TORSION RELATIONS IN FINSLER SPACETIME TANGENT BUNDLE

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Abstract. A set of algebraic relations involving the bundle torsion, gauge curvature field, and four-velocity in the Finsler-spacetime tangent bundle is presented that maintains (1) compatibility with Cartan's theory of Finsler space, (2) the almost complex structure, and (3) the vanishing of the covariant derivative of the almost complex structure. This avoids the much more restrictive condition of vanishing gauge curvature field. A simple solution to the torsion relations is also obtained.

1. Introduction

It was demonstrated recently that the spacetime tangent bundle of a Finsler spacetime [1, 2] is almost complex, and also Kählerian [1, 3] and complex [4] with vanishing covariant derivative of the almost complex structure, provided that the gauge curvature field is vanishing. A vanishing gauge curvature field is equivalent to the condition that the four-velocity tangent-space coordinate be a parallel vector field. The vanishing of the gauge curvature field was also shown to be a sufficient condition for the bundle connection to have a form consistent with Cartan's theory of Finsler space [1, 2]. However, through the introduction of bundle torsion satisfying prescribed conditions, the Finsler-spacetime tangent bundle can be made to remain consistent with Cartan's theory of Finsler space, and remain almost complex with a vanishing covariant derivative of the almost complex structure, without the need to impose the relatively restrictive condition of vanishing gauge curvature field [5]. However, a nonvanishing gauge curvature field precludes that the bundle be complex [5]. A number of implied relations involving the torsion, gauge curvature field, and four-velocity can be demonstrated.

In the present work, we first review the basis for the torsion relations and then obtain a simple solution, in which the only nonvanishing component of the torsion is in the fiber-base-base sector of the bundle, and is given by the negative of the gauge curvature field.

2. Finsler-Spacetime tangent bundle with torsion.

The components of the bundle connection ${}^{(8)}\Gamma^M{}_{AB}$ of the Finsler-spacetime tangent bundle, including bundle torsion, and written in an anholonomic basis adapted to the spacetime connection, are given by [5]

$$ {}^{(8)}\Gamma^\mu{}_{\alpha\beta} = \overline{\left\{ {}^{\mu}{}_{\alpha\beta} \right\}} + {}^{(8)}K^\mu{}_{\alpha\beta}, \tag{1} $$

$$ {}^{(8)}\Gamma^\mu{}_{\alpha b} = \Pi^\mu{}_{\alpha b} + \frac{1}{2} F_{b\alpha}{}^\mu + {}^{(8)}K^\mu{}_{\alpha b}, \tag{2} $$

$$ {}^{(8)}\Gamma^\mu{}_{b\alpha} = \Pi^\mu{}_{\alpha b} + \frac{1}{2} F_{b\alpha}{}^\mu + {}^{(8)}K^\mu{}_{b\alpha}, \tag{3} $$

$$ {}^{(8)}\Gamma^\mu{}_{ab} = \rho_0 v^\lambda \frac{\overline{D}}{Dx^\lambda} \Pi_{ab}{}^\mu + {}^{(8)}K^\mu{}_{ab}, \tag{4} $$

$$ {}^{(8)}\Gamma^m{}_{\alpha\beta} = -\Pi_{\alpha\beta}{}^m + \frac{1}{2} F^m{}_{\alpha\beta} + {}^{(8)}K^m{}_{\alpha\beta}, \tag{5} $$

$$ {}^{(8)}\Gamma^m{}_{\alpha b} = -\rho_0 v^\lambda \frac{\overline{D}}{Dx^\lambda} \Pi_b{}^m{}_\alpha + {}^{(8)}K^m{}_{\alpha b}, \tag{6} $$

$$ {}^{(8)}\Gamma^m{}_{b\alpha} = \overline{\left\{ {}^{m}{}_{b\alpha} \right\}} + {}^{(8)}K^m{}_{b\alpha}, \tag{7} $$

$$ {}^{(8)}\Gamma^m{}_{ab} = \Pi^m{}_{ab} + {}^{(8)}K^m{}_{ab}. \tag{8} $$

Here recall that a generic point in the bundle manifold has coordinates $\{\chi^M; M = 0,1,...,7\} = \{\chi^\mu, \chi^m; \mu = 0,1,2,3; m = 4,5,6,7\} \equiv \{\chi^\mu, \rho_0 v^\mu; \mu = 0,1,2,3\}$, where χ^μ and v^μ are the spacetime and four-velocity coordinates, respectively. Greek indices refer to spacetime and range from 0 to 3; lower case Latin indices refer to four-velocity space and range from 4 to 7; and upper case Latin indices refer to a point in the bundle and range from 0 to 7. Any lower case Latin index n appearing in a canonical spacetime tensor or connection is defined to be $n-4$ implicitly. The length ρ_0 is of the order of the Planck length [6]. Also in the above equations, there appears the spacetime connection

$$ \Gamma^\mu{}_{\alpha\beta} = \overline{\left\{ {}^{\mu}{}_{\alpha\beta} \right\}} = \left\{ {}^{\mu}{}_{\alpha\beta} \right\} - A^\lambda{}_\alpha \Pi_{\lambda\beta}{}^\mu - A^\lambda{}_\beta \Pi_{\lambda\alpha}{}^\mu - A^{\lambda\mu} \Pi_{\lambda\alpha\beta}, \tag{9} $$

in which the gauge potential is given by

$$A^\mu{}_\nu = \rho_0 v^\lambda \Gamma^\mu{}_{\lambda\nu} = \rho_0 v^\lambda \left\{ {}^{\mu}{}_{\lambda\nu} \right\} - \rho_0^2 v^\alpha v^\beta \Pi^\mu{}_{\nu\lambda} \left\{ {}^{\lambda}{}_{\alpha\beta} \right\}, \tag{10}$$

where $\left\{ {}^{\mu}{}_{\alpha\beta} \right\}$ is the ordinary Christoffel symbol

$$\left\{ {}^{\mu}{}_{\alpha\beta} \right\} = \frac{1}{2} g^{\mu\nu} \Big(\frac{\partial}{\partial x^\beta} g_{\nu\alpha} + \frac{\partial}{\partial x^\alpha} g_{\nu\beta} - \frac{\partial}{\partial x^\nu} g_{\alpha\beta} \Big), \tag{11}$$

and $g_{\mu\nu}$ is the spacetime metric tensor. Also, the Christoffel symbol of four-velocity space is given by

$$\Pi^\mu{}_{\alpha\beta} = \frac{1}{2} \rho_0^{-1} g^{\mu\lambda} \frac{\partial}{\partial v^\lambda} g_{\alpha\beta}. \tag{12}$$

Also in the above, ${}^{(8)}K^M{}_{AB}$ is the bundle contorsion

$${}^{(8)}K^M{}_{AB} = \frac{1}{2} \Big(G^{ML} G_{AD} {}^{(8)}\bar{T}^D{}_{BL} + G^{ML} G_{BD} {}^{(8)}\bar{T}^D{}_{AL} + {}^{(8)}\bar{T}^M{}_{AB} \Big), \tag{13}$$

where ${}^{(8)}\bar{T}^M{}_{AB}$ is the bundle torsion, and the bundle metric is given by

$$G_{MN} = \begin{pmatrix} g_{\mu\nu} & 0 \\ 0 & g_{mn} \end{pmatrix} \tag{14}$$

in the adapted anholonomic basis. Also in the above, the gauge curvature field is given by

$$F^\mu{}_{\alpha\beta} = \rho_0 v^\lambda \bar{R}^\mu{}_{\lambda\alpha\beta}, \tag{15}$$

where

$$\bar{R}^\mu{}_{\lambda\alpha\beta} = \Gamma^\mu{}_{\lambda\beta,\alpha} - \Gamma^\mu{}_{\lambda\alpha,\beta} + \Gamma^\mu{}_{\gamma\alpha}\Gamma^\gamma{}_{\lambda\beta} - \Gamma^\mu{}_{\gamma\beta}\Gamma^\gamma{}_{\lambda\alpha} \tag{16}$$

is the spacetime Riemann curvature tensor, written in the adapted basis. Here, the comma followed by a lower case Greek index denotes the operator ${}_{,\nu} \equiv \partial/\partial x^\nu - \rho_0^{-1} A^\beta{}_\nu \partial/\partial v^\beta$, corresponding to the adapted basis. Also in the above $\bar{D}/Dx^\lambda$ denotes the ordinary spacetime convariant derivative with the spacetime connection Eq. (9). The anholonomic basis vectors are defined by

$$\{E_M\} \equiv \{E_\mu, E_m\} \equiv \Big\{ \frac{\partial}{\partial x^\mu} - \rho_0^{-1} A^\beta{}_\mu \frac{\partial}{\partial v^\beta}, \rho_0^{-1} \frac{\partial}{\partial v^\mu} \Big\}. \tag{17}$$

The associated structure coefficients $C_{AB}{}^M$ are defined by the commutator

$$[E_A, E_B] = C_{AB}{}^M E_M, \tag{18}$$

and the only nonvanishing components are

$$C_{\alpha\beta}{}^{m} = -F^{m}{}_{\alpha\beta}, \tag{19}$$

$$C_{\alpha b}{}^{m} = -C_{b\alpha}{}^{m} = \phi^{m}{}_{\alpha b}, \tag{20}$$

where

$$\phi^{\mu}{}_{\alpha\beta} = \rho_0{}^{-1}\frac{\partial}{\partial v^{\beta}}A^{\mu}{}_{\alpha}. \tag{21}$$

In Cartan's theory of Finsler geometry, involving the base manifold only, the connection coefficients are those corresponding to Eqs. (1) and (2). Those of Eq. (1) are identical to one set of connection coefficients appearing in Cartan's theory, provided

$$^{(8)}K^{\mu}{}_{\alpha\beta} = 0. \tag{22}$$

Those appearing in Eq. (2) are identical to the remaining set of connection coefficients of Cartan's theory, only if

$$^{(8)}K^{\mu}{}_{\alpha b} = -\frac{1}{2}F_{b\alpha}{}^{\mu}. \tag{23}$$

If the bundle torsion is not present, then the contorsion is vanishing, and Eq. (23) then requires that the gauge curvature field be vanishing, but a nonvanishing torsion circumvents the latter more severe restriction. From Eqs. (23) and (13) and the antisymmetry of the torsion, it follows that

$$^{(8)}\bar{T}^{\mu}{}_{b\alpha} = -{}^{(8)}\bar{T}^{\mu}{}_{\alpha b} = \frac{1}{2}F_{b\alpha}{}^{\mu} + {}^{(8)}K^{\mu}{}_{b\alpha}. \tag{24}$$

Next define the antisymmetric part of the bundle connection by

$$^{(8)}\Gamma^{M}{}_{\underset{V}{AB}} = -{}^{(8)}\Gamma^{M}{}_{\underset{V}{BA}} \equiv \frac{1}{2}{}^{(8)}\Gamma^{M}{}_{[AB]} = \frac{1}{2}\left(-C_{AB}{}^{M} + {}^{(8)}\bar{T}^{M}{}_{AB}\right). \tag{25}$$

Throughout, we employ the notation $T^{\cdot\cdot}_{\cdot\cdot[\mu..\nu]..} \equiv T^{\cdot\cdot}_{\cdot\cdot\mu..\nu..} - T^{\cdot\cdot}_{\cdot\cdot\nu..\mu..}$. According to Eqs. (13) and (22) and the antisymmetry of the torsion, one also has

$$^{(8)}\bar{T}^{\mu}{}_{\alpha\beta} = {}^{(8)}K^{\mu}{}_{[\alpha\beta]} = 0. \tag{26}$$

Then using Eqs. (25), (18), and (26), we obtain

$$^{(8)}\Gamma^{\mu}{}_{\underset{V}{\alpha\beta}} = 0. \tag{27}$$

Next, if we use Eqs. (25), (18), and (24), we deduce that

$$ {}^{(8)}\underset{V}{\Gamma^{\mu}{}_{\alpha b}} = \frac{1}{2}{}^{(8)}\bar{T}^{\mu}{}_{\alpha b} = -\frac{1}{4}F_{b\alpha}{}^{\mu} - \frac{1}{2}{}^{(8)}K^{\mu}{}_{b\alpha}, \tag{28} $$

and

$$ {}^{(8)}\underset{V}{\Gamma^{\mu}{}_{b\alpha}} = \frac{1}{2}{}^{(8)}\bar{T}^{\mu}{}_{b\alpha} = \frac{1}{4}F_{b\alpha}{}^{\mu} + \frac{1}{2}{}^{(8)}K^{\mu}{}_{b\alpha}. \tag{29} $$

Also, according to Eqs. (25) and (18), one has

$$ {}^{(8)}\underset{V}{\Gamma^{\mu}{}_{ab}} = \frac{1}{2}\bar{T}^{\mu}{}_{ab}. \tag{30} $$

Only the components of the antisymmetric part of the connection Eqs. (27)-(30) are needed explicitly for the considerations that follow.

3. Almost complex structure

The Finsler-spacetime tangent bundle is almost complex, and in the anholonomic basis adapted to the spacetime connection, the almost complex structure is given by [1, 3]

$$ J_{AB} = \begin{pmatrix} 0 & -g_{\alpha b} \\ g_{a\beta} & 0 \end{pmatrix} \tag{31} $$

in the absence of torsion. In the presence of bundle torsion, the bundle connection has an antisymmetric part, and the almost complex structure becomes [5]

$$ J_{AB} = \begin{bmatrix} 2\rho_0{}^{(8)}\underset{V}{\Gamma^{\mu}{}_{\alpha\beta}}v_{\mu} & -g_{\alpha b} + 2\rho_0{}^{(8)}\underset{V}{\Gamma^{\mu}{}_{\alpha b}}v_{\mu} \\ g_{a\beta} + 2\rho_0{}^{(8)}\underset{V}{\Gamma^{\mu}{}_{a\beta}}v_{\mu} & 2\rho_0{}^{(8)}\underset{V}{\Gamma^{\mu}{}_{ab}}v_{\mu} \end{bmatrix}. \tag{32} $$

If we use Eqs. (27)-(30) in Eq. (32), and compare with Eq. (31), we conclude that the almost complex structure (Eq. (31)) is preserved in the presence of torsion, if the following conditions are satisfied:

$$ {}^{(8)}K^{\mu}{}_{b\alpha}v_{\mu} = -\frac{1}{2}v^{\mu}F_{b\alpha\mu} \tag{33} $$

and

$$ {}^{(8)}\bar{T}^{\mu}{}_{ab}v_{\mu} = 0. \tag{34} $$

Next, if one expands the convariant derivative of the almost complex structure $\nabla_E J_A{}^B$ by using Eqs. (31), (14), (1)-(8), (22) and (23), together with the corresponding results of [3] one concludes that

$$ \nabla_E J_A{}^B = 0, \tag{35} $$

provided that the following relations involving the bundle torsion are satisfied (including the other relations obtained above):

$$ {}^{(8)}K^{\mu}{}_{\delta\varepsilon} = 0, \quad {}^{(8)}\bar{T}^{\mu}{}_{\delta\varepsilon} = 0, \quad {}^{(8)}K^{\mu}{}_{\delta e} = -\frac{1}{2}F_{e\delta}{}^{\mu}, \qquad (36-38) $$

$$ {}^{(8)}\bar{T}^{\mu}{}_{\delta e} = -\frac{1}{2}F_{e\delta}{}^{\mu} - {}^{(8)}K^{\mu}{}_{e\delta}, \quad {}^{(8)}K^{\mu}{}_{d\varepsilon}P^{d}_{b}{}_{\mu a} = \frac{1}{2}F_{[ab]\varepsilon}, \qquad (39,40) $$

$$ {}^{(8)}K^{\mu}{}_{d\varepsilon}v_{\mu} = -\frac{1}{2}v^{\mu}F_{d\varepsilon\mu}, \quad {}^{(8)}\bar{T}^{\mu}{}_{d\varepsilon} = \frac{1}{2}F_{d\varepsilon}{}^{\mu} + {}^{(8)}K^{\mu}{}_{d\varepsilon}, \qquad (41,42) $$

$$ {}^{(8)}K^{\mu}{}_{de}P^{db}{}_{\mu a} = 0, \quad {}^{(8)}\bar{T}^{\mu}{}_{de}v_{\mu} = 0, \quad {}^{(8)}K^{m}{}_{\delta\varepsilon}P^{\delta}_{\beta}{}_{m\alpha} = \frac{1}{2}F_{[\alpha\beta]\varepsilon}, \qquad (43-45) $$

$$ {}^{(8)}K^{m}{}_{\delta e}P^{\beta\delta}{}_{m\alpha} = 0, \quad {}^{(8)}K^{m}{}_{d\varepsilon} = 0, \quad {}^{(8)}K^{m}{}_{de} = 0, \qquad (46-48) $$

where

$$ P^{\beta\delta}{}_{\mu\alpha} = \delta^{\beta}{}_{\mu}\delta^{\delta}{}_{\alpha} - g^{\beta\delta}g_{\mu\alpha}. \qquad (49) $$

In summary, Eqs. (36)-(39) and (42) insure compatibility of the bundle connection with Cartan's theory of Finsler space; Eqs. (41) and (44) insure that the almost complex structure is maintained; and Eqs. (40), (43), and (45)-(48) insure that the covariant derivative of the almost complex structure is vanishing.

By means of the following identity [5],

$$ v^{\lambda}F_{[\alpha\beta]\lambda} = 0, \qquad (50) $$

together with Eqs. (39)-(42) and (45), the following additional torsion relations can also be demonstrated [5]:

$$ {}^{(8)}\bar{T}^{\mu}{}_{\delta e}v_{\mu} = 0, \quad {}^{(8)}\bar{T}^{\mu}{}_{d\varepsilon}v_{\mu} = 0, \qquad (51,52) $$

$$ {}^{(8)}\bar{T}^{\mu}{}_{\delta e}P^{e}_{b}{}_{\mu a} = 0, \quad {}^{(8)}\bar{T}^{\mu}{}_{d\varepsilon}P^{d}_{b}{}_{\mu a} = 0, \qquad (53,54) $$

$$ {}^{(8)}K^{\mu}{}_{de}v^{\varepsilon}P^{d}_{b}{}_{\mu a} = 0, \quad {}^{(8)}K^{m}{}_{\delta\varepsilon}v^{\varepsilon}P^{\delta}_{\beta}{}_{m\alpha} = 0. \qquad (55,56) $$

4. Simple solution to torsion relations

A general solution to the torsion relations, Eqs. (36)-(48) and (51)-(56), expressing the components of the bundle torsion explicitly in terms of the gauge curvature field and four-velocity, will be addressed elsewhere. Here we instead seek a simple particular solution.

Begin by considering Eq. (45) with the following ansatz

$$^{(8)}K^{m}{}_{\delta\varepsilon} = \kappa F^{m}{}_{\delta\varepsilon}, \tag{57}$$

as part of a possible self-consistent solution, where κ is a constant. If we substitute Eq. (57) in Eq. (45), it follows, that κ = -1/2, and therefore

$$^{(8)}K^{m}{}_{\delta\varepsilon} = -\frac{1}{2}F^{m}{}_{\delta\varepsilon}. \tag{58}$$

Also, Eq. (41) immediately suggests

$$^{(8)}K^{\mu}{}_{d\varepsilon} = -\frac{1}{2}F_{d\varepsilon}{}^{\mu}. \tag{59}$$

Furthermore, in accordance with Eqs. (43) and (46), we can make the simple ansatz

$$^{(8)}K^{\mu}{}_{de} = 0, \quad ^{(8)}K^{m}{}_{\delta e} = 0. \tag{60,61}$$

Thus, the only nonvanishing components of the contorsion are given by Eqs. (38), (58), and (59), which are assembled here:

$$^{(8)}K^{\mu}{}_{d\varepsilon} = ^{(8)}K^{\mu}{}_{\varepsilon d} = -\frac{1}{2}F_{d\varepsilon}{}^{\mu} \tag{62}$$

and

$$^{(8)}K^{m}{}_{\delta\varepsilon} = -\frac{1}{2}F^{m}{}_{\delta\varepsilon}. \tag{63}$$

All other components of the bundle contorsion are taken to be vanishing (Eqs. (36), (60), (61), (47), and (48)), namely,

$$^{(8)}K^{\mu}{}_{\delta\varepsilon} = {}^{(8)}K^{\mu}{}_{de} = {}^{(8)}K^{m}{}_{\delta e} = {}^{(8)}K^{m}{}_{d\varepsilon} = {}^{(8)}K^{m}{}_{de} = 0. \tag{64}$$

Next we can substitute Eq. (62) in Eq. (39) and obtain

$$^{(8)}\bar{T}^{\mu}{}_{\delta e} = 0. \tag{65}$$

Also, if we substitute Eq. (62) in Eq. (42), we get

$$^{(8)}\bar{T}^{\mu}{}_{d\varepsilon} = 0. \tag{66}$$

Furthermore, in accordance with Eq. (44), we also make the simple ansatz

$$^{(8)}\bar{T}^{\mu}{}_{de} = 0. \tag{67}$$

Next, if we substitute Eqs. (64) and (67) in the expression for $^{(8)}K^{m}{}_{\alpha b}$ given by Eq. (13), we obtain

$$(g^{ml} g_{bd} + \delta^{m}{}_{d} \delta^{l}{}_{b})^{(8)}\bar{T}^{d}{}_{\alpha l} = 0. \tag{68}$$

Equation (68) suggests the simple ansatz

$$^{(8)}\bar{T}^{d}{}_{\alpha l} = 0. \tag{69}$$

Next, if we substitute Eqs. (64), (69), and (67) in the expression for $^{(8)}K^{m}{}_{b\alpha}$ given by Eq. (13), we obtain directly,

$$^{(8)}\bar{T}^{m}{}_{b\alpha} = 0. \tag{70}$$

Furthermore, if we substitute Eq. (64) in the expression for $^{(8)}K^{m}{}_{ab}$ given by Eq. (13), we obtain

$$\left(g^{ml} g_{ad} \delta^{n}{}_{b} + g^{ml} g_{bd} \delta^{n}{}_{a} + \delta^{m}{}_{d} \delta^{n}{}_{a} \delta^{l}{}_{b}\right)^{(8)} \bar{T}^{d}{}_{nl} = 0. \tag{71}$$

Equation (71) suggests the simple ansatz

$$^{(8)}\bar{T}^{d}{}_{nl} = 0. \tag{72}$$

Finally, if we substitute Eqs. (65) and (63) in the expression for $^{(8)}K^{m}{}_{\alpha\beta}$ given by Eq. (13), we get

$$^{(8)}\bar{T}^{m}{}_{\alpha\beta} = -F^{m}{}_{\alpha\beta}. \tag{73}$$

In summary, the only nonvanishing component of the torsion is in the fiber-base-base sector, and is given by Eq. (73). All other components of the bundle torsion are vanishing (Eqs. (37), (65)-(67), (69), (70), and (72)), namely,

$$^{(8)}\bar{T}^{\mu}{}_{\delta\varepsilon} = {}^{(8)}\bar{T}^{\mu}{}_{\delta e} = {}^{(8)}\bar{T}^{\mu}{}_{d\varepsilon} = {}^{(8)}\bar{T}^{\mu}{}_{de} = {}^{(8)}\bar{T}^{m}{}_{\delta e} = {}^{(8)}\bar{T}^{m}{}_{d\varepsilon} = {}^{(8)}\bar{T}^{m}{}_{de} = 0. \tag{74}$$

Equations (36), (37), (43), (44), (46)-(48), and (51)-(54) are trivially satisfied by Eqs. (74) and (64). Equations (39) and (42) are satisfied by Eqs. (74) and (62). Equation (40) is satisfied by Eq. (62) together with Eq. (49). Equation (45) is satisfied by Eq. (63) together with Eq. (49). Equation (55) is satisfied by Eq. (62) together with Eq. (50). Equations (38) and (41) are satisfied by Eq. (62). And finally, Eq. (56) is satisfied by Eq. (63) together with Eq. (50). Thus, all of the torsion relations are satisfied by the simple solution given by Eqs. (73) and (74).

5. Conclusion

The Finsler-spacetime tangent bundle with bundle torsion is compatible with Cartan's theory of Finsler space, and is almost complex with a vanishing covariant derivative of the almost complex structure, provided that the torsion satisfies the relations given by Eqs. (36)-(48) and (51)-(56). A simple particular solution to these torsion relations is given by Eqs. (73) and (74), in which the only nonvanishing component of the torsion is in the fiber-base-base sector of the bundle, and is given by the negative of the gauge curvature field.

References

[1] Brandt H. E.: *Differential Geometry of Spacetime Tangent Bundle*, Internat. J. Theor. Phys. **31** (1992) 575.
[2] Brandt H. E.: *Finsler-Spacetime Tangent Bundle*, Found. Phys. Lett. **5** (1992) 221.
[3] Brandt H. E.: *Kähler Spacetime Tangent Bundle*, Found. Phys. Lett. **5** (1992) 315.
[4] Brandt H. E.: *Complex Spacetime Tangent Bundle*, Found. Phys. Lett. **6** (1993) 245.
[5] Brandt H. E.: *Spacetime Tangent Bundle with Torsion*, Found. Phys. Lett. **6** (1993) 339.
[6] Brandt H. E.: *Structure of Spacetime Tangent Bundle*, Found. Phys. Lett. **4** (1991) 523.