# TWO-DIMENSIONAL NONLINEAR EQUATIONS OF SUPERSYMMETRIC GAUGE THEORIES

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The main results of the report are collected in two theorems, which are preceded by the following reasons of their physical meaning:

The nontrivial dynamics of a physical system in the supersymmetry and supergravity theories for (2 + 2)- dimensions is described by the integrable embeddings of a supermanifold $V_{2|2}$ into the flat enveloping superspace $R_{N|M}$, supplied with the structure of a Lie superalgebra $\mathcal{G}$.

Theorem I [1]. The embeddings of $V_{2|2}$ into $R_{N|M}$ are described by the Lax- type representation:

$$[\partial/\partial z_+ + A_+(z_+,z_-), \partial/\partial z_- + A_-(z_+,z_-)]_- = 0,$$

$z_\pm = \frac{1}{2}(x^1 \mp i x^2)$; $x^{1,2}$ are even coordinates of $V_{2|2}$; $A_\pm$ are even elements of the Grassmann hull of the Lie superalgebra $\mathcal{G}$, with their coefficient functions being the induced connections of $V_{2|2}(x^1, x^2; \theta_+, \theta_-)$.

Theorem II [2]. The integrable embeddings of $V_{2|2}$ into $R_{N|M}$ are described by the exactly integrable supersymmetric nonlinear equations:

$$\mathcal{D}_+(g^{-1}\mathcal{D}_- g) = [Y_-, g^{-1} Y_+ g]_+ .$$

Here $\mathcal{D}_\pm$ are the supersymmetric covariant derivatives; $H$, $Y_\pm$, $X_\pm \equiv (Y_\pm)^2$ are the basis of the $osp(1,2)$ subsuperalgebra ($[H, Y_\pm]_- = \pm Y_\pm$, $[Y_+, Y_-]_+ = H$) of the superalgebra $\mathcal{G}$. The $\mathbb{Z}$ -grading of it ($\mathcal{G} = \oplus_m \mathcal{G}_m, [\mathcal{G}_m, \mathcal{G}_n] \subset \mathcal{G}_{m+n}, m \in \mathbb{Z}$) is realized by the Cartan element $H$, $[H, \mathcal{G}_m] = m\mathcal{G}_m$; $g = \exp \sum_\alpha F_\alpha^0 \Phi_\alpha(z_\pm, \theta_\pm)$, $F_\alpha^0 \in \mathcal{G}_0$, where $\{\Phi_\alpha\}$ is a multiplet of bosonic superfields. The proof of Th.II is based on the algebraic approach[3] and Th.I.

REFERENCES

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Appendix 1. Example.

In particular, for $\mathcal{G} = sl(n|n+1)$ with the Cartan matrix $k_{\alpha\beta} = \delta_{\alpha\beta+1} + \delta_{\alpha+1\beta}$ and $\mathcal{G}_0$ being Abelian the system in Th.II describes a supersymmetric generalization of the two- dimensional Toda lattice:

$$\mathcal{D}_+ \mathcal{D}_- \Phi_\alpha = \exp(k\Phi)_\alpha , \quad 1 \le \alpha \le 2n$$

When $n = 1$, these equations contain both (super) Liouville and sine- Gordon equations.

Appendix 2. Physical Roots

The successive development of the ideas of supersymmetry and supergravity necessitates both to consider the supersymmetric generalizations of nonlinear dynamical equations of gauge theories and to work out efficient methods for integrating them. A convenient poligon for the investigation are additional symmetries, which reduce the number of degrees of freedom to two Bose and two Fermi coordinates ($x^{1,2}; \theta_\pm$). This approach reflects a number of principal features of the real physical picture to no smaller degree than the cylindrically- or spherically- symmetric configurations of the conventional gauge theories. Besides, it allows to use the algebraic method[3] for the integration of two- dimensional nonlinear equations, which is very efficient for the construction of their exact solutions in the classical, as well as in the quantum regions.