Bianchi type III holographic dark energy cosmological model in general relativity

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Abstract

In this paper, we have studied the Bianchi type III universe filled with matter and holographic dark energy (DE) components. In order to obtain a determinant solution, special law of variation for Hubble's parameter proposed by Berman (1983) has been considered. The relationship between holographic dark energy model with the quintessence dark energy has been established. Quintessence potential and dynamics of the quintessence scalar field are reconstructed, which describes the accelerated phase of the expanding universe.

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1 Introduction

Recent cosmological observations (Riess et al. 1998; Perlmutter et al. 1999; Spergel et al. 2003; Tegmark et al.2004; Eisenstein et al.

2005; Kessler et al. 2009; Komatsu et al.2009) indicate that the universe is spatially flat and has an accelerated expansion. These observations lead to a matter called dark energy (DE) which has large negative pressure. The DE can be explained in terms of cosmological constant ($\wedge$), acts like a perfect fluid with an equation of state, satisfying the observational data so far. However, these involve the problems of fine tuning and cosmic coincidence. The simplest candidate for DE is the cosmological constant with equation of state parameter $w = -1$, since it fits the observational data well, but it needs to be extremely fine-tuned to satisfy the present value of DE (Copeland et al. 2006). To solve cosmological constant problem at present epoch, $\wedge$ with a dynamical character is preferred over a constant $\wedge$, especially a time dependent $\wedge$ which has decreased slowly from its large initial value to reach its present small value (Overduin 1998). To further investigate the properties of DE, many dynamical DE models have been proposed, such as quintessence (Barreiro et al. 2000), phantom (Caldwell 2003), tachyon (Padmanabhan & Choudhary 2002; Sen 2002; Bagla et al. 2003), k-essence (Armendariz et al. 2001), dilatonic ghost condensate (Gasperini et al.2001), quartessence (Leon et al. 2010) and so forth.

The nature of DE can also be studied according to some basic quantum gravitational principles, for example, holographic dark energy principle. According to this principle (Susskind 1995), the degrees of freedom in a bounded system should be finite and does not scale by it volume but with its boundary era. Cohen et al.(1995) found that for a system with infrared (long distance) cutoff scale L and ultraviolet (short distance) cutoff scale $\wedge$ without decaying into a black hole, the quantum vacuum energy should be less than or equal to the mass of a black hole i. e.$L^3\rho_\wedge \leq LM_p^2$. Here $\rho_\wedge$ is the vacuum energy density and $M_p = (8\pi G)^{-\frac{1}{2}}$ is the reduced Plank mass. Using this idea in cosmology, one can take L which satisfies this inequality with $\rho_\wedge$ as DE density. The holographic principle is considered as another alternative to the solution of the DE problem. This principle was first put forward by G.'t Hooft (1993) in the context of black hole physics. According to the holographic principle, the entropy of a system scales not with its volume, but with its surface area (Li 2004). In the cosmological context, Fischler and

Susskind (1998) have proposed a new version of the holographic principle, viz. at any time during cosmological evolution, the gravitational entropy within a closed surface should not be always larger than the particle entropy that passes through the past light-cone of that surface. In the context of the DE problem, though the holographic principle proposes a relation between the holographic DE density $\rho_\wedge$ and the Hubble parameter H as $\rho_\wedge = H^2$, it does not contribute to the present accelerated expansion of the universe. Granda and Olivers (2008) proposed a holographic density of the form $\rho_\wedge \cong \alpha H^2 + \beta \dot{H}$, where H is the Hubble parameter and α,β are constants which must satisfy the restrictions imposed by the current observational data. They showed this new model of DE represents the accelerated expansion of the universe and is consistent with the current observational data. Moreoevr, Granda and Olivers (2009) studied the corresponding between the quintessence, tachyon, k-essence and dilaton DE models with this holographic DE model in the flat FRW universe. There exist many cosmological versions of holographic principle in literature (Setare 2006a,b, 2007a,b,c; Wu et al. 2009; Mubasher et al. 2009; Tavakol & Ellis 1999). Esther and Lowe (1999) found that this principle can be replaced by generalized second law of thermodynamics (GSLT) for time dependent backgrounds. This is similar to the cosmological holographic principle given by Fishler and Susskind (1998) for an isotropic open and flat universe with fixed equation of state. Spatially homogeneous and anisotropic cosmological models play a significant role in the description of large scale behavior of the universe and such models have been widely studied by many authors in search of relativistic picture of the early universe. Anisotropic Bianchi type-I, Bianchi type-III, Bianchi type-V dark energy models with the usual perfect fluid have also been extensively studied in the literature (Adhav 2011a,b; Pradhan et al. 2011; Akarsu & Kilinc 2010; Yadav 2011a,b,c; Yadav & Yadav 2010; Kumar & Yadav 2011; Yadav & Saha 2012; Saha & Yadav 2012; Reddy et al. 2013; Samanta 2013a,b).
Recently Saha (2014) investigated a number of cosmological models, namely Bianchi type I, III, V, VI_0, VI, and FRW space time, for the anisotropic models. He used proportionality condition as an additional constraint and found that the matter distribution remains anisotropic in the Bianchi type VI_0, V, III and I models.

Also he observed that at the early stage, the EoS parameter is positive i.e. the universe was matter dominated at the early stage but at later time, the universe is evolving with negative values, i. e. the present epoch. Moreover in this work, he has studied a system of two fluids within the scope of spatially flat and isotropic FRW model. In order to investigate this three different scale factors were used that gives rise to a variable deceleration parameter.
So it will be interesting to study the evolution of holographic DE in an anisotropic model like Bianchi type-III space time. In this paper, we considered the generalized holographic DE cosmological model in anisotropic Bianchi type-III space time to investigate the correspondence with quintessence models of the universe. We obtained the EoS parameter for the holographic DE model in Bianchi type-III space time in section-2. In section-3, we solved the Einstein field equations by considering deceleration parameter to be constant proposed by Berman (1983) and obtained the solution. The correspondence between the holographic DE with quintessence is shown in section-4. Some concluding remarks are given in section-5.

2 Dark energy model in Bianchi type III universe

We consider anisotropic Bianchi type III line element given by

$$ds^2 = dt^2 - a_1^2 dx^2 - a_2^2 e^{-2mx} dy^2 - a_3^2 dz^2 \tag{1}$$

Einstein's field equation is given by

$$R_{ij} - \frac{1}{2} g_{ij} R = -(T_{ij} + \bar{T_{ij}}) \tag{2}$$

where R_{ij} is the Ricci tensor and R is the Ricci scalar.

The energy momentum tensor for matter and holographic dark energy are defined as

$$T_{ij} = \rho_m u_i u_j ; \bar{T}_{ij} = (\rho_\wedge + p_\wedge) u_i u_j - g_{ij} p_\wedge \tag{3}$$

where ρ_m, $\rho_\wedge$ are the energy densities of matter and holographic dark energy and $p_\wedge$ is the pressure of the holographic dark energy.

The field equations for Bianchi tupe III space time are

$$\frac{\dot{a}_1\dot{a}_2}{a_1a_2} + \frac{\dot{a}_1\dot{a}_3}{a_1a_3} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} - \frac{m^2}{a_1^2} = \rho_m + \rho_\wedge \tag{4}$$

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} = -p_\wedge \tag{5}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_1\dot{a}_3}{a_1a_3} = -p_\wedge \tag{6}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1\dot{a}_2}{a_1a_2} - \frac{m^2}{a_1^2} = -p_\wedge \tag{7}$$

$$\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2} = 0 \tag{8}$$

The average scale factor a and the average Hubble's parameter H are defined as

$$a = (a_1a_2a_3)^{\frac{1}{3}} \tag{9}$$

$$H = \frac{1}{3}\Big(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\Big) \tag{10}$$

The scalar expansion θ, deceleration parameter q, shear scalar σ^2 and the average anisotropiy parameter $\triangle$ are defined as

$$\theta = 3H = \Big(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\Big) \tag{11}$$

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -1 - \frac{\dot{H}}{H^2} \tag{12}$$

$$\sigma^2 = \frac{1}{2}\big(H_1^2 + H_2^2 + H_3^2 - \frac{1}{3}\theta^2\big) \tag{13}$$

$$\triangle = \frac{1}{3}\Sigma\Big(\frac{\triangle H_i}{H}\Big)^2; i = 1, 2, 3. \tag{14}$$

where $\triangle H_i = H_i - H; i = 1, 2, 3$. The holgographic dark energy density is given by

$$\rho_\wedge = 3(\alpha H^2 + \beta \dot{H}) \tag{15}$$

with $M_p^{-2} = 8\pi G = 1$
The continuity equation can be obtained as

$$\dot{\rho}_m + \dot{\rho_\wedge} + 3H(\rho_m + \rho_\wedge + p_\wedge) = 0 \tag{16}$$

The continuity equation of the matter is

$$\dot{\rho}_m + 3H\rho_m = 0 \tag{17}$$

The continuity equation of the holographic dark energy is

$$\dot{\rho}_\wedge + 3H(\rho_\wedge + p_\wedge) = 0 \tag{18}$$

The barotropic equation of state is

$$p_\wedge = w_\wedge \rho_\wedge \tag{19}$$

Using eqn. (15) and eqn. (19) in eqn. (18), we obtained

$$w_\wedge = -1 - \frac{2\alpha H\dot{H} + \beta\ddot{H}}{3H(\alpha H^2 + \beta\dot{H})} \tag{20}$$

3 Solutions

From eqn. (8), we obtained

$$a_1 = ka_2 \tag{21}$$

where k is any non-zero constant of integration. The volume scale factor V can be defined as

$$V = a^3 = a_1a_2a_3 \tag{22}$$

The directional Hubble parameter in the direction of x, y and z for the Bianchi type III metric can be defined as follows:

$$H_x = \frac{\dot{a}_1}{a}; H_y = \frac{\dot{a}_2}{a}; H_z = \frac{\dot{a}_3}{a} \tag{23}$$

Now, using eqns. (21) and (23) in eqns. (11) and (13), we obtained

$$\theta = u^i_{;i} = 3H = 2\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_3}{a_3} \tag{24}$$

$$\sigma^2 = \frac{1}{2}\left(\sum H_i^2 - \frac{\theta^2}{3}\right) = \frac{1}{3}\left(H_1 - H_3\right)^2 \tag{25}$$

The shear scalar may be taken to be proportional to the expansion scalar which envisages a linear relationship between the Hubble parameters H_1 and H_3,

$$H_3 = \alpha_1 H_1 \tag{26}$$

which leads to a relation between a_1 and a_3 as

$$a_3 = a_1^{\alpha_1} \tag{27}$$

where α_1 is a constant and is usually assumed to be positive. α_1 takes care of the anisotropic nature of the model. The expansion scalar and the shear scalar now be expressed in terms of the single time varying parameter H_1 as

$$\theta = (\alpha_1 + 2)H_1 \tag{28}$$

$$\sigma^2 = \frac{1}{3}(1-\alpha_1)^2 H_1^2 \tag{29}$$

The field equations (4-8) are highly non linear. Hence, to obtain determinate solution of the system, we can take the help of special law of variation proposed by Berman (1983) for Hubble's parameter that yields constant deceleration parameter models of the universe. It may be noted here that most of the well known models of the Einstein's theory and alternative theories including inflationary models are models with constant deceleration parameter. Therefore, we can assume the average Hubble parameter H to be related to the average scale factor a, by the relation as follows

$$H = na^{-\frac{1}{n}} \tag{30}$$

where n is a non zero constant. On solving eqn.(30), we obtained

$$a(t) = (t+c)^n \tag{31}$$

Using eqns. (21),(22) and (27) and with the help of eqn.(31), one can obtain

$$a_1(t) = k^{\frac{1}{\alpha_1+2}}(t+c)^{\frac{3n}{\alpha_1+2}} \tag{32}$$

$$a_2(t) = k^{\frac{-(\alpha_1+1)}{\alpha_1+2}}(t+c)^{\frac{3n}{\alpha_1+2}} \tag{33}$$

$$a_3(t) = k^{\frac{\alpha_1}{\alpha_1+2}}(t+c)^{\frac{3n\alpha_1}{\alpha_1+2}} \tag{34}$$

The deceleration parameter

$$q = -1 + \frac{1}{n} \tag{35}$$

Since recent observational data indicates that the universe is accelerating and the value of deceleration parameter lies somewhere in the range $-1 < q < 0$, therefore we have $n > 1$ for the accelerating universe.

The metric (1) with the help of eqns. (32),(33) and (34) can now be written as:

$$ds^2 = dt^2 - k^{\frac{1}{\alpha_1+2}}(t+c)^{\frac{3n}{\alpha_1+2}}[dx^2 - k^{-1}e^{-2mx}dy^2] - k^{\frac{\alpha_1}{\alpha_1+2}}(t+c)^{\frac{3n\alpha_1}{\alpha_1+2}}dz^2 \tag{36}$$

The directional Hubble's parameter H_x, H_y and H_z have values given as:

$$H_x = \frac{3n}{(\alpha_1+2)(t+c)} \tag{37}$$

$$H_y = \frac{3n}{(\alpha_1+2)(t+c)} \tag{38}$$

$$H_z = \frac{3n\alpha_1}{(\alpha_1+2)(t+c)} \tag{39}$$

From eqn. (10) the mean Hubble's parameter H has the value given by

$$H = \frac{n}{t+c} \tag{40}$$

Using the directional and mean Hubble's parameter, we obtained

$$\triangle = 3\frac{(\alpha_1-1)^2}{(\alpha_1+2)^2} \tag{41}$$

The dynamical scalars are given by

$$\theta = \frac{3n}{t+c} \tag{42}$$

$$\sigma^2 = \frac{1}{3}(1-\alpha_1)^2\frac{9n^2}{[(\alpha_1+2)(t+c)]^2} \tag{43}$$

The matter density parameter Ω_m and holographic dark energy parameter $\Omega_\wedge$ are given by

$$\Omega_m = \frac{\rho_m}{3H^2}; \Omega_\wedge = \frac{\rho_\wedge}{3H^2} \tag{44}$$

Using eqns (32)-(34), eqn. (40) and eqn. (44), we got

$$\Omega_m + \Omega_\wedge = 3\frac{(2\alpha_1+1)}{(\alpha_1+2)^2} - \frac{m^2}{3n^2 k^{\frac{2}{\alpha_1+2}}(t+c)^{\frac{6n}{\alpha_1+2}-2}} \tag{45}$$

From equation (45), one can observe that the sum of the energy density parameters approaches to $3\frac{2\alpha_1+1}{(\alpha_1+2)^2}$ as $t \to \infty$. So at late times the universe becomes flat. Therefore for sufficiently large time, this model predicts that the anisotropy of the universe will damp out and universe will become isotropic. This result also shows that in the early universe i.e. during the radiation and matter dominated era the universe was anisotropic and the universe approaches to isotropy as dark energy starts to dominate the energy density of the universe. It is interesting to note that for $\alpha_1 = 1$, the universe becomes isotropic and the sum of the energy density parameters approaches to 1. This result is quite different from the result obtained by Pradhan et al. (2011), where they found in their model that the universe does not approach isotropy throughout the evolution.

Using eqn. (40) in eqns.(15), (17) and (20), we found

$$\rho_\wedge = 3\frac{\alpha_1(n\alpha_1 - \beta)}{(t+c)^2} \tag{46}$$

$$\rho_m = \frac{c_t}{(t+c)^{3n}} \tag{47}$$

$$w_\wedge = -1 + \frac{2}{3n} \tag{48}$$

Since recent observational data indicates that the universe is accelerating and the value of the deceleration parameter lies in the range $-1 < q < 0$, so we have $n > 1$ for the accelerating universe. It is interesting to note that, from equation (48), one can observe that $-1 < w_\wedge < -\frac{1}{3}$, for $n > 1$. In this case, the holographic dark energy EoS parameter behaves like quintessence.

4 Correspondence between the holographic and quintessence scalar field model of dark energy

To establish the correspondence between the holographic dark energy with quintessence dark energy models, we compare the EoS and the dark energy density for the corresponding models of dark energy.

The action for the quintessence scalar ϕ is given by

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}g^{ij}\partial_i\phi\partial_j\phi - V(\phi)\right] \tag{49}$$

The energy density and pressure for the quintessence scalar field are respectively represented as:

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \tag{50}$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) \tag{51}$$

where $V(\phi)$ is the quintessence potential.

The equation of state (EoS) for the scalar field is given by

$$\omega_\phi = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)} \tag{52}$$

For the accelerated expansion of the universe, the equation of state parameter for quintessence must be less than $-\frac{1}{3}$.

From eqn. (48) and eqn. (52), one can write

$$-1 + \frac{2}{3n} = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)} \tag{53}$$

Also, comparing eqn. (15) and eqn. (50), we have

$$\rho_\wedge = 3(\alpha H^2 + \beta\dot{H}) = \frac{1}{2}\dot{\phi}^2 + V(\phi) \tag{54}$$

From eqn. (52) and eqn. (53), the kinetic energy term and the quintessence potential in power-law form can be obtained as follows:

$$\phi - \phi_0 = (2(n\alpha - \beta))^{\frac{1}{2}} ln\left(\frac{t+c}{t_0+c}\right) \tag{55}$$

Using eqn. (55) in eqn. (56), we obtained the potential in the exponential form

$$V(\phi) = (3n-1)(n\alpha - \beta)t_0^{-2} exp\left(-\frac{1}{(2(n\alpha-\beta))^{\frac{1}{2}}}(\phi - \phi_0)\right) \tag{56}$$

For $n > 1$, this type of exponential potential can produce an accelerated expansion of the universe. Therefore, one can establish a correspondence between the holographic dark energy and quintessence scalar field, and describe holographic dark energy by making use of quintessence

5 Conclusions

It is well known that, the holographic dark energy cosmological model with quintessence in general relativity play a vital role in the discussion of the accelerated expansion of the universe which is the crux of the problem in the present scenario. Evolution of Bianchi type-III cosmological model is studied in the presence of holographic dark energy and quintessence with negative constant deceleration parameter. Though the present day universe appears to be isotropic, there is no evidence that the early universe was also of the same type. There is a cosmological view that the universe might be inhomogeneous and anisotropic in the very early era and that in the course of its evolution there characteristic have been wiped out under the action of some process or mechanism, and finally an isotropic and homogeneous universe had resulted. Observational data also suggest that dark energy is responsible for gearing up the universe some five billion years ago. But at that time the universe need not to be isotropic. So in this paper we assume the universe to be anisotropic and consider a homogeneous Bianchi type-III universe filled with matter and holographic dark energy. Our solution describes the accelerated expansion of the universe under certain conditions. The EoS parameter of the holographic dark energy behaves like quintessence EoS. For $\alpha_1 = 1$, we obtained the universe is isotropic throughout the evolution, the universes approaches to isotropy as $t \to \infty$ for $\alpha_1 \neq 1$. We also established a correspondence between the holographic dark energy models with the quintessence scalar field dark energy models in Bianchi type-III universe. Quintessence potential and the dynamics of the quintessence scalar field are reconstructed for this anisotropic accelerating model of the universe. Our result shows that the universe has anisotropic in the early stage and at the late time dynamics anisotropy of the universe dams out and the present day universe becomes isotropy as suggested by different observational data (Collins & Hawking 1973). Results and solutions of our model is quite different from the result and solutions obtained by Saha (2014), viz. here we obtained a isotropic model for $\alpha_1 = 1$ throughout the evolution whereas they observed that the model is anisotropic throughout the evolution. Moreover, Saha(2014), results on the LRS Bianchi type I space time filled with perfect fluid and anisotropic dark energy possessing dynamical energy density is remarkable which is in consistent with Bianchi type III model also.

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