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Study of Three-Baryon Systems in a Quark Model for Solution of Hyperon Puzzle

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Abstract: We evaluate the three-body baryon effect in the quark model to solve the hyperon puzzle. As candidates for the extra repulsive effect required to solve the puzzle, we focus on the structural repulsive core caused by the quark-Pauli effect and the color-magnetic term. The result is that in the lowest threshold ΛNN system, no significant structural repulsive core is obtained, but a strong repulsive three-body effect is obtained from the color-magnetic term.

Key words: quark model; three-baryon forces; hyperon puzzle

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0 Introduction

Inside a neutron star, in addition to the degeneracy pressure caused by neutrons and the nuclear force, the effect of the three-body force also plays an important role as an internal pressure that resists the strong gravity.

On the other hand, due to its high density, it is believed that not only nucleons but also hyperons will appear inside the neutron star, and this fact means that the equation of state that describes the interior of the neutron star will be softened. The existence of neutron stars with masses twice the mass of the sun observed in recent years cannot be explained solely by the conventional nuclear force, three-body force, and the hyperon-nucleon interaction in hypernuclear physics. Some studies have hypothesized a three-body baryon force that is equivalent to the three-body force of the nuclear force, but the theoretical basis for this has not been established.

The description of hadron phenomena using the quark model was very effective because it could be treated systematically from the viewpoint of constructors. Its effectiveness has been demonstrated not only in the success of hadron spectroscopy, but also in the description of the nuclear force and the interaction between baryons, including hyperons. Three-body systems or three-body forces have also been evaluated using quark models.

In this paper, we attempt to solve the Hyperon puzzle using the quark model. In Section 1, we examine the quark-Pauli effect in a three-body baryon system containing a hyperon. In Section 2, we evaluate the three-body baryon force from the collar magnetic term in three systems including hyperons. This is summarized in Section 3.

1 Quark-Pauli effects in 3-baryon systems

Just as white dwarfs are interpreted as being supported by electron-induced degeneracy, namely Pauli effect by electrons, neutron stars are also considered to be supported by neutron-induced degeneracy. If the neutron Pauli effect and the repulsive force originating from the two-baryon interaction cannot support the neutron star, it may be supported by the quark degeneracy pressure, namely the quark-Pauli effect.

The quark-Pauli effect is evaluated by solving the eigenvalue problem of the resonating group method(RGM) norm kernel in the three-baryon systems[1]. This method may be intuitively interpreted as follows. Let us consider a two-baryon wave function,

$$\Phi_{B_1B_2}(1;2)=\frac{1}{\sqrt{2}}\left[\phi_{B_1}(1)\phi_{B_2}(2)-\phi_{B_1}(2)\phi_{B_2}(1)\right], \qquad (1)$$

which is normalized, $\langle\Phi_{B_1B_2}|\Phi_{B_1B_2}\rangle=1$. When this two-baryon wave function is re-expressed as a six-quark wave

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function, it becomes as follows using the quark antisymmetrizer A:

$$\Psi_{(3q)_1(3q)_2}(123;456) = A\left\{\frac{1}{\sqrt{2}}\left[\psi_{(3q)_1}(123)\psi_{(3q)_2}(456) - \psi_{(3q)_1}(456)\psi_{(3q)_2}(123)\right]\right\}, \tag{2}$$

which is not normalized, $\langle\Psi_{(3q)_1(3q)_2}|\Psi_{(3q)_1(3q)_2}\rangle = \mu$ with $\mu \neq 1$, due to the effect of antisymmetry under the exchange of quarks among the baryons. Namely, this is the quark-Pauli effect. If μ is explicitly zero, such a two-baryon system means that the existence probability is zero from the viewpoint of six-quark configuration, that is, the Pauli-forbidden state. Even if μ is not explicitly zero, the configuration of a two-baryon system with a value of μ near zero also has a small existence probability, so that a system should be difficult to realize. Since such a state, called an almost Pauli-forbidden state, resembles the characteristics of the wave function of a system with a strong repulsive potential, it can be considered as a structural repulsive effect due to the quark-Pauli effect. Studies of two-baryon interactions have shown that the structural repulsive effect of the quark-Pauli effect is dominant with or without attraction in the interaction potential.

The 9-quark 3-baryon wave function is given by $\Psi_{SYI}((0s)^9;B_1B_2B_3) = \Psi^{(\mathrm{orb})}\Psi^{(\mathrm{SF})}_{SYI}\Psi^{(\mathrm{color})}$, where $\Psi^{(\mathrm{orb})}$ denotes the orbital part with the $(0s)^9$ configuration, $\Psi^{(\mathrm{SF})}_{SYI}$ the spin-flavor part with the spin value S, the hypercharge Y, and the isospin I, and $\Psi^{(\mathrm{color})}$ the color part assuming a color singlet in each baryon. Assuming the eigenfunction of A to be

$$\sum_{B_1B_2B_3} C(SYI;B_1B_2B_3)\Psi_{SYI}\left((0s)^9;B_1B_2B_3\right), \tag{3}$$

we solve the eigenvalue problem

$$\sum_{B'_1B'_2B'_3}\left\langle\Psi_{SYI}\left((0s)^9;B_1B_2B_3\right)|A|\Psi_{SYI}\left((0s)^9;B'_1B'_2B'_3\right)\right\rangle\times C(SYI;B'_1B'_2B'_3) = \mu_{SYI}C(SYI;B_1B_2B_3)\ . \tag{4}$$

Table 1 shows the almost Pauli-forbidden states in the three-baryon system. We can find in this table the feature that many systems have large isospin-value with Σ, namely systems containing Σ^- particle. This is due to the inclusion of the strong quark-Pauli repulsive effect on the $\Sigma^-n\ {}^3S_1$ state, as is well known in the study of the two-baryon system[2-3]. We find that the ΛNN system, which has the lowest threshold of the three baryon systems containing hyperons, is not in a almost Pauli-forbidden state, because the eigenvalues of the ΛNN system are 0, $\frac{200}{243}$, and $\frac{100}{81}$, and they have values that are not close to zero. Therefore, the quark-Pauli effect is not a candidate for solving the hyperon puzzle.

Table 1 Almost Pauli-forbidden states in the three-baryon systems. S denotes the total spin, Y the total hypercharge and I the total isospin.

S	Y	I	$B_1B_2B_3$	Particle basis	Eigen value
$\frac{1}{2}$	2	2	ΣNN	Σ^-nn	$\frac{4}{81}$
$\frac{1}{2}$	1	$\frac{5}{2}$	$\Sigma\Sigma N$	$\Sigma^-\Sigma^-n$	$\frac{4}{81}$
$\frac{1}{2}$	−2	0	$\Xi\Xi\Lambda$ - $\Xi\Xi\Sigma$	-	0, $\frac{4}{81}$
$\frac{1}{2}$	−3	$\frac{1}{2}$	$\Xi\Xi\Xi$	-	$\frac{4}{81}$
$\frac{3}{2}$	2	1	ΣNN	-	$\frac{35}{243}$
$\frac{3}{2}$	1	$\frac{3}{2}$	$\Sigma\Sigma N$ - $\Sigma\Lambda N$	$\Sigma^-\Lambda n$	$\frac{1}{27}$, $\frac{35}{243}$
$\frac{3}{2}$	0	2	$\Xi\Sigma N$	$\Xi^-\Sigma^-n$	$\frac{35}{243}$
$\frac{3}{2}$	−2	0	$\Xi\Xi\Lambda$	-	$\frac{1}{27}$
$\frac{3}{2}$	−2	1	$\Xi\Xi\Sigma$	-	$\frac{35}{243}$

2 Three-baryon forces from color-magnetic interaction

Since the solution of the hyperon puzzle cannot be expected by the structural repulsive effect, we next aim at the solution by the three-body baryon force generated from the quark-quark interaction.

We use the RGM formalism for this purpose. From the study of the barion-baryon interactions in the RGM formalizm, it is known that even between colorless baryons, repulsive effects arise from the color-magnetic interaction between quarks through the quark exchange diagram[2]. The RGM wave function for the nine-quark $B_1B_2B_3$ system can be expressed by

$$\psi = A\left[\phi(SYI;B_1B_2B_3)\chi(\boldsymbol{R}_{12},\boldsymbol{R}_{12-3})\right]\ . \tag{5}$$

For a definition of $\phi(SYI;B_1B_2B_3)$ see Ref. [4]. The wave function χ is assumed to satisfy the following RGM equation

$$\left[\frac{\hbar^2}{2\mu_1}\Delta_{R_{12}} - \frac{\hbar^2}{2\mu_2}\Delta_{R_{12-3}}\right]\chi(\boldsymbol{R}_{12},\boldsymbol{R}_{12-3}) + \int K^{(E)}(\boldsymbol{R}_{12},\boldsymbol{R}_{12-3};\boldsymbol{R}'_{12},\boldsymbol{R}'_{12-3})\times \chi(\boldsymbol{R}'_{12},\boldsymbol{R}'_{12-3})\,\mathrm{d}\boldsymbol{R}'_{12}\,\mathrm{d}\boldsymbol{R}'_{12-3} = \varepsilon\chi(\boldsymbol{R}_{12},\boldsymbol{R}_{12-3})\ , \tag{6}$$

where μ_1 and μ_2 are the appropriate reduced masses, and ε is the total energy minus three baryon masses. The quark exchange kernel in Eq. (6),

$$K^{(E)} = K_T^{(E)} + K_V^{(E)} - (\varepsilon + \varepsilon_{B_1} + \varepsilon_{B_2} + \varepsilon_{B_3})K_1^{(E)}\ , \tag{7}$$

gains contributions from the translationally invariant nine-quark kinetic energy operator, T, from the quark-quark interaction, $V=\sum_{i>j} v_{ij}$, and the norm kernel. In Eq. (7) ε_{B_i} is the internal energy of the i th baryon. The separation into two-body and three-body terms can be made through the specific construction of the antisymmetrizer, A, of Eq. (5) which can be expanded in terms of the permutations P_{ij} and P_{ijk} by using double coset generators[5] as

$$A=\frac{1}{6}\Big\{\Big[1-9(P_{36}+P_{69}+P_{93})+27(P_{369}+P_{396})+54(P_{36}P_{59}+P_{69}P_{83}+P_{93}P_{26})\Big]\Big[\sum_{P=1}^{6}(-1)^{\pi(P)}P\Big]-216P_{26}P_{59}P_{83}\Big\}. \quad (8)$$

The six P include those quark exchanges which are equivalent to baryon exchanges. Of the five basic types of terms in A, the third to fifth category involve the exchange of quark pairs between different baryon pairs and give rise to nonlocal three-body forces in the baryon-baryon interaction[6]. With these terms, the three-body exchange kernel $K^{(E)}_{\Theta 3}$ for the operator $\Theta(=T,V,1)$ can be evaluated in terms of the baryon-separation parameters $\boldsymbol{R}_a$, $\boldsymbol{R}_b$, $\boldsymbol{R}'_a$, $\boldsymbol{R}'_b$ through

$$K^{(E)}_{\Theta 3}(\boldsymbol{R}_a,\boldsymbol{R}_b;\boldsymbol{R}'_a,\boldsymbol{R}'_b)=\langle\phi(SYI;B_1B_2B_3)\delta(\boldsymbol{R}_{B_1B_2}-\boldsymbol{R}_a)\delta(\boldsymbol{R}_{B_1B_2-B_3}-\boldsymbol{R}_b)\times|\Theta A_3|\phi(SYI;B_1B_2B_3)\delta(\boldsymbol{R}_{B_1B_2}-\boldsymbol{R}'_a)\times\delta(\boldsymbol{R}_{B_1B_2-B_3}-\boldsymbol{R}'_b)\rangle, \quad (9)$$

where A_3 represents the third to fifth terms of the Eq. (8).

Here we focus on the three-baryon three-body effect originating from the color-magnetic term $\Theta=-\frac{\alpha_S\pi\hbar^3}{6m^2c}\lambda^c_i\cdot\lambda^c_j\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\delta(r)$ (or $\Theta=cm$), which is well known for giving short-range repulsion between two baryons, as the quark potential, where λ^c_i denotes the color-SU(3) generator, $\boldsymbol{\sigma}_i$ the spin-SU(2) generator, α_S the strong quark-gluon coupling constant, m quark mass. We consider the flavor-SU(3) limit in this work. We adopt a value that seems reasonable as a quark-parameter set; $mc^2=313$ MeV, $\alpha_S=1$ and the oscillator length parameter for the quark wave functions in the baryon $b=0.6$ fm.

We evaluate the nonlocal three-body color-magnetic exchange kernel, $K^{(E)}_{\rm cm3}$, as the effective potential $V_{\rm cm3}$ using the following transformation method:

$$V_{\rm cm3}(\boldsymbol{R}_a,\boldsymbol{R}_b)\chi(\boldsymbol{R}_a,\boldsymbol{R}_b)=\int K^{(E)}_{\rm cm}(\boldsymbol{R}_a,\boldsymbol{R}_b;\boldsymbol{R}'_a,\boldsymbol{R}'_b)\chi(\boldsymbol{R}'_a,\boldsymbol{R}'_b)\,{\rm d}\boldsymbol{R}'_a\,{\rm d}\boldsymbol{R}'_b, \quad (10)$$

$$K^{(E)}_{\rm cm}(\boldsymbol{R}_a,\boldsymbol{R}_b;\boldsymbol{R}'_a,\boldsymbol{R}'_b)=K^{(E)}_{\rm cm3}(\boldsymbol{R}_a,\boldsymbol{R}_b;\boldsymbol{R}'_a,\boldsymbol{R}'_b)-\left(\varepsilon^{\rm cm}_{B_1}+\varepsilon^{\rm cm}_{B_2}+\varepsilon^{\rm cm}_{B_3}\right)K^{(E)}_1, \quad (11)$$

$$\chi(\boldsymbol{R}_a,\boldsymbol{R}_b)=\left(\frac{\sqrt{3}}{\pi\nu_1\nu_2}\right)^{\frac{3}{2}}\exp\left(-\frac{3}{4\nu_1^2}\boldsymbol{R}_a^2-\frac{1}{\nu_2^2}\boldsymbol{R}_b^2\right), \quad (12)$$

where ν_1 and ν_2 are two types of width parameters in Jacobian coordinates composed of three baryons. we assume $\nu_1=1$ fm and $\nu_2=\frac{\sqrt{3}}{2}$ fm, which implies an equilateral triangle.

Figure 1 shows $V_{\rm cm3}$ in Eq. (10) for the case where the three baryons form the equilateral triangle configurations. From the figure, we can find the following behavior of the three-body effect, which is highly flavor-dependent; it is a repulsive force in the NNN system with the isospin $I=1/2$, a strong repulsive force in the ΛNN system, an attractive force in the ΣNN system with $I=2$, and a weak repulsive force in the ΞNN system with $I=3/2$. In particular, the strength of the repulsive force in the ΛNN system contributes as much as the two-body effect generated from the second term in Eq. (8). Considering the consistency with the binding energy of hyper-triton, such a strong ΛNN three-body repulsive force is not realistic. This may be due to the use of the same values as the oscillator length parameter b for the two width parameters ν_1 and ν_2. The relatively strong repulsive three-body effect in the ΛNN system of color-magnetic terms may be a promising candidate for the extra repulsive effect needed to solve the hyperon puzzle.

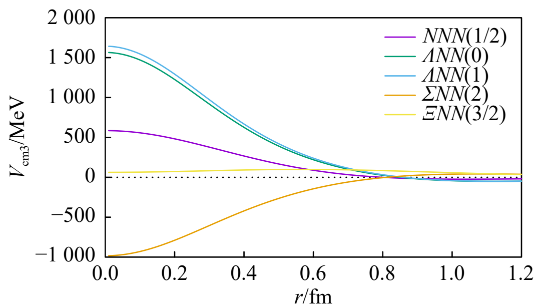

Fig. 1 (color online) $V_{\rm cm3}$ in Eq. (10) for the case where the three baryons form the equilateral triangle configurations. r denotes the length of a piece of equilateral triangle. the bold line denotes NNN (the total isospin $I=\frac{1}{2}$) system, the dashed-line $\Lambda NN(0)$, the solid line $\Lambda NN(1)$, the bold dashed-line $\Sigma NN(2)$, and the dotted line $\Xi NN(\frac{3}{2})$. The small dotted-line denotes the two-body contribution in the second term in Eq. (8) for the $\Lambda NN(1)$. Total spin is $\frac{1}{2}$ in all channels.

3 Summary

We evaluated the three-body baryon effect in the quark model to solve the `hyperon puzzle'. As candidates for the extra repulsive effect required to solve the hyperon puzzle, we focused on the structural repulsive core caused by the quark-Pauli effect and the color-magnetic term, which is a promising interaction between quarks. The result is that in the lowest threshold ΛNN system, no significant structural repulsive core is obtained, but a strong repulsive three-body effect is obtained from the color-magnetic term. This result is inconsistent with the conclusions of Ref. [7], which the intrinsic three-baryon interaction at short distance vanishes for all quantum numbers. Further research is needed to investigate the cause.

As shown in Fig. 1, there is a rich flavor-dependence even for systems containing only one hyperon, and the three-body forces are not necessarily all repulsive. Therefore, it is an interesting question whether a particular configuration involving multiple hyperons is also repulsive, since there could be a large number of Λ-hyperons in a neutron star, for example.

Systematic theoretical studies of three-body forces include the Effective Field Theory[8] and Lattice QCD[9] calculations, but they have yet to describe a consistent three-body baryon force that includes hyperons. It is important to advance our understanding of the 3-baryon force through comparison with these theoretical studies in the future.

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应用夸克模型研究三重子系统及其对解决超子疑难的启示

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摘要: 为解决超子疑难，在夸克模型框架下研究了超子物质的三体力。作为解决这个疑难所需的额外排斥效应的候选者，关注由夸克泡利效应和色磁作用引起的排斥芯。结果表明，在ΛNN系统最小阈值处，不存在明显的排斥芯结构，但色磁作用提供了强烈的三体排斥效应。

关键词: 夸克模型；三体力；超子疑难

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