Polarization test of gravitational waves from compact binary coalescences

Hiroki Takeda*, Yuta Michimura, Kentaro Komori and Masaki Ando
Department of Physics, University of Tokyo, Bunkyo, Tokyo 113-0033, Japan
**E-mail: takeda@granite.phys.s.u-tokyo.ac.jp*

Atsushi Nishizawa
Kobayashi-Maskawa Institute for the Origin of Particles and the Universe,Nagoya University, Nagoya, Aichi 464-8602, Japan

Koji Nagano
KAGRA Observatory, Institute for Cosmic Ray Research, University of Tokyo, Kashiwa, Chiba, 277-8582, Japan

Kazuhiro Hayama
Department of Applied Physics, Fukuoka University, Nanakuma, Fukuoka 814-0180, Japan

The most general gravitational wave is composed of six polarization modes in metric theories of gravity. Polarization modes of gravitational waves can be used for the gravitational-wave tests of general relativity because the properties of polarization modes depend on the specific theory of gravity. We study the separability of the polarizations and the degeneracies between binary and polarization parameters for the inspiral gravitational waves from the compact binary coalescences.

Keywords: Gravitational waves, polarization, compact binary, modified gravity.

1. Introduction

The detection of gravitational waves from compact binary coalescences by Advanced LIGO and Advanced Virgo provides experimental approach to test general relativity [1-3]. In general metric theory of gravity, there are four non-tensorial polarization modes of gravitational waves in addition to two tensor modes allowed in general relativity [4]. Moreover, the number and the properties of the polarization modes reflect the nature of the theory of gravity. Thus, polarization test of gravitational waves make it possible to test general relativity. We focus on the polarization test of the inspiral gravitational waves from compact binary coalescences. We need to consider the source parameters, which are correlated with each other and determine the frequency evolution, to separate the polarizations from compact binary coalescences unlike some other waveforms. Polarization search by GW170814 had been conducted by the substitution of the antenna pattern functions [5]. However, the observational results of the polarizations may be affected by the inclination-angle dependence or the existence of other polarization modes. Thus, we study the separability of the polarizations and the degeneracies between binary and polarization parameters for the inspiral gravitational waves from the compact binary coalescences [6].

2. Angular dependence of a gravitational-wave waveform in modified gravity

In general relativity, the detector signal of I-th detector can be expressed as,

$$h_I = \frac{2}{5}\mathcal{G}_{T,I} h_{\mathrm{GR}}, \tag{1}$$

where h_{GR} is the gravitational-wave waveform in general relativity and $\mathcal{G}_{T,I}$ is the geometrical factor for the tensor modes defined by

$$\begin{aligned}\mathcal{G}_{T,I} := \frac{5}{2}\{&(1+\cos^2\iota)F_{+,I}(\boldsymbol{\theta_s},\boldsymbol{\theta_e}) \\ &+ 2i\cos\iota F_{\times,I}(\boldsymbol{\theta_s},\boldsymbol{\theta_e})\}e^{i\phi_{D,I}(\theta_s,\phi_s,\theta_e,\phi_e)}.\end{aligned} \tag{2}$$

Here ι is the inclination angle, $\boldsymbol{\theta_s} := (\theta_s, \phi_s, \psi_p)$ is the source direction angle parameters (θ_s, ϕ_s) and polarization angle ψ_p, $\boldsymbol{\theta_e} := (\theta_e, \phi_e, \psi)$ is the detector location and orientation angle parameters, and $\phi_{D,I}$ is the Doppler phase. We calculated geometrical factors for non-tensorial modes having different inclination-angle dependence from those of tensor modes by quadrupole formula,

$$\mathcal{G}_{V_x,I} := \sqrt{\frac{525}{56}}\sin 2\iota F_{V_x,I}(\boldsymbol{\theta_s},\boldsymbol{\theta_e})e^{i\phi_{D,I}(\theta_s,\phi_s,\theta_e,\phi_e)}, \tag{3}$$

$$\mathcal{G}_{V_y,I} := \sqrt{\frac{15}{2}}\sin\iota F_{V_y,I}(\boldsymbol{\theta_s},\boldsymbol{\theta_e})e^{i\phi_{D,I}(\theta_s,\phi_s,\theta_e,\phi_e)}, \tag{4}$$

$$\mathcal{G}_{S_2,I} := \sqrt{\frac{225}{8}}\sin^2\iota F_{b,I}(\boldsymbol{\theta_s},\boldsymbol{\theta_e})e^{i\phi_{D,I}(\theta_s,\phi_s,\theta_e,\phi_e)}. \tag{5}$$

The inclination-angle dependence for the scalar dipole radiation is proportional to $\sin\iota$ in modified gravity theories with a scalar degree of freedom [7],

$$\mathcal{G}_{S_1,I} := \sqrt{\frac{45}{2}}\sin\iota F_{b,I}(\boldsymbol{\theta_s},\boldsymbol{\theta_e})e^{i\phi_{D,I}(\theta_s,\phi_s,\theta_e,\phi_e)}. \tag{6}$$

3. Polarization model

We adopt polarization models using the above geometrical factors and adding additional polarization amplitude parameters As. The followings are examples of our polarization models.

Model TS1 is a tensor-scalar dipole model in which a scalar mode having the inclination-angle dependence of dipole radiation is added.

$$h_I = \{\mathcal{G}_{T,I} + A_{S_1}\mathcal{G}_{S_1,I}\}h_{\mathrm{GR}}. \tag{7}$$

Model TS2 is a tensor-scalar quadrupole model in which a scalar mode having the inclination-angle dependence of quadrupole radiation is added.

$$h_I = \{\mathcal{G}_{T,I} + A_{S_2}\mathcal{G}_{S_2,I}\}h_{\mathrm{GR}}. \tag{8}$$

Model TV is a tensor vector model in which the combination of vector x and vector y mode is added.

$$h_I = \{\mathcal{G}_{T,I} + A_{V_x}\mathcal{G}_{V_x,I} + A_{V_y}\mathcal{G}_{V_y,I}\}h_{\rm GR}. \tag{9}$$

4. Results

We estimate model parameters for 500 BBHs or 500 BNSs whose angular parameters are uniformly random, with a detector network such as three detectors aLIGO-AdV(HLV) or four detectors aLIGO-AdV-KAGRA(HLVK) in each polarization model by Fisher analysis. We use the inspiral waveform up to 3 post-Newtonian order in amplitude and 3.5 post-Newtonian order in phase as $h_{\rm GR}$, and consider 11 model parameters in general relativity and additional polarization amplitude parameters. Table 1 is the results of the medians of parameter estimation errors and correlation coefficients. We say that the polarization modes would be separable when the errors of the additional polarization amplitude parameters are less than unity because their fiducial values are set to be unity.

Table 1. Medians of parameter estimation errors and correlation coefficients. Here, d_L is the luminosity distance of the source and $\Delta\Omega_s$ is the sky localization error. Masses of BBH and BNS are $10M_\odot - 10M_\odot$ and $1.4M_\odot - 1.4M_\odot$, respectively. Only correlation coefficients larger than 10% are shown.

	parameter	BBH(HLV)	BBH(HLVK)	BNS(HLV)	BNS(HLVK)
	SNR	33.3	40.2	36.4	44.3
Model TS1	$\Delta \ln d_L$	0.678	0.179	0.359	0.134
	$\Delta\Omega_s[\mathrm{deg}^2]$	4.74	0.912	0.919	0.250
	ΔA_{S1}	1.16	0.284	0.606	0.197
	$C(A_{S1}, \log d_L)$	0.998	0.989	0.996	0.984
	$C(A_{S1}, \cos\iota)$	-0.553	-0.500	-0.231	-0.159
Model TS2	$\Delta \ln d_L$	0.676	0.182	0.358	0.134
	$\Delta\Omega_s[\mathrm{deg}^2]$	4.74	0.913	0.862	0.246
	ΔA_{S2}	1.51	0.385	0.765	0.256
	$C(A_{S2}, \log d_L)$	0.997	0.989	0.996	0.984
	$C(A_{S2}, \cos\iota)$	-0.609	-0.564	-0.246	-0.189
Model TV	$\Delta \ln d_L$	1.98	0.310	1.22	0.193
	$\Delta\Omega_s[\mathrm{deg}^2]$	5.68	0.795	0.813	0.187
	ΔA_{V_x}	2.55	0.420	1.37	0.241
	ΔA_{V_y}	3.91	0.513	2.12	0.298
	$C(A_{V_y}, \log d_L)$	0.999	0.993	0.998	0.991
	$C(A_{V_y}, \cos\iota)$	-0.846	-0.335	-0.307	-0.207
	$C(A_{V_x}, A_{V_y})$	0.987	0.814	0.948	0.624

In all models, the additional polarization amplitude parameters are strongly correlated with the amplitude parameters such as the luminosity distance and the inclination angle. In the model TS1 and TS2 having three polarizations, A_S is determined even by three detectors HLV for BNS. However, it is difficult to separate the additional polarization mode by three detectors HLV for BBH due to the short duration of the signal although the number of the detectors is equal to the polarization modes. In the model TV, the errors of the amplitude parameters are larger than unity with HLV for both BBH and BNS so that four detectors are always necessary to determine two additional polarizations.

Thus, we found that at least the same number of detectors is necessary to separate the polarization modes and obtain the polarization information of gravitational waves in principle. However, even when the number of detectors is equal to the number of the polarization modes, the modes would be inseparable in some cases, depending on the correlation among the amplitude parameters. Therefore, there are two conditions for the separation of polarization modes; (i) the same number of detectors or more as the number of polarization modes and (ii) significant signal-to-noise ratio (SNR) and the long duration of the signal.

5. Discussions

The second condition for the separation of polarizations (ii) indicates that the duration of the signal or frequency band of the detector has essential role in polarization test. Here, we use the networks composed of the second generation gravitational-wave detectors. However, a single next generation gravitational-wave detector such as the Einstein Telescope and the Cosmic Explorer could test several polarization modes because they have great sensitivity at lower frequency so that they could be effectively treated as a detector network including a set of detectors along its trajectory due to the Earth's rotation.

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