\ \omega_c = \frac{3}{2}\frac{C}{R}\gamma^3$$

and

$$\gamma = \frac{E_e}{\omega_0 c^2}.$$

R is the magnetic orbital radius, e the elementary charge K_{113} and K_{213} are modified Bessel functions