DISCUSSION

MARSHAK: You know of course that, if you introduce the B-field to give you a sufficient coupling to construct the baryons from the leptons, then the weak decays are much too strong. How do you look at this now?

MAKI: As far as we described B^+ as a kind of boson field, we meet with the difficulty you just mentioned. We think rather that B^+ cannot be described by a conventional quantized field.

MARSHAK: The second point is connected with the baryon-lepton symmetry principle. As you realise, I would like very much to maintain this principle with 2 neutrinos, but I don't see how your attempt really helps.

FEINBERG: If you accept the evidence for $\Delta S = -\Delta Q$, on the basis of the experiment of Fry *et al.*, then one has an argument that the decay $K^0 \rightarrow \pi^+ + e^- + \nu$ must involve the same neutrino as the decay $\overline{K^0} \rightarrow \pi^+ + e^- + \bar{\nu}$, because otherwise one would not get the interference phenomenon that actually is found in the experiment. Therefore I think if you are proposing that it is a muon-neutrino that occurs in $\Delta S = -\Delta Q$ decays with an electron, and an electron-neutrino which occurs with an electron in the $\Delta S = +\Delta Q$ decays, then in fact you do not explain Fry's experiment.

THIRRING: It seems to me that the second neutrino does not generate a new difficulty for the correspondence of leptons and baryons. It rather removes one difficulty, because up to now, so to speak, the neutrino was only half of a Dirac particle. Now, since the second half is found, it seems convenient to combine them and you have a correspondence of three particles to three particles.

MARSHAK: This was my first thought as soon as I heard about the two neutrinos. But the trouble is that you have to worry about the conservation of lepton number and also retain positive chiralties for the two neutrinos. If you try some scheme like that of Iso, and take a four component neutrino and let the electron neutrino be the positive chirality part, and the neutrino be the charged conjugate of the negative chirality part, then you have to associate the second neutrino with μ^+. Then, if you associate μ^+ with Λ, you have forsaken the baryon-lepton symmetry principle.

MARX: You must have interference effects in the neutrino absorption experiments if you have two masses but if the oscillation is very short, you have no possibility to distinguish the two neutrinos; so you can have some experimental possibility to get a limit for that mass difference.

WEINBERG: The upper limit on the mass difference comes out to be something like 3 Volts though. Also, the $\mu \rightarrow e + \nu$ experiment would be another place to derive another upper limit, because the existence of a matrix element between ν_e and ν_μ would give a non-zero decay rate.

YAMAGUCHI: Yes, I agree. According to the Nagoya Group, the mass difference is something like 1 eV. As for the second point it is true, that the decay $\mu \rightarrow e + \nu$ exists, but you can manage your theory so that this is sufficiently slow, not to disagree with experiment.

SUDARSHAN: While we are all looking for different particles, is there a second kind of muon? In other words is the K-meson muon the same as the π-meson muon?

FEINBERG: I believe the answer to the question is that there is only one muon. The argument is the following: there exist experiments on producing muon pairs by photons which are in agreement with the Bethe-Heitler formula by a few percent. If the K-muon and the π-muon were different then the total cross-section would be twice as great since the cross section for producing each type depends only on its charge, and so is given by the Bethe-Heitler formula and the total cross section would be the sum of the two.

GAUGE INVARIANCE AND VECTOR FIELDS

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(presented by A. N. Tavkhelidze)

Recently the possibilities of constructing theories of elementary particles by analogy with electrodynamics have been extensively discussed. The notion of gauge invariance plays an important role in such theories (papers of Yang and Mills, Utiyama, Sakurai, Salam and Ward, Gell-Mann and Glashow and others [1-22]). The characteristic feature of gauge theories is a deep parallel in treatment of, for example, baryon and electric charges.

The present report is devoted to the discussion of gauge invariance (G.I. below) and some other problems, concerning electrodynamics and similar theories.

1. G.I. cannot be a principle for deducing the existence of vector fields

The gauge invariance renders harmless some degree of freedom of vector fields making the corresponding components completely (or in a restricted sense) arbitrary and having no dynamical manifestations. In electrodynamics, because of the second kind G.I.

$$A_\mu(x) \rightarrow A_\mu(x) + \partial_\mu \Lambda(x) \qquad (1)$$

the four-vector potential A_μ is determined up to the four-gradient of an arbitrary function only. This means that A_μ has an unessential componnet for which there is no equation of motion. To secure the form invariance of the Lagrangian density transformation (1) is accompanied by the first kind gauge transformation of charged fields

$$\psi(x) \rightarrow \exp\left[ie\Lambda(x)\right]\psi(x) \qquad (2)$$

Transformation (2) ensures the independence of charged field phases at different space—time points (" locality ").

There is a tendency to raise the requirement of the first kind G.I. to the level of principle from which it follows with necessity the existence of vector fields [1-14, 16-22] (and also other fields [23-27]). This statement is not correct. In fact, transformation (2) is allowable in all theories with the conserved current j_μ (i.e. when there is invariance under the transformation (2) with $\Lambda = \text{const}$) provided that the equal-time commutator of j_μ and j_μ vanishes. We note the following examples of such theories:

a) The theory of free charged fields. In this theory transformation (2) adds to the Lagrangian the term $j_\mu \partial_\mu \Lambda$. However, it is well known that the new Lagrangian is equivalent to the original one since it has the same physical consequences.

b) The same theory of free fields can be made form-invariant by means of introducing the gradient coupling $j_\mu \partial_\mu B$ with the scalar field $B(x)$ and of prescribing to the latter the law of gauge transformation $B(x) \rightarrow B(x) + \Lambda(x)$ [15].

c) Finally, electrodynamics with Lagrangian $j_\mu A_\mu$ and law of transformation (1) for A_μ.

An apparent advantage of c) compared to b) is a possibility of writing down the equation of motion for the introduced field. However, the component of the vector-potential A_μ which depends on the gauge cannot have equation of motion as well as the field B in b). The components for which the electrodynamical Lagrangian yields equations of motion (Maxwell equation) bear no relation to G.I. From this point of view they are not necessarily [15] contrary to the widespread belief [1-14, 16-22].

A similar conclusion can be made about more general G.I. also [15]. So, the necessity of introduction of vector fields from the requirement of Yang-Mills isotopic G.I. is disproved by the following example: the Lagrangian

$$\begin{aligned} L = & -\bar{\psi}(\gamma_\mu \partial_\mu + m)\psi \\ & -\bar{\psi} \exp(ig\tau_a b_a)\gamma_\mu\left[\partial_\mu \exp(-ig\tau_a b_a)\right]\psi \qquad (3) \end{aligned}$$

is invariant under " local " isotopic transformations $\psi'(x) = \exp\left[ig\tau_a \lambda_a(x)\right]\psi(x)$ if scalar fields $b_a (a = 1,2,3)$ are transformed according to the law

$$\exp\left[ig\tau_a b_a'(x)\right] = \exp\left[ig\tau_a \lambda_a(x)\right] \exp\left[ig\tau_a b_a(x)\right] \qquad (4)$$

The introduced field $b_a(x)$ has three components in accordance with the number of gauge functions $\lambda_a(x)$ and does not lead, as is easily seen, to any dynamical manifestations.

Similar counter-examples can be formulated for a general class of gauge-invariant theories as well [13, 14].

The law of transformation (4) for λ_a independent of x has been considered by Gursey [28] and means that the field b_a is transformed according to the non-linear representation of the three-dimensional rotation group. New isoscalars, e.g.

$$\bar{N} \exp(ig\tau_a b_a), \quad \bar{N}(\tau\pi) \exp(ig\tau_a b_a)$$

and other tensor quantities can be formed with this field. These isoscalars are two-component, each component is isoscalar. Components with one or another charge can be singled out by multiplying by non-transformable matrices such as $\binom{1}{0}$ or $\binom{0}{1}$. It is interesting to note that a suprising rule of violation of isotopic spin in decays of strange particles $\Delta T = 1/2$ may be interpreted by means of the field b_a as a consequence of strict isotopic invariance [28]. Curiously, electromagnetic and weak interactions can be made strictly isotopic invariant also by means of this trick.

2. G.I. theories and the spin on vector field

We return to discussion of G.I. Thus, it is impossible to prove the necessity for introducing vector fields. However, G.I. theories of interaction with an a priori postulated vector field can be understood as the condition for the vector field to describe quanta with spin 1 only, since G.I. makes harmless quanta with zero spin. For example, in Yang and Mills theory a spin 1 for the i-th isocomponent of the vector field is assured by the invariance under rotation by the angle $Ai(x)$ around the i-th axis of isospace. Gell-Mann and Glashow [13] (see also [14]) have pointed out a wide class of theories possessing G.I. for zero mass vector fields to which belongs in particular electrodynamics and Yang and Mills theory. The inclusion of mass terms violates G.I. However, the above theories have an important general property: the vector field describes spin 1 both for zero and non-zero mass. For zero mass this is guaranteed by G.I. and for non-zero by the vanishing of the four-divergence of vector fields (as the consequence of the equation of motion). Thus the interacting vector field in real and virtual states obeys the same supplementary condition which excludes spin 0 as in the case of free field. This is a distinctive feature of all generalized Yang-Mills type theories. In other vector field theories the supplementary condition has a different form for free and interacting fields [29].

3. G.I. theory of a massive neutral vector field

It is possible to formulate in a G.I. manner a theory of massive neutral vector field [15] coupled, e.g. with baryon or hyperon current. This example shows especially clearly that G.I. plays a role similar to that of the supplementary condition $\partial_\mu A_\mu = 0$ in conventional approach. Usually A_μ obeys the equation

$$\Box A_\mu - \partial_\mu \partial_\nu A_\nu - M^2 A_\mu = -j_\mu \tag{5}$$

from which follows the supplementary condition $\partial_\nu A_\nu = 0$ which means that the zero spin component vanishes.

The G.I. formulation [15] is based on the equation

$$(\Box - M^2) A_\mu = -j_\mu \tag{6}$$

which is invariant under the transformation (1) with an evident restriction $(\Box - M^2)\partial_\mu \Lambda = 0$. As a consequence of such G.I. the component with zero spin is found not to be completely arbitrary but obeys the free equation. But it is already sufficient to ensure the diagonality of the S-matrix in the quantum numbers which characterize states of zero spin quanta. So, these quanta do not lead to physical effects. Both formulations are equivalent, they can be reduced to each other and describe interaction of quanta with spin 1 only. Very recently, Schwinger [20] has suggested a hypothesis according to which in a G.I. theory with zero bare rest mass the renormalized mass spectrum can, generally speaking, have a sharp maximum at $M \neq 0$. This problem is important but it requires investigation (regardless of G.I.). We have seen above however that G.I. bears no direct relation to the mass of quanta with spin 1 and from the very beginning it is possible to introduce a non-zero rest mass of vector field.

4. Quantum electrodynamics in terms of field strengths

An unessential component with zero spin causes much trouble in quantum electrodynamics. So, commutation relations for vector potential A_μ can not be given in theory, which is based on Maxwell equations for A_μ. In a conventional approach one restricts the gauge in some degree and obtains, in that way, the equations of motion which determine A_μ more strictly. Because of this such equations permit the quantization. The disadvantages of such formulations are widely known. Therefore a series of attempts have been undertaken to overcome difficulties of quantization of Maxwell equations by fixing commutation relations not for A_μ, but only for gauge independent quantitites [30-35], e.g. for field strengths. However, in such theories either the vector-potential is not completely excluded and the operation with it is difficult, or an explicit Lorentz-invariance is absent.

Recently two different formulations of electrodynamics have been given in which the potentials are excluded completely. One of them has been suggested by De Witt [36] another one by us [37]. In this connection, the statement of Aharonov and Bohm [38] that the potential in quantum theory has an independent significance unlike classical theory, seems to be erroneous.

Our covariant formulation in terms of field strengths is based on the exclusion of a gauge dependent component of A by the gauge transformation

$\psi \rightarrow \exp\left[ie\Box^{-1}\partial_\nu A_\nu\right]\psi$. After this transformation has been made the electrodynamical equations take the form:

$$\begin{gathered}\partial_\mu F_{\mu\nu} = -j_\nu \qquad \partial_\mu \check{F}_{\mu\nu} = 0 \qquad (\check{F}_{\mu\nu} = \tfrac{1}{2}\varepsilon_{\mu\nu\lambda\rho}\, F_{\lambda\rho}) \\ (\gamma_\mu\partial_\mu + m)\psi = ie\gamma_\nu\psi\partial_\mu\Box^{-1}F_{\mu\nu}\end{gathered} \tag{7}$$

and the interaction Lagrangian is $L_I = j_\nu\partial_\mu\Box^{-1}F_{\mu\nu}$. The free Maxwell equations for the field strengths $F_{\mu\nu}$ can be quantized. The corresponding commutation relations for $F_{\mu\nu}$ have been well-known for a long time. Therefore, electrodynamics can be formulated in the interaction representation with the interaction Lagrangian L_I. To virtual photon lines there will correspond the contraction

$$\begin{aligned}&\frac{\partial}{\partial x_\mu}\Box_x^{-1}\overbracket{F_{\mu\nu}(x)\frac{\partial}{\partial y_\lambda}\Box_y^{-1}F}_{\lambda\rho}(y) = \\ &\qquad = \left(\delta_{\nu\rho} + \frac{\partial}{\partial x_\nu}\,\frac{\partial}{\partial y_\rho}\Box^{-1}\right)(-i)\Delta^c(x-y)\end{aligned} \tag{8}$$

The photon states are given by the action on vacuum of the negative-frequency part of $F_{\mu\nu}$. Thus in quantum electrodynamics one succeeds in using only **E** and **H** without using potentials.

The interaction L_I possesses a seeming non-locality and can be written in many equivalent forms since as operator $\Box^{-1}$ one can take an integral operator whose kernel is any Greens function of D'Alembert's operator. However, this ambiguity is related not to an operator describing photons but to the choice of one of the equivalent forms of writing down the S-matrix in terms of these operators. As for the matrix elements they can be unambiguously represented in the following visual form

$$\langle f|\rho|i\rangle \sim \bar{u}\ldots\gamma_{\mu_1}\ldots\gamma_{\mu_n}\ldots uF_{\mu_1 u}(\mathbf{q}_1\,,S_1)\ldots F_{\mu_n u}(\mathbf{q}_n\,,S_n) \tag{9}$$

where $F_{mu} = iE_m$ are c-number solutions of Maxwell's equations and indices $S_k(S_k = 1, 2)$ characterize the spin state of the photon with momentum $\mathbf{q}_k$.

From this formulation a usual calculation scheme may be obtained.

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DISCUSSION

Bludman: I would just like to comment that perhaps the role of gauge invariance has recently been overstressed, and that we have seen recently that gauge invariance need not imply masslessness of the electromagnetic field. My interpretation of Schwinger's recent paper in the Physical Review on mass and gauge invariance is that if photons are non-elementary particles to begin with and do not appear in the Lagrangian, then there is certainly no question of gauge transformations on the photon field. Then gauge invariance on the electromagnetic potential plays no role at all, and implies nothing concerning the mass of the photon.

Ne'eman: Just to add that for the other vector mesons besides the electromagnetic field, we also get too strong conditions through gauge invariance because all couplings would then be F couplings, in the case of a gauge, whereas we have just seen that it is a mixture of F and D in the case of SU_3, for instance.

Feinberg: Since we now agree that gauge invariance has nothing to do with zero mass, perhaps someone can answer why the photon *does* have zero mass?

Yamaguchi: It is a " miserable " experimental fact.

Bludman: How can you know that it really does have zero mass?

Feinberg: The experimental limit on the photon mass is 10^{-49} g. Historically in physics whenever one has had a thing very close to zero, it has usually actually been zero. Therefore I would say that until you do an experiment to find the photon mass it is safe enough to ask the question I did.

ρ-MESONS AND THE YANG-MILLS FIELD

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(presented by J. C. Taylor)

Ever since the formulation by Yang, Mills $^{1)}$ and Shaw $^{2)}$ of a field permitting local isotopic spin rotations, it has been widely held to be in doubt $^{1, 3, 4)}$ whether such a field must have zero mass quanta or not. Schwinger $^{5)}$ has even questioned whether there is any kinematical argument for the masslessness of the photon.

This question has gained importance since the discovery of the ρ-meson, because many people have speculated whether this might not be connected with the Yang-Mills-Shaw field $^{3, 6)}$.

It is the purpose of this note to point out a simple argument, which seems to have been overlooked, that the Yang-Mills-Shaw field (or the electromagnetic