Spin Systematics of Triaxial Nuclei in A=160 Mass Region

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Introduction

When nuclei are spun to very high speeds, they can settle into exotic shapes. One of the most intriguing possibilities is Triaxial Superdeformation (TSD), where the nucleus is both highly elongated and non-axial, lacking symmetry around its rotational axis [1, 2]. Theoretical models predict that TSD structures are stabilized by shell effects created by high-j intruder orbitals at large deformation. In the higher mass region, these structures are particularly influenced by particle-hole excitations involving high-j orbitals, which create stabilizing gaps in the single-particle energy spectrum at non-zero triaxiality. The region is an ideal area for exploring these structures, as stable triaxiality here leads to unique observable effects like nuclear wobbling [3] and distinct signature splitting patterns.

A major hurdle in studying TSD bands, is determining their actual spins. These weakly populated TSD bands are incredibly difficult to produce experimentally, we often only observe the high-spin portion of the rotational sequence, while the gamma-ray transitions linking them back to known, low-spin states are missing [4, 5]. This "floating band" problem means the band-head spin (I_0) is unknown. Precise measurement of rotational properties is essential to verify theoretical predictions and distinguish TSD structures from ND bands. However, without an accurate spin assignment, key physical quantities like the kinematic moment of inertia cannot be calculated precisely. This directly hinders the interpretation of band crossing frequencies and comparisons with theoretical predictions of nuclear shape and pairing correlations.

To solve this ambiguity, we analyze the internal pattern of the rotational sequence. The Variable Moment of Inertia (VMI) model, first proposed by Mariscotti [6], is particularly well-suited for this task. By applying modified versions of the VMI model [7, 8], we can extrapolate from the observed transitions to find the most probable band-head spin. This paper uses this VMI approach to analyze TSD bands found in ^{160}Er and ^{160}Yb [9, 10]. We then use the Ratio of Transition Energy Over Spin (RTEOS) to verify these spin assignments and explore the underlying physics of these triaxial structures.

Theoretical Framework

The analysis of rotational bands in this work employs the modified Variable Moment of Inertia (VMI) model, specifically adapted for Triaxial Superdeformed (TSD) structures, detailed in Ref. [7]. This framework characterizes a band using the band-head moment of inertia (J_0) and a restoring force constant (C).

The energy for a level with spin I, within a specific rotational band characterized by a band-head spin I_0 and band-head energy E_0, is given by [6, 7]:

$$E_I = E_0 + \frac{1}{2J_I}[I(I+1) - I_0(I_0+1)] + \frac{1}{2}C(J_I - J_0)^2 \tag{1}$$

The theoretical transition energy $E_\gamma(I \to I-2)$ used for fitting experimental data is calculated as:

$$E_\gamma(I \to I-2) = \frac{1}{2J_0}[I(I+1) - (I-2)(I-1)] + \frac{1}{8CJ_0^4}\Big([I(I+1)]^2 - [(I-2)(I-1)]^2\Big) \tag{2}$$

The band-head spin I_0 is determined by minimizing the Root Mean Square Deviation (RMS) (χ) between calculated transition energies (E_γ^{cal}) and experimental transition energies (E_γ^{exp}) for a set of trial spins. The RMSD is calculated as [7]:

$$\chi = \left[\frac{1}{n}\sum_i \left(\frac{E_\gamma^{\mathrm{cal}}(I_i) - E_\gamma^{\mathrm{exp}}(I_i)}{E_\gamma^{\mathrm{exp}}(I_i)}\right)^2\right]^{1/2} \tag{3}$$

where n is the number of transitions considered. The spin value I_0 that yields the minimum χ value is adopted as the correct band-head spin.

The results are verified using the Ratio of Transition Energy Over Spin (RTEOS).

$$RTEOS = \frac{E_\gamma(I \to I-2)}{2I} \tag{4}$$

Results and Discussion

To analyze the rotational properties of the Triaxial Superdeformed (TSD) bands, we first established the band-head spin (I_0) for ^{160}Er(b1), ^{160}Er(b2), ^{160}Er(b3) and

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^{160}Yb(b1). This determination relies on minimizing the RMS Deviation (χ) between the experimental transition energies and the theoretically calculated values. Figure 1 presents the χ plots for all four bands. Each band exhibits a distinct parabolic minimum at a specific spin value, allowing for an unambiguous assignment of I_0.

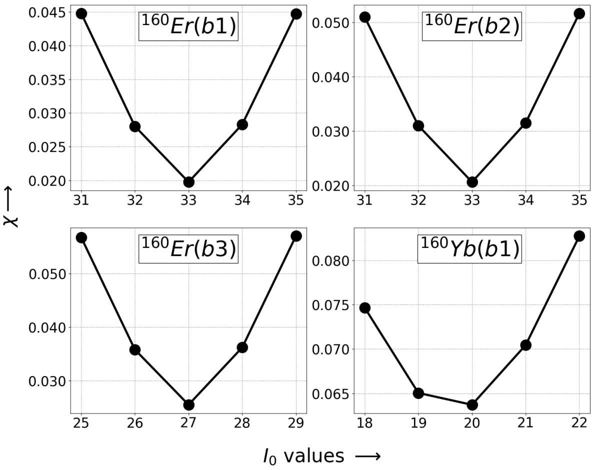

Figure 1: RMSD (χ) minimization plots for TSD bands in ^{160}Er and ^{160}Yb. The minimum χ value for each band corresponds to the adopted band-head spin I_0.

The spin assignments are subsequently verified using the RTEOS ratio, as shown in Figure 2. The observed near-horizontal trend for the adopted I_0 values in Figure 2 confirms the validity of the spin assignments from the χ-minimization.

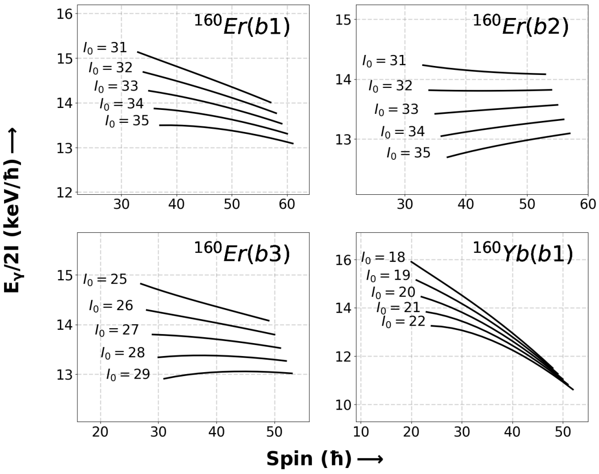

Figure 2: RTEOS ratio plots for TSD bands in ^{160}Er and ^{160}Yb. The consistency of the ^{160}Er bands supports a common TSD origin, while the trend in ^{160}Yb shows the effect of the $\nu i_{13/2}$ band crossing.

Furthermore, Figure 2 reveals the rotational systematics. The TSD bands in ^{160}Er exhibit highly consistent RTEOS values, indicating nearly identical kinematic moments of inertia ($J^{(1)}$). This strongly supports the interpretation that bands b1, b2, and b3 originate from a common TSD minimum and share the same rigid triaxial core structure ($\gamma \approx 20°$). Conversely, the plot for ^{160}Yb visualizes the band crossing attributed to the $\nu i_{13/2}$ alignment. This crossing manifests as a characteristic down-bend in the plot at high rotational frequency, corresponding to a sudden increase in $J^{(1)}$ as the quasiparticles align. This analysis provides a quantitative measure of the alignment gain within the TSD potential well.

Conclusion

A systematic analysis of TSD bands in ^{160}Er and ^{160}Yb was conducted using the modified VMI model. The band-head spins were determined by minimizing the RMS (χ), and assignments were subsequently verified using the RTEOS. The results confirmed consistent rotational behavior for multiple TSD bands in ^{160}Er and successfully characterized the $\nu i_{13/2}$ band crossing in ^{160}Yb. This work demonstrates that the combination of χ-fitting and RTEOS analysis provides a robust method for validating theoretical interpretations of high-spin triaxial structures.

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