Classification of Dark Energy Models

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Abstract. Accelerated expansion is well-defined from the observational redshift. It is found from Type Ia Supernova data that the major share of the energy of the universe is yet to be explained and that prompted the search for dark energy and dark matter. Dark energy is expected to be around 68% of the total energy of the universe. To match with experimental observations different theoretical models of dark energy are proposed over the years. Here it is attempted to classify the models of current scenario. **Keywords:** Cosmological constant, time evolution, Dark energy models, Quintessence, Classification.

1 Introduction

To begin with a theory of gravitation, Einstein introduced the field equation as

$$G^{\mu}_{\nu} = R^{\mu}_{\nu} - \frac{1}{2} g^{\mu}_{\nu} R = -8\pi G T^{\mu}_{\nu}. \tag{1}$$

Here G^{μ}_{ν} is the Einstein tensor, R^{μ}_{ν} is the Ricci tensor and R is the Ricci scalar. T^{μ}_{ν} is the energy-momentum tensor. Einstein incorporated a term called cosmological constant, Λ to state a static universe . A homogeneous, isotropic and expanding universe can be expressed mathematically by

$$ds^2 = -c^2 dt^2 + a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 Sin^2\theta d\phi^2 \right] \tag{2}$$

From Einstein's equation one can easily derive two other dynamical equations for Hubble parameter H. Therefore we have

$$H^2 = \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G\rho}{3} - \frac{K}{a^2} \tag{3}$$

and

$$\dot{H} = -4\pi G(p + \rho) + \frac{K}{a^2} \tag{4}$$

Here $H \left(= \frac{\dot{a}}{a} \right)$ is the Hubble parameter, ρ is the total energy density and p is the total pressure. This helps us to write

$$\frac{\ddot{a}}{a} = \frac{-4\pi G}{3} (\rho + 3p). \tag{5}$$

In the above equation a negative $(\rho + 3p)$ confirms an acelerated universe [1, 2]. The ratio of the pressure to the density of the vacuum energy is termed as equation of state $w = \frac{p}{\rho}$. After the discovery of expanding universe by Hubble, the cosmolgical constant was abondoned. At the end of the previous century, the observational data from Type Ia Supernova, the so-called standard candles as considered presently provided the evidence of the accelerating expansion[3]. This makes scope to re-visit and revise the position of the Cosmological Constant[3]. Another observation by the other Supernovae project team showed the evidence for a non-zero cosmological constant [4]. Thus the evidence of expanding universe and hence the evidence of dark energy is established.

The job of a Cosmologist could be easier if everything would just be fitting correctly to confirm the cosmological constant as the dark energy. But the fine tuning problem makes the claim for Λ weaker. There arises the question of other possible candidates and hence the scalar field or Quintessence models, Modified Gravity models, Chaplygin gas model, Phantom Dark Energy Models etc. came into the picture.

2 Classifying Models

Theroretical approaches began almost immediately to search for the corresponding theoretical explanation of the dark energy; i.e. in terms of a empirical model. Models were introduced following the observational behaviour and the corresponding data. Classifications of different models are approached in several ways.

Regularly used dark energy models are classified in so many ways [5, 6, 7, 8, 9]. One of them claims to classify the models into five different categories[10]. They are a)Cosmological constant model, b) Dark Energy models with chosen w, the equation of state, c) Chaplygin gas models, d) Holographic dark energy models and e) Modified gravity models. Analysis of Type Ia supernova data shows that current data do not exclude any model classes. However, investigations using various datasets indicate that non-phantom barotropic models with a positive sound speed are excluded at a 95% confidence level.

Ten typical and popular dark energy models are considered and compared based on their ability to fit current observational data[11]. The classification of dark energy models includes Cosmological constant model[12],Constant equation of state model ,Chevalliear–Polarski–Linder model [13, 14], Generalized Chaplygin gas model[15], New generalized Chaplygin gas model[16], Holographic dark energy model[17], New agegraphic dark energy model[18], Ricci dark energy model[19], Dvali- Gabadadze- Porrati model[20] and alpha dark energy model[21].

2.1 Dynamical Dark Energy Models

Dynamical dark energy models can be classified into three classes broadly. They are a) Kinematic models, b) Hydrodynamic models and c) Field-theoretic models [5]. Kinematic models describe the dark energy to be a function of either cosmic time or the scale factor [5]. So if we consider Λ to be the dark energy, one can say $\Lambda = f(a)$ or $\Lambda(t)$, this t is cosmic time. Thus we can write

$$\Lambda = 8\pi G\rho_\Lambda. \tag{6}$$

In case of hydrodynamic models the dark energy is generally described by a bariotropic fluid with stating the equation of state $w = \frac{p}{\rho}$[5]. Connecting this with kinematic models one can write

$$w = \frac{p}{\rho} = -\left(1 + \frac{1}{3}\frac{dln\rho_\Lambda}{dln\rho}\right) \tag{7}$$

The field theoretic models are expressed using phenomenological Lagrangian [5].

2.2 Equation of State parametrized dark energy

When the hydrodynamic models say about a constant equation of state, there are non-constant equation of state models also [6]. In such case the equation of state is given by

$$w_x(a) = w_0 + w_a(1 - a). \tag{8}$$

This is called parametrization of dark energy by a linear eqyuation of state. [13] So depending on the nature of the equation of state (w) also the classification of dark energy can be done [6]. The models are classified as $w \geq -1$ or $w < -1$ [6]. Basically the cosmological constant Λ is assumed to be with $w = -1$. Hence $w \geq -1$ and $w < -1$ are important as we get struck with limitations to confirm cosmological conastant as the probable dark energy model. Quintessence models are designed with $w < -1$ [7]. Quintessence models are based on the concept of time-varying scalar field [8]. The evolution equation of the scalar field models are expressed by

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \tag{9}$$

The term $3H\dot{\phi}$ is a friction term which classifies the scalar models in thawing and freezing models [8]. The scalar models with potential ϕ^n when $n > 0$ and $e^{-\phi}$ are known to be called as thawing models [6, 7]. On the contrary, the freezing models can be noted with ϕ^{-n} or $\phi^{-n}e^{\phi^2}$ type of potential [6, 8].

In the study by Emine Canan Günay Demirel, the classification of dark energy models in higher dimensional FRW models [22] is based on state parameters (r, s) and exponential acceleration considerations. The key points of the classification are as follows:

Cosmological Constant Model: For state parameters r = 1 and s = 0, the study identifies the cosmological constant as a dark energy candidate.

Phantom Energy Model: When state parameters are such that r <1 and s >0, the study classifies Phantom energy as a dark energy candidate.

The research provides insights into how different state parameters can lead to the classification of dark energy models in the context of higher dimensional FRW models.

A very general classification can be done for models of dark energy based on lagrangian and equation of state as

1) Quintessence
2) Chaplygin Gas Models
3) Phantom Dark Energy Models
4) Dark Energy - Dark Matter Interaction Models
5) Modified Gravity Models
6) Dark Energy Models from Quantum Effects
7) Extra-Dimensions Models
8) Holographic Dark Energy Models
9) Oscillating Dark Energy Models
10) Scalar-Tensor Dark Energy Models

3 Models in Brief

3.1 Cosmological Constant

Cosmological Constant model is also known as ΛCDM model. With cosmological constant Einstein equation takes the shape

$$G^{\mu}_{\nu} = R^{\mu}_{\nu} - \frac{1}{2}g^{\mu}_{\nu}R - \Lambda g_{\mu\nu} = -8\pi G T^{\mu}_{\nu}$$

For cosmological constant, the density is equivalent to $\frac{\Lambda}{8\pi G}$. Then the corresponding density parameter is given by $\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{crit}}$, where $\rho_{crit} = \frac{3H^2}{8\pi G}$ [1, 23, 24].

3.2 Quintessence

Due to the fine tuning problem related to Cosmological constant, alternate approaches are thought and the scalar field potential dark energy models or Quintessence models are frontrunners of them. This is dynamic process and with evolving as the universe expands. It can be either attractive or repulsive, depending on the balance between its kinetic and potential energy. The term ‘quintessence’ was coined to express scalar field [25].

The Lagrangian density for quintessence involves a scalar field [26], typically denoted as (ϕ). We denote the kinetic energy term as $X = \frac{1}{2}(\partial_\mu \phi)^2$ and the potential energy term as $V(\phi)$ essence Lagrangian density is given by:

$$\mathcal{L}_Q = X - V(\phi). \tag{10}$$

Here X represents the kinetic energy associated with the scalar field and $V(\phi)$ is the potential energy for the scalar field ϕ. Several forms of the scalar field potentials $V(\phi)$ are used by the authors over the years [27]. Out of them $V_0 e^{-\lambda\phi}$, $m^2\phi^2$, $\lambda\phi^4$, $\dfrac{V_0}{\phi^\alpha}$ etc are very commonly used.

Using the equation of state parameter w, one can predict the evolution of the quintessence field and its contribution to the overall energy density of the universe[28, 29]. But the equation of state alone does not provide specific forms for the potential or dynamics of the quintessence field.

The exponential Potential $V_0 e^{\lambda\phi}$ is associated with [30] $w_q = -1+\frac{1}{3}\frac{\lambda^2}{H^2}$. Similarly, $V_0\phi^{-n}$ is associated with $w_q = -1 + \frac{1}{3}\frac{n}{\phi^2}$ [30]. The other scalar potentials are also related to corresponding equation of state parametrs.

3.3 Chaplygin Gas Model

The Lagrangian density for the Chaplygin gas model is given by:

$$\mathcal{L}_{\mathrm{CG}} = -A\sqrt{1-\theta^2} \tag{11}$$

Here: A is a positive constant. θ represents the velocity of the Chaplygin gas.

Interestingly, the original Chaplygin gas model is equivalent to the Dirac-Born-Infeld description of a Nambu-Goto membrane [31]. However, string theory D-branes introduce additional features like an Abelian gauge field, coupling to the dilaton, and coupling to the Kalb-Ramond antisymmetric tensor field. The equation of state for the Chaplygin gas is given by:

$$p_{\mathrm{CG}} = -\frac{A}{\rho_{\mathrm{CG}}^\alpha} \tag{12}$$

Where: p_{CG} is the pressure. ρ_{CG} is the energy density. α is a parameter in the range $0 \le \alpha \le 1$.

3.4 Phantom dark energy model

In a phantom dark energy model, the Lagrangian density is associated with a scalar field ϕ. The Lagrangian density for the phantom field can be expressed as:

$$\mathcal{L}_{\mathrm{phantom}} = -\frac{1}{2}\dot{\phi}^2 - V(\phi) \tag{13}$$

Here: $\dot{\phi}$ represents the time derivative of the scalar field. $V(\phi)$ is the potential energy associated with the phantom field.

The negative kinetic term in the Lagrangian contributes to the phantom behavior, leading to an equation of state parameter $w_p < -1$ [32].

The equation of state parameter for the phantom dark energy model is given by:

$$w_p = \frac{p_{\mathrm{phantom}}}{\rho_{\mathrm{phantom}}} = -1 - \frac{2\dot{\phi}^2}{3V(\phi)} \tag{14}$$

Where: p_{phantom} is the pressure associated with the phantom field. ρ_{phantom} is the energy density of the phantom field.

3.5 Dark Energy - Dark Matter Interaction Models

In the context of an interacting dark energy model, we consider the gravitational interaction between the matter fields—specifically, between the barotropic fluid (representing dark matter) and the dark energy component. The Lagrangian density for this interacting scenario can be expressed as:

$$\mathcal{L}_{\mathrm{interacting}} = -\frac{1}{2}\dot{\phi}^2 - V(\phi) \tag{15}$$

Here: $\dot{\phi}$ represents the time derivative of the scalar field associated with dark energy. $V(\phi)$ corresponds to the potential energy of the scalar field.

This Lagrangian captures the dynamics of the interacting dark energy model within the framework of spatially homogeneous and isotropic Friedmann-Robertson-Walker space-time [33].

The equation of state (EoS) parameter for dark energy in this interacting scenario is given by:

$$w_{\text{interacting}} = \frac{p_{\text{dark energy}}}{\rho_{\text{dark energy}}} = -1 - \frac{2\dot{\phi}^2}{3V(\phi)} \tag{16}$$

Where: $p_{\text{dark energy}}$ represents the pressure associated with dark energy. $\rho_{\text{dark energy}}$ denotes the energy density of dark energy. The equation of state parameter for this interacting model satisfies $w < -\frac{1}{3}$.

3.6 Modified Gravity Model

In the context of modified gravity theories [34], the Lagrangian density plays a crucial role in describing the gravitational dynamics. Let's consider a specific form of modified gravity known as $f(R, L_m)$ gravity. In this theory, the Lagrangian density is an arbitrary function of the Ricci scalar (R) and the trace of the energy-momentum tensor (T). The Lagrangian density for this model can be expressed as:

$$\mathcal{L}_{\text{modified}} = -gf(R, T) + L_m \tag{17}$$

Here: R represents the Ricci scalar. T denotes the trace of the energy-momentum tensor. L_m stands for the matter Lagrangian density.

This modified Lagrangian captures deviations from standard general relativity and allows for a broader exploration of gravitational effects [35].

The equation of state (EoS) parameter is a fundamental quantity that relates pressure (p) to energy density (ρ). In the context of modified gravity, we can express the EoS parameter as:

$$p = w\rho \tag{18}$$

Where: p represents the isotropic pressure. ρ represents the energy density of the perfect fluid.

Lagrangian Density for modified gravity models involves modifications to the Einstein-Hilbert action, often including additional scalar fields or functions of the Ricci scalar.The Lagrangian density for modified gravity models can vary depending on the specific theory. Here are a few examples:

a. $f(R)$ Gravity: In $f(R)$ gravity, the Lagrangian density is given by:

$$\mathcal{L}_{f(R)} = \sqrt{-g}\,(f(R) + \mathcal{L}_m) \tag{19}$$

Here: $f(R)$ is an arbitrary function of the Ricci scalar R. $\mathcal{L}_m$ represents the matter Lagrangian density.

The $f(R)$ gravity model introduces modifications to the Einstein-Hilbert action, allowing for deviations from standard general relativity.

b. DGP Gravity: For the Dvali-Gabadadze-Porrati (DGP) model, the Lagrangian density includes both the Einstein-Hilbert term and the DGP term:

$$\mathcal{L}_{\text{DGP}} = \sqrt{-g}\left(R - \frac{1}{2}m^2 R_c\right) + \mathcal{L}_m \tag{20}$$

Here: m is a parameter related to the crossover scale. R_c is the critical curvature scale.

The DGP model provides an alternative explanation for cosmic acceleration by introducing a higher-dimensional brane-world scenario [33].

The equation of state parameter relates pressure (p) to energy density (ρ) as $p = w\rho$. In the context of scalar field dark energy models, such as $f(R)$ gravity, the EoS parameter may differ from the standard smooth dark energy case.

3.7 Dark Energy Models from Quantum Effects

Lagrangian Density for dark energy models influenced by quantum effects may include higher-order kinetic terms or quantum potential terms. These models explore the behavior of dark energy at fundamental

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quantum scales. The unified Lagrangian density that combines the behaviors of tachyon, quintessence, and phantom scalar fields can be written as

$$\mathcal{L}_{\text{unified}} = f(\alpha)\left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right) \tag{21}$$

$f(\alpha)$ is a function that accommodates different scenarios and captures the distinct characteristics of these scalar fields. ϕ represents the scalar field. $V(\phi)$ is the potential energy term associated with the scalar fields [25].

The equation of state parameter (w) relates pressure (p) to energy density (ρ):

$$w = \frac{p}{\rho}$$

For dark energy models influenced by quantum effects, the specific form of w depends on the chosen scalar field dynamics and potential.

3.8 Extra-Dimensions Models

In the context of higher-dimensional spacetime, we can modify the Einstein-Hilbert action to incorporate the influence of extra dimensions. The D-dimensional Lagrangian density for gravity (including matter) takes the form:

$$\mathcal{L} = \sqrt{-g}\,(R - 2\Lambda + \mathcal{L}_m) \tag{22}$$

g represents the determinant of the metric tensor $g_{\mu\nu}$. R is the Ricci scalar. Λ denotes the cosmological constant. $\mathcal{L}_m$ is the matter Lagrangian density.[36]. The energy conservation equation for the Chaplygin gas, from which the equation of state can be determined and written as:

$$\frac{d(\rho a^3)}{dt} = -p\frac{d(a^3)}{dt} \tag{23}$$

3.9 Holographic Dark Energy Models

The Holographic Dark Energy (HDE) model arises from applying the holographic principle to the dark energy problem. The energy density of HDE can be expressed as:

$$\rho_{\text{HDE}} = BL^2 d^{-4} \tag{24}$$

where: B is a constant model parameter. L represents the characteristic length scale (such as the future event horizon). d is the number of spatial dimensions (usually $d = 3$ for our 3D universe).

The equation of state parameter (w_{HDE}) describes the relationship between pressure (p_{HDE}) and energy density (ρ_{HDE}):

$$w_{\text{HDE}} = \frac{p_{\text{HDE}}}{\rho_{\text{HDE}}} \tag{25}$$

The specific form of w_{HDE} depends on the chosen characteristic length scale and the underlying theory[17].

3.10 Oscillating Dark Energy Models

For Oscillatory Dark Energy Models the Lagrangian density often includes a potential that varies with time, such as a cosine potential[37]. The general form of the Lagrangian density for such a model can be expressed as:

$$\mathcal{L}_{\text{osc}} = \frac{1}{2}\dot{\phi}^2 - V_0\left[1 - \cos\left(\frac{\phi}{f}\right)\right] \tag{26}$$

where ϕ is the scalar field, V_0 is the amplitude of the potential, and f is the decay constant associated with the field. Researchers have used various forms of Lagrangian density in oscillatory dark energy models. Here are some examples:

Quadratic Potential:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 \tag{27}$$

This is a simple quadratic potential, where ϕ is the scalar field and m^2 is the mass term. This potential is often used to model quintessence, which is a type of dark energy that can drive the acceleration of the universe[9].

3.11 Scalar-Tensor Dark Energy Models
For Scalar-Tensor Dark Energy Models, the Lagrangian density is a bit more complex as it involves coupling between the scalar field and the tensor field of gravity. A simplified version of the Lagrangian density for a scalar-tensor model with a quintessence-like scalar field could be written as:

$$\mathcal{L}_{\rm st} = \frac{1}{2}\left(M_{\rm Pl}^2 R - g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right) - V(\phi)] \tag{28}$$

Here, $M_{\rm Pl}$ is the reduced Planck mass, R is the Ricci scalar, $g^{\mu\nu}$ is the inverse metric tensor, and $V(\phi)$ is the potential of the scalar field ϕ.

These Lagrangian densities form the foundation for analyzing the dynamics of the scalar fields in these models and their impact on the evolution of the universe[38]. The specific form of the potential $V(\phi)$ and the interaction terms can vary based on the model being considered and can lead to different cosmological implications.

4 Conclusion
Different classification schemes of the dark energy models are discussed in this article depending on the parameters and the concept of the dark energy models. Close observation of these models show the importance of equation of state parameter. Continuing with this one can also express the physical significance also. Here the classification is done with analysis of the properties and they are in agreement with regularly used models.

References
[1] E. J. Copeland et. al., *Int. J. Mod. Phys. D*, **15**, 1753 (2006)

[2] S. M. Caroll, *Living Rev. Rel.* **4**, 1 (2001).

[3] A. G. Riess et al., *Astron. J* **116**, 1009 (1998)

[4] S. Perlmutter et al., *Astrophys. J.* **517**, 565 (1999)

[5] V. Sahni, *Chaos, Solitons and Factals*, **16**, 527 (2003)

[6] V. Barger et. al.,*Physics Letters B* **635**, 61 (2006)

[7] R. R. Cadwell and E. V. Linder, *Phys. Rev. Lett.* **95**, 141301 (2005)

[8] J. Yoo and W. Watanabe, *Int. J. Mod. Phys. D* **21**, 1230002 (2012)

[9] M. Sami, *Models of Dark Energy*

[10] S. Wen, S Wang and X. Luo, *Journal of Cosmology and Astroparticle Physics* **7**,011(2018)

[11] X. Zhang, Xu Yao Yue, *European Physical Journal C***76**, 588(2016)

[12] O. Avsajanishvili et. al,*Universe* **10**, 122 (2024)

[13] M. Chevallier and D. Polarski, *Int. J. Mod. Phys. D***10** 213 (2001)

[14] E. V. Linder, *Phys. Rev. D***73**, 063010(2006)

[15] M. C. Bento, O. Bertolami, and A. Sen, *Phys. Rev. D***66** 043507(2002)

[16] X. Zhang, F. Q. Wu, and J. Zhang, *JCAP* 0601 003 (2006)

[17] M. Li, *Phys. Lett. B***603** 1 (2004)

[18] H. Wei, R.G. Cai, *Phys.Lett. B***660** 113 (2008)

[19] C. Gao, X. Chen, Y.G. Shen , *Phys. Rev. D***79** 043511 (2009)

[20] R. G. Dvali, G. Gabadadze, and M. Porrati, *Phys. Lett. B* 485 208 (2000)

[21] G. Dvali, M.S. Turner, *arXiv:0301510[astro-ph]*

[22] E. C. G Demirel, *International Journal of Engineering Science and Application* **4**, 6(2022)

[23] A. Harvey, *Eur. J. Phys.*, **30**, 877 (2009)

[24] V. Sahni and A. Starobinsky, *Int. J. Mod. Phys. D*, **9**, 373 (2000)

[25] R. R. Caldwell et. al, *Phys. Rev. Lett.* **80** (1998)

[26] E. V. Linder, *Phys.Rev. Lett.* **90** 091301 (2003)

[27] V. Sahni, *Class. Quantum Grav.*, **19**, 3435 (2002)

[28] R. R. Caldwell, R. Dave, and P.J. Steinhardt, *Physical Review Letters* **80**(8), 1582–1585(1998)

[29] J. C. Liu, *Preprint* rs.3.rs-69462/v1

[30] J. C. H. Liu, *Eur. Phys. J. C* (2022) 82:165

[31] N. Bili , G. B. Tupper and D. V. Raoul, *arxiv.org/pdf/gr-qc/0610104v2.pdf*

[32] A. Bouali et. al, *arXiv:1905.07304 [astro-ph]*

[33] R. K. Mishra et. al., *Elsevier* **40**,101211(2023)

[34] S. Tsujikawa, *Modified Gravity Models of Dark Energy. In: Wolschin, G. (eds) Lectures on Cosmology. Lecture Notes in Physics*, **800**, (2010)

[35] A. Shukla, R. Raushan and R. Chaubey, *arXiv:2308.06519v2(2024)*

[36] J. M. Overduin and P. S. Wesson, *Physics Reports* 283(5–6), 303–380(1998)

[37] Albin Joseph , Rajib Saha, *MNRAS* 511, 1637–1646 (2022)

[38] Xu Tengpeng et. atl., *arxiv:astro-ph*/2109.02453v3