Massive gluons in Curci-Ferrari model for describing infrared QCD

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Abstract. At large energy quarks and gluons behave as free particles and therefore the standard perturbative analysis of QCD gives very good results. However, this is not the situation in the low energy regime of QCD which is generally treated with nonperturbative methods. In spite of this, lattice simulations observe that the expansion parameter is not large in the gluonic sector. In particular, the coupling constant do not reach a Landau pole in the infrared as it is expected by standard perturbation theory. On top of this, lattice simulations find that the gluon propagator behaves as a massive propagator in the infrared. Motivated by these observations we use a model that includes a mass for the gluon (Curci-Ferrari) that can reproduce the same kind of behaviour for the gluon propagator and also a similar expansion parameter as the one found by the lattice. In this proceeding we show some of the results of quenched correlation functions obtained by using that small parameter for computing one and two loops corrections. At the end, we compare them with lattice data obtaining very good results.

1 Introduction

The low-energy regime of QCD is much more difficult to study than the high-energy regime where perturbation theory gives very good approximations due to asymptotic freedom. In order to perform analytic calculations of correlation functions in QCD it is necessary to fix the gauge. The most widely used gauge fixing procedure is the Faddeev-Popov (FP) procedure. In this procedure, auxiliary fields called "ghosts" are introduced which do not appear in the physical observables but play an important role in performing the perturbative calculations. Those allow us to successfully reproduce the values obtained in experiments involving the strong force at high energies.

Unfortunately, the standard perturbation theory is not a valid tool to deal with the low energy regime. This is so, in particular, because the standard perturbation theory based on the FP Lagrangian predicts that the coupling constant grows indefinitely in this region (Landau pole) and thus becomes inconsistent in that regime. This fact allows us to deduce that the perturbative expansion and the FP Lagrangian cannot work together. Therefore there are semi-analytical methods to study the infrared based, for example, on the Dyson-Schwinger (DS) equations (see e.g. [1–13]) or on the functional Renormalization Group (fRG) [14–19]. However, one could also question the fact that the correct gauge Lagrangian to study the

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low-energy regime is the FP Lagrangian. This questioning is possible due to some technical difficulties presented by the Faddeev-Popov procedure used to fix the gauge and observed by Gribov [20] . This problem is known as the "Gribov copies" problem and becomes harmless in the high energy regime. Due to the Gribov problem, at low energies, it is still not known from first principles which gauge fixed Lagrangian should be used. Some methods that try to overcome these difficulties use the Gribov-Zwanziger action [21, 22], where the domain with Gribov copies is constrained. However, this action implies the introduction of a large number of auxiliary fields making the computations with this method very complicated and only accessible for correlation functions of few fields. Unfortunately, the Gribov-Zwanziger action does not completely solve the Gribov problem.

Due to the scarcity of fully analytical methods to deal with the low-energy regime, numerical simulations play a great role in gaining insight into the behavior of the theory in this regime [12, 23–30]. The improvement in computational power in the last decades allowed numerical simulations to claim that the gluon behaves like a massive particle at low energies [25, 31, 32]. Moreover, numerical simulations also show that, contrary to what is predicted by the standard perturbation theory, the coupling constant at low energies remains at moderate values [24, 32–38]. Both observations led us to advocate the use of a possible perturbative expansion using a slightly modified gauge fixed Lagrangian by the addition of a gluon mass term. It is important to stress that the addition of this mass is done in the already gauge fixed Lagrangian.

The FP Lagrangian with massive gluons in the Landau gauge is a particular case of the Curci-Ferrari Lagrangian (CF) [39]. The CF Lagrangian is renormalizable but it does not satisfies unitarity in the standard way. However, numerical simulations show that at low energies the gluon propagator violates unitarity [40]. These numerical observations allow us not to discard the CF model and allow us to study its consequences.

In this proceeding we are going to present a summary of the results obtained perturbatively in Landau Curci-Ferrari model to for the study of infrared propagators and vertices in the case of quenched quarks.

2 Curci-Ferrari model in Landau gauge.

In this section we present Curci-Ferrari model in Landau gauge in its description in the Euclidean space. All the computation are done in the Euclidean space since at the end we are going to compare the results for correlation functions with lattice simulations. In Landau gauge the CF Lagrangian is obtained from Faddeev-Popov Lagrangian by adding a gluon mass term and therefore follows the following expression:

$$\mathcal{L} = \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + ih^a\partial_\mu A^a_\mu + \partial_\mu \bar{c}^a (D_\mu c)^a + \frac{m^2}{2}A^a_\mu A^a_\mu.$$

where the strengh tensor and the covariant derivatives take the form:

$$\begin{aligned} F^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu, \\ (D_\mu c)^a &= \partial_\mu c^a + g f^{abc} A^b_\mu c^c. \end{aligned}$$

This gauge-fixed Lagrangian, that includes a gluon mass term, yields almost the same Feynman rules as those obtained with the standard procedure. In particular, it gives a gluon propagator which is transverse in Landau gauge and only the prefactor is modified by the addition of a mass.

$$G^{ab}_{\mu\nu}(p) = \delta^{ab} P^{\perp}_{\mu\nu}(p) \frac{1}{p^2 + m^2}, \tag{1}$$

where we have introduced the transverse projector:

$$P^{\perp}_{\mu\nu}(p) = \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}. \tag{2}$$

This structure remains in the full propagator so we can define the gluon function $D(p)$ from the full gluon propagator, $G^{ab}_{\mu\nu}(p)$, as:

$$G^{ab}_{\mu\nu}(p) = P^{\perp}_{\mu\nu}(p)\delta_{ab}D(p). \tag{3}$$

Similarly, the dressing function of the ghost field, named $J(p)$, is obtained from the ghost propagator as

$$G^{ab}(p) = \delta_{ab}J(p)/p^2, \tag{4}$$

With this notation the 2-point vertex functions are read as

$$\Gamma^{(2)}_{A}(p) = D^{-1}(p) \qquad \text{and} \qquad \Gamma^{(2)}_{\bar{c}c}(p) = p^2 J^{-1}(p). \tag{5}$$

When doing perturbation theory and in order to avoid infinite quantities we must renormalize the theory. For this purpose we redefine the renormalized coupling constant, mass and fields, which are related to the bare ones (which we denote now using the subscript B) including the multiplicative renormalization factors Z:

$$A^{a\mu}_B = \sqrt{Z_A}A^{a\mu}, \quad c^a_B = \sqrt{Z_c}c^a, \quad \bar{c}^a_B = \sqrt{Z_c}\bar{c}^a, \quad \lambda_B = Z_g^2\lambda, \quad m^2_B = Z_{m^2}m^2, \tag{6}$$

with $\lambda = \frac{g^2 N_c}{16\pi^2}$.

Those renormalization factors can be deduced from a chosen renormalization scheme. It is important to mention that CF model allows to define a renormalization scheme that is infrared safe, meaning that the running coupling constant does not reach a Landau pole as it is observed by lattice simulations.

In the infrared-safe (IS) scheme the renormalization conditions are:

$$\begin{aligned}
\Gamma^{(2)}_{AA}(p=\mu,\mu) &= \mu^2 + m^2(\mu),\\
\Gamma^{(2)}_{C\bar{C}}(p=\mu,\mu) &= \mu^2,\\
Z_g\sqrt{Z_A}Z_c &= 1,\\
Z_{m^2}Z_AZ_c &= 1
\end{aligned}$$

Using these definitions for the renormalization factors we can then compute $\beta-$ functions and the field anomalous dimension which governs the running of the coupling, the gluon mass and the field renormalization factors as

$$\beta_g(g,m^2) = \mu\frac{dg}{d\mu}\Big|_{g_B,m_B^2}, \tag{7}$$

$$\beta_{m^2}(g,m^2) = \mu\frac{dm^2}{d\mu}\Big|_{g_B,m_B^2}, \tag{8}$$

$$\gamma_A(g,m^2) = \mu\frac{d\log Z_A}{d\mu}\Big|_{g_B,m_B^2}, \tag{9}$$

$$\gamma_c(g,m^2) = \mu\frac{d\log Z_c}{d\mu}\Big|_{g_B,m_B^2}. \tag{10}$$

It is important to note that IS scheme allows to write the β−functions in terms of the anomalous dimensions of the propagators.

$$\beta_g(g,m^2) = g\left(\frac{1}{2}\gamma_A(g,m^2) + \gamma_c(g,m^2)\right), \tag{11}$$

$$\beta_{m^2}(g,m^2) = m^2(\gamma_A(g,m^2) + \gamma_c(g,m^2)). \tag{12}$$

Once we computed the propagators we can then compute the anomalous dimension which at the end will give us the running of the coupling constant $g(\mu)$ and the running gluon mass $m(\mu)$ by integrating te β−functions with initial conditions given a some scale μ_0.

Applying the renormalization group equation to the vertex functions we obtain

$$\Gamma_A^{(2)}(p,\mu_0,g,m^2) = \frac{p^2 + m^2(p)}{z_A(p)}, \tag{13}$$

$$\Gamma_{\bar{c}c}^{(2)}(p,\mu_0,g,m^2) = \frac{p^2}{z_c(p)}. \tag{14}$$

where

$$\log z_A(\mu) = \int_{\mu_0}^{\mu}\frac{d\mu'}{\mu'}\gamma_A\left(g(\mu'),m^2(\mu')\right),$$

$$\log z_c(\mu) = \int_{\mu_0}^{\mu}\frac{d\mu'}{\mu'}\gamma_c\left(g(\mu'),m^2(\mu')\right). \tag{15}$$

In this particular renormalization scheme, $z_A(p)$ and $z_c(p)$ are simple expressions of $g(\mu)$ and $m(\mu)$:

$$z_A(\mu) = \frac{m^4(\mu)}{m^4(\mu_0)}\frac{g^2(\mu_0)}{g^2(\mu)}, \tag{16}$$

$$z_c(\mu) = \frac{g^2(\mu)}{g^2(\mu_0)}\frac{m^2(\mu_0)}{m^2(\mu)}. \tag{17}$$

It is important to note the dressing of the propagators are exact in the infrared scheme:

$$G(p) = \frac{1}{(p^2 + m^2(p)}\frac{m^4(\mu)}{m^4(\mu_0)}\frac{g^2(\mu_0)}{g^2(\mu)}, \tag{18}$$

$$F(p) = \frac{g^2(\mu)}{g^2(\mu_0)}\frac{m^2(\mu_0)}{m^2(\mu)}, \tag{19}$$

while the role of the approximation is reflected in the calculation of the running the g and m.

2.1 Propagators

The use of perturbation theory within Curci-Ferrari model provides a self-consistent way of computing propagators and vertices. In particular we have computed one and two-loop corrections for Yang-Mills propagators in the IS scheme [41–43]. One-loop diagrams can be written analytically while two-loop corrections are computed using TSIL [44] package once integrals are reduced to master integrals. For computing propagators we only have two free parameters, $g(\mu_0)$ and $m(\mu_0)$, that appear in expressions (18)-(19). Both parameters are fixed in order to minimize the joined error of both propagators. In this case, we choose $\mu_0 = 1$GeV. When comparing with lattice data there is a multiplicative factor which relates the different renormalization scheme, this factor is also taken into account when choosing the parameters in order to minimize the error of both propagators. The optimal parameter are:

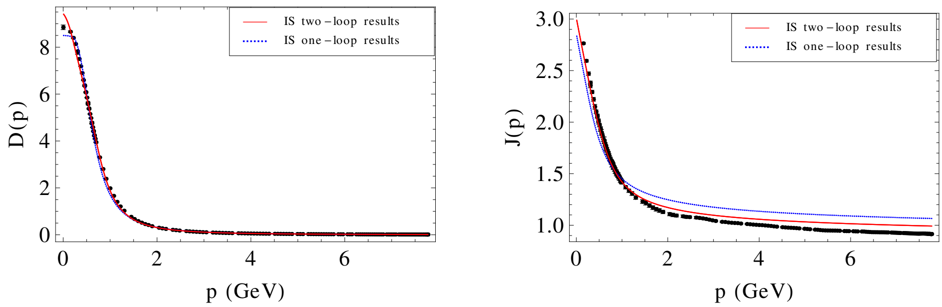

Figure 1. Gluon propagator and ghost dressing function.

order	λ_0	m_0 (MeV)
1-loop	0.30	350
2-loop	0.27	320

We show in Fig. 2.1 how the leading correction is sufficient to obtain a very good fit of both propagators. Moreover, it also shows that the next to leading order correction, although small, it improves the comparison. The latter is an indication that perturbation theory within this model with massive gluons is a valid tool to study these quantities even in the infrared. This brings great advantages because it allows to estimate the order of the corrections and to see which diagrams contribute the most with relatively simple calculations. As we will see below, the same is true for more complicated quantities that depend on several momenta. These fits seem to indicate that at least much of the physics of large distances, at least in the ghost-gluon sector, can be understood with a mass term. Moreover, it opens the door to ask whether this mass term does not indeed arise from a correct gauge fixation at low energies. Some attempts in this direction can be seen in, for instance [45, 46].

3 Three point correlation functions

In this section we briefly show some results for the ghost gluon vertex and the three-gluon vertex. The one-loop correction has been computed in arbitrary dimension, number of color and kinematics in [42]. While two-loop corrections for both vertices were computed in the one gluon vanishing momentum configuration and in four dimension. The two-loops results for the ghost-gluon vertex are deeply discussed in [47] while the next to leading order corrections for the three gluon vertex are introduced in [48].

For the ghost-gluon vertex it is convenient to define the function $G^{c\bar{c}A}(p,k,r)$ as:

$$G^{c\bar{c}A}(p,k,r) = \frac{k_\nu P^\perp_{\mu\nu}(r)\Gamma_\mu(p,k,r)}{k_\nu P^\perp_{\mu\nu}(r)k_\mu} \tag{20}$$

using $\Gamma^{(3)}_{c^a\bar{c}^b A^c_\mu}(p,k,r) = -ig_0 f^{abc}\Gamma_\mu(p,k,r)$.

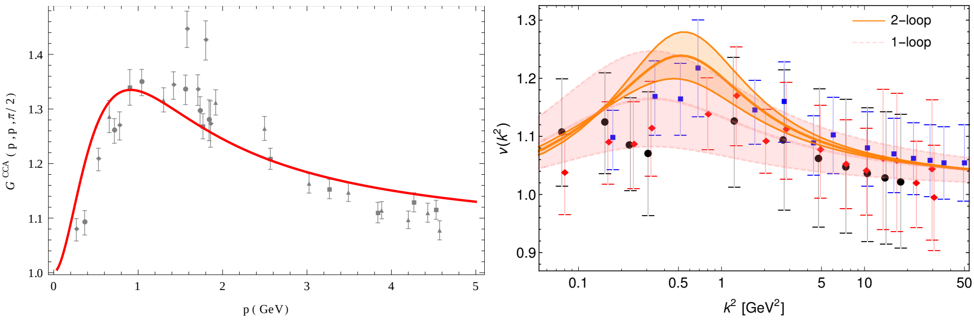

Figure 2. (left) ghost-gluon vertex in the two orthogonal and equal norm momenta configuration (data from:[25]) (right) ghost-gluon vertex in the vanishing gluon momentum configuration (data from [32])

The tensor structure of the three-gluon vertex is much more complicated and can be written as:

$$\begin{aligned}\Gamma_{\mu\nu\rho}(p,k,r) &= A(p^2,k^2,r^2)\delta_{\mu\nu}(p-k)_\rho + B(p^2,k^2,r^2)\delta_{\mu\nu}(p+k)_\rho \\ &- C(p^2,k^2,r^2)(\delta_{\mu\nu}p.k - p_\nu k_\mu)(p-k)_\rho + \frac{1}{3}S(p^2,k^2,r^2)(p_\rho k_\mu r_\nu + p_\nu k_\rho r_\mu) \\ &+ F(p^2,k^2,r^2)(\delta_{\mu\nu}p.k - p_\nu k_\mu)(p_\rho k.r - k_\rho p.r) \\ &+ H(p^2,k^2,r^2)\left[-\delta_{\mu\nu}(p_\rho k.r - k_\rho p.r) + \frac{1}{3}(p_\rho k_\mu r_\nu - p_\nu k_\rho r_\mu)\right] \\ &+ \text{cyclic permutations}\end{aligned}$$

In Fig. 3 we show the results for the one gluon vanishing momentum configuration comparing one loop with two-loops results. It can be seen that the zero crossing observed by lattice simulations, see e.g. [25, 49–51], is shifted to the deep infrared at next to leading order. This is supported by the numerical simulations of [28] which show smaller error bars in the infrared.

$$\Gamma(p^2,\mu) = \frac{\Gamma^{\text{tree}}_{\mu'\nu'\rho}(p)P^{\perp}_{\mu'\mu}(p)P^{\perp}_{\nu'\nu}(p)\Gamma_{\mu\nu\rho}(p,\mu)}{\Gamma^{\text{tree}}_{\mu'\nu'\rho}(p)P^{\perp}_{\mu'\mu}(p)P^{\perp}_{\nu'\nu}(p)\Gamma^{\text{tree}}_{\mu\nu\rho}(p)},$$

It is important to stress that in order to reproduce the vertices no extra fitting of the parameters need to be done and all the information needed can be extract from propagators. However, in the case of the three-gluon vertex, even though the parameters m_0 and λ_0 are already fixed we fit the multiplicative factor again in order to take into account possible differences in the lattice renormalization schemes.

4 Conclusions

In the proceeding we briefly introduce Curci-Ferrari model in Landau gauge which was studied in depth in [52]. This model includes a gluon mass term which regularises the infrared and allows to do perturbation theory. One and two-loops corrections for propagators and vertices in the quenched case are successfully compared with lattice data. In particular, as the results of numerical simulations for the three-gluon vertex present larger error bars, having this semi-analytical calculation is a great input to understand the infrared properties of QCD.

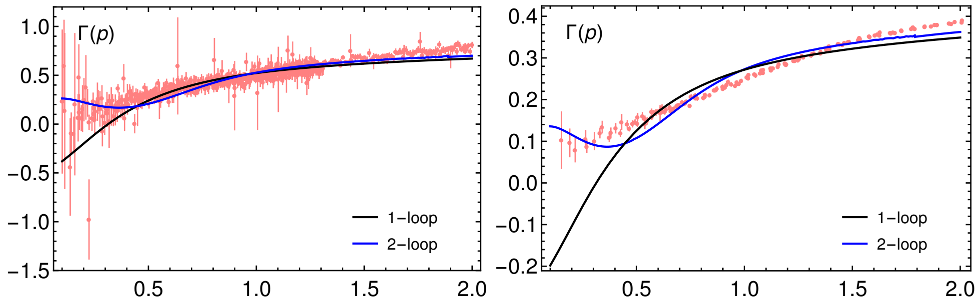

Figure 3. Three-gluon vertex in the one vanishing momentum configuration compared with lattice data from [12] (left) and [28] (right).

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