Unitary Extension of Exotic Massive 3D Gravity from Bigravity

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We obtain a new 3D gravity model from two copies of parity-odd Einstein-Cartan theories. Using Hamiltonian analysis, we demonstrate that the only local degrees of freedom are two massive spin-2 modes. Unitarity of the model in anti–de Sitter and Minkowski backgrounds can be satisfied for vast choices of the parameters without fine-tuning. The recent "exotic massive 3D gravity" model arises as a limiting case of the new model. We also show that there exist trajectories on the parameter space of the new model which cross the boundary between unitary and nonunitary regions. At the crossing point, one massive graviton decouples resulting in a unitary model with just one bulk degree of freedom but two positive central charges at odds with the usual expectation that the critical model has at least one vanishing central charge. Given the fact that a suitable nonrelativistic version of bigravity has been used as an effective theory for gapped spin-2 fractional quantum Hall states, our model may have interesting applications in condensed matter physics.

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Three-dimensional massive gravity possesses many surprises that are not shared by its higher-dimensional cousins. There exists a particular combination of linear and quadratic curvature terms such that the theory is perturbatively unitary and power-counting renormalizable about the Minkowski background [1]. At the linearized level, this theory reproduces the Fierz-Pauli theory propagating a pair of massive gravitons. It is general coordinate invariant, differing from the background dependent de Rham–Gabadadze–Tolley massive gravity [2] and thus gaining its name "new massive gravity" (NMG). The maximally symmetric vacua of NMG also contains the anti–de Sitter (AdS) space. Unitarity in the AdS background requires in addition to the ghost-free dynamical degrees of freedom (d.o.f.) the positivity of the central charges present in the asymptotic symmetry algebra. This is a requirement from the correspondence between quantum gravity in the AdS_3 background and two-dimensional conformal field theory (CFT) in which the central charges in the gravity model are identified with those in the dual CFT. NMG, however, is not a satisfactory model to apply the $\mathrm{AdS}_3/\mathrm{CFT}_2$ correspondence, as the condition under which the massive gravitons are unitary implies negative central charges.

Another surprise in 3D gravity is that many interesting models can be reformulated using frame-field formalism as a multiflavor Chern-Simons- (CS) like theory [3]. Different from usual CS theories, CS-like theories can bear dynamical d.o.f., as the structure constants disobey Jacobi identity. The resolution of the bulk-boundary clash for NMG in AdS_3 becomes straightforward, once adopting the frame-field formalism and losing the requirement that the massive gravitons interact only through a finite number of curvature terms. As a consequence, NMG is extended to the "zweidreibein gravity" (ZDG) [4] and further generalizations [5] which are also parity even. The main building blocks of ZDG are two copies of parity-even cosmological Einstein-Cartan theories glued together by a cubic potential involving the two dreibeins. Thus ZDG can be viewed as a variant of bigravity. By properly choosing parameters, ZDG can satisfy both criteria for a unitary AdS_3 quantum gravity, thus serving as an attractive toy model of lower-dimensional quantum gravity defined via $\mathrm{AdS}_3/\mathrm{CFT}_2$ correspondence.

Besides the more mathematically orientated goal of constructing a well-defined lower-dimensional quantum gravity model, 3D spatially covariant bigravity has recently been utilized as a tool to build effective theories for the spin-2 gapped collective excitations observed in certain fractional quantum Hall states (FQHS) [6–8]. This latest application of bigravity in condensed matter physics is of particular interest, as it directly links two broad areas in physics, gravity, and condensed matter physics.

NMG and its two-frame fields extension ZDG are standard gravity models in the sense that the same parity property is shared by the equations of motion and the Lagrangian. Interestingly, there also exist "exotic" 3D gravity models of which the equations of motion are parity even, while the actions are parity odd. The recently proposed "exotic massive gravity" (EMG) [9] is of just

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such a kind whose perturbative spectrum coincides with that of the NMG; see, e.g., Refs. [10–13] for recent discussions on solutions of EMG and generalizations. Similar to NMG, the full physical spectrum of EMG contains either ghostlike massive gravitons or negative entropy Bañados-Teitelboim-Zanelli black holes. Simple extensions of EMG by adding parity violating terms in the Lagrangian turn out to be not sufficient to recover unitarity [9]. However, just as the ZDG example, there could exist a unitary extension of EMG using two-frame fields. As we will show below, such a unitary extension (in both AdS$_3$ and Minkowski background) indeed exists. Moreover, we have also found a trajectory on the parameter space interpolating between a unitary model and a nonunitary generalized EMG living on the boundary of the parameter space. We emphasize that the previously discovered parameter flow in the ZDG model [14] resides entirely in the nonunitary region and thus differs from ours *per se*. On the trajectory, there is a special point separating the unitary models from the nonunitary ones. Counterintuitively, at the dividing point, the theory is not critical in the usual sense; namely, at least one central charge vanishes [15] with the presence of logarithmic modes [16,17]. Instead, it describes a unitary CS-like theory based on one frame field and two spin connections. The unitary extension of EMG retains the exotic feature of EMG as its kinetic terms are composed of two copies of parity-odd exotic Einstein-Cartan theories [18,19] in which the role of the curvature term is played by the gravitational CS term. Given the connection between bigravity and FQHS, it is conceivable that the new model obtained here together with the parity-even models paves the way for a fully covariant nonlinear theory describing the bulk gapped collective spin-2 excitations present in a hidden sector of quantum Hall states exhibiting a parity-violating pattern.

We begin our construction by introducing the basic fields in the frame-field formalism. We choose the same set of fields as the ZDG model, because more fields may introduce unwanted d.o.f. These are a pair of Lorentz vector-valued 1-forms e_I^a ($I = 1$, 2 and $a = 0$, 1, 2) and a pair of Lorentz vector-valued connection 1-forms ω_I^a, out of which one can build torsion and the Lorentz Chern-Simons actions:

$$\begin{aligned} T_I^a(\omega_J) &= de_I^a + \epsilon^{abc}\omega_{Jb}e_{Ic}, \\ L_{\mathrm{CS}}(\omega_I) &= \frac{1}{2}\left(\omega_{Ia}d\omega_I^a + \frac{1}{3}\epsilon^{abc}\omega_{Ia}\omega_{Ib}\omega_{Ic}\right). \end{aligned} \tag{1}$$

The model we start with takes the form

$$\mathcal{L}_{\mathrm{GEZDG}} = \mathcal{L}_{1-} + \mathcal{L}_{2-} + \mathcal{L}_{12-} + \mathcal{L}_{v+}, \tag{2}$$

where the subscript GEZDG stands for generalized exotic zwei-dreibein gravity. The "±" in the subscript refers to the parity of the action. $\mathcal{L}_{1-}$ and $\mathcal{L}_{2-}$ are two exotic Einstein-Cartan Lagrangians constructed from the Lorentz CS term and the torsion term,

$$\mathcal{L}_{I-} = \alpha_I L_{\mathrm{CS}}(\omega_I) + \beta_I e_{Ia}T_I^a(\omega_I), \tag{3}$$

where $\{\alpha_I, \beta_I\}$ are dimensionful parameters. The two copies of exotic Einstein-Cartan theories are coupled via two other parity-odd terms,

$$\mathcal{L}_{12-} = \beta_3 e_{1a}T_1^a(\omega_2) + \beta_4 e_{2a}T_2^a(\omega_1), \tag{4}$$

where $\{\beta_3, \beta_4\}$ are new parameters. Without $\mathcal{L}_{12-}$, $\{e_1, e_2\}$ are both invertible as independent dreibeins. As we will see from the Hamiltonian analysis below, the interaction terms Eq. (4) single out a unique invertible dreibein for the model to have the designed number of d.o.f. The first three terms in the Lagrangian are all parity odd. However, the odd parity implies that in the linear theory, the kinetic terms of the two massive modes have the opposite signs, meaning one of them must be ghostlike. To recover unitarity, we need to add parity-even terms,

$$\mathcal{L}_{v+} = \epsilon^{abc}(\gamma_1 e_{1a}e_{1b}e_{1c} + \gamma_2 e_{2a}e_{2b}e_{2c}), \tag{5}$$

which carry two more parameters, $\{\gamma_1, \gamma_2\}$. This explains the origin of the name for our model. It should also be noted that the curvature term is not present in the Lagrangian, which is a special feature of the exotic model.

We now proceed to count the d.o.f. There are in total 4 Lorentz vector-valued 1-forms in Eq. (2). Their temporal components are the Lagrange multipliers; thus, the physical phase space is 24 dimensional spanned by the spatial components of the 1-forms. As the Lagrangian is first order in time derivative, the 24 phase space variables already form 12 canonical pairs. Varying the Lagrangian with respect to the 12 temporal components, one obtains 12 primary constraints. The integrability conditions of the first-order equations of motion give rise to new algebraic conditions on the fields. We find that upon imposing $\beta_3 = 0$ and the invertibility of e_2^a, the integrability conditions lead to only two secondary constraints,

$$\epsilon^{ij}e_{1ia}e_{2j}^a = 0, \qquad \epsilon^{ij}e_{2ia}(\omega_{2j}^a - \omega_{1j}^a) = 0, \tag{6}$$

and solutions to the Lagrangian multipliers associated with the temporal components of e_1^a, $\omega_1^a - \omega_2^a$ for generic choice of the parameters. Using the procedure given in Ref. [3], one can check that among the 14 constraints, 6 of them are first class while the rest are second class. We are then left with a $24 - 12 - 8 = 4$ dimensional phase space (per space point) indicating 2 d.o.f. in the usual sense. It should also be mentioned that the action Eq. (2) is symmetric under $\{e_1^a, \omega_1^a\} \leftrightarrow \{e_2^a, \omega_2^a\}$ modulo relabeling the parameters. Thus there is a totally equivalent choice by imposing $\beta_4 = 0$ and invertibility of e_1^a. From now on, we will focus on the model Eq. (2) with $\beta_3 = 0$ while keeping other parameters generic.

The linearized spectrum about the maximally symmetric AdS$_3$ vacuum should be compatible with the results from nonperturbative Hamiltonian analysis, namely, the model is free of the Boulware-Deser ghost [20,21] propagating only two physical massive spin-2 modes. The field equations can be readily derived from Eq. (2) from which we can read off the AdS vacuum,

$$e_I = a_I \bar{e}, \qquad \omega_I = \bar{\omega} + b_I \bar{e}, \tag{7}$$

provided that the parameters satisfy the relation

$$\alpha_1 = -\frac{2(\beta_1 a_1^2 + \beta_4 a_2^2)}{\Lambda + b_1^2}, \qquad \alpha_2 = -\frac{2\beta_2 a_2^2}{\Lambda + b_2^2},$$
$$\gamma_1 = -\frac{2b_1\beta_1}{3a_1}, \qquad \gamma_2 = -\frac{2(b_1\beta_4 + b_2\beta_2)}{3a_2}, \tag{8}$$

where $\Lambda \equiv -1/\ell^2$ is the cosmological constant, $\bar{e}$ and $\bar{\omega}$ are the dreibein and spin connection of the unit-radius AdS$_3$ metric, and a_I and b_I are constants. Fluctuations about the AdS$_3$ vacuum are characterized by a small expansion parameter κ as follows:

$$\omega_I = \omega + b_I e + \kappa v_I, \qquad e_I = a_I(e + \kappa k_I). \tag{9}$$

Diagonalizing the quadratic action Eq. (2) about AdS$_3$ vacuum is straightforward and results in

$$\begin{aligned}\mathcal{L}^{(2)}_{\text{GEZDG}} = &-\frac{K_-}{M_-}(\phi_{-a} D\phi^a_- - M_- \epsilon_{abc}\bar{e}^a \phi^b_- \phi^c_-)\\ &+\frac{K_+}{M_+}(\phi_{+a} D\phi^a_+ + M_+ \epsilon_{abc}\bar{e}^a \phi^b_+ \phi^c_+)\\ &+ a_+(f_{+a} D f^a_+ + \ell^{-1}\epsilon_{abc}\bar{e}^a f^b_+ f^c_+)\\ &- a_-(f_{-a} D f^a_- - \ell^{-1}\epsilon_{abc}\bar{e}^a f^b_- f^c_-),\end{aligned} \tag{10}$$

where $\{\phi_{-a}, \phi_{+a}\}$ form a pair of massive gravitons, and $\{f_{-a}, f_{+a}\}$ are the usual massless gravitons. The coefficients in front of the kinetic terms read

$$\begin{aligned}K_\pm &= M_\pm(b_1 \pm M_\pm)(\ell^2 M_\pm^2 - 1)\Delta,\\ a_\pm &= \frac{\alpha_1 b_1 \ell + \alpha_2 b_2 \ell \pm \alpha_1 \pm \alpha_2}{\ell^2},\\ \Delta &= (b_2 - b_1)(M_-^2 - M_+^2)\beta_2 a_2^2 \ell^{-2}.\end{aligned} \tag{11}$$

The mass eigenvalues $M_\pm$ can of course be solved in terms of the parameters in the Lagrangian. However, the expressions are not convenient for further analysis. Instead, we find that it is more handy to treat $\{M_\pm, b_1, b_2, a_1, a_2, \beta_2, \ell\}$ as free parameters, recasting the original parameters $\{\alpha_1, \alpha_2, \beta_1, \beta_4, \gamma_1, \gamma_2\}$ in the Lagrangian as functions of them using Eq. (8) and the eigenvalue equation from which $M_\pm$ is solved. In AdS$_3$ vacuum, no-tachyon and no-ghost conditions implies

$$(\ell M_\pm)^2 > 1, \qquad K_\pm > 0. \tag{12}$$

Choosing the Brown-Henneaux boundary condition [22] in AdS$_3$, the necessary condition for nonperturbative unitarity is the positivity of central charges appearing in the asymptotic Virasoro ⊕ Virasoro symmetry algebra. The central charges can be computed directly from the CS-like action using the method given in Ref. [23]. The final results are

$$c_\pm = \frac{3\ell}{2G}\left(b_1\alpha_1 + b_2\alpha_2 \pm \frac{1}{\ell}(\alpha_1 + \alpha_2)\right), \tag{13}$$

which implies that

$$c_+ = \frac{3\ell}{2G} a_+, \qquad c_- = \frac{3\ell}{2G} a_-. \tag{14}$$

In terms of the new set of parameters, $a_\pm$ take the form

$$a_\pm = \frac{2(b_1 - b_2)(\ell M_- \pm 1)(\ell M_+ \mp 1)\beta_2 a_2^2}{(b_2\ell \mp 1)\delta},$$
$$\delta = (b_1 - b_2)(1 - \ell^2 M_+ M_-) + (M_+ - M_-)(1 - \ell^2 b_1 b_2). \tag{15}$$

The tachyon-free, ghost-free, and $c_\pm > 0$ conditions can be simultaneously satisfied by various choices of the parameters. However, we will leave the systematic study for future work. Instead, we focus on one such region as

$$M_\pm \ell > 1, \qquad M_- > M_+, \qquad b_1 > b_2,$$
$$b_1 + M_+ < 0, \qquad \delta < 0, \qquad \beta_2 > 0, \tag{16}$$

about which interesting physics will be discussed. a_1 and a_2 are unconstrained in Eq. (16). One representative from this unitary region takes the form

$$M_- = 4, \qquad M_+ = 2, \qquad b_1 = -2.5,$$
$$b_2 = -15, \qquad \beta_2 = 1, \qquad a_1 = a_2 = 1, \tag{17}$$

where various values are given in AdS units with $\ell = 1$. The spectrum analysis about Minkowski vacuum can be obtained from the AdS$_3$ results by taking the $\ell \to \infty$ limit. As there is no Bañados-Teitelboim-Zanelli black hole in Minkowski space, the unitarity condition becomes less stringent. Only $K_\pm > 0$ is required, which can be easily satisfied.

The GEZDG Eq. (2) model with $\beta_3 = 0$ is in fact related to the generalized EMG by taking a scaling limit in both the fields and the parameters. Specifically, in taking the scaling limit, we redefine the fields,

$$e_1=e,\quad \omega_1=\omega-\frac{1}{m^2}f,\quad e_2=e+\frac{\lambda}{2}h,\quad \omega_2=\omega, \tag{18}$$

and the parameters,

$$\begin{aligned} &\alpha_1=\frac{\nu}{m^2}, \qquad \alpha_2=-1, \qquad \beta_1=\frac{\nu}{2}-\frac{1}{\lambda},\\ &\beta_2=-\frac{\nu}{2}, \qquad \beta_4=\frac{1}{\lambda}, \qquad \gamma_1+\gamma_2=\frac{\nu m^2}{3\mu}. \end{aligned} \tag{19}$$

The $\lambda \to 0$ limit then reproduces the generalized EMG model [9].

In fact, one can construct infinitely many trajectories connecting a point in the unitary region, such as the one given in Eq. (16), to a tachyon-free generalized EMG model. One exemplary trajectory is exhibited here on which $\ell = 1$ and $\{M_\pm, b_1, b_2, a_1, a_2, \beta_2\}$ are parametrized with the dependence on the flow parameter λ,

$$\begin{aligned} M_\pm &= (1-\lambda^2)\mathring{M}_\pm + \lambda^3 M_\pm^*,\\ b_1 &= b_1^*\lambda^3 + (1-\lambda^3)[b_2-(\beta_2\lambda+1)(\mathring{M}_+-\mathring{M}_-)],\\ b_2 &= b_2^*\lambda^3 + (1-\lambda^2)(\mathring{M}_+-\mathring{M}_-+1),\\ a_1 &= a_1^*,\\ a_2 &= a_1(1-\lambda^2)\left(1-\frac{\nu\lambda}{4}\right)+a_2^*\lambda^2,\\ \beta_2 &= \beta_2^*\lambda^2-\frac{\nu}{2}(1-\lambda^2), \end{aligned} \tag{20}$$

so that near $\lambda = 0$, the leading behaviors of parameters above take the form Eq. (19) up to terms linear or higher order in λ. When $\lambda = 1$, the trajectory reaches the unitary model defined by the parameters Eq. (17). The starred parameters represent the values in a unitary region and $\mathring{M}_\pm$ are adjustable parameters. In the $\lambda \to 0$ limit, we find that along the above trajectory, GEZDG reaches a generalized EMG whose defining parameters $\{\nu, m^2, \mu\}$ are expressed in terms of $\{\mathring{M}_\pm, a_1\}$ as

$$\begin{aligned} m^2 &= \frac{(\mathring{M}_- -1)(\mathring{M}_+ +1)}{a_1^2}, \qquad \mu = \frac{2(\mathring{M}_- -1)(\mathring{M}_+ +1)}{a_1(2\mathring{M}_+ - 2\mathring{M}_- -1)},\\ \nu &= -\frac{(\mathring{M}_+ - \mathring{M}_-)(\mathring{M}_+ - \mathring{M}_- +2)}{a_1^2}. \end{aligned} \tag{21}$$

We recall that the cosmological constant of the generalized EMG is given by [9]

$$\mathring{\Lambda} = -\left(\nu+\frac{m^4}{\mu^2}\right) = \frac{12(\mathring{M}_+ - \mathring{M}_-)-1}{4a_1^2}, \tag{22}$$

which is negative as long as we choose $\mathring{M}_- > \mathring{M}_+$. This means the limiting generalized EMG model admits AdS vacuum about which the tachyon-free condition can also be satisfied if

$$m^2|\mu| - m^2\sqrt{-\mathring{\Lambda}} + |\mu|\mathring{\Lambda} > 0. \tag{23}$$

Upon setting $\mathring{M}_+ = \mathring{M}_- - c$ with $c > 0$ and substituting in Eq. (21), the above condition becomes

$$-4c\mathring{M}_- - 8c + (2c+1)\sqrt{12c+1} + 4\mathring{M}_-^2 - 5 > 0, \tag{24}$$

which can be easily obeyed for large enough $\mathring{M}_-$. For instance, if $c = 3$, the required condition is satisfied for any value of $\mathring{M}_-$. With these results in hand, we may now see explicitly how the central charges $c_\pm$ and $K_\pm$ change along the trajectory as λ runs between 0 and 1. In Figs. 1–3, the parameters take the values given by Eq. (21) and

$$\mathring{M}_- = 12, \qquad \mathring{M}_+ = 9. \tag{25}$$

In Fig. 1, it is evident that central charges are smooth along the trajectory and are always positive. In Fig. 2, we plot K_+ weighted by K_-. The reason for this is that the magnitude of $K_\pm$ become quite large near $\lambda = 0$ while their ratio is still modest. This ratio makes sense only when K_- stays positive along the trajectory, which we have also confirmed. We also notice that K_+ crosses zero at $\lambda \sim 0.85$. At this point, K_- and $c_\pm$ are still positive. This is very different from the usual intuition that the unitary and nonunitary models are separated by critical points at which one of the central charges vanishes [15]. This intriguing feature guides

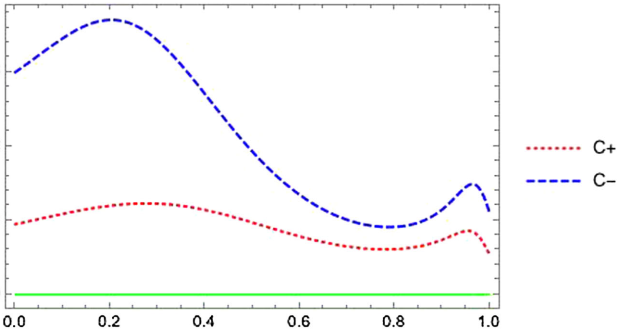

FIG. 1. The flow of two central charges from a unitary GEZDG to a nonunitary generalized EMG model along the trajectory Eq. (20).

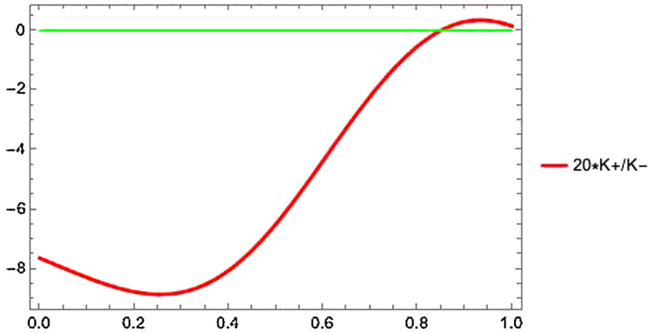

FIG. 2. The flow of the ratio K_+/K_- from a unitary GEZDG to a nonunitary generalized EMG model along the trajectory Eq. (20). At $\lambda = 1$, $20K_+/K_- \approx 0.15$.

us to take a closer look at the crossing point on the trajectory.

From Fig. 3, we see that while β_4 stays positive along the entire trajectory, β_1 vanishes at $\lambda \sim 0.85$. Meanwhile, the relation given in Eq. (8) implies $\gamma_1 = 0$ when $\beta_1 = 0$. Setting $\beta_1 = \gamma_1 = 0$ in the GEZDG action with $\beta_3 = 0$, we see that e_{1a} decouples, and the resulting theory is a three-flavor model given by

$$\mathcal{L}_{3f} = \alpha_1 L_{\mathrm{CS}}(\omega_1) + \alpha_2 L_{\mathrm{CS}}(\omega_2) + \beta_2 e_{2a} T_2^a(\omega_2) + \beta_4 e_{2a} T_2^a(\omega_1) + \gamma_2 \epsilon_{abc} e_2^a e_2^b e_2^c. \tag{26}$$

The plots we show above indicate that it also admits unitary regions. Hamiltonian analysis reveals a single secondary constraint,

$$\epsilon^{ij} e_{2ia}(\omega_{2j}^a - \omega_{1j}^a) = 0, \tag{27}$$

indicating that the model propagates a single massive graviton. We have also checked that the new three-flavor model is inequivalent to “topologically massive gravity” [24] or “minimal massive gravity” [25]. The AdS vacuum is now given by

$$e_2 = a_2 \bar{e}, \qquad \omega_1 = \bar{\omega} + b_1 \bar{e}, \qquad \omega_2 = \bar{\omega} + b_2 \bar{e}, \tag{28}$$

together with parameter relations similar to those in Eq. (8) upon setting $\beta_1 = \gamma_1 = 0$. The quadratic action for fluctuations about the AdS$_3$ background takes the form

$$\begin{aligned}\mathcal{L}_{3f}^{(2)} = &-\frac{A}{M}(\phi_a D\phi_1^a - M\epsilon_{abc}\bar{e}^a\phi^b\phi^c) \\ &+ a_+(f_{+a} D f_+^a + \ell^{-1}\epsilon_{abc}\bar{e}^a f_+^b f_+^c) \\ &- a_-(f_{-a} D f_-^a - \ell^{-1}\epsilon_{abc}\bar{e}^a f_-^b f_-^c),\end{aligned} \tag{29}$$

where $a_\pm$ are still related to central charges via Eq. (14). In terms of the new parameters $\{M, b_1, b_2, a_2, \beta_2, \ell\}$, $a_\pm$ and A are given as

$$\begin{aligned}a_\pm &= \frac{2a_2^2 \ell \beta_2 (b_2 - b_1)(M\ell \pm 1)}{(b_2\ell \mp 1)(b_1\ell \mp 1)(b_2 + M)}, \\ A &= M(b_1 - b_2)(b_1 + M)(\ell^2 M^2 - 1).\end{aligned} \tag{30}$$

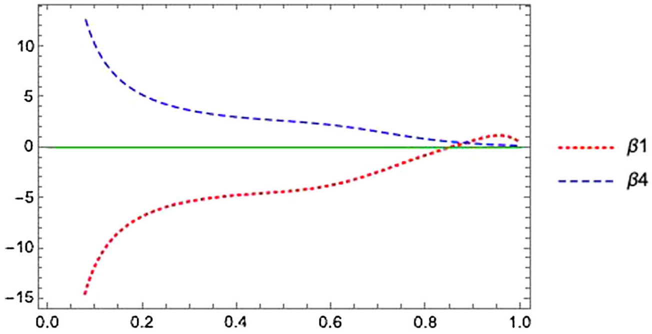

FIG. 3. Parameters β_1 and β_4 flow from a unitary GEZDG to a nonunitary generalized EMG model along the trajectory Eq. (20). At $\lambda = 1$, $\beta_1 \approx 0.53$, $\beta_4 \approx 0.19$.

The bulk-boundary unitarity is achieved when

$$A > 0, \qquad a_\pm > 0, \tag{31}$$

which can be satisfied for various choices of parameters. In AdS units, one example is given for the choice of parameters below:

$$b_1 = -3, \quad b_2 = -12, \quad M = 5, \quad \beta_2 = a_2 = 1. \tag{32}$$

In this Letter, we report a new unitary four-flavor CS-like model obtained from merging together two copies of parity-odd exotic Einstein-Cartan theories. The generalized EMG model appears as a limiting case of the new model. Regarding the connection between bigravity and effective theory for the gapped spin-2 FQHS, the exotic nature of the new model may describe certain novel phenomena in fractional quantum Hall effects once a nonrelativistic limit is properly taken. From the effective field theory point of view, it is crucial to understand the boundary on the parameter space separating the unitary models from the nonunitary ones. Therefore, it should be interesting to carry out a systematic study of all unitary four-flavor CS-like theories with a bigravity origin. Besides the standard and exotic four-flavor CS-like theories, there exists a third kind of mixed nature by coupling a parity-even theory to a parity-odd one. Unitary models of this type have not been investigated to date. Finally, incorporating supersymmetry in the CS-like theory is also interesting. It has been proposed that global supersymmetry may emerge in certain condensed matter systems; see, e.g., Ref. [26]. If in certain FQHS emergent local supersymmetry can be realized, its effective theory must be a supersymmetric bigravity model, thereby revitalizing supergravity in a new arena.

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