STUDY OF T_{cc}^{+} AND HIDDEN CHARMONIUM 1^{+-} AND 0^{++} TETRAQUARKS IN QCD SUM RULES

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ABSTRACT

STUDY OF T_{CC}^{+} AND HIDDEN CHARMONIUM 1^{+-} AND 0^{++} TETRAQUARKS IN QCD SUM RULES

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Recently, exotic hadrons have become a very important and interesting topic for particle physics, especially after the observation of the exotic particle $X(3872)$. The T_{cc}^{+} tetraquark, which has a very similar structure to $X(3872)$, was observed in the LHCB experiment of CERN in 2021. However, this new particle has different properties than the previously observed exotic hadrons, as it contains two heavy quarks and no heavy antiquark. In this study, the masses of the double open charm T_{cc}^{+} tetraquark, and hidden charm states with $J^{PC} = 1^{+-}$ and $J^{PC} = 0^{++}$, and with the motivation taken from heavy-quark spin symmetry, their mass relations are studied. QCD sum rules method, which allows successful results in hadron phenomenology, is used in the calculations. As a result of the analyses, by determining the most appropriate values of the parameters required for the QCD sum rules, the mass of the T_{cc}^{+} tetraquark is obtained in a manner comparable to the experiments and as predicted by heavy-quark spin symmetry, the mass differences of the 1^{+-} and 0^{++} states are found to be small enough, only a few MeV, as expected.

Keywords: Exotic hadron, $X(3872)$, T_{cc}^{+} tetraquark, double open charm, $J^{PC} = 1^{+-}$ and $J^{PC} = 0^{++}$ states, hidden charm, heavy-quark spin symmetry, QCD, QCD Sum Rules

ÖZ

T_{cc}^{+} VE GİZLİ ÇARMONYUM 1^{+-} VE 0^{++} TETRAKUARKLARIN KRD TOPLAM KURALLARINDA ÇALIŞILMASI

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Son zamanlarda, egzotik hadronlar, özellikle $X(3872)$ egzotik parçacığının gözlemlenmesinden sonra, parçacık fiziği için çok önemli ve ilginç bir konu haline gelmiştir. $X(3872)$ ile oldukça benzer bir yapıya sahip olan T_{cc}^{+} tetrakuark, 2021 yılında CERN'in LHCB deneyinde gözlemlenmiştir. Ancak bu yeni parçacık, iki ağır kuark içerdiği ve antikuark içermediği için, daha önce gözlemlenen egzotik hadronlardan farklı özelliklere sahiptir. Bu çalışmada açık çift çarm T_{cc}^{+} tetrakuark ve gizli çarm $J^{PC} = 1^{+-}$ ve $J^{PC} = 0^{++}$ durumlarının kütleleri ve ağır kuark spin simetriden alınan motivasyonla, onların kütle ilişkileri çalışılmıştır. Hesaplamalarda hadron fenomolojisinde başarılı sonuçlar elde edilmesine olanak sağlayan KRD toplam kuralları yöntemi kullanılmıştır. Analizler sonucunda KRD toplam kuralları için gerekli olan parametrelerin en uygun değerleri belirlenerek, T_{cc}^{+} tetrakuarkın kütlesi, deneylerle kıyaslanabilir bir şekilde elde edilmiş ve ağır kuark spin simetrisinin öngördüğü gibi, 1^{+-} ve 0^{++} durumlarının kütle farkının yeterince küçük, sadece bir kaç MeV civarında olduğu görülmüştür.

Anahtar Kelimeler: Egzotik hadron, $X(3872)$, T_{cc}^{+} tetrakuark, açık çift çarm, $J^{PC} = 1^{+-}$ ve $J^{PC} = 0^{++}$ durumları, gizli çarm, ağır kuark spin simetri, KRD, KRD Toplam Kuralları

To my Family and especially to my Husband..

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ABBREVIATIONS	
QCD	Quantum Chromo Dynamics
EPSHEP	European Physical Society Conference on High Energy Physics
LHCb	Large Hadron Collider beauty
CDF	The Collider Detector at Fermilab The Beijing Spectrometer III (BES III)
BES III	The Beijing Spectrometer III
SM	Standard Model
QED	Quantum Electrodynamics
OPE	Operator Product Expansion

CHAPTER 1

INTRODUCTION

Subatomic particles that are composed of quarks and gluons are known as hadrons. The quark model proposed by Murray Gell-Mann and Georg Zweig in 1964 is a quite successful model for explaining hadrons and their properties. According to this model, conventional hadrons can be classified into two groups: mesons, consisting of one valence quark and one valence antiquark, and baryons, composed of three valence quarks (three quarks or three antiquarks) [7, 8]. Since the early days when the quark model was introduced, there has been a prediction/theory that there were more quark and antiquark states than mesons and baryons. However, since there has been not yet enough experimental data to support this theory/prediction, this topic remained a mysterious and important subject of study for particle physics in those years [9, 10].

Recently, hadron states with quantum properties different from conventional hadrons have been observed in independent experiments and research conducted at many accelerator centers such as LHCb, BaBar, CLEO, Belle, CDF, D0, and BESIII [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21]. In the ongoing process, the emergence of Quantum Chromo Dynamics (QCD), which has properties such as asymptotic freedom and confinement, has opened the way for more detailed theoretical research on these particles [22, 23, 24].

Hadrons that do not meet the definition of conventional hadrons are called exotic hadrons [25, 26, 27]. One such example is hadrons with valence-gluon content. Although the structure of exotic hadrons is very different from conventional hadrons, there is no need for a new theory to explain them. Instead, it is thought to be sufficient to make serious progress in the already existing strong interaction theory [28].

Just like ordinary hadrons, exotic hadrons are divided into two categories: fermions like ordinary baryons and bosons like ordinary mesons. This scheme further classifies pentaquarks that comprise five valence quarks (one being an antiquark), as exotic baryons, whereas tetraquarks (which comprise four valence quarks: two quarks and two antiquarks) and hexaquarks (which contain six quarks that consist of dibaryons or three pairs of quarks-antiquarks) are called exotic mesons [29, 30].

Therefore, exotic hadrons are divided into some categories according to the quark and gluon states they contain. Some selected quark configurations for exotic hadrons are given in Figure 1.1. In the figure, a tetraquark state consists of four valence quarks, specifically they are include two quarks and two antiquarks. The hadro-quarkonium model states that there is a $Q\bar{Q}$ structure formed by heavy quarks in the center and around it there are $q\bar{q}$ quarkoniums formed by light quarks [31, 32]. In the hadronic molecule model, a heavy quark and a light antiquark $Q\bar{q}$ and a heavy antiquark and a light quark $q\bar{Q}$ are thought to come together to form a molecule. The hybrid model states that a heavy quark and its antiquark form a bound state, and the gluon in the valence band acts actively in the bound state. The states in the glueball model are composite particles formed by gluons and do not contain any quark structure.

As mentioned above, the most crucial development with respect to exotic hadrons was the observation of a new particle with unexpected properties in 2003 [11]. The

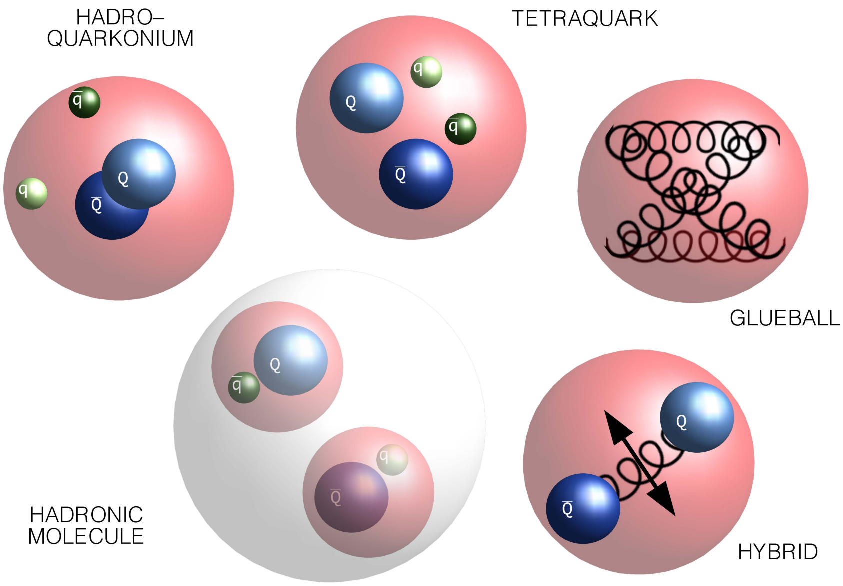

Figure 1.1: Some selected quark configurations for exotic hadrons [5]

particle called $X(3872)$ has an important place in particle physics, as it is the first tetraquark observed. This new particle was first discovered in the $B^+ \to J/\Psi\pi^+\pi^-K^+$ decay by the Belle collaboration and was quickly confirmed by the CDF, D0 and BaBar collaborations very soon thereafter and was observed most recently by the LHCb collaboration in 2012 [11, 33, 34, 35, 36]. It decays to $J/\Psi\pi^+\pi^-$ with a very small natural width for a state above the $D\bar{D}$ threshold with spin-parity quantum number $J^{PC} = 1^{++}$. Using the world average as reference, $X(3872)$ with a mass of 3871.69 ± 0.17 MeV is very close to the threshold $\bar{D}D^{*0}$ of 3872.68 ± 0.07 MeV. Therefore, this particle with a mass and width less than 2 MeV does not match any of the theoretically predicted charmonium states [37, 38, 39, 40]. The discovery of $X(3872)$ ushered in a new era of exotic states, and after its observation, subsequently, studies on this subject have been intensified and new states with many unusual properties, including various charged states, were observed.

European Physical Society Conference on High Energy Physics (EPSHEP), a new tetraquark was presented as a discovery by the CERN's LHCb experiment in 2021. Named T_{cc}^{+}, this new particle has two charm quarks and an up and down antiquark with quantum numbers $J^P = 1^+$, $I = 0$, which is found in the invariant mass spectrum $D^0 D^0 \pi^+$ with a mass around 3875 MeV [41, 42].

Recently, many tetraquarks have been discovered. Counting the one which contained two charm quarks and two charm antiquarks, this would be the first time that a tetraquark was discovered having two charm quarks, and no anti- charm. This is referred to as an open charm by physicists, but in this situation it would be considered a "double open charm".

The T_{cc}^{+} is also the first tetraquark to be observed, which contains two heavy quarks and two light antiquarks. Similarly, the Ξ_{cc}^{++} baryon contains two c quarks and a u quark; the similarity in the contents of these two states causes a relationship between the properties of them [43]. One of the similar properties is the mass, and from the measured mass of the Ξ_{cc}^{++} baryon [43, 44, 45, 46], it is stated that the mass of the T_{cc}^{+} tetraquark is close to the sum of the masses of the D^{*+} and D^0 mesons, as supported in [47]. According to theoretical estimates for the mass of the ground state of T_{cc}^{+} tetraquark with spin-parity quantum numbers $J^P = 1^+$ and isospin $I = 0$ with respect to the $D^{*+}D^0$ mass threshold δm is

$$\delta m \equiv m_{T_{cc}^{+}} - (m_{D^{*+}} + m_{D^0}) = -273 \pm 61 \pm 5^{+11}_{-14} \text{ keV}, \tag{1.1}$$

where $m_{D^{*+}}$ and m_{D^0} denote the known masses of the $D^{*+}(c\bar{d})$ and $D^0(c\bar{u})$ mesons respectively, and $m_{T_{cc}^{+}}$ is the mass of the T_{cc}^{+} tetraquark; this state is the narrowest exotic state observed to date [41, 42]. T_{cc}^{+} tetraquark decays into particles that are easily detected, and when they are combined with the small amount of energy that is available in the decay, it results in fine precision on its mass that helps to study these fascinating particle quantum numbers. These also may generate a strict test for the existing theoretical models, which could lead to a potential probe of effects.

In this study, the masses of the double open charm T_{cc}^{+} tetraquark and the hidden charm states $J^{PC} = 1^{+-}$ which have a similar interpolating current with T_{cc}^{+} and its spin-symmetry partner $J^{PC} = 0^{++}$, and their mass relations with the motivation taken from the heavy-quark spin symmetry, are studied. For this purpose, in the 2^{nd} Chapter, under the title "Theoretical Foundations", general information in the Standard Model (SM) is summarized and explanations are made with field theories. In the 3^{rd} Chapter, necessary definitions are made for the QCD sum rules method used in this study. In the 4^{th} Chapter, analytical expressions have been obtained for the mass expressions of the particles that are the subject of the thesis, and in the 5^{th} Chapter, the numerical results obtained by performing numerical analyzes for these analytical expressions are given. In the 6^{th} Chapter, the numerical data obtained are physically interpreted by comparing them with the results of similar studies on the same subject.

CHAPTER 2

THEORETICAL FOUNDATIONS

2.1 The Standard Model of Particle Physics

The particle physics at its most fundamental level tries to find out what the building blocks of matter are, explains how these basic structures are formed, and describes how the parts interact with each other. Particle physics is also called high-energy physics because it is not possible to investigate the properties of fundamental particles under normal conditions in nature, and therefore high energies are needed. For this purpose, many theoretical and experimental researches have been carried out in the particle physics, and as a result of these researches, a basic theory called the SM has emerged.

Elementary particles in SM can be grouped under two main headings: Fermions, which consist of leptons and quarks with half-number spins, form the basic structure of matter and comply with Fermi-Dirac statistics, and bosons, which mediate fundamental interactions, including the spin 0 Higgs boson that gives mass to the particles, have integer spins and comply with Bose-Einstein statistics.

So far, six different types of leptons have been found in nature, known as "flavors": Electron, electron neutrino, muon, muon neutrino, tau, tau neutrino, and antileptons corresponding to these leptons. Of these leptons, electrons (e) and electron neutrinos (ν_e) that form electronic leptons, are called the first generation; muons (μ) and muon neutrinos (ν_μ) that form muonic leptons, are called the second generation; and taus (τ) and tau neutrinos (ν_τ) form tauonic leptons, are called the third generation.

Each generation of leptons has a quantum number associated with it, called the lepton number. From neutrino oscillations, it is known that these individual lepton numbers are not conserved but the total lepton number is conserved perturbatively. The first of these lepton numbers is the number of electrons L_e and for any situation

$$L_e \equiv N(e^-) - N(e^+) + N(\nu_e) - N(\bar{\nu}_e) \tag{2.1}$$

is as defined. In Equation 2.1, $N(e^-)$, $N(e^+)$, $N(\nu_e)$, $N(\bar{\nu}_e)$ are the numbers of electrons, positrons, electron neutrino and anti-electron neutrino present, respectively. For single particle states, $L_e = 1$ for e^- and ν_e, $L_e = -1$ for e^+ and $\bar{\nu}_e$, and $L_e = 0$ for all other particles. Similarly, the following definitions are made for the number of muon L_μ and tau L_τ, respectively [48]:

$$L_\mu \equiv N(\mu^-) - N(\mu^+) + N(\nu_\mu) - N(\bar{\nu}_\mu), \tag{2.2}$$

$$L_\tau \equiv N(\tau^-) - N(\tau^+) + N(\nu_\tau) - N(\bar{\nu}_\tau). \tag{2.3}$$

Six types of leptons and six types, or "flavors" of quarks have been observed to date, and the types and some known properties of these leptons and quarks are given in detail in Table 2.1 and Table 2.2, respectively [49, 50, 6].

There are four types of fundamental interaction (forces) known in nature: weak, electromagnetic, strong, and gravitational interaction. The areas of influence of these forces are different from each other. The ranges of the strong force and the weak

Table 2.1: Properties of the Leptons

Particle	**Symbol**	**Anti-Particle**	**Rest Mass** MeV	**Charge**	**Lifetime (seconds)**
Electron	e^-	e^+	0.511	-1	Stable
Neutrino (Electron)	v_e	$\bar{v}_e$	$0\,(< 7 \times 10^{-6})$	0	Stable
Muon	μ^-	μ^+	105.7	-1	2.20×10^{-6}
Neutrino (Muon)	v_μ	$\bar{v}_\mu$	$0(< 0.27)$	0	Stable
Tau	τ^-	τ^+	1777	-1	2.96×10^{-13}
Neutrino (Tau)	v_τ	$\bar{v}_\tau$	$0(< 31)$	0	Stable

Table 2.2: Properties of the Quarks

Quark	**Symbol**	**Charge**	**Spin**	**Isospin**	**Baryon Number**	**Rest Mass**
up	u	+2/3	1/2	+1/2	1/3	$2.16^{+0.48}_{-0.26}$ MeV
down	d	-1/3	1/2	-1/2	1/3	$4.67^{+0.48}_{-0.17}$ MeV
charm	c	+2/3	1/2	0	1/3	1.27 ± 0.02 GeV
strange	s	-1/3	1/2	0	1/3	$93.4^{+8.6}_{-3.4}$ MeV
top	t	+2/3	1/2	0	1/3	172.69 ± 0.30 GeV
bottom	b	-1/3	1/2	0	1/3	$4.18^{+0.03}_{-0.02}$ GeV

force are very short and have an influence only on the distances shorter than the size of protons.

SM is a gauge theory with the gauge group $SU(3)_C \bigotimes SU(2)_L \bigotimes U(1)_Y$ that defines the fundamental symmetries of particle physics. This gauge group is the union of three separate groups: $U(1)_Y$ associated with the electromagnetic force, $SU(2)_L$

associated with the weak force, and $SU(3)_C$ associated with the strong force. These groups are of great importance for explaining how particles interact for each interaction and for describing how conservation laws arise through symmetries.

The gravitational interaction is the gravitational force to which all objects with mass or energy are exposed in direct proportion to these masses. This interaction, defined scientifically by Isaac Newton in the $17th$ century, is the first force to be studied scientifically. The intermediate vector bosons that carry the gravitational force are the hypothetical gravitons with spin 2 and mass 0. The range of interaction of gravitational interactions is infinite [51, 52].

The electromagnetic interaction takes place between only electrically charged particles, connecting electrons to nuclei to form atoms and then keeping the atoms together, contributing to the formation of molecules and matter. The intermediate vector carrier bosons of the electromagnetic interaction are photons with spin 1 and mass 0. The electromagnetic interaction is an interaction with an infinite interaction range. The field theory of this interaction in particle physics is quantum electrodynamics (QED) [53, 54, 55, 56].

Another way to call the weak force is the nuclear weak interaction. The weak interaction is responsible for the instability of many particles and some atomic nuclei and therefore for radioactive decay. The effective range of the weak interaction is quite short, approximately 10^{-18} m. The weak force is carried through the vector bosons W^+, W^- and Z^0, which were discovered at CERN in 1983 by Carlo Rubbia and Simon van der Meer [57]. $W^+, W^-$and Z^0 vector bosons are particles with a mass of around 90 GeV [58, 59, 60, 61, 62, 63].

The strong interaction is the force between subatomic particles found in the atomic nucleus, thus keeping the nuclei together. The carrier intermediate vector bosons of the interaction are massless gluons that have a color charge. Due to these color charges, the gluons interact with themselves. As its name suggests, it is the strongest of the four fundamental forces and has an interaction range of around 10^{-15} m [64].

SM can successfully explain all of these interactions except gravity [65, 66]. Although there is no definitive information about the gravitational force, the other three fundamental forces arise as a result of the interaction of force-carrying particles (bosons). All fundamental forces have their own carrier particles, and these particles are listed with their properties in Table 2.3.

Table 2.3: Force and force carrier particles and their main properties [1, 2]

Force	**Associated Property**	**Range**	**Carrier Particle**	**Spin**	**Relative Strength**
Gravitational	Mass	Infinite but weakens with distance	Graviton	2	10^{-36}
Electromagnetic	Electric charge	Infinite but weakens with distance	Photon	1	1
Strong	Color charge	$\approx 10^{-15}$ meters (distance between protons in atomic nucleus)	Gluon	1	102
Weak	Weak charge	$\approx 10^{-18}$ meters $(1/1000^{\text{th}}$ proton diameter)	W^+ W^- & Z	1 1 1	10^{-7}

2.2 The Quantum Chromodynamics

QCD is a quantum field theory of strong interactions that describes the interaction of quarks and gluons, which form the building blocks of hadrons. QCD was first proposed by Chen Ning Yang and Robert Mills in the 1950s [67]. In QCD, unlike photons in QED, carrier vector bosons can emit carrier vector bosons outside themselves, and this feature has led to increased interest in research aimed at understanding the nature of the strong interaction. Thus, in 1973, physicists named Murray Gell-Mann, Harald Fritzsch, and Heinrich Leutwyler developed QCD, inspired by the concept of "color" that creates a strong interaction field [68].

In QCD, quark fields are represented by the fundamental representation of the Yang-Mills gauge theory with gauge group $SU(3)_C$, consisting of complex 3-dimensional matrices, while antiquark fields are represented in the conjugate representation. The index "C" here is for the color charge, which is a new quantum number specifically defined for QCD theory. Although the color charges mentioned are known as red (r), $blue$ (b), $green$ (g) for quarks, and their antis for antiquarks, this color concept has no relation with the colors known in daily life and is just a kind of naming [69, 70].

The Lagrangian density of QCD is

$$\mathcal{L} = \bar{\psi}^i_f \left(i\gamma^\mu\right) \left(D_\mu\right)_{ij} \psi^j_f - m_f \bar{\psi}^i_f \psi_{fi} - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}, \tag{2.4}$$

where ψ^i_f represents a quark field as

$$\psi^i_f = \begin{pmatrix} \psi^{\mathrm{r}}_f \\ \psi^{\mathrm{g}}_f \\ \psi^{\mathrm{b}}_f \end{pmatrix},$$

with flavor f, $\psi_f \in \{u, d, s, c, t, b\}$ and color indices i, $i = 1, 2, 3$, corresponding to r, g and b, μ and ν denote Lorentz vector indices with $\mu, \nu = 0, 1, 2, 3$, γ^μ are the Dirac-gamma matrices consisting of 4×4 matrices, m_f are the non-zero quark masses generated by SM Higgs or similar mechanisms, D_μ is the covariant derivative and $G^a_{\mu\nu}$ is the gluon field strength tensor [71]. In QCD, covariant derivative and gluon-field strength tensor are usually defined as

$$(D_\mu)_{ij} = \delta_{ij}\partial_\mu - ig_s T^a_{ij} A^a_\mu, \tag{2.5}$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_s f^{abc} A^b_\mu A^c_\nu, \tag{2.6}$$

where g_s is the strong interaction coupling constant, A^a_μ represents the gluon fields, f^{abc} ($a, b, c \in [1, \ldots, 8]$) are the structure constants and T^a_{ij} represents the generators of the $SU(3)_C$ symmetry group with

$$T^a_{ij} = \frac{1}{2}\lambda^a_{ij} \tag{2.7}$$

where λ^a refers to 3×3 hermitian and traceless Gell-Mann matrices, with a ($a \in [1, \ldots, 8]$), there are 8 independent generators of $SU(3)_C$ and, hence, there are 8 different gluons with different color combinations, corresponding to each generator [71, 72, 73, 74].

The Lagrangian of QCD has non-Abelian local symmetry, and the strong interaction coupling constant, although called a constant, is not actually a constant and varies according to distance (momentum). This means that QCD, unlike other quantum field theories, has two very important new features: asymptotic freedom and confinement [75, 76].

Although the phrase confinement is often used in this way, it is actually "color confinement". Confinement states that colored isolated systems cannot exist and therefore colored particles such as quarks and gluons cannot be observed under normal conditions. Due to the color charges of the gluons that enable the interaction between the quarks, a gluon cloud forms around the quarks. As large distances are reached, these gluon clouds and, therefore, the interactions between the quarks increase. As a result of increasing interactions, quarks behave as if they were imprisoned in hadrons.However, although there is no definitive proof for this, these fields, also called chromoelectric fields, resulting from the color charge between two static quarks, are distributed in tubelike structures. These structures are called "flux tubes" [77, 78, 79, 80, 81, 82]. According to this model, there is a linear potential between static color charges resulting from these tube-like structure charges, which appear to arise naturally. This can be considered as numerical evidence for color confinement [83, 84].

Therefore, the confinement has been seen as a necessity because no quark has been observed in isolation, although many properties of quarks such as their masses, electric charges, and color charges have been determined in many experiments carried out so far. For example, when two quarks are required to be separated from each other in high-energy scattering reactions, the energy of the force fields increases at long distances, and thanks to this increased energy, new quarks are formed from the gap. The initial quarks tend to come together with these new quarks to form hadrons, which do not have a net color charge, and behave more like fundamental entities than the quarks that form them [85].

The asymptotic freedom term is used to describe the behavior of quarks at short distances (high energy or high momentum). As the distance between the quarks decreases, the effect of the gluon clouds, and therefore the interaction between the quarks, weakens, and the quarks behave as if they are free. Since this decrease in interaction changes asymptotically depending on the distance between the quarks, this situation is called "asymptotic freedom" [86, 85]. Asymptotic freedom was first predicted in 1973 by D. Politzer, D. Gross, and F. Wilczek unaware of each other, and they were awarded the Nobel Prize in Physics in 2004.

In QCD, dependence of the strong coupling constant α_s on the monentum is described as [87, 88, 89]:

$$\alpha_s\left(Q^2\right)=\frac{12\pi}{\left(33-2n_f\right)\ln\left(Q^2/\Lambda_{\mathrm{QCD}}^2\right)}. \tag{2.8}$$

Here, n_f is the quark flavor number, Q is the 4-dimensional momentum, and Λ_{QCD} is the QCD energy reference scale. The value of Λ_{QCD} obtained experimentally is approximately $200\ \mathrm{MeV}$. This value is taken as a reference in classifying quarks as heavy and light. Quarks with a mass below this value are classified as light quarks (u, d, s), and quarks with a mass above this value are classified as heavy quarks (c, b, t) [90, 91].

Due to the asymptotic freedom property of QCD, at energies lower than Λ_{QCD} or at long distances, perturbative QCD is no longer useful. The changes of $\alpha_s\left(Q^2\right)$ according to the energy scale Q [GeV] are shown in Figure 2.1. In summary, in QCD, for short distances (or large momentum), perturbative expansion is possible with respect to the running coupling constant α_s. Due to the asymptotic freedom property of this method, the perturbation theory can be used in this region. For long distances (or small momentums), on the other hand, quark-gluon interactions are strong; therefore, non-perturbative effects are important. Thus, a non-perturbative approach is required.

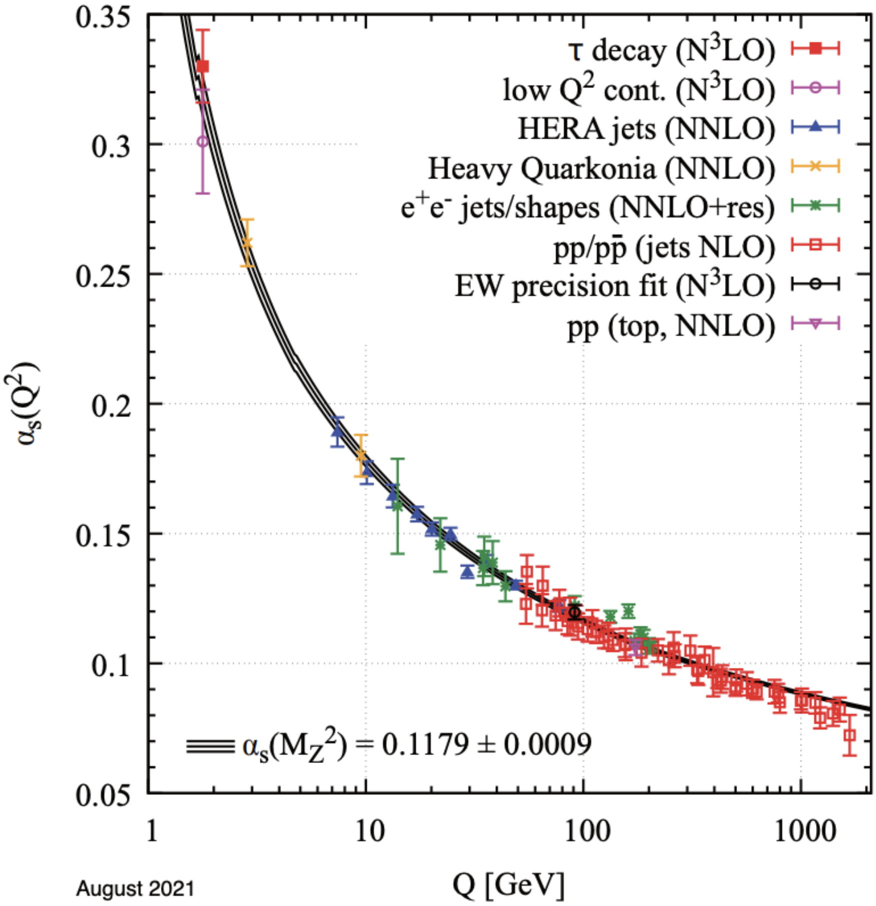

Figure 2.1: Summary of measurements of $\alpha_s(Q^2)$ as a function of the energy scale Q [GeV] [6]

Therefore, non-perturbative models are needed to describe physical quantities at low energy levels.

One of these methods is QCD sum rules. QCD sum rules method is a powerful method that associates QCD parameters with hadronic properties. As one of the most popular methods used to study hadrons, this method can also be used to study exotic particles. Mass is one of the significant properties of the particles and QCD sum rules can be used to study the masses of hadrons.

2.2.1 Heavy-Quark Spin Symmetry

Hadrons consisting of heavy and light quarks have a more simple structure than hadrons that do not contain heavy quarks. The heavy quark is surrounded by the interaction cloud formed by the strong interaction of other particles (light quarks and antiquarks, gluons) in the hadron. In this case, the size of the hadron, which is $\mathcal{O}(1/\Lambda_{QCD})$, is much larger than the Compton wavelength of the heavy quark, $1/m_Q$, so it would be appropriate to make some simplifications.

In the heavy quark limit, the spin of the heavy quark decouples from the dynamics. Hence, systems in which light quarks have the same configuration are degenerate. The difference in such hadrons can be thought of as resulting from the effects $1/m_Q$ due to the heavy quark they contain [92, 93, 94, 95, 96].

For example, considering a hadron containing a heavy quark at speed v, when this heavy quark is replaced by a heavy quark of another flavor or spin at speed u, the configuration of light degrees of freedom of the hadron does not change, because these two heavy quarks have the same color field. Heavy-quark symmetry is only an approximate symmetry. However, quark masses are not actually infinity and some corrections for this symmetry may be needed [97].

CHAPTER 3

THE QCD SUM RULES METHOD

The QCD sum rules method, which is widely used in hadron phenomenology, is a powerful method developed by M. A. Shifman, A. I. Vainshtein and V. I. Zakharov for mesons in 1979 [98] and generalized to baryons by B. L. Ioffe in 1981 [99]. In this method, hadrons are represented by their interpolating currents.

The QCD sum rules method relates QCD parameters such as quark masses, condensates, etc. to hadron properties such as mass, decay constant, form factor, etc. The first step, which is considered as the starting point of this method, is the construction of the interpolating current in terms of quark fields such that it has the same quantum numbers as the hadron under study. Using the interpolating currents, a suitable correlation function is constructed. The two-point correlation function is used to calculate properties such as masses, decay constants, etc. [100].

3.1 Two Point Correlation Function

In order to understand the properties of hadrons in vacuum, quarks are placed in the QCD vacuum at the $x = 0$ space-time point, and their evolution is examined. For this purpose, the following two-point correlation functions can be used for scalar and (axial) vector particles, respectively [101, 102]:

$$\Pi(p^2) = i \int d^4x e^{ipx} \left\langle \Omega \left| \mathcal{T} \left\{ J(x) J^\dagger(0) \right\} \right| \Omega \right\rangle , \tag{3.1}$$

$$\Pi_{\mu\nu}(p^2) = i \int d^4x e^{ipx} \left\langle \Omega \left| \mathcal{T} \left\{ J_\mu(x) J_\nu^\dagger(0) \right\} \right| \Omega \right\rangle , \tag{3.2}$$

where p is the four momentum of the hadron, $|\Omega\rangle$ is the physical non-perturbative hadronic vacuum, $J(x)$ and $J_\mu(x)$ are the interpolating currents for the scalar and axial vectors, respectively, and $\mathcal{T}$ is the time ordering operator.

The time ordering operator reorders the operators in its argument on the basis of the earlier time and for the products of two operators, it is defined as:

$$\begin{aligned} \mathcal{T}\left\{(X\left(x_1\right)Y\left(x_2\right)\right\} &= \begin{cases} \xi Y\left(x_2\right)X\left(x_1\right) & t_1 < t_2 \\ X\left(x_1\right)Y\left(x_2\right) & t_2 < t_1 \end{cases} \\ &= \left[\theta\left(t_2 - t_1\right)\xi Y\left(x_2\right)X\left(x_1\right) + \theta\left(t_1 - t_2\right)X\left(x_1\right)Y\left(x_2\right)\right], \end{aligned} \tag{3.3}$$

where $t_1 = x_1^0, t_2 = x_2^0$ and X, Y are two arbitrary operators, and $\theta(t)$ is the Heaviside step function. The value of ξ changes depending on whether the operators X and Y are fermionic or bosonic. If they are bosonic, then $\xi = 1$, but if they are fermionic, then $\xi = -1$.

For conventional mesons, the interpolating currents in the correlation function are expressed as;

$$J(x) = \bar{\psi}_i^a(x)\Gamma\psi_j^a(x), \tag{3.4}$$

and for conventional baryons, these currents can be expressed as;

$$J(x) = \varepsilon^{abc}\left[\psi_i^a(x)\Gamma_1\psi_j^b(x)\Gamma_2\psi_k^c(x)\right]. \tag{3.5}$$

Here, i, j, k and a, b, c represent the flavor and color of the quarks, respectively. ε^{abc} is the Levi-Civita tensor. Γ and Γ_i can be any of $\mathbb{1}, \gamma_5, \gamma_\mu, \gamma_5\gamma_\mu$ and $\sigma_{\mu\nu}$, which are the Dirac matrices and can also involve derivatives.

The main quantum numbers taken into account in the selection of the interpolating current are the total angular momentum J, parity P, charge conjugation parity C (if the particle under study has C parity) and the flavor quantum number. For conventional conventional mesons, the parity P is $P = (-1)^{l+1}$ and the parity C is $C = (-1)^{l+s}$ where l and s are the orbital and spin angular momentum of the quark-antiquark pair that makes the meson. In terms of l and s, the total spin J of the meson can have any value in the parity range $|l - s| \leq J \leq |l + s|$. Possible interpolating currents without derivatives for mesons according to the relevant quantum numbers of the quark-antiquark pairs are given in Table 3.1. In Table 3.1, the i and j indices

Table 3.1: Interpolating currents of different types of mesons according to their corresponding quantum numbers [3]

Meson	$\mathbf{J^{PC}}$	**S**	**L**	**Hermitian Quark Current Operators**
Pseudoscalar	0^{-+}	0	0	$P_{ij} = \bar{\psi}_j i\gamma^5\psi_i$
Vector	1^{--}	1	0	$V^\mu_{ij} = \bar{\psi}_j\gamma^\mu\psi_i$
Scalar	0^{++}	1	1	$S_{ij} = \bar{\psi}_j\psi_i$
Axial Vector	1^{++}	1	1	$A^\mu_{ij} = \bar{\psi}_j\gamma^5\gamma^\mu\psi_i$

represent the corresponding quark fields.

The correlation function is written in terms of hadronic degrees of freedom in the $p^2 > 0$ region, which is called the hadronic (phenomenological) part, and quark-gluon degrees of freedom in the $p^2 \ll 0$ region, using the Operator Product Expansion (OPE), which is called the QCD (theoretical) part. Then, the physical quantity to be calculated is obtained by matching these two parts using analytic continuity [103].

3.1.1 Hadronic (Phenomenological) Side of the Correlation Function

The value of p^2 determines the behavior of the correlation function. The hadronic side is defined in the $p^2 > 0$ region. In this region, the correlation function can be written in terms of hadronic properties.

On the hadronic side, the correlation function is calculated in terms of hadronic parameters. For this purpose, if the time ordering operator in Equation (3.2) is written explicitly, $\Pi^{Had}\left(p^2\right)$ will take the form

$$\Pi^{Had}\left(p^2\right) = i \int d^4x e^{ipx} \left\langle \Omega \left| \left(\theta\left(x^0\right) J^\dagger(x) J(0) + \theta\left(-x^0\right) J(0) J^\dagger(x) \right) \right| \Omega \right\rangle . \quad (3.6)$$

The resolution of identity in terms of Hamiltonian eigenstates is inserted. Hadron states that have the same quantum numbers as the interpolating current contribute to the correlation function (equation 3.7).

$$\begin{aligned} \mathbb{1} = & \sum_{\substack{\text{Eigenstates} \\ \text{of Hamiltonian}}} |n\rangle\langle n| \\ = & \underbrace{|\Omega\rangle\langle\Omega|}_{\text{vacuum}} + \underbrace{\sum\!\!\!\!\!\!\int |h_1(q_1)\rangle\langle h_1(q_1)|}_{\text{States with one hadron}} + \underbrace{\sum\!\!\!\!\!\!\int |h_1(q_1)h_2(q_2)\rangle\langle h_1(q_1)h_2(q_2)|}_{\text{States with two hadron}} + \underbrace{\cdots\cdots}_{\text{Higher States}} \\ = & |\Omega\rangle\langle\Omega| + \sum\!\!\!\!\!\!\int_h d^4q \frac{1}{(2\pi)^4} (2\pi) \theta\left(q^0\right) \delta\left(q^2 - m_h^2\right) |h(q)\rangle\langle h(q)| + \cdots . \end{aligned} \quad (3.7)$$

Here, $h_1, h_2, ..$ represent the hadron states, m_h represents the mass of the hadron, and also $\sum\!\!\!\!\!\!\int$ represents a sum over all the hadrons and their discrete quantum numbers as well as an integral over their continuous quantum numbers such as their momentum. By inserting equation 3.7 into Equation 3.6 between two operators;

$$\begin{aligned}\Pi^{Had}\left(p^2\right) = i\int d^4x e^{ipx} \sum\!\!\!\!\!\!\int_h \frac{d^4q}{(2\pi)^4}(2\pi)\delta\left(q^2-m_h^2\right)\theta\left(q^0\right) \\ \times\left\{\theta\left(x^0\right)\langle 0|J(x)|h(q)\rangle\left\langle h(q)\left|J^{+}(0)\right|0\right\rangle\right. \\ \left.+\,\theta\left(-x^0\right)\left\langle 0\left|J^{+}(0)\right|h(q)\right\rangle\langle h(q)|J(x)|0\rangle\right\}\end{aligned} \tag{3.8}$$

can be obtained. Here, when $c=\hbar=1$ is used, $J(x)$ can be written in terms of $J(0)$ using the translation operator.

$$J(x) = e^{i\hat{p}x}J(0)e^{-i\hat{p}x}, \tag{3.9}$$

Hence, the matrix elements can be written as:

$$\begin{aligned}\langle 0|J(x)|h(q)\rangle &= \langle 0\,|J(0)|\,h(q)\rangle\, e^{-iqx}, \\ \langle h(q)|J(x)|0\rangle &= e^{iqx}\,\langle h(q)\,|J(0)|\,0\rangle\,.\end{aligned} \tag{3.10}$$

Using equation 3.10, the following result is obtained:

$$\begin{aligned}\Pi^{Had}\left(p^2\right) =& i\sum\!\!\!\!\!\!\int_h d^3x\frac{d^4q}{(2\pi)^4}(2\pi)\delta\left(q^2-m_h^2\right)\theta\left(q^0\right) \\ &\left\{\int_0^\infty dx^0 e^{ipx-iqx}\theta\left(x^0\right)\left\langle\Omega\left|J^\dagger(0)\right|h(q)\right\rangle\langle h(q)|J(0)|\Omega\rangle\right. \\ &\left.+\int_{-\infty}^0 dx^0 e^{ipx+iqx}\theta\left(-x^0\right)\langle\Omega|J(0)|h(q)\rangle\left\langle h(q)\left|J^\dagger(0)\right|\Omega\right\rangle\right\},\end{aligned} \tag{3.11}$$

taking space integrals $\Pi^{Had}\left(p^2\right)$ can be written as

$$\Pi^{Had}\left(p^2\right) = i\int dx_0 e^{ip_0x_0} \sum\!\!\!\!\!\!\int_h \frac{d^4q}{(2\pi)^4}(2\pi)\delta\left(q^2-m_h^2\right)\theta\left(q^0\right)$$

$$\times\left\{\theta\left(x^0\right)e^{-iq_0x_0}(2\pi)^3\delta^3(\vec{p}+\vec{q})\left\langle\Omega\left|J^\dagger(0)\right|h(q)\right\rangle\langle h(q)|J(0)|\Omega\rangle\right. \tag{3.12}$$

$$\left.+\,\theta\left(x^0\right)e^{iq_0x_0}(2\pi)^3\delta^3(\vec{p}-\vec{q})\langle\Omega|J(0)|h(q)\rangle\left\langle h(q)\left|J^\dagger(0)\right|\Omega\right\rangle\right\}.$$

Using the properties of the Dirac-Delta function, the q integrals can be evaluated explicitly:

$$\Pi^{Had}\left(p^2\right) = i\sum_h \frac{1}{2\sqrt{\vec{p}^2+m_h^2}}$$

$$\times\left\{\frac{-1}{i(p^0-q^0+i\epsilon)}\left\langle\Omega\left|J^\dagger(0)\right|h(q^0,-\vec{p})\right\rangle\langle h(q^0,-\vec{p})|J(0)|\Omega\rangle\right. \tag{3.13}$$

$$\left.+\,\langle\Omega|J(0)|h(q^0,\vec{p})\rangle\left\langle h(q^0,\vec{p})\left|J^\dagger(0)\right|\Omega\right\rangle\frac{1}{i(p^0+q^0-i\epsilon)}\right\},$$

where, x^0 integral is also evaluated. To ensure the convergence of x^0 integrals, $\epsilon = 0^+$ are inserted in the exponents. If the quantum numbers of operator J are the same as the relevant meson m and then $h = m$ contributes to the first term and $h = \bar{m}$ contributes to the second term when taking the sum over all the mesons. But if $m = \bar{m}$, $h = m$ contributes to both terms. Therefore, the product of operator matrix elements for particles and antiparticles is equal to each other. Then, Equation 3.13 can be written as

$$\Pi^{Had}\left(p^2\right) = \sum_h \frac{1}{2\sqrt{\vec{p}^2+m_h^2}}\left|\left\langle\Omega\left|J(0)\right|h(q^0,0)\right\rangle\right|^2$$

$$\times\left\{\frac{1}{i(p^0+q^0-i\epsilon)}-\frac{1}{i(p^0-q^0+i\epsilon)}\right\}. \tag{3.14}$$

The sum in Equation 3.14 contains only particles and not antiparticles. In addition, Equation 3.14 is written in the frame where $\vec{p} = 0$. This is possible because this side is for the kinematical region $p^2 > 0$. After further simplifications,

$$\Pi^{Had}\left(p^2\right) = \sum_h \frac{\left|\langle\Omega\left|J(0)\right|h(p)\rangle\right|^2}{m_h^2 - p^2}, \tag{3.15}$$

is obtained by taking the limit $\epsilon \to 0$. In this case, the following expression is found for the correlation function $\Pi^{Had}(p^2)$ for the hadronic side (in the region $p^2 > 0$):

$$\Pi^{Had}\left(p^2\right) = \frac{\left|\langle\Omega\left|J(0)\right|h_G(p)\rangle\right|^2}{m_{h_G}^2 - p^2} + \text{Higher States} + \cdots \tag{3.16}$$

Here, h_G represents the energy of the lowest-mass hadron corresponding to the interpolating current operator.

3.1.2 QCD (Theoretical) Side of the Correlation Function

In order to calculate the QCD part of the correlation function, the Wilson operator product expansion of the time-order product of two or more interpolating currents is calculated.

3.1.2.1 Wilson Operator Product Expansion

OPE was developed by K.G.Wilson in 1969 to separate short and long distances in the relevant physical process [104, 105, 106]. This is achieved by writing the time-order product in the correlation function as:

$$\mathcal{T}\left\{J(x)J^\dagger(0)\right\} \overset{x\to 0}{=} \sum_i C_i\left(x^2\right)O_i, \qquad i = 0, 1, 2, \ldots \quad . \tag{3.17}$$

Here, C_i are the Wilson coefficients, providing information about short distance physics. O_i are the local gauge-invariant operators ordered by dimension, containing contributions from non-perturbative low energies (long distances). The perturbative contribution is provided by the unit operator in the lowest dimension ($d = 0$). Since the QCD vacuum is colorless, the gauge invariant operators for $d = 1, 2$ dimensions do not contribute to the correlation function. O_i operators for higher dimensions up to $i = 6$ are listed in Table 3.2.

Table 3.2: Some non-zero local operators up to 6 dimensions [4]

Operator	**Dimension**
$O_0 = I$ (Unit Operator)	$d = 0$
$O_3 = \bar{\psi}\psi$	$d = 3$
$O_4 = m_\psi \bar{\psi}\psi$	$d = 4$
$O_4 = G^a_{\mu\nu} G^{a\mu v}$	$d = 4$
$O_5 = \bar{\psi}\sigma_{\mu\nu}\frac{\lambda^a}{2}G^{a\mu v}\psi$	$d = 5$
$O_6 = \left(\bar{\psi}\Gamma_r\psi\right)\left(\bar{\psi}\Gamma_s\psi\right)$	$d = 6$
$O_6 = f_{abc}G^a_{\mu\nu}G^{bv}_{\sigma}G^{c\sigma\mu}$	$d = 6$

Here, the term $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$ is defined in terms of gamma matrices.

When Equation 3.17 is inserted into the two-point correlation function in Equation 3.2, it can be expressed as

$$\begin{aligned} \Pi^{OPE}\left(p^2\right) &= i\int d^4x e^{ipx} \sum_i C_i\left(x^2\right)\left\langle \Omega \left| \hat{O}_i \right| \Omega \right\rangle \\ &= \sum_i C_i\left(p^2\right)\left\langle \Omega \left| \hat{O}_i \right| \Omega \right\rangle . \end{aligned} \quad (3.18)$$

The term "$\left\langle \Omega \left| \hat{O}_i \right| \Omega \right\rangle$" is defined as the expected vacuum value, or the condensates, of the QCD operators, characterizing non-perturbative effects.

3.1.3 Analytic Continuity and Dispersion Relation

The correlation function has been expressed in terms of the hadronic parameters for $p^2 > 0$ and in terms of the QCD parameters for $p^2 \ll 0$. Since these results are from different regions, it is not possible to equate them. In order to relate these two expressions, the analytical continuity of the correlation function is used. Since the correlation function is an analytic function of p^2, it can be expressed using the Cauchy formula. Looking at the obtained Equation 3.16; the expression has poles at positive values of p^2 due to the expression $m_{h_G}^2 - p^2$ in the denominator, and also has a brunch cut from a threshold value on the positive axis, due to multi-hadronic states. For a given negative $p^2 < 0$ value using the C_1 contour, as shown in Figure 3.1 the correlation function can be written as:

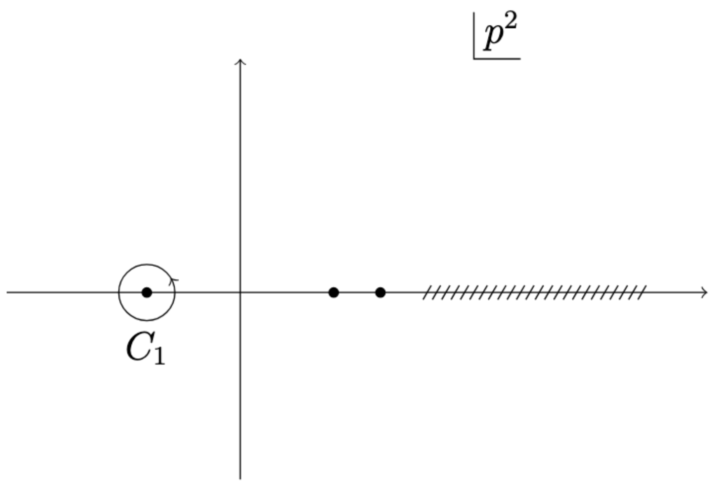

Figure 3.1: C_1 contour in the $z = p^2$ complex plane. The dark points in the figure show the pole points, i.e. the states of the hadrons, and the zigzag lines show the branch cuts.

$$\Pi\left(p^2\right) = \frac{1}{2\pi i}\oint_{C_1} ds \frac{\Pi(s)}{s - p^2}. \tag{3.19}$$

The integrand in Equation 3.19 has a single pole at $s = p^2$, within the contour C_1, its residue is the value of $\Pi(p^2)$. According to complex analysis, the contour can be deformed as desired, as long as no pole or branch cut is crossed. It is possible to deform the contour C_1 and draw it as shown in Figure 3.2, and thus the equivalent of the integral given in Equation 3.19 can be written for the new contour as follows:

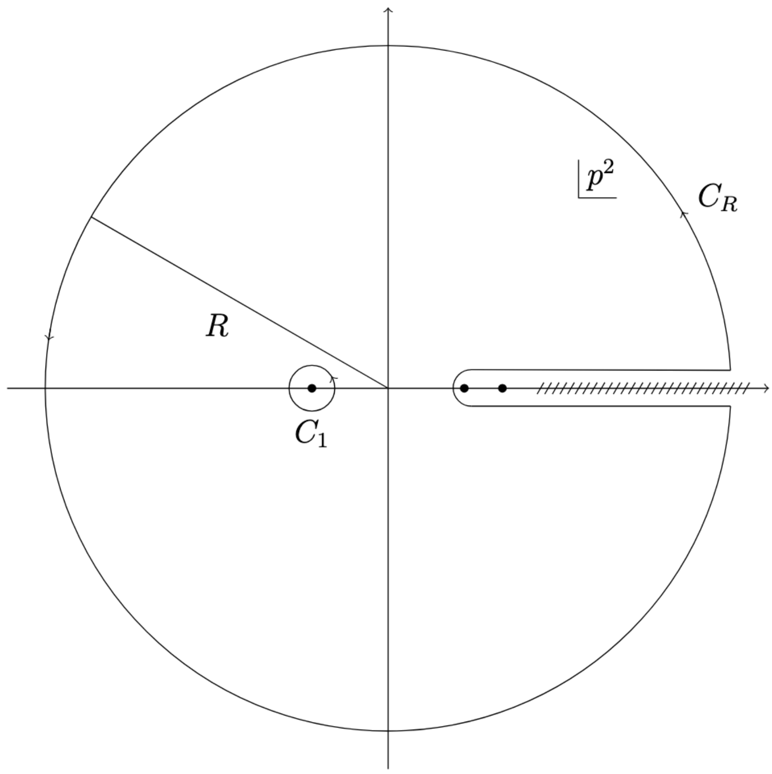

Figure 3.2: C_R contour in the $z = p^2$ complex plane. The dark points in the figure show the pole points, i.e. the states of the hadrons, and the zigzag lines show the brunch cuts.

$$\Pi\left(p^2\right) = \frac{1}{2\pi i} \oint_{C_R} ds \frac{\Pi(s)}{s - p^2}. \tag{3.20}$$

In Figure 3.2, contour can be splitted as:

$$\Pi\left(p^2\right)=\frac{1}{2\pi i}\int_{|s|=R}ds\frac{\Pi(s)}{s-p^2}+\frac{1}{2\pi i}\int_{R}^{s_0^h}ds\frac{\Pi(s-i\epsilon)}{s-p^2-i\epsilon}+\frac{1}{2\pi i}\int_{s_0^h}^{R}ds\frac{\Pi(s+i\epsilon)}{s-p^2+i\epsilon}$$

$$=\frac{1}{2\pi i}\oint_{|s|=R}ds\frac{\Pi(s)}{s-p^2}+\frac{1}{\pi}\int_{s_0^h}^{R}ds\frac{1}{s-p^2}\frac{\Pi(s+i\epsilon)-\Pi(s-i\epsilon)}{2i}, \qquad (3.21)$$

where s_0^h is the threshold for the creation of real states. Taking the limit $\epsilon \to 0$ in Equation 3.21 the Schwarz reflection principle will be used. According to this principle the discontinuity of an analytic function on the positive real axis is equal to its imaginary part if it takes real values on the negative real axis [107, 108, 109]:

$$\Pi(s+i\epsilon)-\Pi(s-i\epsilon)=2i\,\mathrm{Im}\,\Pi(s). \qquad (3.22)$$

When the radius R of the circular part of the contour goes to infinity, the fraction can be Taylor expanded in terms of (p^2/s) as:

$$\frac{1}{s-p^2}=\sum_{n=0}^{\infty}\frac{(p^2)^n}{s^{n+1}}. \qquad (3.23)$$

Using Equation 3.23, Equation 3.21 can be written as

$$\Pi(p^2)=\sum_{n=0}^{\infty}\frac{(p^2)^n}{2\pi i}\int_{|s|\to\infty}ds\frac{\Pi(s)}{s^{n+1}}+\int_{s_0^h}^{\infty}ds\frac{\mathrm{Im}\,\Pi(s)}{\pi}. \qquad (3.24)$$

In the summation over the n value in the first term of the left side of the Equation 3.24, after a certain n value, $\Pi(p^2)$ goes to zero in the $|s|\to\infty$ limit and only a finite polynomial in terms of p^2, called *subtraction terms*, will remain. In this case, the dispersion relation can be expressed as

$$\Pi(p^2) = \frac{1}{\pi}\int_{s_0^h}^{\infty} ds \frac{\mathrm{Im}\,\Pi(s)}{(s-p^2)} + \textit{Subtraction Terms.} \tag{3.25}$$

Here, $\mathrm{Im}\,\Pi(s)/\pi \equiv \rho(s)$ is the spectral density. Using this dispersion relation, $\Pi^{Had}(p^2)$ for positive values of p^2 and $\Pi^{OPE}(p^2)$ at negative values of p^2 can be connected to each other.

The point to be noted about Equation 3.25 is that using this expression, the value of $\Pi(p^2)$ in the $p^2 \ll 0$ region can be calculated using the expression $\mathrm{Im}\,\Pi(s)$ in the $s > 0$ region. Taking $\langle \Omega\,|J(0)|\,h(p)\rangle = \lambda_h$ and using equations 3.16 and 3.25, the following equation can be written for the hadronic side of the correlation function $\Pi^{Had}(p^2)$:

$$\Pi^{Had}\left(p^2\right) = \frac{|\lambda_h|^2}{m_{h_G}^2 - p^2} + \int_{s_0^h}^{\infty} ds \frac{\rho^{Had}(s)}{s-p^2} + \textit{Subtraction Terms}\,. \tag{3.26}$$

Here, $\rho^{Had}(s)$ is the spectral density of the hadronic side that includes the spectral properties of hadrons.

On the other hand, by evaluating the correlation function for the OPE side defined for negative values of p^2 in the large Euclidean deep momentum limit, the following expression is obtained:

$$\Pi^{OPE}\left(p^2\right) = \int_{0}^{\infty} ds \frac{\rho^{\mathrm{OPE}}(s)}{s-p^2} + \textit{Subtraction Terms}\,. \tag{3.27}$$

Here, $\rho^{OPE}(s)$ is the spectral density of the QCD side that includes the spectral properties of quarks and gluons.

The correlation function obtained in two different regions (Equations 3.26 and 3.27) can be matched, and hence the relation to match the results of the obtained correlation function in two different regions, using Equations 3.26 and 3.27,

$$
\int_0^\infty ds \frac{\rho^{\text{OPE}}(s)}{s-p^2} + \textit{Subtraction Terms} = \frac{|\lambda_h|^2}{m_{h_G}^2 - p^2} + \int_{s_0^h}^\infty ds \frac{\rho^{Had}(s)}{s-p^2} + \textit{Subtraction Terms}, \tag{3.28}
$$

can be written. However, it can be seen from Equation 3.28 that it cannot be used to extract useful information due to unknown subtraction terms. These subtraction terms consist of polynomials in p^2 whose degree is unknown. By taking an infinite number of derivatives, any polynomial can be eliminated. This can be achieved by Borel transform in Equation 3.28.

3.1.4 Borel Transformation

The Borel transformation is defined as [110]:

$$
\Pi\left(M^2\right) = \hat{\mathcal{B}}_{M^2}\left[\Pi\left(p^2\right)\right] = \lim_{\substack{-p^2, n\to\infty \\ -p^2/n \to M^2}} \frac{\left(-p^2\right)^{n+1}}{n!} \left(\frac{d}{dp^2}\right)^n \Pi\left(p^2\right). \tag{3.29}
$$

As mentioned above, the Borel transformation eliminates polynomials of unknown degree by taking infinite derivatives. It also suppresses the contribution of higher states by converting the expression $\frac{1}{s-p^2} \to e^{-s/M^2}$.

Using the definition of Borel transformations, some explicit transformations are obtained as follows [111].

$$\hat{\mathcal{B}}_{M^2}\left[\left(-p^2\right)^t\right]=0 \text{ for } t\geqslant 0, \tag{3.30}$$

$$\hat{\mathcal{B}}_{M^2}\left[\frac{1}{\left(-p^2\right)^t}\right]=\frac{1}{(t-1)!}\left(\frac{1}{M^2}\right)^{t-1}, \tag{3.31}$$

$$\hat{\mathcal{B}}_{M^2}\left[\frac{1}{(m^2-p^2)^t}\right]=\frac{1}{(t-1)!}\frac{e^{-m^2/M^2}}{(M^2)^{t-1}}, \tag{3.32}$$

$$,\hat{\mathcal{B}}_{M^2}\left[\left(-p^2\right)^t\log\left(-p^2/\Lambda^2\right)\right]=t!\left(-M^2\right)^{t+1}. \tag{3.33}$$

Considering all these situations, when the Borel transformation is applied to the phenomenological and OPE parts of the correlation function, the corresponding results are obtained. For the phenomenological part,

$$\hat{\mathcal{B}}_{M^2}\left[\Pi^{Had}\left(p^2\right)\right]=\lambda_h^2 e^{-m_h^2/M^2}+\int_{s_0^h}^{\infty} ds\rho^{had}(s)e^{-s/M^2} \tag{3.34}$$

and for the OPE part,

$$\hat{\mathcal{B}}_{M^2}\left[\Pi^{OPE}\left(p^2\right)\right]=\int_0^{\infty} ds\rho^{\mathrm{OPE}}(s)e^{-s/M^2}. \tag{3.35}$$

Hence, Equation 3.28 becomes

$$\lambda_h^2 e^{-m_h^2/M^2}+\int_{s_0^h}^{\infty} ds\rho^{had}(s)e^{-s/M^2}=\int_0^{\infty} ds\rho^{\mathrm{OPE}}(s)e^{-s/M^2}. \tag{3.36}$$

According to the local quark-hadron duality assumption, the integral on the left hand side of Equation 3.36 can be written in terms of ρ^{OPE} as [112],

$$\int_{s_0^h}^{\infty} ds\rho^{Had}(s)e^{-s/M^2} = \int_{s_0}^{\infty} ds\rho^{\mathrm{OPE}}(s)e^{-s/M^2}, \tag{3.37}$$

where s_0 is called the continuum threshold. Substituting Equation 3.37 into Equation 3.36, the sum rules is obtained as

$$\lambda_h^2 e^{-m_h^2/M^2} = \int_0^{s_0} ds\rho^{\mathrm{OPE}}(s)e^{-s/M^2}. \tag{3.38}$$

Using this expression, the mass of the hadron in the ground state can be calculated by taking the derivative of Equation 3.38 with respect to the parameter $-1/M^2$ and then dividing it by Equation 3.38:

$$m_h^2 = \frac{\int_0^{s_0} dss\rho^{\mathrm{OPE}}(s)e^{-s/M^2}}{\int_0^{s_0} ds\rho^{\mathrm{OPE}}(s)e^{-s/M^2}}. \tag{3.39}$$

Once m_h^2 is obtained, it can be used in Equation 3.39 to calculate the decay constant λ_h^2 as

$$\lambda_h^2 = e^{m_h^2/M^2} \int_0^{s_0} ds\rho^{\mathrm{OPE}}(s)e^{-s/M^2}. \tag{3.40}$$

CHAPTER 4

QCD SUM RULES FOR THE MASS OF TETRAQUARKS

4.1 THE MASS OF T_{cc}^{+} TETRAQUARK

This section consists of detailed calculations to obtain the mass of the ground state of the T_{cc}^{+} tetraquark. As mentioned in the Chapter 3, application of QCD sum rules to calculate the mass of the given hadron follows three important steps: The first is the calculation of the hadronic side or the phenomenological side used to express the correlation function in hadronic degrees of freedom in the $p^2 > 0$ region. In the second step, the correlation function is expressed in terms of the gluon and quark properties in the $p^2 \ll 0$ region and is called the QCD side or the theoretical side. In the final step, these two expressions are matched using analytical continuity.

Then, using Equation 3.2, the two-point correlation function for the T_{cc}^{+} tetraquark can be written as:

$$\Pi_{\mu\nu,T_{cc}^{+}}\left(p^2\right) = i\int d^4x e^{ipx}\left\langle \Omega \left| \mathcal{T}\left\{ J_{\mu}^{T_{cc}^{+}}(x) J_{\nu}^{T_{cc}^{+}\dagger}(0) \right\} \right| \Omega \right\rangle . \tag{4.1}$$

In order to calculate Equation 4.1, it is first necessary to determine the interpolating current. The spin-parity quantum numbers of the state T_{cc}^{+} tetraquark is determined as $J^P = 1^+$ and the measured mass of the T_{cc}^{+} tetraquark is located at $(-273 \pm$

$61\pm\ 5^{+11}_{-14})$ keV just below the D^0D^{+*} mass threshold [41, 42]. For this reason, the molecular picture is quite attractive for studying the properties of the T^+_{cc} state. Thus, considering the quantum flavor contents and spin numbers of the D mesons, where $D^\circ\ :\ \mathrm{c\bar{u}}\,(0)$, $D^+\ :\ c\bar{d}\,(0)$, $D^{*+}\ :\ c\bar{d}\,(1)$, $D^{*0}\ :\ \mathrm{c\bar{u}}\,(1)$, the following equation is obtained.

$$
\begin{aligned}
\left|D^0D^{*+}-D^+D^{*0}\right\rangle &\equiv |I=0, \quad I_3=0\rangle \\
\left|D^0D^{*+}-D^+D^{*0}\right\rangle &\equiv |(c\bar{u})\,(0)\rangle\otimes\left|\left(c\bar{d}\right)(1)\right\rangle-\left|\left(c\bar{d}\right)(0)\right\rangle\otimes|(c\bar{u})\,(1)\rangle \\
&\equiv |c_\uparrow\bar{u}_\downarrow-c_\downarrow\bar{u}_\uparrow\rangle\otimes\left|c_\uparrow\bar{d}_\uparrow\right\rangle-\left|c_\uparrow\bar{d}_\downarrow-c_\downarrow\bar{d}_\uparrow\right\rangle\otimes|c_\uparrow\bar{u}_\uparrow\rangle \\
&\equiv \left|c_\uparrow\bar{u}_\downarrow c_\uparrow\bar{d}_\uparrow\right\rangle-\left|c_\downarrow\bar{u}_\uparrow c_\uparrow\bar{d}_\uparrow\right\rangle-\left|c_\uparrow\bar{d}_\downarrow c_\uparrow\bar{u}_\uparrow\right\rangle+\left|c_\downarrow\bar{d}_\uparrow c_\uparrow\bar{u}_\uparrow\right\rangle \\
&\equiv c_\uparrow c_\uparrow\left(\bar{u}_\downarrow\bar{d}_\uparrow-\bar{u}_\uparrow\bar{d}_\downarrow\right)+\bar{u}_\uparrow\bar{d}_\uparrow\left(c_\uparrow c_\downarrow-c_\downarrow c_\uparrow\right) \\
\left|D^0D^{*+}-D^+D^{*0}\right\rangle &\equiv -\left(cc\right)(1)\left(\bar{u}\bar{d}\right)(0)+\left(\bar{u}\bar{d}\right)(1)\left(cc\right)(0)
\end{aligned}
\tag{4.2}
$$

As can be seen from Equation 4.2, in the molecular picture, there are two cases: One of them, the total spin of the heavy quarks is 1, and the total spin of the light quarks is 0 and the other, the total spin of the heavy quarks is 0 and the total spin of the heavy quarks is 1. In these cases, when the total spin of the heavy quarks is 0, the color wave function must be symmetric, since the wave function of the diquark must be antisymmetric according to the Pauli Exclusion Principle. However, since color interactions are not attractive in the symmetric case, the probability of the symmetric wavefunction forming a diquark is low [113, 114, 115, 116]. Therefore, in this study, the construction of the interpolating current, the component for which the heavy quarks have total spin 1 is considered.

Taking into account the fact that the observed particle has positive parity, $P=+$, the interpolating current $J^{T^+_{cc}}_\mu(x)$ for the T^+_{cc} tetraquark can be written as follows:

$$J_{\mu}^{T_{cc}}(x) = \left[\left(c^{aT} C \gamma_{\mu} c^{b}\right)(x)\left(\bar{u}^{c} C \gamma_{5} \bar{d}^{dT}\right)(x)\right] \varepsilon^{abe} \varepsilon^{cde}, \tag{4.3}$$

where a, b, c are colors, ε^{abc} is the Levi-Civita tensor, γ_5, and γ_μ are the Dirac matrices, u, d and c represent u quark, d quark, c quark fields, respectively, and C is charge conjugation operator.

4.1.1 Hadronic Side of the T_{cc}^{+} Tetraquark Correlation Function

To calculate the hadronic representation of the T_{cc}^{+} tetraquark correlation function, starting from Equation 3.16, the following equation is obtained.

$$\Pi_{\mu\nu, T_{cc}^{+}}^{Had}\left(p^{2}\right) = \sum_{h} \frac{\left\langle \Omega \left| J_{\mu}^{T_{cc}^{+}} \right| h(p) \right\rangle \left\langle h(p) \left| J_{\nu}^{T_{cc}^{+}\dagger} \right| \Omega \right\rangle}{p^{2} - m_{h}^{2}} + \cdots$$

$$= \sum_{0} \frac{\left\langle \Omega \left| J_{\mu}^{T_{cc}^{+}} \right| 0^{+}(p) \right\rangle \left\langle 0^{+}(p) \left| J_{\nu}^{T_{cc}^{+}\dagger} \right| \Omega \right\rangle}{p^{2} - m_{0}^{2}+}$$

$$+ \sum_{1} \frac{\left\langle \Omega \left| J_{\mu}^{T_{cc}^{+}} \right| 1^{+}(\varepsilon, p) \right\rangle \left\langle 1^{+}(\varepsilon, p) \left| J_{\nu}^{T_{cc}^{+}\dagger} \right| \Omega \right\rangle}{p^{2} - m_{1^{+}}^{2}} + \cdots . \tag{4.4}$$

Note that the chosen current can couple to states with $S = 0$ as well as states with $S = 1$. Using the following matrix elements,

$$\left\langle S = 0^{+}(p) \left| J_{\nu}^{T_{cc}^{+}\dagger} \right| \Omega \right\rangle = \lambda_{0}^{+} p_{\nu}, \tag{4.5}$$

$$\left\langle S = 1^{+}(\varepsilon, p) \left| J_{\nu}^{T_{cc}^{+}\dagger} \right| \Omega \right\rangle = \lambda_{1}^{+} \varepsilon_{\nu}^{*}, \tag{4.6}$$

in Equation 4.4, the hadronic representation becomes:

$$\Pi^{Had}_{\mu\nu,T^+_{cc}}\left(p^2\right) = \frac{|\lambda_{0^+}|^2 p_\mu p_\nu}{p^2 - m^2_{0^+}} + \frac{|\lambda_{1^+}|^2 \sum_s \varepsilon^*_\mu \varepsilon_\nu}{p^2 - m^2_{1^+}}. \tag{4.7}$$

In Equation 4.7, only the contributions of the lowest-mass scalar and axial-vector mesons are explicitly stated, and other terms are not shown. Then using the sum over all polarizations,

$$\sum_s \varepsilon^*_\mu(p)\varepsilon_\nu(p) = -g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}, \tag{4.8}$$

Equation 4.7 is expressed as:

$$\Pi^{Had}_{\mu\nu,T^+_{cc}}\left(p^2\right) = \frac{|\lambda_{0^+}|^2 p_\mu p_\nu}{p^2 - m^2_{0^+}} + \frac{|\lambda_{1^+}|^2 \left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}\right)}{p^2 - m^2_{1^+}}. \tag{4.9}$$

If the definitions

$$\tilde{\Pi}^0_{T^+_{cc}}\left(p^2\right) \equiv \sum \frac{\lambda^2_{0^+}}{p^2 - m^2_{0^+}}, \tag{4.10}$$

and

$$\tilde{\Pi}^1_{T^+_{cc}}\left(p^2\right) = \sum \frac{\lambda^2_{1^+}}{p^2 - m^2_{1^+}}, \tag{4.11}$$

are used, Equation 4.9 can be written as:

$$\Pi^{Had}_{\mu\nu,T^+_{cc}}\left(p^2\right) = \tilde{\Pi}^0_{T^+_{cc}}\left(p^2\right) p_\mu p_\nu + \tilde{\Pi}^1_{T^+_{cc}}\left(p^2\right)\left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}\right). \tag{4.12}$$

Note that two masses of axial vectors appear as the poles of $\tilde{\Pi}^1_{T^+_{cc}}\left(p^2\right)$, which is the coefficient of $-g_{\mu\nu}$ in the correlation function. Hence, only $\Pi^1_{T^+_{cc}}\left(p^2\right)$ will be studied.

4.1.2 Theoretical Side of the T_{cc}^{+} Tetraquark Correlation Function

To obtain the expression of the correlation function in terms of the QCD parameters, first the interpolating current expression in Equation 4.3 is inserted into the correlation function in Equation 4.1.

$$\Pi_{\mu\nu,T_{cc}^{+}}\left(p^2\right) = i\int d^4x e^{ipx}\left\langle\Omega\left|\mathcal{T}\left\{\left[\left(c^{aT}C\gamma_\mu c^b\right)(x)\left(\bar{u}^c C\gamma_5\bar{d}^{dT}\right)(x)\right]\ \varepsilon^{abe}\varepsilon^{cde}\right.\right.\right.$$

$$\times\ \left.\left.\left.\left[\left(d^{d'}\gamma_5 C u^{c'T}\right)(0)\left(\bar{c}^{b'}\gamma_\nu C\bar{c}^{a'T}\right)(0)\right]\varepsilon^{a'b'e'}\varepsilon^{c'd'e'}\right\}\right|\Omega\right\rangle. \tag{4.13}$$

Using Wick's theorem, time-ordered products can be written in terms of normal-order products as follow:

$$\begin{aligned}
\Pi_{\mu\nu,T_{cc}^{+}}\left(p^2\right) &= i\int d^4x e^{ipx}\varepsilon^{abe}\varepsilon^{cde}\varepsilon^{a'b'e'}\varepsilon^{c'd'e'}\\
&\times\left\{\left\langle\Omega\right|:\left\{[(c^{aT}C\gamma_\mu c^b)(x)(\bar{u}^c C\gamma_5\bar{d}^{dT})(x)][(d^{d'}\gamma_5 C u^{c'T})(0)(\bar{c}^{b'}\gamma_\nu C\bar{c}^{a'T})(0)\right\}:\left|\Omega\right\rangle\right.\\
&+\left\langle\Omega\right|:\left\{[(c^{aT}C\gamma_\mu c^b)(x)(\bar{u}^c C\gamma_5\,\bar{d}^{dT})(x)][(d^{d'}\,\gamma_5 C u^{c'T})(0)(\bar{c}^{b'}\gamma_\nu C\bar{c}^{a'T})(0)\right\}:\left|\Omega\right\rangle\\
&+\left\langle\Omega\right|:\left\{[(c^{aT}C\gamma_\mu\,c^b)(x)(\bar{u}^c C\gamma_5\,\bar{d}^{dT})(x)][(d^{d'}\,\gamma_5 C u^{c'T})(0)(\bar{c}^{b'}\gamma_\nu C\bar{c}^{a'T})(0)\right\}:\left|\Omega\right\rangle\\
&+\left\langle\Omega\right|:\left\{[(c^{aT}C\gamma_\mu\,c^b)(x)(\bar{u}^c C\gamma_5\,\bar{d}^{dT})(x)][(d^{d'}\gamma_5 C u^{c'T})(0)(\bar{c}^{b'}\,\gamma_\nu C\bar{c}^{a'T})(0)\right\}:\left|\Omega\right\rangle\\
&+\left\langle\Omega\right|:\left\{[(c^{aT}C\gamma_\mu\,c^b)(x)(\bar{u}^c C\gamma_5\,\bar{d}^{dT})(x)][(d^{d'}\,\gamma_5 C u^{c'T})(0)(\bar{c}^{b'}\,\gamma_\nu C\bar{c}^{a'T})(0)\right\}:\left|\Omega\right\rangle\\
&+\left.\text{(all possible contractions)}\right\}
\end{aligned} \tag{4.14}$$

Here, the sign "$\sqcap$" represents the contractions of the relevant quarks, and only one example of each of the situations involving one contraction, two contractions, three contractions, and four contractions is shown. The "$\langle\Omega| \; : \quad : \; |\Omega\rangle$" notation is used to denote normal ordering in quantum field theory and indicates that the creation operators precede the annihilation operators. For example, if a and b represent the annihilation operators, and $a^\dagger$ and $b^\dagger$ represent the creation operators, normal ordering is defined as:

$$\left\langle\Omega\left|: a a^{\dagger} b b^{\dagger}:\right| \Omega\right\rangle=\eta^{N}\left\langle\Omega\left|a^{\dagger} b^{\dagger} a b\right| \Omega\right\rangle . \tag{4.15}$$

Here, N is the number of commutations required to convert the original ordering to the final ordering, $\eta = -1$ and $\eta = 1$ are used for the fermionic and bosonic operators, respectively. In cases where there are c quarks that are not contracted with each other, the corresponding terms are equal to zero since there are no heavy quarks in the QCD vacuum.

Contributions to the correlation function in Equation 4.14 include perturbative and non-perturbative contributions. Situations in which all quark fields are contracted are called perturbative contributions. In Figure 4.1, the mentioned contributions taken into account in this study are shown schematically in terms of Feynman diagrams.

In Figure 4.1, the diagram (a) corresponds to the perturbative part which is the contribution of the zero-dimensional operator in OPE. The other diagrams show the contributions from non-perturbative parts sorted according to the mass dimension of the corresponding operator in OPE. The diagram (b) shows the contribution of $d = 3$ operator containing the light quark propagator $\langle\bar{q}q\rangle$, the diagrams (c1) and (c2) show the contribution of two-gluon operator's containing $\langle g_s^2 G^2\rangle$ condensate, the diagrams (d1) and (d2) diagrams show the contribution of 5-dimensional operator consisting of the light quark-gluon mixture containing $\langle \bar{q} q g_s G \rangle$ condensate, the diagram (e) shows the contribution of the operator, two light quarks consisting of $\langle \bar{q} q \bar{q} q \rangle$ condensates. The contributions of the 8-, 10-, 12- and 14- dimensional mixed condensates are also represented in Figure 4.1 by diagrams (f1) and (f2), (g1) and (g2), (h1) and (h2), and (i1) and (i2), respectively.

For the T_{cc}^{+} tetraquark, there are two terms with four contractions forming the pertubative part. One of these terms, which is also present in Equation 4.14, can be written as

$$\Pi^{4,1}_{\mu\nu,T_{cc}^{+}}(p^2) = i \int d^4x e^{ipx} \varepsilon^{abe} \varepsilon^{cde} \varepsilon^{a'b'e'} \varepsilon^{c'd'e'}$$

$$\left\langle \Omega \middle| : \left\{ [(c^{aT} C\gamma_\mu c^b)(x)(\bar{u}^c C\gamma_5 \bar{d}^{dT})(x)][(d^{d'} \gamma_5 C u^{c'T})(0)(\bar{c}^{b'} \gamma_\nu C \bar{c}^{a'T})(0)] \right\} : \middle| \Omega \right\rangle. \tag{4.16}$$

Expressing the matrix products in terms of summations over spinor indices, Equation 4.16 can be written as:

$$\Pi^{4,1}_{\mu\nu,T_{cc}^{+}}(p^2) = i \int d^4x e^{ipx} (C\gamma_\mu)_{ij} (\gamma_\nu C)_{kl} (C\gamma_5)_{\alpha\beta} (\gamma_5 C)_{\gamma\delta} \varepsilon^{abe} \varepsilon^{cde} \varepsilon^{a'b'e'} \varepsilon^{c'd'e'}$$

$$\times \left\langle \Omega \middle| : \left\{ \left[(c_i^{aT} c_j^b)(x)(\bar{c}_k^{b'} \bar{c}_l^{a'T})(0) \right] \left[(\bar{u}_\alpha^c \bar{d}_\beta^{dT})(x)(d_\gamma^{d'} u_\delta^{c'T})(0) \right] \right\} : \middle| \Omega \right\rangle, \tag{4.17}$$

where the number "4" in the superscript "4, 1" represents terms that contain four contractions, and the number 1 represents the first of the terms with four contractions. To evaluate Equation 4.17, the following definitions are needed.

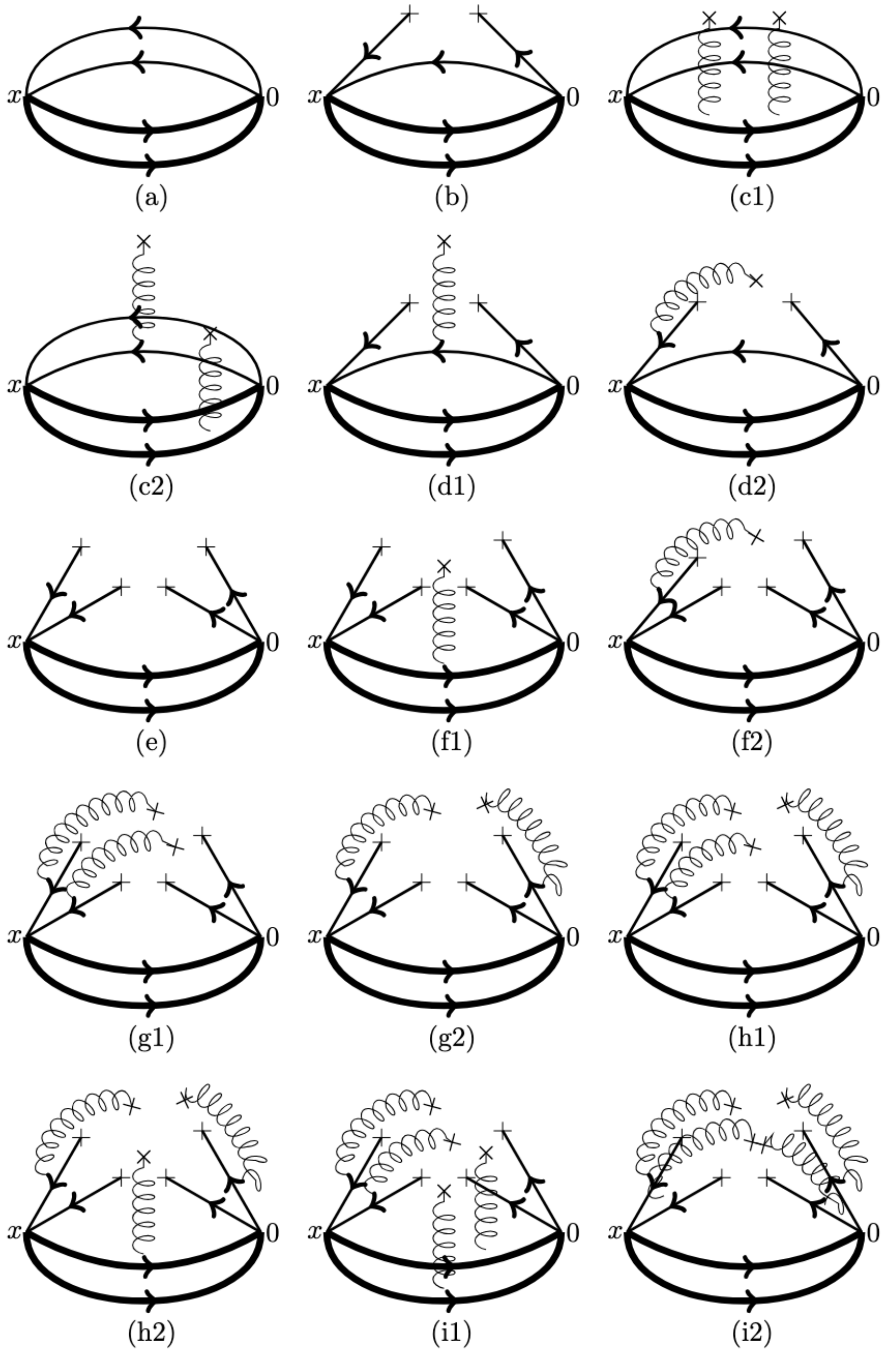

Figure 4.1: Feynman diagrams of the contributions to the correlation function considered in the T_{cc}^{+} tetraquark mass calculation are given by selecting only one of those expressing similar cases. Here, the thick(thin) lines represent the corresponding heavy(light) quarks and the spirals represent the soft gluons.

$$
\begin{aligned}
&\langle\Omega|\, \overbracket{c_i^{aT}(x)\,\bar{c}_l^{a'T}}(0)|\Omega\rangle \equiv iS_{il}^c(x)\delta^{aa'},\\
&\langle\Omega|\, \overbracket{c_j^{b}(x)\,\bar{c}_k^{b'}}\,(0)\,|\Omega\rangle \equiv iS_{jk}^c(x)\delta^{bb'},\\
&\langle\Omega|\, \overbracket{\bar{u}_\alpha^{c}(x)\,u_\delta^{c'T}}\,(0)\,|\Omega\rangle \equiv i(-1)S_{\delta\alpha}^u(-x)\delta^{cc'},\\
&\langle\Omega|\, \overbracket{\bar{d}_\beta^{dT}(x)\,d_\gamma^{d'}}\,(0)\,|\Omega\rangle \equiv i(-1)S_{\gamma\beta}^d(-x)\delta^{dd'},
\end{aligned}
\tag{4.18}
$$

where $S_{\delta\alpha}^u(x)$ and $S_{\gamma\beta}^d(-x)$ are light-quark propagators and $S_{il}^c(x)$ and $S_{jk}^c(x)$ are heavy-quark propagators, and they are generally defined as

$$
iS_{\alpha\beta}^{q,ab}(x) \equiv \left\langle\Omega\left|\mathcal{T}\left\{q_\alpha^a(x)\bar{q}_\beta^b(0)\right\}\right|\Omega\right\rangle \tag{4.19}
$$

for the light quarks and

$$
iS_{\alpha\beta}^{Q,ab}(x) \equiv \left\langle\Omega\left|\mathcal{T}\left\{Q_\alpha^a(x)\bar{Q}_\beta^b(0)\right\}\right|\Omega\right\rangle \tag{4.20}
$$

for the heavy quarks. These propagators include perturbative and non-perturbative contributions. The parts of these propagators that contain only perturbative contributions are indicated with the subscript free $S_{free}^q(x)$, and they are expressed with Equations 4.21 and 4.22 for light and heavy quarks [103, 117, 110], respectively.

$$
S_{free}^q(x) = \left[\frac{i\,\not{x}}{2\pi^2 x^4} - \frac{m_q}{4\pi^2 x^2}\right], \tag{4.21}
$$

$$
S_{free}^Q(x) = \left[\frac{m_Q^2}{4\pi^2}\frac{K_1\left(m_Q\sqrt{-x^2}\right)}{\sqrt{-x^2}} - i\frac{m_Q^2\,\not{x}}{4\pi^2 x^2}K_2\left(m_Q\sqrt{-x^2}\right)\right]. \tag{4.22}
$$

Here, $\not{x} = x_\alpha\gamma^\alpha$ and $K_n\left(m_Q\sqrt{-x^2}\right)$ are the modified Bessel functions of the second kind, which can be expressed as [118]:

$$K_n\left(m_Q\sqrt{-x^2}\right)=\frac{(m_Q)^n}{\left(-x^2\right)^{n/2}2^{n+1}}\int_0^\infty dtt^{(-n-1)}\exp\left[-\frac{m_Q^2}{4t}+tx^2\right]. \tag{4.23}$$

The propagators are inserted into Equation 4.16 to obtain:

$$\begin{aligned}\Pi^{4,1}_{\mu\nu,T_{cc}^+}\left(p^2\right)&=i\int d^4xe^{ipx}(C\gamma_\mu)_{ij}(\gamma_\nu C)_{kl}(C\gamma_5)_{\alpha\beta}(\gamma_5C)_{\gamma\delta}\varepsilon^{abe}\varepsilon^{cde}\varepsilon^{a'b'e'}\varepsilon^{c'd'e'}\\&\times S^c_{il}(x)\delta^{aa'}S^c_{jk}(x)\delta^{bb'}(-1)S^u_{\delta\alpha}(-x)\delta^{cc'}(-1)S^d_{\gamma\beta}(-x)\delta^{dd'}\\&=i\int d^4xe^{ipx}\delta^{aa'}\delta^{bb'}\delta^{cc'}\delta^{dd'}\varepsilon^{abe}\varepsilon^{cde}\varepsilon^{a'b'e'}\varepsilon^{c'd'e'}\\&\times\left[S^c_{il}(x)(\gamma_\nu C)^T_{lk}S^{cT}_{kj}(x)(C\gamma_\mu)^T_{ji}\right]\left[S^u_{\delta\alpha}(-x)(C\gamma_5)_{\alpha\beta}S^{dT}_{\beta\gamma}(-x)(\gamma_5C)_{\gamma\delta}\right].\end{aligned} \tag{4.24}$$

The following expressions are needed to evaluate the last equation.

$$\varepsilon^{abc}\varepsilon^{abc'}=2!\delta^{cc'}, \tag{4.25}$$

$$\delta^{cc'}\delta^{cc'}=3, \tag{4.26}$$

$$\left[S^c_{il}(x)(\gamma_\nu C)^T_{lk}S^{cT}_{kj}(x)(C\gamma_\mu)^T_{ji}\right]=Tr\left[S^c(x)(\gamma_\nu C)^TS^{cT}(x)(C\gamma_\mu)^T\right], \tag{4.27}$$

$$\left[S^u_{\delta\alpha}(-x)(C\gamma_5)_{\alpha\beta}S^{dT}_{\beta\gamma}(-x)(\gamma_5C)_{\gamma\delta}\right]=Tr\left[S^u(-x)(C\gamma_5)S^{dT}(-x)(\gamma_5C)\right]. \tag{4.28}$$

Using these equations, Equation 4.24 can be rewritten as

$$\begin{aligned}\Pi^{4,1}_{\mu\nu,T_{cc}^+}\left(p^2\right)&=12i\int d^4xe^{ipx}\\&\times Tr\left[S^c(x)(\gamma_\nu C)^TS^{cT}(x)(C\gamma_\mu)^T\right]Tr\left[S^u(-x)(C\gamma_5)S^{dT}(-x)(\gamma_5C)\right].\end{aligned} \tag{4.29}$$

Equation 4.29 can be simplified using the properties of the charge conjugation matrix C given in the Appendix A.0.2:

$$\Pi^{4,1}_{\mu\nu,T^{+}_{cc}}\left(p^2\right) = 12i\int d^4x e^{ipx}$$
$$\times Tr\left[S^c(x)\gamma_\nu S^c(-x)\gamma_\mu\right] Tr\left[S^u(-x)(C\gamma_5)S^{dT}(-x)(\gamma_5 C)\right]. \quad (4.30)$$

In order to simplify the notation, the following definitions are used:

$$A_0^Q\left(x^2\right) \equiv \left[\frac{m_Q^2}{4\pi^2}\frac{K_1\left(m_Q\sqrt{-x^2}\right)}{\sqrt{-x^2}}\right], \qquad B_0^Q\left(x^2\right) \equiv -i\left[\frac{m_Q^2}{4\pi^2 x^2}K_2\left(m_Q\sqrt{-x^2}\right)\right], \quad (4.31)$$

$$A_0^q\left(x^2\right) \equiv -\left[\frac{m_q}{4\pi^2 x^2}\right], \qquad B_0^q\left(x^2\right) \equiv \left[\frac{i}{2\pi^2 x^4}\right]. \quad (4.32)$$

The expressions of propagators in terms of these definitions are given as:

$$S^Q_{free}(x) = A_0^Q\left(x^2\right) + B_0^Q\left(x^2\right)\not{x}, \quad (4.33)$$

$$S^q_{free}(x) = A_0^q\left(x^2\right) + B_0^q\left(x^2\right)\not{x}. \quad (4.34)$$

In terms of these newly defined functions, Equation 4.30 can be written as:

$$\Pi^{4,1}_{\mu\nu,T^{+}_{cc}}\left(p^2\right) = -12i\int d^4x e^{ipx}$$
$$\times Tr\left[(A_0^c\left(x^2\right) + B_0^c\left(x^2\right)\not{x})\gamma_\nu\left(A_0^c\left(x^2\right) - B_0^c\left(x^2\right)\not{x}\right)\gamma_\mu\right]$$
$$\times Tr\left[(A_0^u\left(x^2\right) - B_0^u\left(x^2\right)\not{x})(C\gamma_5)(A_0^d\left(x^2\right) - B_0^d\left(x^2\right)\not{x})(\gamma_5 C)\right] \quad (4.35)$$

Here, taking advantage of the properties of the gamma matrices given in Appendix A.0.1, the results for the traces are obtained as:

$$Tr\left[(A_0^c\left(x^2\right)+B_0^c\left(x^2\right)\ \not{x})\gamma_\nu\left(A_0^c\left(x^2\right)-B_0^c\left(x^2\right)\ \not{x}\right)\gamma_\mu\right]=
4\left(A_0^c\left(x^2\right)^2+B_0^c\left(x^2\right)^2x^2\right)g_{\mu\nu}-8B_0^c\left(x^2\right)^2x_\nu x_\mu, \quad (4.36)$$

$$Tr\left[(A_0^u\left(x^2\right)-B_0^u\left(x^2\right)\ \not{x})(C\gamma_5)(A_0^d\left(x^2\right)-B_0^d\left(x^2\right)\ \not{x}^T)(\gamma_5 C)\right]=
-4A_0^u\left(x^2\right)A_0^d\left(x^2\right)-4B_0^u\left(x^2\right)B_0^d\left(x^2\right)x^2. \quad (4.37)$$

When the traces in Equations 4.36 and 4.37 are inserted into Equation 4.35, the following expression is obtained.

$$\begin{aligned}
\Pi^{4,1}_{\mu\nu,T_{cc}^+}\left(p^2\right)&=(3i)2^6\int d^4xe^{ipx}A_0^c\left(x^2\right)^2A_0^u\left(x^2\right)A_0^d\left(x^2\right)g_{\mu\nu}\\
&+(3i)2^6\int d^4xe^{ipx}x^2A_0^c\left(x^2\right)^2B_0^u\left(x^2\right)B_0^d\left(x^2\right)g_{\mu\nu}\\
&+(3i)2^6\int d^4xe^{ipx}x^2B_0^c\left(x^2\right)^2A_0^u\left(x^2\right)A_0^d\left(x^2\right)g_{\mu\nu}\\
&+(3i)2^6\int d^4xe^{ipx}x^4B_0^c\left(x^2\right)^2B_0^u\left(x^2\right)B_0^d\left(x^2\right)g_{\mu\nu}\\
&-(3i)2^7\int d^4xe^{ipx}B_0^c\left(x^2\right)^2A_0^u\left(x^2\right)A_0^d\left(x^2\right)x_\nu x_\mu\\
&-(3i)2^7\int d^4xe^{ipx}B_0^c\left(x^2\right)^2B_0^u\left(x^2\right)B_0^d\left(x^2\right)x_\nu x_\mu \qquad (4.38)
\end{aligned}$$

Here, every integral is named by the i indice $(i=1,2,3...,6)$ such as $\Pi^{4,1}_{\mu\nu i}\left(p^2\right)$. In this case, the solution of the first integral $\Pi^{4,1}_{\mu\nu1}\left(p^2\right)$ is as:

$$\Pi^{4,1}_{\mu\nu1,,T_{cc}^+}\left(p^2\right)=(3i)2^6\int d^4xe^{ipx}A_0^c\left(x^2\right)^2A_0^u\left(x^2\right)A_0^d\left(x^2\right)g_{\mu\nu}. \quad (4.39)$$

By substituting the expressions of the relevant constants and Bessel functions,

$$\Pi^{4,1}_{\mu\nu 1,T^+_{cc}}\left(p^2\right) = (3i)2^6 \int d^4x e^{ipx}\left[\frac{i\not{x}}{2\pi^2 x^4}\right]^2$$

$$\times\left[\frac{m_c^2}{4\pi^2\sqrt{-x^2}}\frac{m_c}{\sqrt{-x^2}2^2}\int_0^\infty dt t^{(-2)}\exp\left[-\frac{m_Q^2}{4t}+tx^2\right]\right]^2 g_{\mu\nu} \quad (4.40)$$

can be written. Then, making the necessary adjustments, the integral takes the form

$$\Pi^{4,1}_{\mu\nu 1,T^+_{cc}}\left(p^2\right) = \frac{-3im_c^6 g_{\mu\nu}}{\pi^8 2^6}\int d^4x e^{ipx}\frac{1}{\left(-x^2\right)^5}$$

$$\times\int_0^\infty dt_1 t_1^{-2}\exp\left[\frac{-m_c^2}{4t_1}+t_1x^2\right]\int_0^\infty dt_2 t_2^{-2}\exp\left[\frac{-m_c^2}{4t_2}+t_2x^2\right]. \quad (4.41)$$

Consider a general integral defined as

$$I_1 = \int d^4x\frac{e^{ipx}}{\left(-x^2\right)^n}\int_0^\infty dt_1 t_1^{-m}\exp\left[\frac{-m_c^2}{4t_1}+t_1x^2\right]\int_0^\infty dt_2 t_2^{-l}\exp\left[\frac{-m_c^2}{4t_2}+t_2x^2\right]$$

$$= \int d^4x\frac{e^{ipx}}{\left(-x^2\right)^n}\int_0^\infty dt_1\int_0^\infty dt_2 t_1^{-m}t_2^{-l}\exp\left[-\left(\frac{m_c^2}{4t_1}+\frac{m_c^2}{4t_2}\right)+(t_1+t_2)x^2\right], \quad (4.42)$$

where $d^4x = dx_0dx_1dx_2dx_3 = dtdxdydz$ and to do the calculation, it is deemed appropriate to switch from Minkowskian to Euclidean space. For this purpose, the Wick rotation is applied. In Wick rotation $t \to it, \quad x_0 \to ix_0, \quad p_0 \to ip_0, \quad d^4x = id^4x$, and hence $x^2 \to -x^2, \quad p^2 \to -p^2$ and $px \to -px$. Then, Wick's rotation of Equation 4.42 becomes:

$$I_1 = i \int d^4x \frac{e^{-ipx}}{(x^2)^n} \int_0^\infty dt_1 \int_0^\infty dt_2 t_1^{-m} t_2^{-l} \exp\left[-\left(\frac{m_c^2}{4t_1} + \frac{m_c^2}{4t_2} \right) - (t_1 + t_2) x^2 \right]. \tag{4.43}$$

Using the Schwinger representation expressed as follows in Appendix B.3

$$\frac{1}{A^n} = \frac{1}{\Gamma(n)} \int_0^\infty dt t^{n-1} \exp(-At), \tag{4.44}$$

the integral I_1 takes the form:

$$I_1 = \frac{i}{\Gamma(n)} \int_0^\infty dt_1 \int_0^\infty dt_2 \int_0^\infty dt_3 t_1^{-m} t_2^{-l} t_3^{n-1} \exp\left[-\left(\frac{m_c^2}{4t_1} + \frac{m_c^2}{4t_2} \right) \right]$$

$$\times \int d^4x \exp\left[-ipx - (t_1 + t_2 + t_3) x^2 \right]. \tag{4.45}$$

Defining a new variable K for the integral d^4x in the second row of the Equation 4.45,

$$K = \int_{-\infty}^{\infty} d^4x \exp\left[-ipx - (t_1 + t_2 + t_3) x^2 \right]$$

$$= \int_{-\infty}^{\infty} dx_0 \exp\left[-ip_0 x_0 - (t_1 + t_2 + t_3) x_0^2 \right] \int_{-\infty}^{\infty} dx \exp\left[-ipx - (t_1 + t_2 + t_3) x^2 \right]$$

$$\times \int_{-\infty}^{\infty} dy \exp\left[-ipx - (t_1 + t_2 + t_3) y^2 \right] \int_{-\infty}^{\infty} dz \exp\left[-ipx - (t_1 + t_2 + t_3) z^2 \right] \tag{4.46}$$

is written and one of these integrals,

$$\int dx_0 \exp\left[-ipx_0 - (t_1 + t_2 + t_3) x_0^2 \right] \tag{4.47}$$

is chosen. To make this integral similar to the Gaussian integral, $\alpha = -ip_{x_0}, \quad \beta = t_1 + t_2 + t_3$ definitions are made, and

$$\int_{-\infty}^{\infty} dx_0 \exp\left[\alpha x_0 - \beta x_0^2\right] = \int_{-\infty}^{\infty} dx_0 \exp\left[-\beta\left(x_0 - \frac{\alpha}{2\beta}\right)^2 + \frac{\alpha^2}{4\beta}\right] \tag{4.48}$$

is found. Since this form has not yet been fully transformed into a Gaussian integral, another transformation needs to be applied. Defining $y = x_0 - \frac{\alpha}{2\beta}$, the desired form ,

$$\int_{-\infty}^{\infty} dy \exp\left[-\beta y^2\right] \exp\left[\frac{\alpha^2}{4\beta}\right] = \sqrt{\frac{\pi}{t_1 + t_2 + t_3}} \exp\left[\frac{-p_0^2}{4\left(t_1 + t_2 + t_3\right)}\right], \tag{4.49}$$

is obtained. The same operations are also applied to the other integrals given in Equation 4.46 . As a result of these transformations, Equation 4.45 can be rewritten as

$$I_1 = \frac{i\pi^2}{\Gamma(n)} \int_0^{\infty} dt_1 \int_0^{\infty} dt_2 \int_0^{\infty} dt_3 t_1^{-m} t_2^{-l} t_3^{n-1} \left(t_1 + t_2 + t_3\right)^{-2}$$

$$\times \exp\left[-\left(\frac{m_c^2}{4t_1} + \frac{m_c^2}{4t_2} + \frac{p^2}{4\left(t_1 + t_2 + t_3\right)}\right)\right]. \tag{4.50}$$

To simplify Equation 4.50, a change of variables is carried out by defining $t_1 = xt$, $t_2 = yt$, $t_3 = (1 - x - y)t$. Also, switching back to the Minkowskian space, using $(p^{\rightarrow} - p^2)$, after which

$$\int d^4x \frac{e^{ipx}}{(-x^2)^n} \int_0^\infty dt_1 t_1^{-m} \exp\left[\frac{-m_c^2}{4t_1} + t_1 x^2\right] \int_0^\infty dt_2 t_2^{-l} \exp\left[\frac{-m_c^2}{4t_2} + t_2 x^2\right]$$

$$= \frac{i\pi^2}{\Gamma(n)} \int_0^1 dx \int_0^{1-x} dy \, (1-x-y)^{n-1} x^{-m} y^{-l} \int_0^\infty dt t^{n-m-l-1}$$

$$\times \exp\left[-\left(\frac{m_c^2}{4xt} + \frac{m_c^2}{4yt}\right) + \frac{p^2}{4t}\right] \tag{4.51}$$

is obtained. In the last case, to simplify the notation in Equation 4.41, $s(x, y) = m_c^2/x + m_c^2/y$ is defined and the following expression is obtained for $\Pi^{4,1}_{\mu\nu 1, T_{cc}^+}(p^2)$.

$$\Pi^{4,1}_{\mu\nu 1, T_{cc}^+}\left(p^2\right) = \frac{-m_c^6 m_u m_d g_{\mu\nu}}{\pi^6 2^7}$$

$$\times \int_0^1 dx \int_0^{1-x} dy (1-x-y)^3 x^{-2} y^{-2} \int dt t^{-3} \exp\left[\frac{-s(x,y) + p^2}{4t}\right] \tag{4.52}$$

Applying similar methods to other integrals in Equation 4.38, the following expression is obtained.

$$\Pi^{4,1}_{\mu\nu, T_{cc}^+}\left(p^2\right) = \frac{-m_c^6 m_u m_d g_{\mu\nu}}{\pi^6 2^7}$$

$$\times \int_0^1 dx \int_0^{1-x} dy (1-x-y)^3 x^{-2} y^{-2} \int dt t^{-3} \exp\left[\frac{-s(x,y) + p^2}{4t}\right]$$

$$+ \frac{m_c^6 g_{\mu\nu}}{\pi^6 2^7} \int_0^1 dx \int_0^{1-x} dy (1-x-y)^3 x^{-2} y^{-2} \int dt t^{-3} \exp\left[\frac{-s(x,y) + p^2}{4t}\right]$$

$$-\frac{m_c^8 m_u m_d g_{\mu\nu}}{\pi^6 2^{11}} \int_0^1 dx \int_0^{1-x} dy (1-x-y)^4 x^{-3} y^{-3} \int dt t^{-4} \exp\left[\frac{-s(x,y)+p^2}{4t}\right]$$

$$-\frac{m_c^8 g_{\mu\nu}}{5\pi^6 2^9} \int_0^1 dx \int_0^{1-x} dy (1-x-y)^5 x^{-3} y^{-3} \int dt t^{-4} \exp\left[\frac{-s(x,y)+p^2}{4t}\right]$$

$$+\frac{m_c^8 m_u m_d}{5\pi^6 2^{11}} \int_0^1 dx \int_0^{1-x} dy (1-x-y)^5 x^{-3} y^{-3}$$

$$\times \int_0^{\infty} dt t^{-5} \left(g_{\mu\nu} + \frac{p_\mu p_\nu}{2t}\right) \exp\left[\frac{-s(x,y)+p^2}{4t}\right]$$

$$-\frac{3 m_c^8 m_u m_d}{15\pi^6 2^{11}} \int_0^1 dx \int_0^{1-x} dy (1-x-y)^6 x^{-3} y^{-3}$$

$$\times \int dt t^{-3} \left(g_{\mu\nu} + \frac{p_\mu p_\nu}{2t}\right) \exp\left[\frac{-s(x,y)+p^2}{4t}\right] \quad (4.53)$$

As mentioned above, there are two terms with 4 contactions that constitute the perturbative part of the correlation function. One of them is given in Equation 4.53. The second term contributing to $\Pi^{4,1}_{\mu\nu,T^+_{cc}}(p^2)$ can be calculated following the same steps. It is obtained that $\Pi^{4,2}_{\mu\nu,T^+_{cc}}(p^2) = \Pi^{4,1}_{\mu\nu,T^+_{cc}}(p^2)$. Hence,

$$\Pi^{pert}_{\mu\nu,T^+_{cc}}\left(p^2\right) = \Pi^{4,1}_{\mu\nu,T^+_{cc}}\left(p^2\right) + \Pi^{4,2}_{\mu\nu,T^+_{cc}}\left(p^2\right) = 2\Pi^{4,1}_{\mu\nu,T^+_{cc}}\left(p^2\right). \quad (4.54)$$

The contribution of the $d = 3$ operators arises from terms in Equation 4.14 with 3 contractions. Consider one of the possible terms with three contractions in Equation 4.14

$$\Pi^{3,1}_{\mu\nu 1,T^+_{cc}}\left(p^2\right) = i\int d^4x e^{ipx}(C\gamma_\mu)_{ij}(\gamma_\nu C)_{kl}(C\gamma_5)_{\alpha\beta}(\gamma_5 C)_{\gamma\delta}\varepsilon^{abe}\varepsilon^{cde}\varepsilon^{a'b'e'}\varepsilon^{c'd'e'}$$

$$\times\left\langle\Omega\left|:\left\{\left[(c_i^{aT}\, c_j^{b})\,(x)\,(\bar{c}_k^{b'}\,\bar{c}_l^{a'T})\,(0)\right]\left[(\bar{u}_\alpha^{c}\bar{d}_\beta^{dT})\,(x)\,(d_\gamma^{d'}\,u_\delta^{c'T})\,(0)\right]\right\}:\right|\Omega\right\rangle. \tag{4.55}$$

Using the propagators defined in Equation 4.55 can be rewritten as

$$\Pi^{3,1}_{\mu\nu 1,T^+_{cc}}\left(p^2\right) = i\int d^4x e^{ipx}(C\gamma_\mu)_{ij}(\gamma_\nu C)_{kl}(C\gamma_5)_{\alpha\beta}(\gamma_5 C)_{\gamma\delta}\varepsilon^{abe}\varepsilon^{cde}\varepsilon^{a'b'e'}\varepsilon^{c'd'e'}$$
$$\times\, iS^c_{il}(x)\delta^{aa'}iS^c_{jk}(x)\delta^{bb'}(-1)iS^u_{\delta\alpha}(-x)\delta^{cc'}\langle\Omega|:\bar{d}^{dT}_\beta\,(x)\,d^{d'T}_\delta\,(0):|\Omega\rangle. \tag{4.56}$$

In perturbation theory, when an annihilation operator is applied to vacuum, the result will be 0, but it will not equal to 0 in the non-perturbative regime of the QCD. In order to simplify the computations, the Fock-Schwinger gauge $x^\mu A_\mu = 0$ is used.

For convenience, the matrix element $\langle\Omega|\,:\,\bar{q}^a_\alpha\,(x)\,q^b_\beta(0)\,:\,|\Omega\rangle$ is considered. The Taylor expansion of $\bar{q}^a_\alpha(x)$ is written as

$$\bar{q}^a_\alpha(x) = \bar{q}^a_\alpha(0) + \bar{q}^a_\alpha(0)\overleftarrow{\partial_\mu}x^\mu + \frac{1}{2}\bar{q}^a_\alpha(0)\overleftarrow{\partial_\mu}\overleftarrow{\partial_\nu}x^\mu x^\nu$$

$$+\frac{1}{6}\bar{q}^a_\alpha(0)\overleftarrow{\partial_\mu}\overleftarrow{\partial_\nu}\overleftarrow{\partial_\alpha}x^\mu x^\nu x^\alpha + \frac{1}{24}\bar{q}^a_\alpha(0)\overleftarrow{\partial_\mu}\overleftarrow{\partial_\nu}\overleftarrow{\partial_\alpha}\overleftarrow{\partial_\delta}x^\mu x^\nu x^\alpha x^\delta + \cdots$$

$$= \bar{q}^a_\alpha\,(0) + (\bar{q}a_\alpha\overleftarrow{\mathcal{D}}_\mu)^a x^\mu + \cdots \tag{4.57}$$

where in the last step, the Fock-Schwinger gauge is used to write $\overleftarrow{\partial}_\mu x^\mu = \overleftarrow{\mathcal{D}}_\mu x^\mu$. Using Equation 4.57, the matrix element $\langle\Omega| : \bar{q}^a_\alpha(x)\, q^b_\beta(0) : |\Omega\rangle$ can be expressed as

$$\langle\Omega| : \bar{q}^a_\alpha(x)\, q^b_\beta(0) : |\Omega\rangle = \langle\Omega| : \Big[\bar{q}^a_\alpha(0) + \left(\bar{q}_\alpha(0)\overleftarrow{\mathcal{D}}_\mu\right)^a x^\mu + \frac{1}{2}\left(\bar{q}_\alpha(0)\overleftarrow{\mathcal{D}}_\mu\overleftarrow{\mathcal{D}}_\nu\right)^a x^\mu x^\nu$$

$$+\frac{1}{6}\left(\bar{q}_\alpha(0)\overleftarrow{\mathcal{D}_\mu}\overleftarrow{\mathcal{D}_\nu}\overleftarrow{\mathcal{D}_\alpha}\right)^a x^\mu x^\nu x^\alpha$$

$$+\frac{1}{24}\left(\bar{q}_\alpha(0)\overleftarrow{\mathcal{D}_\mu}\overleftarrow{\mathcal{D}_\nu}\overleftarrow{\mathcal{D}}_\alpha\overleftarrow{\partial_\delta}\right)^a x^\mu x^\nu x^\alpha x^\delta + \cdots\Big]\, q^b_\beta(0) : |\Omega\rangle$$

$$= \underbrace{\langle\Omega| : (\bar{q}^a_\alpha(0) q^b_\beta(0) : |\Omega\rangle}_{\text{First Term}} + \underbrace{\langle\Omega| : \left(\bar{q}_\alpha(0)\overleftarrow{\mathcal{D}}_\mu\right)^a x^\mu q^b_\beta(0) : |\Omega\rangle}_{\text{Second Term}}$$

$$+ \underbrace{\frac{1}{2}\langle\Omega| : \left(\bar{q}_\alpha(0)\overleftarrow{\mathcal{D}}_\mu\overleftarrow{\mathcal{D}}_\nu\right)^a x^\mu x^\nu q^b_\beta(0) : |\Omega\rangle}_{\text{Third Term}}$$

$$+ \underbrace{\frac{1}{6}\langle\Omega| : \left(\bar{q}_\alpha(0)\overleftarrow{\mathcal{D}}_\mu\overleftarrow{\mathcal{D}_\nu}\overleftarrow{\mathcal{D}}_\gamma\right)^a x^\mu x^\nu x^\gamma q^b_\beta(0) : |\Omega\rangle}_{\text{Fourth Term}}$$

$$+ \underbrace{\frac{1}{24}\langle\Omega| : \left(\bar{q}_\alpha(0)\overleftarrow{\mathcal{D}_\mu}\overleftarrow{\mathcal{D}_\nu}\overleftarrow{\mathcal{D}_\gamma}\overleftarrow{\mathcal{D}_\delta}\right)^a x^\mu x^\nu x^\gamma x^\delta q^b_\beta(0) : |\Omega\rangle}_{\text{Fifth Term}} + \cdots . \tag{4.58}$$

Starting from the first term $\langle\Omega| : (\bar{q}^a_\alpha(0) q^b_\beta(0) : |\Omega\rangle$, the a and b indices must be the same because the vacuum is colorless for this term and, also, due to parity and rotational symmetry, it is expected that $\langle\Omega| : (\bar{q}^a_\alpha(0) q^b_\beta(0) : |\Omega\rangle$ is proportional to the unit matrix in the spinor space. Taking into account all these considerations, the matrix element can be parametrized as:

$$\langle\Omega| : \bar{q}^a_\alpha(0) q^b_\beta(0) : |\Omega\rangle = A\delta^{ab}\delta_{\alpha\beta}. \tag{4.59}$$

To obtain A this term is multiplied by, $\delta_{ba}\delta^{\beta\alpha}$ and the following equation is obtained:

$$\delta_{ba}\delta^{\beta\alpha}\langle\Omega| : \bar{q}^a_\alpha(0) q^b_\beta(0) : |\Omega\rangle = A\delta_{ba}\delta^{\beta\alpha}\delta^{ab}\delta_{\alpha\beta}. \tag{4.60}$$

The terms in Equation 4.60 are

$$\delta^{ab}\delta_{ba} = N_c = 3, \tag{4.61}$$

$$\delta_{\alpha\beta}\delta^{\beta\alpha} = 4. \tag{4.62}$$

From Equation 4.60 it is concluded that

$$A = \frac{1}{12}\langle\bar{q}q\rangle, \tag{4.63}$$

where $\langle\bar{q}q\rangle \equiv \langle\Omega| : \bar{q}\,(0)\,q\,(0) : |\Omega\rangle$. As a result, the following equation is obtained.

$$\langle\Omega| : \bar{q}^a_\alpha(0) q^b_\beta(0) : |\Omega\rangle \equiv \frac{1}{12}\langle\bar{q}q\rangle\delta^{ab}\delta_{\alpha\beta}. \tag{4.64}$$

If the second term in Equation 4.58 is to be calculated, considering that the second term has a Lorentz index and two spinor indices, it can be written as

$$\langle\Omega| : (\bar{q}_\alpha(0)\overleftarrow{D_\mu})^a q^b_\beta(0) : |\Omega\rangle = B\delta^{ab}(\gamma_\mu)_{\alpha\beta}. \tag{4.65}$$

Since γ_μ is the only quantity with a Lorentz index on which the matrix element can depend. For this term, because of the chosen Fock-Schwinger gauge, simplifications are possible with the help of the equation of motion of quark fields, $\bar{q}(\not{D} - im_q) = 0$

.

Multiplying this equation by $\delta_{ba}(\gamma_\mu)^{\beta\alpha}$ on both sides, the equation can be written as

$$\delta_{ba}(\gamma_\mu)^{\beta\alpha}\langle\Omega| : (\bar{q}_\alpha(0)\overleftarrow{\mathcal{D}_\mu})^a q^b_\beta(0) : |\Omega\rangle = B\delta^{ab}\delta_{ba}(\gamma_\mu)^{\beta\alpha}(\gamma_\mu)_{\alpha\beta}. \tag{4.66}$$

Using Equation 4.61 and the relation $Tr[\gamma_\mu\gamma^\mu] = 16$ in Equation 4.66, the following result is obtained for the value B:

$$B = \frac{im_q}{48}\langle\bar{q}q\rangle. \tag{4.67}$$

In this case, the following equation is obtained.

$$\langle\Omega| : (\bar{q}_\alpha(0)\overleftarrow{\mathcal{D}_\mu})^a q^b_\beta(0) : |\Omega\rangle = \frac{im_q}{48}\langle\bar{q}q\rangle\delta^{ab}(\gamma_\mu)_{\alpha\beta}. \tag{4.68}$$

The third term in Equation 4.58 can be written as

$$\frac{1}{2}\langle\Omega| : (\bar{q}_\alpha(0)\overleftarrow{\mathcal{D}}_{\{\mu}\overleftarrow{\mathcal{D}}_{\nu\}})^a x^\mu x^\nu q^b_\beta(0) : |\Omega\rangle. \tag{4.69}$$

Here, curly braces imply that the indices within them must be symmetrized.

Multiplying the relation,

$$\overleftarrow{\mathcal{D}}_\mu\overleftarrow{\mathcal{D}}_\nu = \frac{1}{2}(\overleftarrow{\mathcal{D}}_\mu\overleftarrow{\mathcal{D}}_\nu + \overleftarrow{\mathcal{D}}_\nu\overleftarrow{\mathcal{D}}_\mu) + \frac{1}{2}(\overleftarrow{\mathcal{D}}_\mu\overleftarrow{\mathcal{D}}_\nu - \overleftarrow{\mathcal{D}}_\nu\overleftarrow{\mathcal{D}}_\mu) \tag{4.70}$$

with the expression $x^\mu x^\nu$,

$$\begin{aligned} x^\mu x^\nu\overleftarrow{\mathcal{D}}_\mu\overleftarrow{\mathcal{D}}_\nu &= \frac{1}{2}(\overleftarrow{\mathcal{D}}_\mu\overleftarrow{\mathcal{D}}_\nu + \overleftarrow{\mathcal{D}}_\nu\overleftarrow{\mathcal{D}}_\mu)x^\mu x^\nu + \frac{1}{2}(\overleftarrow{\mathcal{D}}_\mu\overleftarrow{\mathcal{D}}_\nu - \overleftarrow{\mathcal{D}}_\nu\overleftarrow{\mathcal{D}}_\mu)x^\mu x^\nu \\ &= \frac{1}{2}(\overleftarrow{\mathcal{D}}_\mu\overleftarrow{\mathcal{D}}_\nu + \overleftarrow{\mathcal{D}}_\nu\overleftarrow{\mathcal{D}}_\mu)x^\mu x^\nu \end{aligned} \tag{4.71}$$

is obtained. This equation is used in Equation 4.69 and this result in

$$\begin{aligned} &x^\mu x^\nu\langle\Omega| : (\bar{q}_\alpha(0)\overleftarrow{\mathcal{D}}_{\{\mu}\overleftarrow{\mathcal{D}}_{\nu\}})^a q^b_\beta(0) : |\Omega\rangle \\ &\qquad = x^\mu x^\nu\langle\Omega| : [\bar{q}_\alpha(0)(\overleftarrow{\mathcal{D}}_\mu\overleftarrow{\mathcal{D}}_\nu + \overleftarrow{\mathcal{D}}_\nu\overleftarrow{\mathcal{D}}_\mu)]^a q^b_\beta(0) : |\Omega\rangle \end{aligned} \tag{4.72}$$

In Equation 4.72, there are two Lorentz indices as well as two spinor indices on the left side. Consequently, the following equality can be written.

$$\frac{1}{2}\langle\Omega| : [\bar{q}_\alpha(0)(\overleftarrow{\mathcal{D}_\mu}\overleftarrow{\mathcal{D}_\nu} + \overleftarrow{\mathcal{D}_\nu}\overleftarrow{\mathcal{D}_\mu})]^a q^b_\beta(0) : |\Omega\rangle = C g_{\mu\nu}\delta^{ab}\delta_{\alpha\beta} \tag{4.73}$$

Note that since the left-hand side is symmetric under the exchange of μ and ν, $\sigma_{\mu\nu}$ cannot appear on the right-hand side. Multiplying with $g^{\mu\nu}\delta_{ba}\delta^{\beta\alpha}$ and using Equations 4.61 and 4.62 and $g_{\mu\nu}g^{\mu\nu} = 4$, Equation 4.74 is obtained.

$$C = \frac{1}{96}x^2\langle\Omega| : \bar{q}(0)\overleftarrow{\mathcal{D}}^2 q(0) : |\Omega\rangle \tag{4.74}$$

Using the following relation

$$\mathcal{D}^2 = \not{\mathcal{D}}^2 + \frac{g_s}{2}G_{\mu\nu}\sigma^{\mu\nu}, \tag{4.75}$$

the C value can be written as:

$$\begin{aligned} C &= \frac{1}{96}x^2\langle\Omega| : \bar{q}(0)(\not{\mathcal{D}}^2 + \frac{g_s}{2}G_{\mu\nu}\sigma^{\mu\nu})q(0) : |\Omega\rangle \\ &= \frac{1}{96}x^2\langle\Omega| : \bar{q}(0)\not{\mathcal{D}}^2 q(0) : |\Omega\rangle + \frac{1}{96}x^2\langle\Omega| : \bar{q}(0)\frac{g_s}{2}G_{\mu\nu}\sigma^{\mu\nu})q(0) : |\Omega\rangle. \end{aligned} \tag{4.76}$$

Since the first term of this equation is of the order of m_q^2, it can be neglected for light quarks and one obtains:

$$C = \frac{1}{192}x^2\langle\Omega| : \bar{q}(0)g_s G_{\mu\nu}\sigma^{\mu\nu})q(0) : |\Omega\rangle. \tag{4.77}$$

Thus, the result for C is

$$C \equiv \frac{1}{192}x^2\langle\bar{q}g_s\sigma G q\rangle, \tag{4.78}$$

where $\langle\bar{q}g_s\sigma Gq\rangle \equiv \langle\Omega| : \bar{q}(0)g_sG_{\mu\nu}\sigma^{\mu\nu})q(0) : |\Omega\rangle$. The value obtained for C is substituted in Equation 4.73 and the third term is found as

$$\frac{1}{2}\langle\Omega| : [\bar{q}_\alpha(0)(\overleftarrow{\mathcal{D}_\mu}\overleftarrow{\mathcal{D}_\nu} + \overleftarrow{\mathcal{D}_\nu}\overleftarrow{\mathcal{D}_\mu})]^a q^b_\beta(0) : |\Omega\rangle \equiv \frac{1}{192}x^2\langle\bar{q}g_s\sigma Gq\rangle g_{\mu\nu}\delta^{ab}\delta_{\alpha\beta}. \quad (4.79)$$

The mixed condensate $\langle\bar{q}g_s\sigma Gq\rangle$ is usually parametrized in terms of $\langle\bar{q}q\rangle$ as

$$\langle\bar{q}g_s\sigma Gq\rangle \equiv m_0^2\langle\bar{q}q\rangle. \quad (4.80)$$

Next step, for the fourth term; similar to the preceding terms, an equation of the form below can be formulated for this term by taking into account the Lorentz indices and color spinor indices.

$$\frac{1}{6}\langle\Omega| : (\bar{q}_\alpha(0)\overleftarrow{\mathcal{D}}_{\{\mu}\overleftarrow{\mathcal{D}_\nu}\overleftarrow{\mathcal{D}}_{\gamma\}})^a q^b_\beta(0) : |\Omega\rangle = F\delta^{ab}(g_{\mu\nu}\gamma_\gamma + g_{\mu\gamma}\gamma_\nu + g_{\nu\gamma}\gamma_\mu)^{\alpha\beta}. \quad (4.81)$$

Following steps similar to the previous cases, the value of F is determined as

$$F = \frac{im_q}{576}\langle\bar{q}g_s\sigma Gq\rangle. \quad (4.82)$$

Equation 4.80 is written here and replaced in Equation 4.81, the fourth term is obtained as

$$\frac{1}{6}\langle\Omega| : (\bar{q}_\alpha(0)\overleftarrow{\partial_\mu}\overleftarrow{\partial_\nu}\overleftarrow{\partial_\gamma})^a x^\mu x^\nu x^\gamma q^b_\beta(0) : |\Omega\rangle \equiv \frac{im_q}{1152}m_0^2\delta^{ab}\langle\bar{q}q\rangle x^2 \not{x}. \quad (4.83)$$

Thus, the first four contributions to the matrix element given in Equation 4.58 arising from the Taylor expansion have been derived. Likewise, it will be feasible to compute the subsequent terms.

$$\langle\Omega| : \bar{q}^a_\alpha(x)\, q^b_\beta(0) : |\Omega\rangle = \frac{1}{12}\langle\bar{q}q\rangle\delta^{ab}\delta_{\alpha\beta} + \frac{im_q}{48}\langle\bar{q}q\rangle\delta^{ab}\not{x}$$

$$+\frac{1}{192}m_0^2\langle\bar{q}q\rangle x^2\delta^{ab}\delta_{\alpha\beta} + \frac{im_q}{1152}m_0^2\delta^{ab}\langle\bar{q}q\rangle x^2\not{x} + \cdots . \quad (4.84)$$

As can be seen, the contributions from each term can be calculated by performing this calculation for three contractions. In the same way, for the complete calculation of the non-perturbative part, the remaining two and one contraction must be calculated in terms of the condensates.

However, there is an easier method for non-perturbative calculation. In that method, it is possible to obtain all possible cases, except for a few exceptions, by writing the full propagator and using it to calculate the correlation function. The full propagators for light quarks and heavy quarks are written in Equations 4.85 and 4.86 [117, 110, 119], respectively.

$$S^{ab,q}_{full}(x) = i\frac{\not{x}}{2\pi^2x^4}\delta^{ab} - \frac{m_q}{4\pi^2x^2}\delta^{ab} - \frac{\langle q\bar{q}\rangle}{12}\left(1 - i\frac{m_q}{4}\not{x}\right)\delta^{ab}$$

$$-\frac{x^2}{192}m_0^2\langle q\bar{q}\rangle\left(1 - i\frac{m_q}{6}\not{x}\right)\delta^{ab}$$

$$-ig_s\int_0^1 du\left[\frac{\not{x}}{16\pi^2x^2}G^{ab}_{\alpha\beta}(ux)\sigma^{\alpha\beta} - ux_\mu G^{ab}_{\alpha\beta}(ux)\gamma^\nu\frac{i}{4\pi^2x^2}\right.$$

$$\left.-i\frac{m_q}{32\pi^2}G^{ab}_{\alpha\beta}\sigma^{\alpha\beta}\left(\ln\left(\frac{-x^2\Lambda^2}{4}\right) + 2\gamma_E\right)\right]\cdots , \quad (4.85)$$

$$S^{ab,Q}_{full}(x) = \left[\frac{m_Q^2}{4\pi^2}\frac{K_1\left(m_Q\sqrt{-x^2}\right)}{\sqrt{-x^2}}\right] - i\left[\frac{m_Q^2}{4\pi^2 x^2}K_2\left(m_Q\sqrt{-x^2}\right)\right] \not{x}$$

$$- ig_s \int \frac{d^4k}{(2\pi)^4}e^{-ikx}\int_0^1 dv \left[\frac{\not{k}+m_Q}{\left(m_Q^2-k^2\right)^2}G^{ab}_{\alpha\beta}(vx)\sigma^{\alpha\beta}\right.$$

$$\left. + \frac{1}{m_Q^2-k^2}vx_\alpha G^{ab}_{\alpha\beta}\gamma^\beta\right] + \cdots . \tag{4.86}$$

Previously, to make the calculations more understandable, the free propagator is redefined in terms of functions $A(x^2)$ and $B(x^2)$. Considering the Feynman diagrams discussed in this study, the same method is continued for the full propagator, and in addition to the previous definitions, contributions are made as follows.

$$A_1^Q\left(x^2\right) = -\langle g_s^2G^2\rangle \frac{x^2}{3.2^7\pi^2}\left[\frac{K_1\left(m_Q\sqrt{-x^2}\right)}{\sqrt{-x^2}}\right] = -\langle g_s^2G^2\rangle\frac{A_0^Q\left(x^2\right)x^2}{3.2^5m_Q^2}, \tag{4.87}$$

$$A_2^Q\left(x^2\right) = -i\langle g_s^2G^2\rangle \frac{m_Qx^2}{3^2.2^8\pi^2}\left[\frac{K_2\left(m_Q\sqrt{-x^2}\right)}{\sqrt{-x^2}}\right] = -i\langle g_s^2G^2\rangle\frac{B_0^Q\left(x^2\right)x^4}{3^2.2^6m_Q}, \tag{4.88}$$

$$B_1^Q\left(x^2\right) = i\langle g_s^2G^2\rangle \frac{m_Qx^2}{3^2.2^8\pi^2}\left[\frac{K_1\left(m_Q\sqrt{-x^2}\right)}{\sqrt{-x^2}}\right] = i\langle g_s^2G^2\rangle\frac{A_0^Q\left(x^2\right)x^2}{3^2.2^6m_Q}, \tag{4.89}$$

$$A_1^q = -\frac{\langle\bar{q}q\rangle}{12}, \qquad A_2^q = -\frac{x^2}{192}m_0^2\langle\bar{q}q\rangle, \qquad A_3^q = -\frac{g_s^2}{4\pi^2}\int_0^1 du\frac{ux^\mu}{x^2}G_{\mu\nu}\left(ux\right)\gamma^\nu, \tag{4.90}$$

$$B_1^q\left(x^2\right)=im_q\frac{\langle\bar{q}q\rangle}{48};\qquad B_2^q\left(x^2\right)=i\frac{m_q m_0^2 x^2}{1152}\langle\bar{q}q\rangle. \tag{4.91}$$

And so, for the variables A and B depending on x, in terms of all these terms can be written as

$$A^Q\left(x^2\right)=A_0^Q\left(x^2\right)+A_1^Q\left(x^2\right)+A_2^Q\left(x^2\right), \tag{4.92}$$

$$B^Q\left(x^2\right)=B_0^Q\left(x^2\right)+B_1^Q\left(x^2\right), \tag{4.93}$$

$$A^q\left(x^2\right)=A_0^q\left(x^2\right)+A_1^q\left(x^2\right)+A_2^q\left(x^2\right)+A_3^q\left(x^2\right), \tag{4.94}$$

$$B^q\left(x^2\right)=B_0^q\left(x^2\right),+B_1^q\left(x^2\right)+B_2^q\left(x^2\right). \tag{4.95}$$

In this case, the expressions of full propagators in terms of these new definitions are

$$S_{full}^Q(x)=A^Q\left(x^2\right)+B^Q\left(x^2\right)\not{x}, \tag{4.96}$$

$$S_{full}^q(x)=A^q\left(x^2\right)+B^q\left(x^2\right)\not{x}. \tag{4.97}$$

Adding new contributions to the functions A and B in the integrals in Equation 4.38, it becomes possible to evaluate the diagrams except for some special cases. Since $m_u=m_d=0$ in this study, the functions $A_0^q\left(x^2\right)$, $A_3^q\left(x^2\right)$, $B_1^q\left(x^2\right)$, $B_2^q\left(x^2\right)$ are zero and therefore will not be taken into account.

In this study, the first integral of Equation 4.38 will be called $\Pi^1_{\mu\nu,T_{cc}^+}\left(p^2\right)$ and will be calculated in detail. For this purpose, the following expression is obtained by placing new functions in $\Pi^1_{\mu\nu,T_{cc}^+}\left(p^2\right)$.

$$
\begin{aligned}
\Pi^{1}_{\mu\nu,T_{cc}^{+}}\left(p^{2}\right) &= (3i)2^{6}\int d^{4}xe^{ipx}\left[A_{0}^{c}\left(x^{2}\right)+A_{1}^{c}\left(x^{2}\right)+A_{2}^{c}\left(x^{2}\right)\right]^{2} \\
&\times\left[A_{1}^{u}\left(x^{2}\right)+A_{2}^{u}\left(x^{2}\right)+A_{3}^{u}\left(x^{2}\right)\right]\left[A_{1}^{d}\left(x^{2}\right)+A_{2}^{d}\left(x^{2}\right)+A_{3}^{d}\left(x^{2}\right)\right]g_{\mu\nu} \\
&= (3i)2^{6}\int d^{4}xe^{ipx}\left\{\ A_{0}^{c}\left(x^{2}\right)^{2}A_{1}^{u}\left(x^{2}\right)A_{1}^{d}\left(x^{2}\right)\right. \\
&+A_{0}^{c}\left(x^{2}\right)^{2}A_{2}^{u}\left(x^{2}\right)A_{2}^{d}\left(x^{2}\right)+A_{0}^{c}\left(x^{2}\right)^{2}A_{3}^{u}\left(x^{2}\right)A_{3}^{d}\left(x^{2}\right) \\
&+2A_{0}^{c}\left(x^{2}\right)^{2}A_{1}^{u}\left(x^{2}\right)A_{2}^{d}\left(x^{2}\right)+2A_{0}^{c}\left(x^{2}\right)^{2}A_{1}^{u}\left(x^{2}\right)A_{3}^{d}\left(x^{2}\right) \\
&+2A_{0}^{c}\left(x^{2}\right)^{2}A_{2}^{u}\left(x^{2}\right)A_{3}^{d}\left(x^{2}\right)+A_{1}^{c}\left(x^{2}\right)^{2}A_{1}^{u}\left(x^{2}\right)A_{1}^{d}\left(x^{2}\right) \\
&+A_{1}^{c}\left(x^{2}\right)^{2}A_{2}^{u}\left(x^{2}\right)A_{2}^{d}\left(x^{2}\right)+A_{1}^{c}\left(x^{2}\right)^{2}A_{3}^{u}\left(x^{2}\right)A_{3}^{d}\left(x^{2}\right) \\
&+2A_{1}^{c}\left(x^{2}\right)^{2}A_{1}^{u}\left(x^{2}\right)A_{2}^{d}\left(x^{2}\right)+2A_{1}^{c}\left(x^{2}\right)^{2}A_{1}^{u}\left(x^{2}\right)A_{3}^{d}\left(x^{2}\right) \\
&+2A_{1}^{c}\left(x^{2}\right)^{2}A_{2}^{u}\left(x^{2}\right)A_{3}^{d}\left(x^{2}\right)+A_{2}^{c}\left(x^{2}\right)^{2}a_{1}^{u}\left(x^{2}\right)A_{1}^{d}\left(x^{2}\right) \\
&+A_{2}^{c}\left(x^{2}\right)^{2}A_{2}^{u}\left(x^{2}\right)A_{2}^{d}\left(x^{2}\right)+A_{2}^{c}\left(x^{2}\right)^{2}A_{3}^{u}\left(x^{2}\right)A_{3}^{d}\left(x^{2}\right) \\
&+2A_{2}^{c}\left(x^{2}\right)^{2}A_{1}^{u}\left(x^{2}\right)A_{2}^{d}\left(x^{2}\right)+2A_{2}^{c}\left(x^{2}\right)^{2}A_{1}^{u}\left(x^{2}\right)A_{3}^{d}\left(x^{2}\right) \\
&+2A_{2}^{c}\left(x^{2}\right)^{2}A_{2}^{u}\left(x^{2}\right)A_{3}^{d}\left(x^{2}\right)+2A_{0}^{c}\left(x^{2}\right)A_{1}^{c}\left(x^{2}\right)A_{1}^{u}\left(x^{2}\right)A_{1}^{d}\left(x^{2}\right) \\
&+2A_{0}^{c}\left(x^{2}\right)A_{1}^{c}\left(x^{2}\right)A_{2}^{u}\left(x^{2}\right)A_{2}^{d}\left(x^{2}\right)+2A_{0}^{c}\left(x^{2}\right)A_{1}^{c}\left(x^{2}\right)A_{3}^{u}\left(x^{2}\right)A_{3}^{d}\left(x^{2}\right) \\
&+4A_{0}^{c}\left(x^{2}\right)A_{1}^{c}\left(x^{2}\right)A_{1}^{u}\left(x^{2}\right)A_{2}^{d}\left(x^{2}\right)+4A_{0}^{c}\left(x^{2}\right)A_{1}^{c}\left(x^{2}\right)A_{1}^{u}\left(x^{2}\right)A_{3}^{d}\left(x^{2}\right) \\
&+4A_{0}^{c}\left(x^{2}\right)A_{1}^{c}\left(x^{2}\right)A_{2}^{u}\left(x^{2}\right)A_{3}^{d}\left(x^{2}\right)+2A_{0}^{c}\left(x^{2}\right)A_{2}^{c}\left(x^{2}\right)A_{1}^{u}\left(x^{2}\right)A_{1}^{d}\left(x^{2}\right) \\
&+2A_{0}^{c}\left(x^{2}\right)A_{2}^{c}\left(x^{2}\right)A_{2}^{u}\left(x^{2}\right)D_{2}^{d}\left(x^{2}\right)+2A_{0}^{c}\left(x^{2}\right)A_{2}^{c}\left(x^{2}\right)A_{3}^{u}\left(x^{2}\right)A_{3}^{d}\left(x^{2}\right) \\
&+4A_{0}^{c}\left(x^{2}\right)A_{2}^{c}\left(x^{2}\right)A_{1}^{u}\left(x^{2}\right)A_{2}^{d}\left(x^{2}\right)+4A_{0}^{c}\left(x^{2}\right)A_{2}^{c}\left(x^{2}\right)A_{1}^{u}\left(x^{2}\right)A_{3}^{d}\left(x^{2}\right) \\
&+4A_{0}^{c}\left(x^{2}\right)A_{2}^{c}\left(x^{2}\right)A_{2}^{u}\left(x^{2}\right)A_{3}^{d}\left(x^{2}\right)+2A_{0}^{c}\left(x^{2}\right)A_{3}^{c}\left(x^{2}\right)A_{1}^{u}\left(x^{2}\right)A_{1}^{d}\left(x^{2}\right) \\
&+2A_{0}^{c}\left(x^{2}\right)A_{3}^{c}\left(x^{2}\right)A_{2}^{u}\left(x^{2}\right)A_{2}^{d}\left(x^{2}\right)+2A_{0}^{c}\left(x^{2}\right)A_{3}^{c}\left(x^{2}\right)A_{3}^{u}\left(x^{2}\right)A_{3}^{d}\left(x^{2}\right) \\
&+4A_{0}^{c}\left(x^{2}\right)A_{3}^{c}\left(x^{2}\right)A_{1}^{u}\left(x^{2}\right)A_{2}^{d}\left(x^{2}\right)+4A_{0}^{c}\left(x^{2}\right)A_{3}^{c}\left(x^{2}\right)A_{1}^{u}\left(x^{2}\right)A_{3}^{d}\left(x^{2}\right) \\
&\left.+4A_{0}^{c}\left(x^{2}\right)A_{3}^{c}\left(x^{2}\right)A_{2}^{u}\left(x^{2}\right)A_{3}^{d}\left(x^{2}\right)\ \right\}g_{\mu\nu}. \qquad (4.98)
\end{aligned}
$$

Since the Feynman diagrams containing 4 gluons of these terms are not taken into account, the expressions $A_1^c\left(x^2\right)^2$, $A_2^c\left(x^2\right)^2$, $B_1^c\left(x^2\right)$, $B_2^c\left(x^2\right)$ containing these contributions are not taken into account, and the following results can be obtained following the same steps used in the calculation of the perturbative part.

$$\Pi^1_{\mu\nu,T_{cc}^+}\left(p^2\right) = -\frac{m_c^6\langle\bar{u}u\rangle\langle\bar{d}d\rangle g_{\mu\nu}}{3.2^6\pi^2}\int_0^1 dx\int_0^{1-x} dy(1-x-y)x^{-2}y^{-2}$$

$$\times\int dtt^{-3}\exp\left[\frac{-s(x,y)+p^2}{4t}\right]$$

$$+\frac{m_c^6 m_0^2\langle\bar{u}u\rangle\langle\bar{d}d\rangle g_{\mu\nu}}{3.2^{10}\pi^2}\int_0^1 dx\int_0^{1-x} dyx^{-2}y^{-2}$$

$$\times\int dtt^{-4}\exp\left[\frac{-s(x,y)+p^2}{4t}\right]$$

$$+\frac{m_c^6 m_0^2\langle\bar{u}u\rangle\langle\bar{d}d\rangle g_{\mu\nu}}{3.2^{10}\pi^2}\int_0^1 dx\int_0^{1-x} dyx^{-2}y^{-2}$$

$$\times\int dtt^{-4}\exp\left[\frac{-s(x,y)+p^2}{4t}\right]$$

$$-\frac{m_c^6 m_0^4\langle\bar{u}u\rangle\langle\bar{d}d\rangle g_{\mu\nu}}{3.2^{14}\pi^2}\int_0^1 dxx^{-2}(1-x)^{-2}$$

$$\times\int dtt^{-5}\exp\left[\frac{-\tilde{s}(x)+p^2}{4t}\right]$$

$$-\frac{m_c^4\langle\bar{u}u\rangle\langle\bar{d}d\rangle\langle g_s^2G^2\rangle\; g_{\mu\nu}}{3^2.2^{10}\pi^2}\int_0^1 dx\int_0^{1-x} dyx^{-2}y^{-2}$$

$$\times\int dtt^{-4}\exp\left[\frac{-s(x,y)+p^2}{4t}\right]$$

$$-\frac{m_c^4 m_0^4\langle\bar{u}u\rangle\langle\bar{d}d\rangle\langle g_s^2G^2\rangle\; g_{\mu\nu}}{3^2.2^{17}\pi^2}\int_0^1 dxx^{-2}(1-x)^{-2}$$

$$\times\int dtt^{-6}\exp\left[\frac{-\tilde{s}(x)+p^2}{4t}\right]$$

$$-\frac{m_c^4 m_0^4\langle\bar{u}u\rangle\langle\bar{d}d\rangle\langle g_s^2G^2\rangle\; p^2 g_{\mu\nu}}{3^2.2^{20}\pi^2}\int_0^1 dxx^{-2}(1-x)^{-2}$$

$$\times\int dtt^{-7}\exp\left[\frac{-\tilde{s}(x)+p^2}{4t}\right]$$

$$+\frac{m_c^4 m_0^2\langle\bar{u}u\rangle\langle\bar{d}d\rangle\langle g_s^2G^2\rangle\; g_{\mu\nu}}{3^2.2^{14}\pi^2}\int_0^1 dxx^{-2}(1-x)^{-2}$$

$$\times\int dtt^{-5}\exp\left[\frac{-\tilde{s}(x)+p^2}{4t}\right]$$

$$+\frac{m_c^4 m_0^2\langle\bar{u}u\rangle\langle\bar{d}d\rangle\langle g_s^2G^2\rangle\; g_{\mu\nu}}{3^2.2^{14}\pi^2}\int_0^1 dxx^{-2}(1-x)^{-2}$$

$$\times\int dtt^{-5}\exp\left[\frac{-\tilde{s}(x)+p^2}{4t}\right]$$

$$+\frac{m_c^4 m_0^2 \langle\bar{u}u\rangle\langle\bar{d}d\rangle\langle g_s^2G^2\rangle\; g_{\mu\nu}}{3^3.2^{12}\pi^2}\int_0^1 dx\int_0^{1-x} dy x^{-2}y^{-3}\int dt t^{-5}\exp\left[\frac{-s(x,y)+p^2}{4t}\right]$$

$$+\frac{m_c^6 m_0^4 \langle\bar{u}u\rangle\langle\bar{d}d\rangle\langle g_s^2G^2\rangle\; g_{\mu\nu}}{3^3.2^{19}\pi^2}\int_0^1 dx x^{-2}(1-x)^{-3}\int dt t^{-7}\exp\left[\frac{-\tilde{s}(x)+p^2}{4t}\right]$$

$$+\frac{m_c^6 m_0^4 \langle\bar{u}u\rangle\langle\bar{d}d\rangle\langle g_s^2G^2\rangle p^2\; g_{\mu\nu}}{3^3.2^{22}\pi^2}\int_0^1 dx x^{-2}(1-x)^{-3}\int dt t^{-8}\exp\left[\frac{-\tilde{s}(x)+p^2}{4t}\right]$$

$$-\frac{m_c^6 m_0^2 \langle\bar{u}u\rangle\langle\bar{d}d\rangle\langle g_s^2G^2\rangle\; g_{\mu\nu}}{3^3.2^{16}\pi^2}\int_0^1 dx x^{-2}(1-x)^{-3}\int dt t^{-6}\exp\left[\frac{-\tilde{s}(x)+p^2}{4t}\right]$$

$$-\frac{m_c^6 m_0^2 \langle\bar{u}u\rangle\langle\bar{d}d\rangle\langle g_s^2G^2\rangle\; g_{\mu\nu}}{3^3.2^{16}\pi^2}\int_0^1 dx x^{-2}(1-x)^{-3}\int dt t^{-6}\exp\left[\frac{-\tilde{s}(x)+p^2}{4t}\right]. \tag{4.99}$$

Here, $\tilde{s}(x) = m_c^2/x + m_c^2/(1-x)$. The remaining terms are named $\Pi^i_{\mu\nu,T_{cc}^+}(p^2)$ where $(i = 1, 2, ..6)$ are given below.

Later, other integrals in Equation 4.38, called $\Pi^i_{\mu\nu,T_{cc}^+}(p^2)$ where $(i = 1, 2, ..6)$ were evaluated with similar methods and included in the calculation.

$$\Pi^2_{\mu\nu,T_{cc}^+}(p^2) = -\frac{m_c^8\langle\bar{u}u\rangle\langle\bar{d}d\rangle g_{\mu\nu}}{3.2^9\pi^2}\int_0^1 dx\int_0^{1-x} dy(1-x-y)^2 x^{-3}y^{-3}$$

$$\times\int dt t^{-4}\exp\left[\frac{-s(x,y)+p^2}{4t}\right]$$

$$+\frac{m_c^8 m_0^2\langle\bar{u}u\rangle\langle\bar{d}d\rangle g_{\mu\nu}}{3.2^{12}\pi^2}\int_0^1 dx\int_0^{1-x} dy(1-x-y)x^{-3}y^{-3}$$

$$\times\int dtt^{-5}\exp\left[\frac{-s(x,y)+p^2}{4t}\right]$$

$$+\frac{m_c^8 m_0^2\langle\bar{u}u\rangle\langle\bar{d}d\rangle g_{\mu\nu}}{3.2^{12}\pi^2}\int_0^1 dx\int_0^{1-x} dy(1-x-y)x^{-3}y^{-3}$$

$$\times\int dtt^{-5}\exp\left[\frac{-s(x,y)+p^2}{4t}\right]$$

$$-\frac{m_c^8 m_0^4\langle\bar{u}u\rangle\langle\bar{d}d\rangle g_{\mu\nu}}{3.2^{16}\pi^4}\int_0^1 dx\int_0^{1-x} dyx^{-3}y^{-3}\int dtt^{-6}\exp\left[\frac{-s(x,y)+p^2}{4t}\right]$$

$$+\frac{m_c^6\langle\bar{u}u\rangle\langle\bar{d}d\rangle\langle g_s^2G^2\rangle g_{\mu\nu}}{3^3.2^{12}\pi^2}\int_0^1 dx\int_0^{1-x} dyx^{-2}y^{-3}$$

$$\times\int dtt^{-5}\exp\left[\frac{-s(x,y)+p^2}{4t}\right]$$

$$-\frac{m_c^6 m_0^2\langle\bar{u}u\rangle\langle\bar{d}d\rangle\langle g_s^2G^2\rangle g_{\mu\nu}}{3^3.2^{16}\pi^2}\int_0^1 dxx^{-2}(1-x)^{-3}$$

$$\times\int dtt^{-6}\exp\left[\frac{-\tilde{s}(x)+p^2}{4t}\right]$$

$$-\frac{m_c^6 m_0^2\langle\bar{u}u\rangle\langle\bar{d}d\rangle\langle g_s^2G^2\rangle g_{\mu\nu}}{3^3.2^{16}\pi^2}\int_0^1 dxx^{-2}(1-x)^{-3}$$

$$\times\int dtt^{-6}\exp\left[\frac{-\tilde{s}(x)+p^2}{4t}\right]$$

$$
-\frac{m_c^6 m_0^4 \langle \bar{u}u\rangle\langle \bar{d}d\rangle\langle g_s^2 G^2\rangle g_{\mu\nu}}{3^3.2^{19}\pi^2}\int_0^1 dx x^{-2}(1-x)^{-3}
$$

$$
\times \int dt t^{-7}\exp\left[\frac{-\tilde{s}(x)+p^2}{4t}\right]
$$

$$
-\frac{m_c^6 m_0^4 \langle \bar{u}u\rangle\langle \bar{d}d\rangle\langle g_s^2 G^2\rangle p^2 g_{\mu\nu}}{3^3.2^{22}\pi^2}\int_0^1 dx x^{-2}(1-x)^{-3}
$$

$$
\times \int dt t^{-8}\exp\left[\frac{-\tilde{s}(x)+p^2}{4t}\right], \qquad (4.100)
$$

$$
\Pi^3_{\mu\nu,T_{cc}^+}\left(p^2\right) = -\frac{m_c^6 g_{\mu\nu}}{2^7\pi^6}\int_0^1 dx\int_0^{1-x} dy(1-x-y)^4 x^{-2}y^{-2}
$$

$$
\times \int dt \exp\left[\frac{-s(x,y)+p^2}{4t}\right]
$$

$$
-\frac{m_c^4\langle g_s^2 G^2\rangle g_{\mu\nu}}{3.2^9\pi^6}\int_0^1 dx\int_0^{1-x} dy(1-x-y)^3 x^{-2}y^{-2}
$$

$$
\times \int dt t^{-1}\exp\left[\frac{-s(x,y)+p^2}{4t}\right]
$$

$$
+\frac{m_c^6\langle g_s^2 G^2\rangle g_{\mu\nu}}{3^2.2^{11}\pi^6}\int_0^1 dx\int_0^{1-x} dy(1-x-y)^3 x^{-2}y^{-3}
$$

$$
\times \int dt t^{-2}\exp\left[\frac{-s(x,y)+p^2}{4t}\right], \qquad (4.101)
$$

$$\Pi^{4}_{\mu\nu,T^{+}_{cc}}\left(p^2\right) = -\frac{m_c^8 g_{\mu\nu}}{5.2^9\pi^6}\int_0^1 dx\int_0^{1-x} dy(1-x-y)^5 x^{-3}y^{-3}$$

$$\times\int dt t^{-1}\exp\left[\frac{-s(x,y)+p^2}{4t}\right]$$

$$+\frac{m_c^6\langle g_s^2G^2\rangle g_{\mu\nu}}{3^2.2^{11}\pi^6}\int_0^1 dx\int_0^{1-x} dy(1-x-y)^3 x^{-2}y^{-3}$$

$$\times\int dt t^{-2}\exp\left[\frac{-s(x,y)+p^2}{4t}\right], \tag{4.102}$$

$$\Pi^{5}_{\mu\nu,T^{+}_{cc}}\left(p^2\right) = \frac{m_c^8\langle\bar{u}u\rangle\langle\bar{d}d\rangle}{3^2.2^9\pi^4}\int_0^1 dx\int_0^{1-x} dy(1-x-y)^3 x^{-3}y^{-3}$$

$$\times\int dt t^{-4}\left(g_{\mu\nu}+\frac{p_\mu p_\nu}{2t}\right)\exp\left[\frac{-s(x,y)+p^2}{4t}\right]$$

$$-\frac{m_c^8 m_0^2\langle\bar{u}u\rangle\langle\bar{d}d\rangle}{3.2^{13}\pi^2}\int_0^1 dx\int_0^{1-x} dy(1-x-y)^2 x^{-3}y^{-3}$$

$$\times\int dt t^{-5}\left(g_{\mu\nu}+\frac{p_\mu p_\nu}{2t}\right)\exp\left[\frac{-s(x,y)+p^2}{4t}\right]$$

$$\times\int dt t^{-5}\left(g_{\mu\nu}+\frac{p_\mu p_\nu}{2t}\right)\exp\left[\frac{-s(x,y)+p^2}{4t}\right]$$

$$+\frac{m_c^8 m_0^4\langle\bar{u}u\rangle\langle\bar{d}d\rangle}{3.2^{16}\pi^2}\int_0^1 dx\int_0^{1-x} dy(1-x-y)^2 x^{-3}y^{-3}$$

$$\times\int dt t^{-6}\left(g_{\mu\nu}+\frac{p_\mu p_\nu}{2t}\right)\exp\left[\frac{-s(x,y)+p^2}{4t}\right]$$

$$-\frac{m_c^6\langle\bar{u}u\rangle\langle\bar{d}d\rangle\langle g_s^2G^2\rangle g_{\mu\nu}}{3^3.2^{12}\pi^2}\int_0^1 dx\int_0^{1-x}dy(1-x-y)x^{-2}y^{-3}$$

$$\times\int dtt^{-5}\left(g_{\mu\nu}+\frac{p_\mu p_\nu}{2t}\right)\exp\left[\frac{-s(x,y)+p^2}{4t}\right]$$

$$-\frac{m_c^6m_0^4\langle\bar{u}u\rangle\langle\bar{d}d\rangle\langle g_s^2G^2\rangle}{3^3.2^{20}\pi^2}\int_0^1 dx(1-x)^{-3}x^{-2}$$

$$\times\int dtt^{-7}\left(g_{\mu\nu}+\frac{p_\mu p_\nu}{2t}\right)\exp\left[\frac{-s(x,y)+p^2}{4t}\right]$$

$$+\frac{m_c^6m_0^2\langle\bar{u}u\rangle\langle\bar{d}d\rangle\langle g_s^2G^2\rangle g_{\mu\nu}}{3^3.2^{16}\pi^2}\int_0^1 dx\int_0^{1-x}dyx^{-2}y^{-3}$$

$$\times\int dtt^{-6}\left(g_{\mu\nu}+\frac{p_\mu p_\nu}{2t}\right)\exp\left[\frac{-s(x,y)+p^2}{4t}\right]$$

$$+\frac{m_c^6m_0^2\langle\bar{u}u\rangle\langle\bar{d}d\rangle\langle g_s^2G^2\rangle g_{\mu\nu}}{3^3.2^{16}\pi^2}\int_0^1 dx\int_0^{1-x}dyx^{-2}y^{-3}$$

$$\times\int dtt^{-6}\left(g_{\mu\nu}+\frac{p_\mu p_\nu}{2t}\right)\exp\left[\frac{-s(x,y)+p^2}{4t}\right], \tag{4.103}$$

$$\Pi^6_{\mu\nu,T_{cc}^+}\left(p^2\right)=\frac{3m_c^8}{5.3^2.2^{10}\pi^6}\int_0^1 dx\int_0^{1-x}dy(1-x-y)^6x^{-3}y^{-3}$$

$$\times\int dtt^{-1}\left(g_{\mu\nu}+\frac{p_\mu p_\nu}{2t}\right)\exp\left[\frac{-s(x,y)+p^2}{4t}\right]$$

$$-\frac{m_c^6 m_0^2\langle\bar{u}u\rangle\langle\bar{d}d\rangle\langle g_s^2G^2\rangle}{3^3.2^{16}\pi^2}\int_0^1 dx\int_0^{1-x} dyx^{-2}y^{-3}$$

$$\times\int dtt^{-6}\left(g_{\mu\nu}+\frac{p_\mu p_\nu}{2t}\right)\exp\left[\frac{-s(x,y)+p^2}{4t}\right]. \tag{4.104}$$

Since there are two possible cases for the contraction of c quarks, all results must be multiplied by 2. Thus, the contributions obtained using the full propagator for T_{cc}^+, called $\Pi^{full}_{\mu\nu,T_{cc}^+}(p^2)$, can be written as follows:

$$\Pi^{full}_{\mu\nu,T_{cc}^+}\left(p^2\right)=2\left[\Pi^1_{\mu\nu,T_{cc}^+}\left(p^2\right)+\Pi^2_{\mu\nu,T_{cc}^+}\left(p^2\right)+\Pi^3_{\mu\nu,T_{cc}^+}\left(p^2\right)\right.$$
$$\left.+\Pi^4_{\mu\nu,T_{cc}^+}\left(p^2\right)+\Pi^5_{\mu\nu,T_{cc}^+}\left(p^2\right)+\Pi^6_{\mu\nu,T_{cc}^+}\left(p^2\right)\right]. \tag{4.105}$$

In addition to these results, there are special situations that cannot be obtained using a full propagator. One of them is the situation where gluon emissions from heavy quarks form condensate. As an example, specifying only the important steps, a solution for this situation is made as follows.

To evaluate the Feynman diagram of this process given in Figure 4.1 (c1), the correlation function,

$$\Pi^{int}_{\mu\nu,T_{cc}^+}(p^2)=i\int d^4xe^{ipx}\left\langle\Omega\left|\mathcal{T}\left\{J_\mu(x)J_\nu^\dagger(0)e^{i\int d^4y\mathcal{L}_{\text{int}}(y)}\right\}\right|\Omega\right\rangle \tag{4.106}$$

is used. Here, $\mathcal{L}_{\text{int}}$ is the interaction term.

The interaction term is placed inside the correlation function and written as follows, using the necessary propagators for the relevant contractions:

In the Fock Schwinger gauge, the gluon field can be written as

$$G_{\alpha}^{N}\left(y\right)=-\frac{1}{2}G_{\alpha\beta}^{N}\left(0\right)y^{\beta}+\cdots, \tag{4.107}$$

and using

$$\langle\Omega|:g_s^2G_{\alpha\mu}^{N}\left(0\right)G_{\beta\nu}^{M}\left(0\right):|\Omega\rangle=\frac{\delta^{NM}}{96}\left(g_{\alpha\beta}g_{\mu\nu}-g_{\alpha\nu}g_{\mu\beta}\right)\langle g_s^2G^2\rangle \quad , \tag{4.108}$$

$$S_{ab}^{G_{\alpha\beta}^{N}}\left(x\right)=\frac{it_{cd}}{2}\int d^4yS_{ac}^{0}\left(x-y\right)\gamma^{\alpha}y^{\beta}S_{db}^{0}\left(y\right), \tag{4.109}$$

one obtains

$$\Pi_{\mu\nu,T_{cc}^{+}}^{G^cG^c}\left(p^2\right)=-\frac{i}{3.2^4}\langle g_s^2G^2\rangle\int d^4xe^{ipx}\varepsilon^{abe}\varepsilon^{a'b'e'}\delta^{ee'}\left(g_{\xi\Gamma}g_{\omega\kappa}-g_{\xi\kappa}g_{\omega\Gamma}\right)$$

$$\times Tr\left[S^{u}\left(-x\right)\left(C\gamma_5\right)S^{d^T}\left(-x\right)\left(\gamma_5C\right)\right]Tr\left[S^{G_{\xi\omega}^{N}}\left(x\right)\left(\gamma_{\nu}C\right)S^{G_{\Gamma\kappa}^{M^T}}\left(x\right)\left(C\gamma_{\mu}\right)^{T}\right]. \tag{4.110}$$

The propagator expression for gluon emission in this equation is

$$S_{ab}^{G_{\alpha\beta}^{N}}\left(x\right)=-\frac{m_{q,Q}}{32\pi^2}\times\left\{im_{q,Q}\left(\sigma^{\alpha\beta}\not{x}+\not{x}\sigma^{\alpha\beta}\right)\frac{K_1\left(m_{q,Q}\sqrt{-x^2}\right)}{m_{q,Q}\sqrt{-x^2}\Big)}+2\sigma^{\alpha\beta}K_0\left(m_{q,Q}\sqrt{-x^2}\right)\right\}. \tag{4.111}$$

Furthermore, the evaluation of one of the traces in Equation 4.110 is given in Equation 4.37. For the other trace,

$$Tr\left[S^{G_{\xi\omega}^{N}}\left(x\right)\left(\gamma_{\nu}C\right)S^{G_{\Gamma\kappa}^{M^T}}\left(x\right)\left(C\gamma_{\mu}\right)^{T}\right]\left(g_{\alpha\beta}g_{\mu\nu}-g_{\alpha\nu}g_{\mu\beta}\right)\left(g_{\xi\Gamma}g_{\omega\kappa}-g_{\xi\kappa}g_{\omega\Gamma}\right)$$

$$=\frac{t_{aa'}^{N}t_{bb'}^{N}}{m_c^2}\left(x^2g_{\mu\nu}+2x_{\mu}x_{\nu}\right)A_0^{c}\left(x^2\right)^2. \tag{4.112}$$

can be obtained. Here,

$$\varepsilon^{abe}\varepsilon^{a'b'e'}t_{aa'}t_{bb'}\delta^{ee'} = -4 \tag{4.113}$$

is also needed. Hence

$$\begin{aligned}
\Pi^{G^cG^c}_{\mu\nu,T^+_{cc}}\left(p^2\right) = &-\frac{i}{3.2m_c^2}\langle g_s^2G^2\rangle\int d^4xe^{ipx}A_0^c\left(x^2\right)^2A^u\left(x^2\right)A^d\left(x^2\right)x^2g_{\mu\nu}\\
&-\frac{i}{3m_c^2}\langle g_s^2G^2\rangle\int d^4xe^{ipx}A_0^c\left(x^2\right)^2A^u\left(x^2\right)A^d\left(x^2\right)x_\mu x_\nu\\
&-\frac{i}{3.2m_c^2}\langle g_s^2G^2\rangle\int d^4xe^{ipx}A_0^c\left(x^2\right)^2B^u\left(x^2\right)B^d\left(x^2\right)x^4g_{\mu\nu}\\
&-\frac{i}{3m_c^2}\langle g_s^2G^2\rangle\int d^4xe^{ipx}A_0^c\left(x^2\right)^2B^u\left(x^2\right)B^d\left(x^2\right)x^2x_\mu x_\nu.
\end{aligned} \tag{4.114}$$

In these integrals, again assuming $m_q \to 0$ and taking the contributions from states with at most two gluons, the following results are obtained.

$$\begin{aligned}
\Pi^{G^cG^c}_{\mu\nu,T^+_{cc}}\left(p^2\right) = &-\frac{m_c^4\langle g_s^2G^2\rangle}{3^2.2^{12}\pi^6}\int_0^1 dx\int_0^{1-x} dy(1-x-y)^3x^{-2}y^{-2}\\
&\times\int dtt^{-1}\exp\left[\frac{-s(x,y)+p^2}{4t}\right]\\
&-\frac{m_c^4\langle g_s^2G^2\rangle g_{\mu\nu}}{3^2.2^{14}\pi^6}\int_0^1 dx\int_0^{1-x} dy(1-x-y)^4x^{-2}y^{-2}\\
&\times\int dtt^{-1}\left(g_{\mu\nu}+\frac{p_\mu p_\nu}{2t}\right)\exp\left[\frac{-s(x,y)+p^2}{4t}\right].
\end{aligned} \tag{4.115}$$

Calculations similar to this calculation are made for other contributions, and contributions that did not yield zero are added to the results.

As stated in Chapter 3, to evaluate a certain physical quantity in the QCD sum rules method, the hadronic and theoretical parts must be calculated, and the results obtained by applying the Borel transformation must be matched.

Accordingly, in this section, just as an example, the following result is obtained from the results obtained for the perturbative part by applying the Borel transformation and continuum subtraction to the contributing terms.

$$\begin{aligned}
\Pi^{pert,\hat{\mathcal{B}}}_{\mu\nu,T_{cc}^{+}}\left(M^2,s_0\right) = &-\frac{m_c^6 g_{\mu\nu}}{2^6\pi^6}\int_0^{s_0} ds e^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x} dy(1-x-y)^4 x^{-2}y^{-2} \\
&\times (s-s(x,y))\theta(s-s(x,y)) \\
&-\frac{m_c^8 g_{\mu\nu}}{5.2^6\pi^6}\int_0^{s_0} ds e^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x} dy(1-x-y)^5 x^{-3}y^{-3}\theta(s-s(x,y)) \\
&+\frac{m_c^8 g_{\mu\nu}}{2^9\pi^6}\int_0^{s_0} ds e^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x} dy(1-x-y)^6 x^{-3}y^{-3}\theta(s-s(x,y)) \\
&+\frac{m_c^8 p_\mu p_\nu}{2^8\pi^6}\int_0^1 dx\int_0^{1-x} dy(1-x-y)^6 x^{-3}y^{-3}e^{-\frac{s(x,y)}{M^2}}\theta(s_0-s(x,y))
\end{aligned} \tag{4.116}$$

In addition, the Borel transformations of all results have been obtained with similar methods and included in the study. Explicit expressions for the correlation function for T_{cc}^{+} are given in Appendix C.1.

4.2 MASS CALCULATION OF THE HIDDEN CHARMONIUM 1^{+-} AND 0^{++} TETRAQUARKS

In this section, the 1^{+-} and 0^{++} tetraquarks will be examined. Since the calculations for T_{cc}^{+} are shown in detail in Section 4.1, the outlines of the calculations for these particles will be mentioned in this section. In order to facilitate follow-up and comparison, the same definitions and symbols are used, and existing definitions are not restated.

Using Equations 3.2 and 3.1, the two-point correlation functions for the 1^{+-} and 0^{++} tetraquarks can be given by, respectively,

$$\Pi_{\mu\nu,1^{+-}}\left(p^2\right) = i\int d^4x e^{ipx}\left\langle\Omega\left|\mathcal{T}\left\{J_{\mu}^{1^{+-}}(x)J_{\nu}^{1^{+-}\dagger}(0)\right\}\right|\Omega\right\rangle, \tag{4.117}$$

$$\Pi_{0^{++}}\left(p^2\right) = i\int d^4x e^{ipx}\left\langle\Omega\left|\mathcal{T}\left\{J^{0^{++}}(x)J^{0^{++}\dagger}(0)\right\}\right|\Omega\right\rangle. \tag{4.118}$$

Here, for 1^{+-} and 0^{++} tetraquarks, interpolating currents are chosen as

$$J_{\mu}^{1^{+-}}(x) = \left[\left(\bar{c}^a\gamma_\mu c^c\right)(x)\left(\bar{q_1}^b\gamma_5 {q_2}^d\right)(x)\right]\varepsilon^{abe}\varepsilon^{cde}, \tag{4.119}$$

$$J^{0^{++}}(x) = \left[\left(\bar{c}^a\gamma_5 c^c\right)(x)\left(\bar{q_1}^b\gamma_5 {q_2}^d\right)(x)\right]\varepsilon^{abe}\varepsilon^{cde}. \tag{4.120}$$

C parity assignments correspond to the case when $q_1 = q_2$. In this work, the $q_1 = u$ and $q_2 = d$ quark is taken, but the C parity assignments are kept as if $q_1 = q_2$.

4.2.1 Hadronic Side of the 1^{+-} and 0^{++} Tetraquarks Correlation Functions

Since 1^{+-} is an axial vector particle similar to the T_{cc}^{+} tetraquark, the hadronic side of 1^{+-} is very similar to T_{cc}^{+}. Therefore, as in the T_{cc}^{+} tetraquark, the hadronic side of 1^{+-} can be written as

$$\Pi^{Had}_{\mu\nu,1^{+-}}\left(p^2\right) = \frac{|\lambda_0|^2 p_\mu p_\nu}{p^2 - m_0^2} + \frac{|\lambda_1|^2\left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}\right)}{p^2 - m_{1^{+-}}^2}. \tag{4.121}$$

The hadronic side of the 0^{++} particle is given as

$$\begin{aligned}\Pi^{Had}_{0^{++}}\left(p^2\right) &= \sum_h \frac{\left\langle \Omega \left| J^{0^{++}} \right| h(p) \right\rangle \left\langle h(p) \left| J^{0^{++}\dagger} \right| \Omega \right\rangle}{p^2 - m_h^2} + \cdots \\ &= \sum_0 \frac{\left\langle \Omega \left| J^{0^{++}} \right| 0(p) \right\rangle \left\langle 0(p) \left| J^{0^{++}\dagger} \right| \Omega \right\rangle}{p^2 - m_0^2} + \cdots .\end{aligned} \tag{4.122}$$

Defining

$$\left\langle S = 0 \left| J^{0^{++}\dagger} \right| \Omega \right\rangle = \lambda_0 \tag{4.123}$$

for the hadronic side of the particle 0^{++}, $\Pi^{Had}_{0^{++}}\left(p^2\right)$ can be written as

$$\Pi^{Had}_{0^{++}}\left(p^2\right) = \frac{|\lambda_0|^2}{p^2 - m_{0^{++}}^2}. \tag{4.124}$$

4.2.2 Theoretical Side of the 1^{+-} and 0^{++} Tetraquarks

By inserting the interpolating currents for both tetraquarks into the relevant correlation function, the function can be written as for 1^{+-}

$$\begin{aligned}\Pi_{\mu\nu,1^{+-}}\left(p^2\right) = i\int d^4x e^{ipx}\left\langle \Omega \right| \mathcal{T}\left\{\left[\left(\bar{c}^a\gamma_\mu c^c\right)(x)\left(\bar{u}^b\gamma_5 d^d\right)(x)\right]\ \varepsilon^{abe}\varepsilon^{cde}\varepsilon^{a'b'e'}\varepsilon^{c'd'e'}\right. \\ \times\ \left[\left(\bar{d}^{d'}\gamma_5 u^{b'}\right)(0)\left(\bar{c}^{c'}\gamma_\mu c^{a'}\right)(0)\right]\varepsilon^{a'b'e'}\varepsilon^{c'd'e'}\left.\right\}\left|\,\Omega\right\rangle,\end{aligned}$$

and 0^{++}

$$\Pi_{0^{++}}\left(p^2\right) = i\int d^4x e^{ipx}\left\langle\Omega\left|\mathcal{T}\left\{\left[\left(\bar{c}^a\gamma_5 c^c\right)(x)\left(\bar{u}^b\gamma_5 d^d\right)(x)\right]\ \varepsilon^{abe}\varepsilon^{cde}\varepsilon^{a'b'e'}\varepsilon^{c'd'e'}\right.\right.\right.$$

$$\times\ \left[\left(\bar{d}^{d'}\gamma_5 u^{b'}\right)(0)\left(\bar{c}^{c'}\gamma_5 c^{a'}\right)(0)\right]\varepsilon^{a'b'e'}\varepsilon^{c'd'e'}\Big\}\Big|\,\Omega\Big\rangle.$$

Wick's rotation is applied to these correlation functions, and by repeating the steps detailed for T_{cc}^{+} in Section 4.1, the following equations are obtained for 1^{+-} and 0^{++}, respectively.

$$\Pi^4_{\mu\nu,1^{+-}}\left(p^2\right) = 12i\int d^4x e^{ipx}$$
$$Tr\left[S^c(-x)\gamma_\mu S^c(x)\gamma_\nu\right]Tr\left[S^u(-x)(\gamma_5)S^d(x)(\gamma_5)\right], \qquad (4.125)$$

$$\Pi^4_{0^{++}}\left(p^2\right) = 12i\int d^4x e^{ipx}$$
$$Tr\left[S^c(-x)\gamma_5 S^c(x)\gamma_5\right]Tr\left[S^u(-x)(\gamma_5)S^d(x)(\gamma_5)\right]. \qquad (4.126)$$

Here, the superscripts "4" refer to the situation with 4 contractions, as in Section 4.1.

To simplify, using the definitions in equations 4.31 and 4.32, equations 4.125 and 4.126 become

$$\begin{aligned}\Pi^4_{\mu\nu,1^{+-}}\left(p^2\right) = &(3i)2^6\int d^4x e^{ipx}A_0^c\left(x^2\right)^2A_0^u\left(x^2\right)A_0^d\left(x^2\right)g_{\mu\nu}\\ &+(3i)2^6\int d^4x e^{ipx}x^2A_0^c\left(x^2\right)^2B_0^u\left(x^2\right)B_0^d\left(x^2\right)g_{\mu\nu}\\ &+(3i)2^6\int d^4x e^{ipx}x^2B_0^c\left(x^2\right)^2A_0^u\left(x^2\right)A_0^d\left(x^2\right)g_{\mu\nu}\\ &+(3i)2^6\int d^4x e^{ipx}x^4B_0^c\left(x^2\right)^2B_0^u\left(x^2\right)B_0^d\left(x^2\right)g_{\mu\nu}\\ &-(3i)2^7\int d^4x e^{ipx}B_0^c\left(x^2\right)^2A_0^u\left(x^2\right)A_0^d\left(x^2\right)x_\nu x_\mu\\ &-(3i)2^7\int d^4x e^{ipx}B_0^c\left(x^2\right)^2B_0^u\left(x^2\right)B_0^d\left(x^2\right)x_\nu x_\mu, \qquad (4.127)\end{aligned}$$

$$\Pi^4_{0^{++}}\left(p^2\right) = (3i)2^6\int d^4xe^{ipx}A^c_0\left(x^2\right)^2A^u_0\left(x^2\right)A^d_0\left(x^2\right)$$
$$+ (3i)2^6\int d^4xe^{ipx}x^2A^c_0\left(x^2\right)^2B^u_0\left(x^2\right)B^d_0\left(x^2\right)$$
$$+ (3i)2^6\int d^4xe^{ipx}x^2B^c_0\left(x^2\right)^2A^u_0\left(x^2\right)A^d_0\left(x^2\right)$$
$$+ (3i)2^6\int d^4xe^{ipx}x^4B^c_0\left(x^2\right)^2B^u_0\left(x^2\right)B^d_0\left(x^2\right). \tag{4.128}$$

respectively. As seen in Equation 4.127, these integrals obtained for 1^{+-} are the same as the integrals in Equation 4.38 given for T^+_{cc}. In addition, the results of 0^{++} appear the same as the first four integrals, except for the coefficient $g_{\mu\nu}$.

Thus, except for some special cases, the results for 1^{+-} are the same as for T^+_{cc}.

One of the situations that causes the difference in the calculations of the correlation function of 1^{+-} compared to T^+_{cc} is that the c quarks in T^+_{cc} can contract in two ways, while in the charmonium case 1^{+-}, a c quark and an $\bar{c}$ antiquark can contract in only one way.

Thus, the results obtained using the full propagator for 1^{+-} can be written in terms of the results obtained for T^+_{cc} as

$$\Pi^{full}_{\mu\nu,1^{+-}}\left(p^2\right) = \frac{1}{2}\Pi^{full}_{\mu\nu,T^+_{cc}}\left(p^2\right). \tag{4.129}$$

Similarly, it is possible to write the results of 0^{++} obtained from the full propagator in terms of the results of T^+_{cc}. These results of 0^{++} are related to the first 4 terms of the results T^+_{cc} in Equation 4.105 and can be expressed exactly as

$$2g_{\mu\nu}\Pi^{full}_{0^{++}}\left(p^2\right) = \Pi^1_{\mu\nu,T^+_{cc}}\left(p^2\right) + \Pi^2_{\mu\nu,T^+_{cc}}\left(p^2\right) + \Pi^3_{\mu\nu,T^+_{cc}}\left(p^2\right) + \Pi^4_{\mu\nu,T^+_{cc}}\left(p^2\right). \tag{4.130}$$

The other reason for the difference between the results is due to the color factors in the interaction terms. For T_{cc}^{+}, color factors are present in the identity in Appendix A.37, while for 1^{+-} and 0^{++}, the identity in Appendix A.40 appears. Therefore, as can be seen from these identities, there is also an additional coefficient $(1/2)$ between the interaction terms of T_{cc}^{+} and the particles 1^{+-}, and 0^{++}.

As in the T_{cc}^{+} tetraquark, after all these cases were obtained and collected, the Borel transformation was applied and the obtained results are analysed numerically. These analytical results obtained for the 1^{+-} and the 0^{++} tetraquarks are given in Appendix D.1 and in Appendix E.1, respectively.

CHAPTER 5

NUMERICAL ANALYSIS

In this section, a numerical analysis of the sum rules obtained in previous sections is presented. The QCD sum rules method has many input parameters, such as quark masses, quark and gluon condensates, etc. Numerical values of these parameters are needed to perform numerical calculations. The numerical values of the parameters used in this thesis are listed in Table 5.1 [6, 98, 120, 121].

Table 5.1: Input parameters used in calculations

Parameters	**Values**
m_u	0
m_d	0
m_c	1.27 ± 0.02 GeV
$\langle \bar{u}u \rangle$	$(-0.24 \pm 0.01)^3$ GeV^3
$\langle \bar{d}d \rangle$	$(-0.24 \pm 0.01)^3$ GeV^3
m_0^2	(0.8 ± 0.1) GeV^2
$\langle g_s^2 G^2 \rangle$	$4\pi^2(0.012 \pm 0.004)$ GeV^4

The two-point QCD sum rules method includes, in addition to these input parameters, two auxiliary parameters called the Borel parameter M^2 and the continuum threshold s_0. Since these auxiliary parameters are not physical quantities, the physical quantities to be calculated must be independent of them. Due to the approximation used, a slight dependence is acceptable.

It is not possible to find regions where the physical parameters are completely independent of the continuum threshold. The continuum threshold is related to the energy threshold for multi hadron states and also the excited states.

The value of s_0 is usually chosen based on the mass of the hadron under consideration as

$$(m_h + 0.3 \text{ GeV})^2 \leq s_0 \leq (m_h + 0.5 \text{ GeV})^2. \tag{5.1}$$

When the Borel transformation is applied to the correlation function, it is seen that the resulting integral has an exponential expression. Due to this expression, the largest contribution to the integral comes from the values of $s \lessapprox M^2$, and it is expected that the contribution of the ground state is dominant if M^2 is chosen as small as possible.

However, choosing the M^2 value below a certain limit causes the contribution of the condensates to increase. Hence, a range must be determined for the possible values of M^2. Therefore, M^2 should be as large as possible.

For the upper limit of M^2, the pole contribution to the correlation function for multiquark systems, the following ratio must be the largest it can be. The maximum possible value for M^2 can be determined by considering the pole contribution fraction (PC) defined as

$$PC = \frac{\Pi^{\hat{\mathcal{B}}}[M^2, s_0]}{\Pi^{\hat{\mathcal{B}}}[M^2, \infty]} = \frac{\int_{s_{min}}^{s_0} ds\rho(s)e^{-\frac{s}{M^2}}}{\int_{s_{min}}^{\infty} ds\rho(s)e^{-\frac{s}{M^2}}}, \tag{5.2}$$

where $\Pi^{\hat{\mathcal{B}}}[M^2, s_0]$ is the Borel transformation of the correlation function.

On the other side, the lower limit of M^2 is obtained from the analysis of the fractional contributions of the condensates

$$R^d\left(M^2\right) = \frac{\Pi^d[M^2, s_0]}{\Pi[M^2, s_0]}, \tag{5.3}$$

where $\Pi^d[M^2, s_0]$ represents the contribution of the d dimensional condensate to the correlation function. For $d = 0$, this ratio should be as large as possible, and should be as small as possible as d gets larger. This equation expresses the convergence of the OPE and the resulting QCD sum rules.

5.1 Numerical Analysis of the Sum Rules for T_{cc}^{+} Tetraquark

To determine the minimum value of M^2 for the T_{cc}^{+} tetraquark, $R^d(M^2)$ in Equation 5.3 is plotted as a function of the Borel parameter M^2 in Figure 5.1.

In Figure 5.1, $d = 0$ represents the contribution of the perturbative part of the contribution and $d = 4, 6...$ represent the convergences of the condensates of higher dimensions. In this study, the contributions to the correlation function are from $0, 4, 6, 8, 10, 12$ and 14 dimensional condensates. Note that there is no contribution from the $d = 3$ operators.

Looking at Figure 5.1, it can be seen that although the contributions from the $d = 6$ and $d = 8$ dimensions seem large, they cancel out each other; however, the contribution from the perturbative part is still dominant. Thus, as can be seen from Figure 5.1, $M^2 \geq 2.5$ GeV^2 is enough to guarantee convergence.

Furthermore, the contribution of each dimension to the correlation function is cumu-

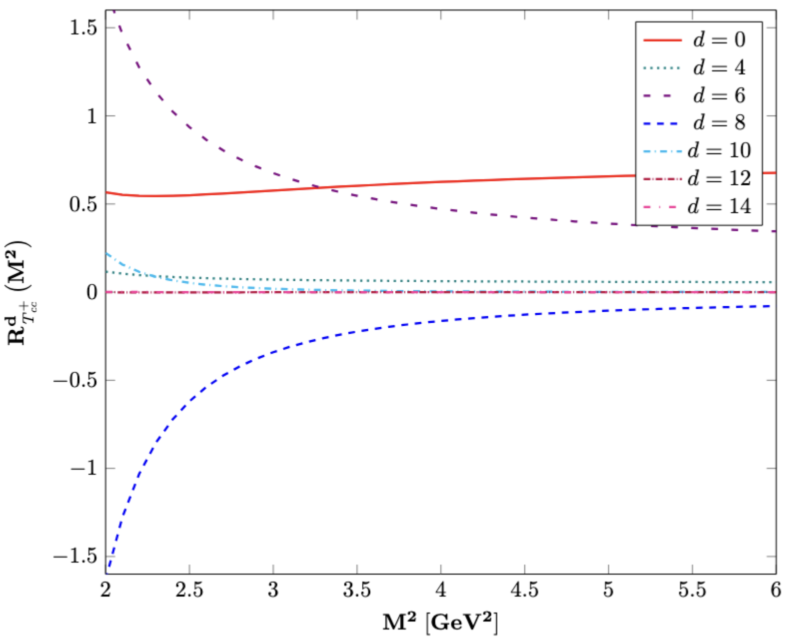

Figure 5.1: The convergence of the contribution of each dimension separately obtained as a function of M^2 at $\mathrm{s_0} = \mathbf{19.0\ GeV^2}$ value for $\mathbf{T^+_{cc}}$ tetraquark

latively added to create the Figure 5.2.

As can be seen in Figure 5.2, the contribution of the perturbative part is sufficiently dominant.

In order to obtain the upper limit for M^2, the pole contributions of the correlation function T^+_{cc} tetraquark can be calculated using Equation 5.2. For the values $s_0 = 18.0, 19.0$ and $20.0\ \mathrm{GeV}^2$, this contribution is shown in Figure 5.3.

Upon examination of Figure 5.3, it can be seen that up to $M^2 = 3.5\ \mathrm{GeV}^2$, the pole contribution is still more than about 30%, and it is an acceptable value as the maximum value of M^2.

In this case, as a result of the convergence and pole contributions, the appropriate interval chosen for the Borel parameter M^2 is $M^2 = [2.5 - 3.5]\ \mathrm{GeV}^2$.

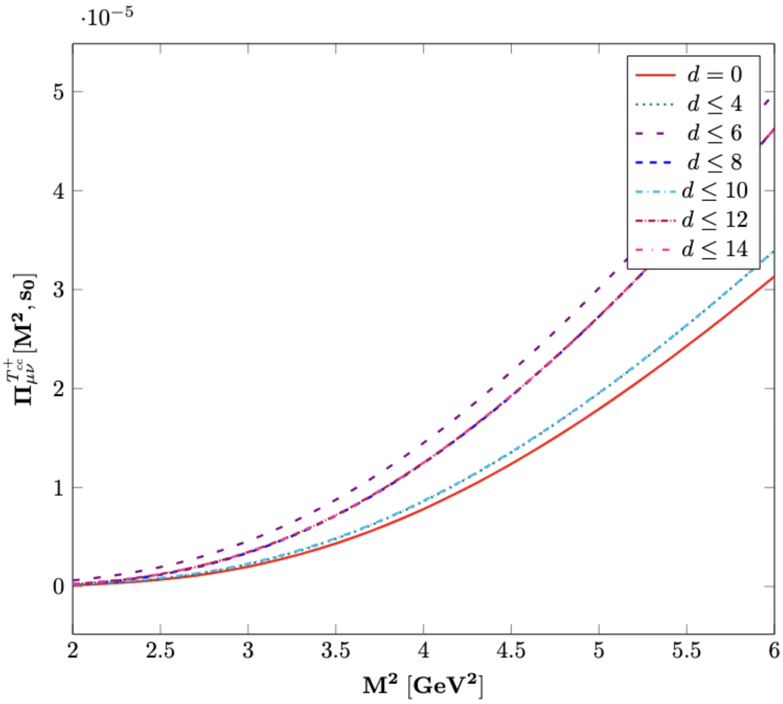

Figure 5.2: The cumulative contribution of each dimension to the correlation function as a function of M^2 at $\mathbf{s_0 = 19.0\ GeV^2}$ value for $\mathbf{T_{cc}^+}$ tetraquark

5.1.1 Mass Calculation of T_{cc}^+ Tetraquark

As mentioned in the 3^{rd} chapter, the mass can be obtained using Equation 3.39 which can also be written as

$$m^2_{T_{cc}^+}[M^2, s_0] = \frac{\frac{d\left(\tilde{\Pi}^1_{\mathcal{B}}[M^2, s_0]\right)}{d(\frac{-1}{M^2})}}{\tilde{\Pi}^1_{\mathcal{B}}[M^2, s_0]}. \tag{5.4}$$

Here, $\tilde{\Pi}^1_{\mathcal{B}}(M^2)$ is the Borel transform of the part related to the T_{cc}^+ tetraquark of the correlation function specified in section 4.1.1.

In Figure 5.4, the M^2 dependence of the predicted mass for T_{cc}^+ is shown for $s_0 = 18.0,\ 19.0,$ and $20.0\ \text{GeV}^2$ for the central values of the input parameters shown in

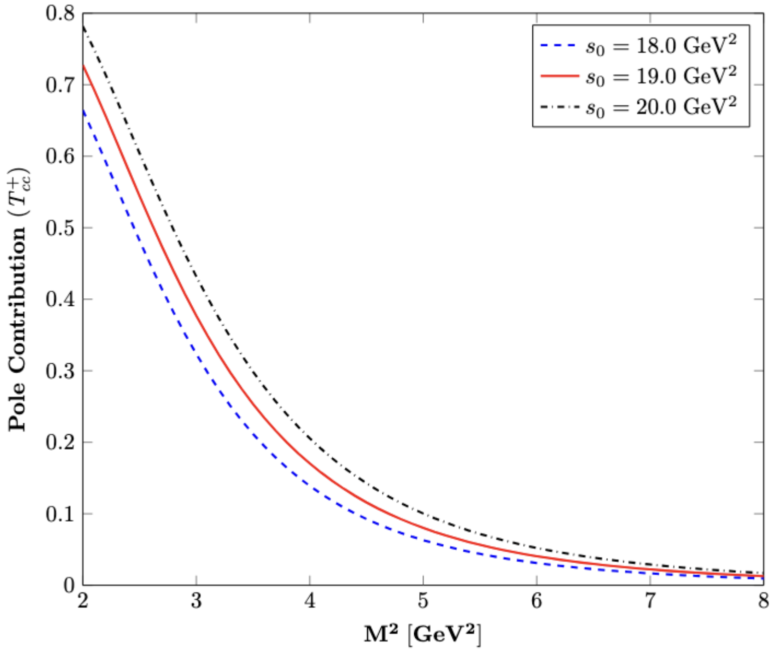

Figure 5.3: The pole contribution to the correlation function of the $\mathbf{T}_{\mathbf{cc}}^{+}$ tetraquark as a function of Borel parameter M^2 for different s_0 values

Table 5.1

As can be noticed in Figure 5.4, the mass of the T_{cc}^{+} tetraquark is almost independent of the Borel parameter M^2 in the working region and when s_0 changes in the range of $s_0 = [18.0 - 20.0]$ GeV^2, the mass value changes at most by 5%.

Borel parameter M^2 and the continuum threshold s_0 are not the only uncertain expressions in the calculations; there are also uncertainties in the condensate values. To determine the uncertainty in the mass predictions due to uncertainties in all the parameters a Monte Carlo analysis is performed. The procedure is presented in detail in [122]. The histogram of the values of the mass predictions is shown in Figure 5.5.

Upon examination of Figure 5.5, it is clearly seen that the distribution forms a distinct peak, with a mean value $\mu = 3.91$ GeV and a standard deviation value $\sigma = 0.05$ GeV.

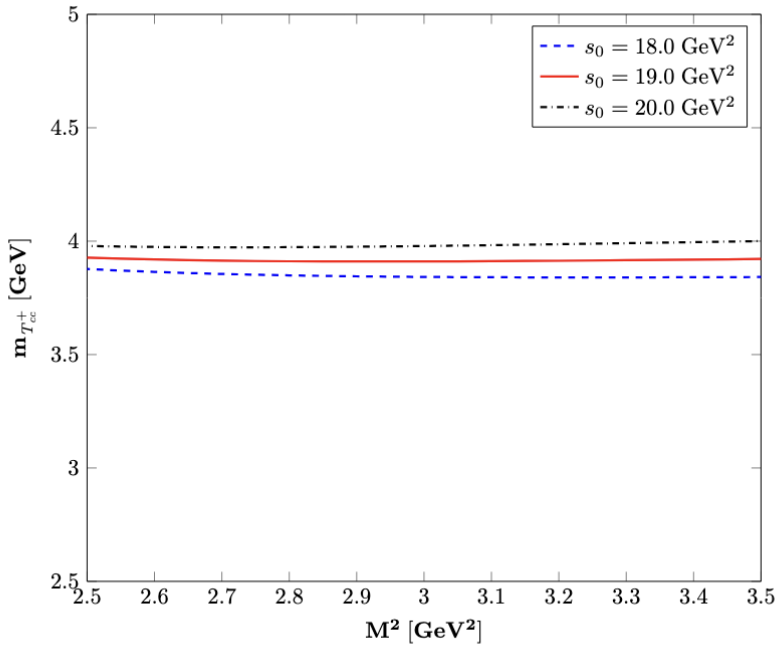

Figure 5.4: The mass obtained for T_{cc}^+ tetraquark as a function of M^2 for different s_0 values

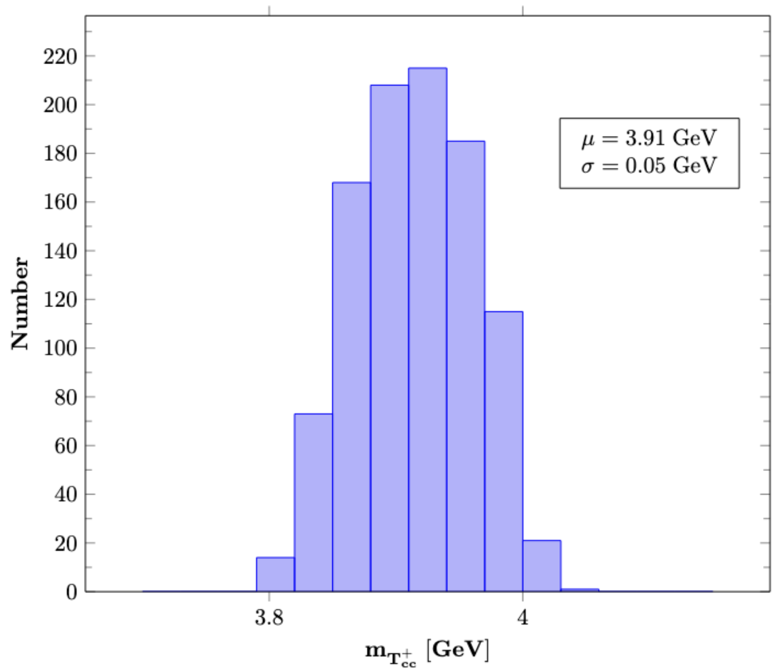

Figure 5.5: Distribution of the $\mathbf{m}_{\mathbf{T}_{cc}^+}$ for $M^2 = [2.5 - 3.5]$ GeV^2 and $s_0 = [18.0 - 20.0]$ GeV^2 values

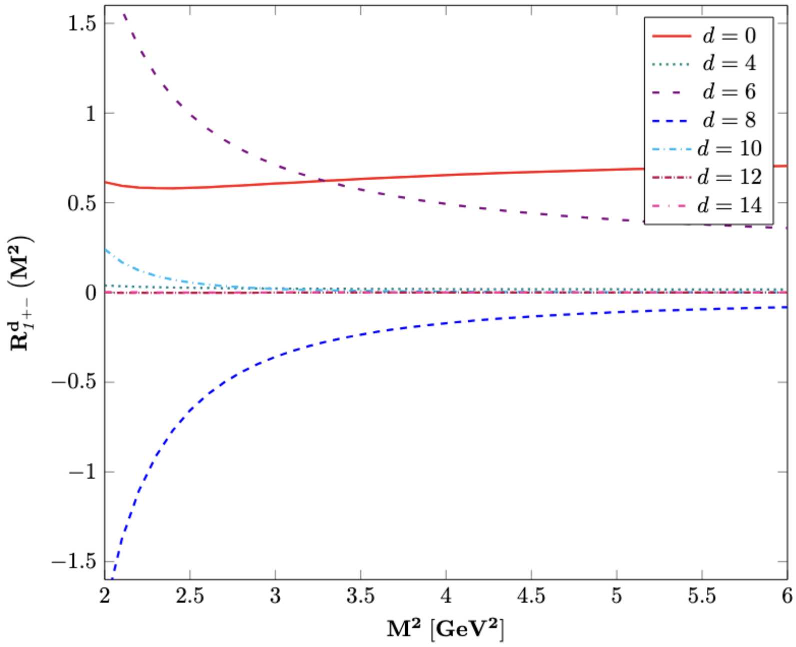

Figure 5.6: The convergence of the contribution of each dimension separately obtained as a function of M^2 at $\mathbf{s_0} = \mathbf{19.0\ GeV^2}$ value for $\mathbf{1^{+-}}$ tetraquark

Therefore, the mass is predicted to be $m_{T_{cc}^+} = 3.91 \pm 0.10$ where the shown error is 2σ so that $\sim 95\%$ of the predictions lie within this range.

5.2 Numerical Analysis of 1^{+-} and 0^{++} Tetraquarks

For the 1^{+-} and the 0^{++} tetraquarks, the analysis is similar to the analysis of the T_{cc}^+ tetraquark. As in the T_{cc}^+ tetraquark, the contribution of each condensate to the correlation function is examined to determine the minimum Borel parameter value M^2 for the 1^{+-} and the 0^{++} tetraquarks. The convergence of the contribution of the perturbative part and other condensates of the contribution to the correlation function of the 1^{+-} and the 0^{++} tetraquarks are shown in Figure 5.6 and in Figure 5.7, respectively.

Looking at Figures 5.6 and 5.7, it can be seen that for the 1^{+-} and 0^{++} tetraquarks, it

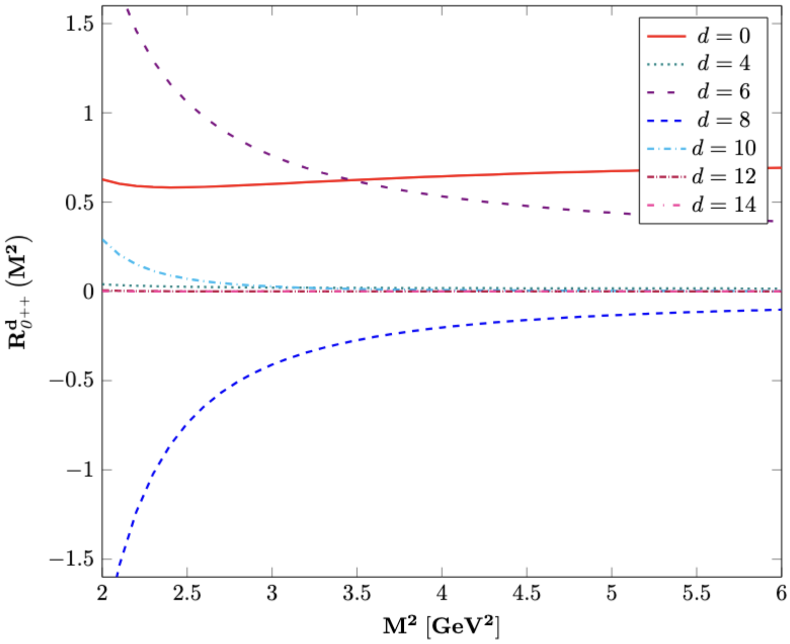

Figure 5.7: The convergence of the contribution of each dimension separately obtained as a function of M^2 at $\mathbf{s_0 = 19.0\ GeV^2}$ value for $\mathbf{0}^{++}$ tetraquark

would be appropriate to choose $M^2 \geq 2.5$ GeV^2 to ensure convergence, as in the T_{cc}^+ tetraquark.

Furthermore, the contribution of each dimension to the correlation function was investigated by adding the dimension cumulatively, and these are shown in Figure 5.8 and in Figure 5.9 for 1^{+-} and 0^{++}, respectively.

Taking into account Figure 5.8 for 1^{+-} and Figure 5.9 for 0^{++}, it is clearly seen that perturbate contributions to the correlation function are dominant for both particles.

For particles 1^{+-} and 0^{++}, it is also necessary to look at the pole contribution to obtain the appropriate maximum value of M^2. The pole contributions are shown in Figure 5.10 and in Figure 5.11 for 1^{+-} and 0^{++}, respectively.

As seen in Figures 5.10 and Figure 5.11, at the value $M^2 = 3.5$ GeV^2 as in the T_{cc}^+

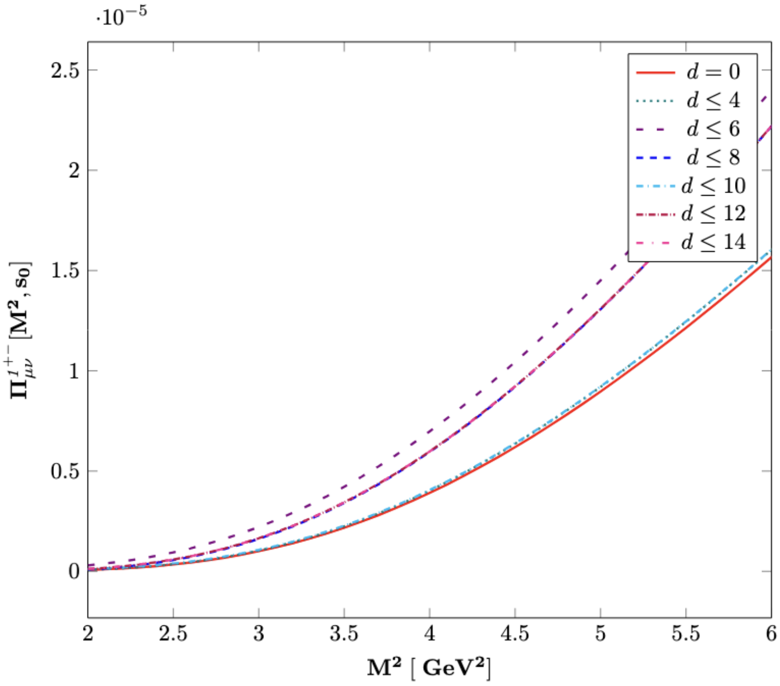

Figure 5.8: The cumulative contribution of each dimension to the correlation function as a function of M^2 at $\mathbf{s_0} = \mathbf{19.0\ GeV^2}$ value for $\mathbf{1^{+-}}$ tetraquark

tetraquark, the pole contribution is greater than 30% for 1^{+-} and 0^{++}, and therefore the upper bound of the working region for $M^2 \leqslant 3.5\ \mathrm{GeV}^2$.

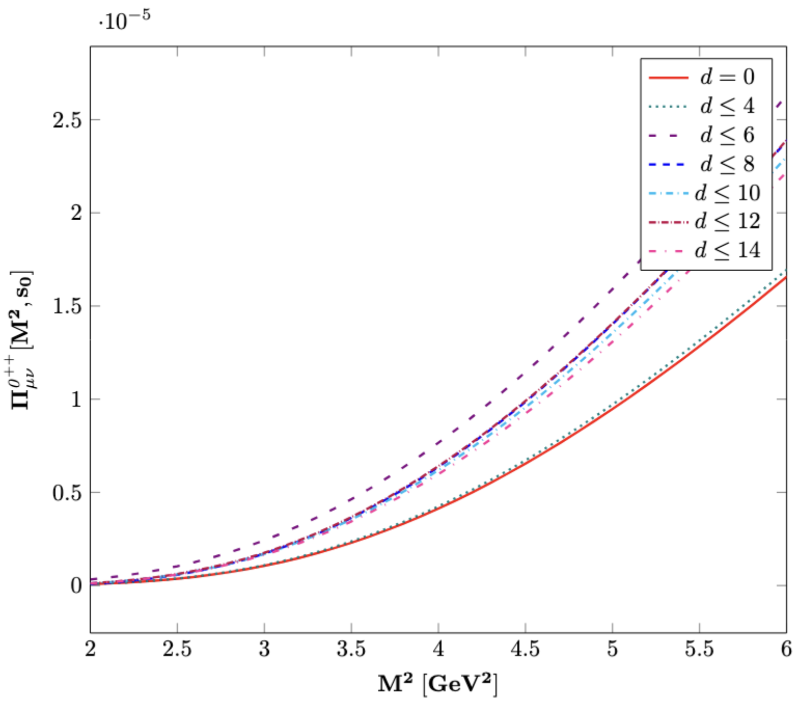

Figure 5.9: The cumulative contribution of each dimension to the correlation function as a function of M^2 at $s_0 = \mathbf{19.0\ GeV^2}$ value for $\mathbf{0^{++}}$ tetraquark

5.2.1 Mass Calculation of 1^{+-} and 0^{++} Tetraquarks

The Borel mass dependence of the predicted masses for the 1^{+-} and 0^{++} tetraquarks are shown in Figures 5.12 and 5.13, respectively.

Looking at Figure 5.12, it can be seen that for 1^{+-}, the mass value in the range of $M^2 = [2.5 - 3.5]$ GeV^2 is almost independent of the Borel parameter M^2 and the continuum threshold, as desired.

Likewise, examining Figure 5.13 for 0^{++}, the mass is independent of the Borel parameter in the working region $M^2 = [2.5 - 3.5]$ GeV^2.

As a result of the analysis, it is seen that the working region for 1^{+-} and 0^{++} is the same as T^+_{cc}. Therefore, in order to obtain the uncertainty arising from other

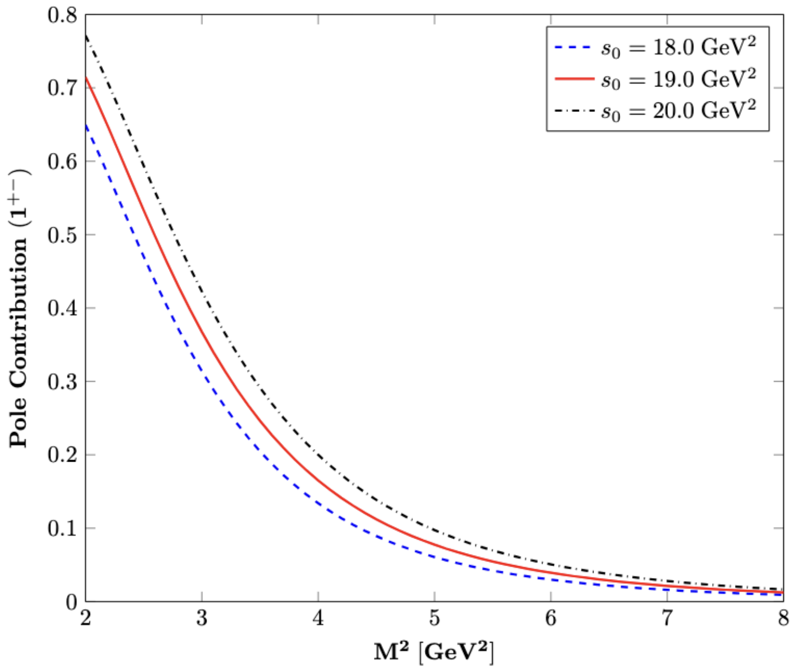

Figure 5.10: The pole contribution to the correlation function of the $\mathbf{1^{+-}}$ tetraquark as a function of Borel parameter M^2 for different s_0 values

parameters, as well as from M^2 and s_0 dependencies, random values are chosen for the input parameters within their uncertainties, and masses are calculated using these random values.

In this regard, histogram graphs showing the distribution of the masses in Figure 5.14 and 5.15 are shown for 1^{+-} and 0^{++}, respectively.

Therefore, the masses of these tetraquarks are predicted as:

$$m_{1^{+-}} = 3.93 \pm 0.10 \text{ GeV} \tag{5.5}$$

$$m_{0^{++}} = 3.94 \pm 0.10 \text{ GeV} \tag{5.6}$$

Upon examination of Figures 5.14 and 5.15, it can easily be seen that they have precise peak distributions, with a mean value $\mu = 3.93$ GeV and a standard deviation

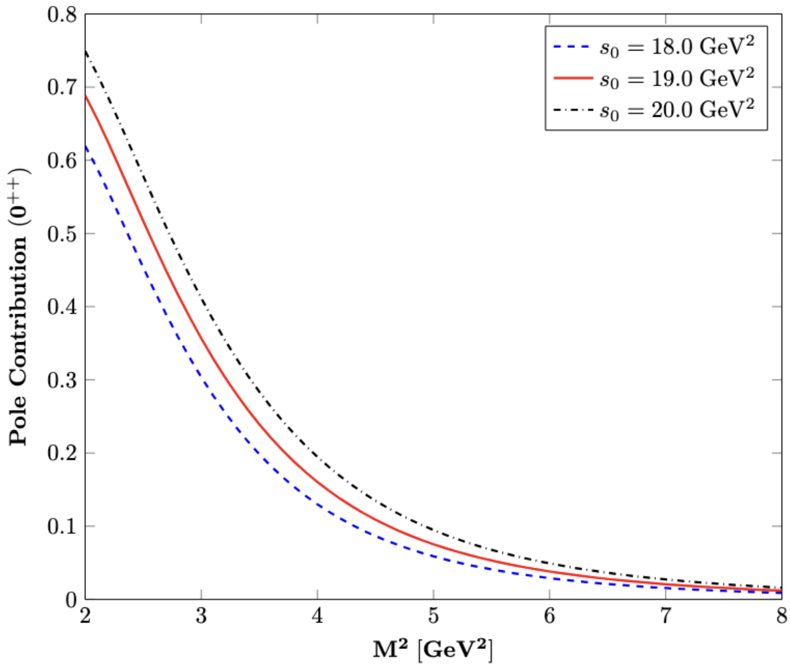

Figure 5.11: The pole contribution to the correlation function of the $\mathbf{0^{++}}$ tetraquark as a function of Borel parameter M^2 for different s_0 values

value $\sigma = 0.05$ GeV, and a mean value $\mu = 3.94$ GeV and a standard deviation value $\sigma = 0.05$ GeV, for 1^{+-} and 0^{++}, respectively.

In order to reduce uncertainty, instead of calculating the mass of each particle individually, it is possible to evaluate each particle by taking advantage of heavy-quark symmetry and looking at the mass difference with its symmetry partners. In line with this information, the difference between the particles 1^{+-} and 0^{++} is shown in Figure 5.16 using the same data set, used for the uncertainty in mass calculations.

Note that in calculating the mass difference, the same random values for the parameters are used in both the mass of 1^{+-} and 0^{++}.

For the mass difference of the particles 1^{+-} and 0^{+-}, looking at Figure 5.17, it is seen that the distribution has a distinct peak, with value $\delta m = (m_{0^{++}} - m_{1^{+-}}) =$

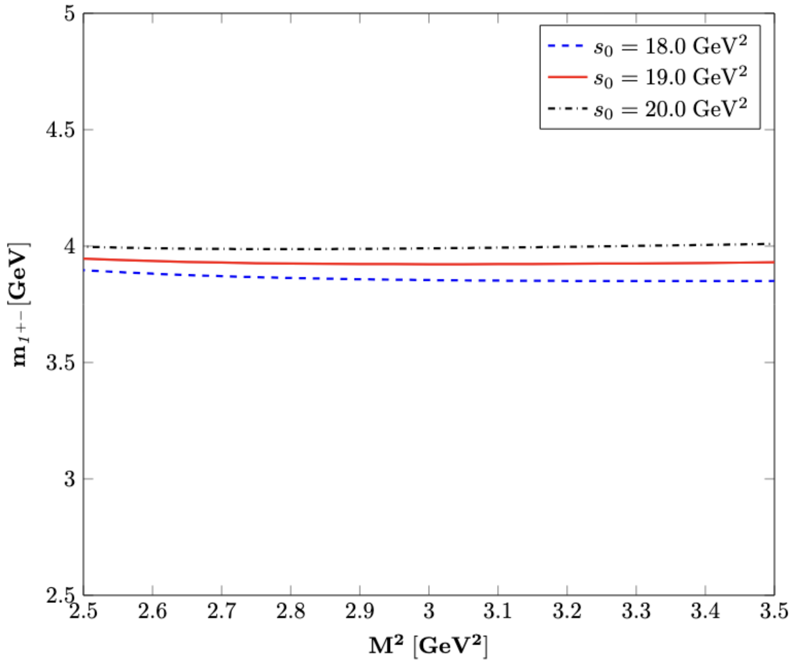

Figure 5.12: The mass obtained for $\mathbf{1}^{+-}$ tetraquark as a function of M^2 for different s_0 values

14.65 ± 6.14 MeV.

Furthermore, in order to make a prospective interpretation, the difference between the particles T_{cc}^{+} and 1^{+-}, which behave very similarly, is examined, and the result is shown in Figure 5.17.

Taking into account Figure 5.17, it is seen that the mass difference of the particles 1^{+-} and T_{cc}^{+} behaves as expected, but there are slight deviations and a peak distribution, as in the mass difference of 1^{+-} and 0^{++}. Therefore, Figure 5.17 with the value $\delta m = (m_{1^{+-}} - m_{T_{cc}^{+}}) = 11.52^{+9.08}_{-5.18}$ MeV has a distribution skewed to the left.

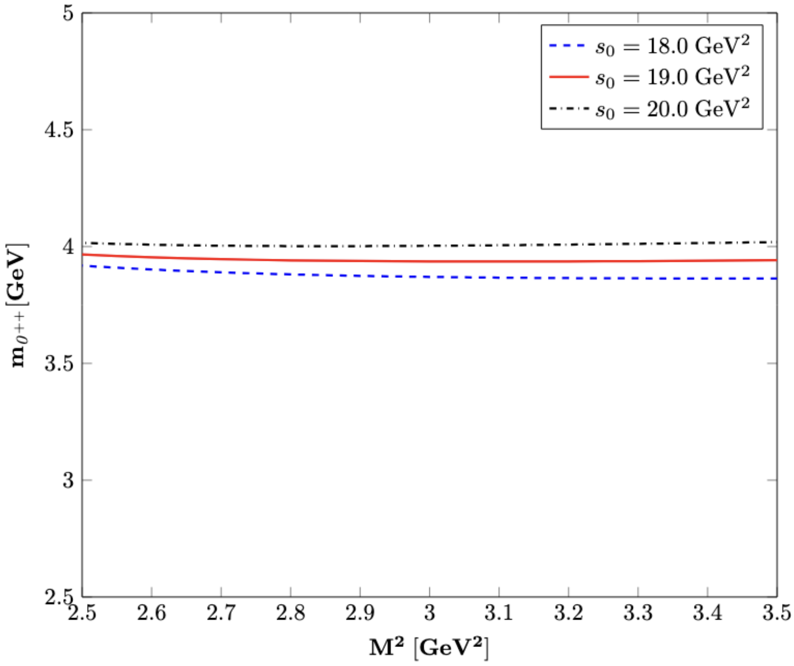

Figure 5.13: The mass obtained for $\mathbf{0}^{++}$ tetraquark as a function of M^2 for different s_0 values

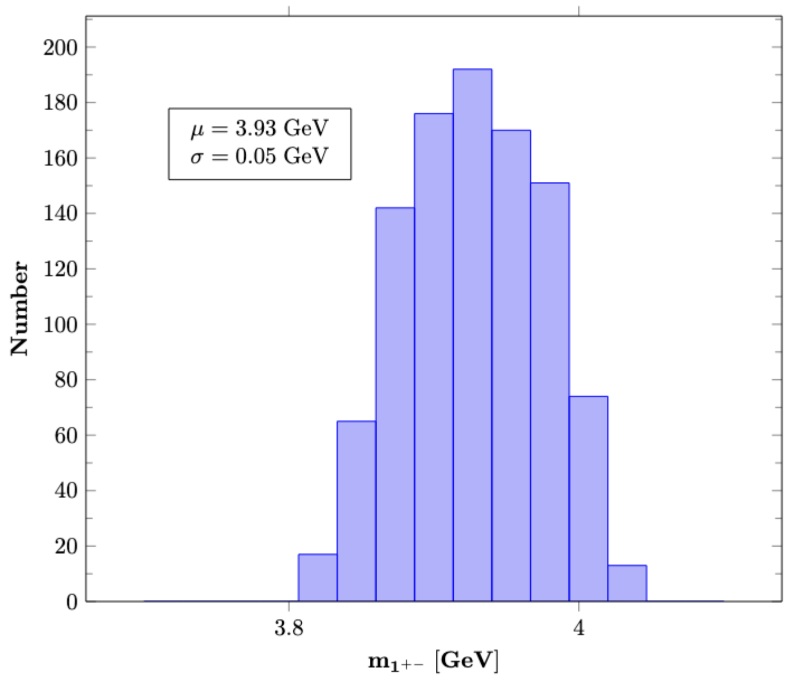

Figure 5.14: Distribution of the $\mathbf{m_{1^{+-}}}$ for $M^2 = [2.5 - 3.5]$ GeV^2 and $s_0 = [18.0 - 20.0]$ GeV^2 values

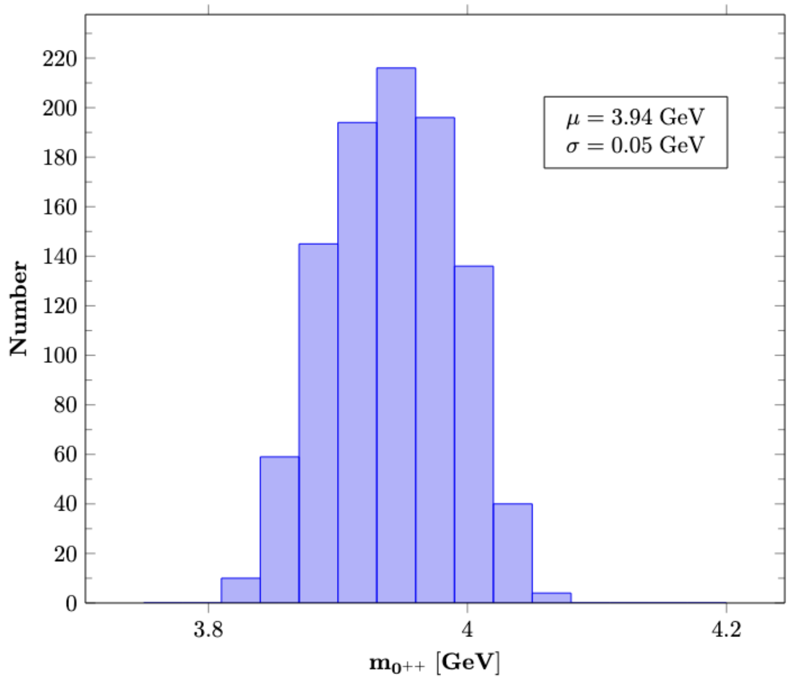

Figure 5.15: Distribution of the $\mathbf{m_{0^{++}}}$ for $M^2 = [2.5 - 3.5]$ GeV^2 and $s_0 = [18.0 - 20.0]$ GeV^2 values

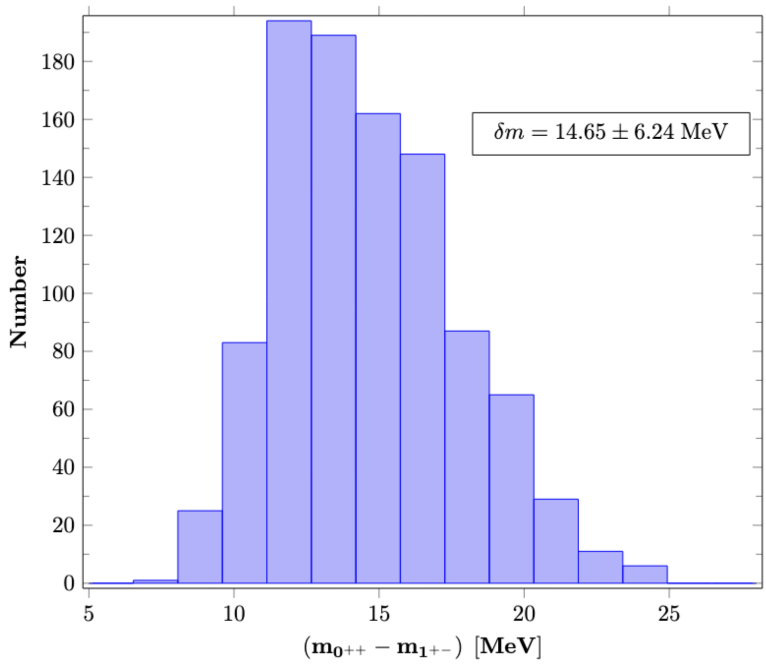

Figure 5.16: Distribution of the $(\mathbf{m_{0^{+-}}} - \mathbf{m_{1^{+-}}})$ for $M^2 = [2.5 - 3.5]$ GeV^2 and $s_0 = [18.0 - 20.0]$ GeV^2 values

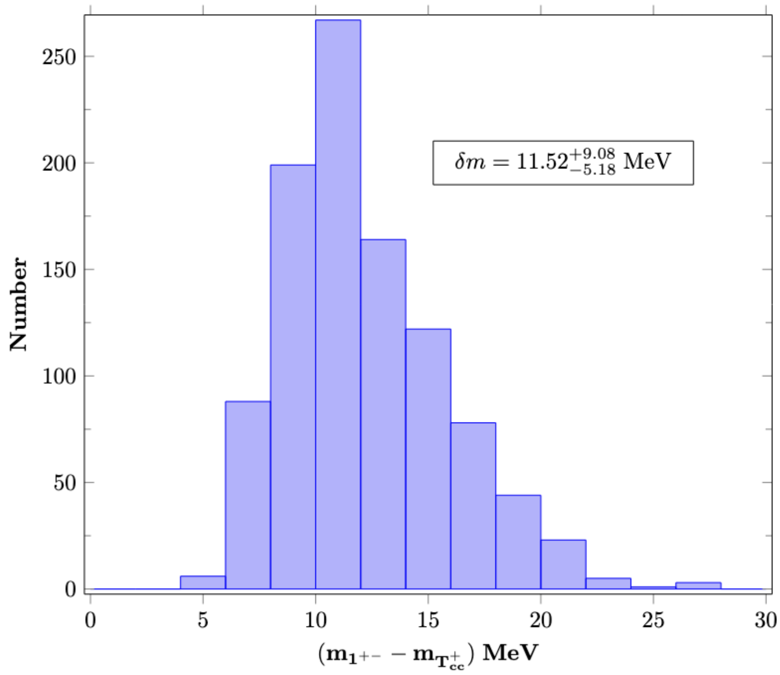

Figure 5.17: Distribution of the $(\mathbf{m_{1^{+-}}} - \mathbf{m_{T_{cc}^+}})$ for $M^2 = [2.5 - 3.5]$ GeV^2 and $s_0 = [18.0 - 20.0]$ GeV^2 values

CHAPTER 6

CONCLUSIONS

In this study, first, general information about particle physics and QCD theory was given. Then, the QCD sum rules method, a theoretically strong and reliable non-perturbative approach, was discussed and applied to calculate the masses of the T_{cc}^+, 0^{++}, and 1^{+-} tetraquarks. The working regions for the Borel parameters are determined to be $M^2 = [2.5 - 3.5]$ GeV^2 and for s_0 to be $s_0 = 19.0 \pm 1.0$ GeV^2.

In addition to these values, to eliminate the uncertainties arising from the uncertainties in the condensates, a data set consisting of random variables is used, and as a result, the mass of T_{cc}^+ tetraquark is found to be $m_{T_{cc}^+} = 3.91 \pm 0.10$ GeV. It is clearly seen that this result is compatible with the experimental value $m_{T_{cc}^+} = 3.88$ GeV [19, 42]. The error of the experimental value is 0.8%.

The predictions obtained from theoretical studies using different methods for the calculation of the mass (or delta mass difference from the two-meson threshold) of the T_{cc}^+ tetraquark are listed in Table 6.1. Upon examination of the values in Table 6.1, it is seen that the mass value obtained from this study is consistent with other theoretical results.

All analyses performed for T_{cc}^+ are also performed for 0^{++} and 1^{+-}, and it is seen

Table 6.1: The predictions of the mass of the T_{cc}^{+} tetraquark obtained using different theoretical methods

Works Methods	$\mathbf{m}$ or $\delta\mathbf{m}$[MeV]
This Study	3914 ± 94
QCD Sum Rules [123]	3868 ± 124
QCD Sum Rules [124]	4000 ± 200
Double Ratios of Sum Rules (DRSR) [125]	3872.2 ± 39.5
Quark Model [47]	3882 ± 12
Heavy Quark Limit [126]	3978
QCD Sum Rules [127]	3900 ± 90
Lattice QCD [128]	$\delta m = -23 \pm 11$
Lattice QCD [129]	3947 ± 11
Chromomagnetic (CMI) Model [130]	3929.3
Constituent Quark Model[131]	$\delta m = -23$

that the working regions suitable for T_{cc}^{+} are also suitable for these tetraquarks. In this direction, in order to make a more accurate comparison, the data set created for T_{cc}^{+} is also used for these particles, and for the range $M^2 = [2.5 - 3.5]$ GeV^2 and value $s_0 = 19.0 \pm 1.0$ GeV^2, $m_{1^{+-}} = 3.93 \pm 0.10$ GeV, and $m_{0^{++}} = 3.94 \pm 0.10$ GeV is obtained for 1^{+-} and 0^{++}, respectively.

In addition, due to the heavy-quark symmetry, in order to reduce the uncertainty in the mass calculation, the mass differences have also been examined using the same data set and for this purpose the mass difference of the particles 1^{+-} and 0^{++} is obtained as $\delta m = (m_{0^{++}} - m_{1^{+-}}) = 14.65 \pm 6.24$ MeV. As expected by the heavy-quark spin symmetry, it is clearly seen from the result obtained that the uncertainty that will arise by obtaining the 1^{+-} and 0^{++} masses separately decreases by calculating the mass difference, and even it only takes values around a few MeV.

To be able to comment further, the mass difference of T_{cc}^{+} and 1^{+-} states is also examined, based on the similarity of their quark contents and structures, and a fairly uniform distribution is obtained, $\delta m = (m_{T_{cc}^{+}} - m_{1^{+-}}) = 11.52^{+9.08}_{-5.18}$ MeV. This value is quite small, as expected, since the T_{cc}^{+} and 1^{+-} particles have very similar structures.

In this study, the motivation was taken from heavy-quark spin symmetry, but it should be noted here that heavy-quark spin symmetry is also an approximate symmetry, because in reality the mass of heavy quarks is not infinite. Especially since the subject of this study is c heavy quarks, their masses are not very large. In this case, some corrections are needed. In the future, it is possible to study heavy-quark spin symmetry in more detail, to do studies on these corrections, and to obtain more accurate results.

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Appendix A

GENERAL IDENTITIES

A.0.1 Gamma Matrices and Properties

In Dirac representation, the four contravariant gamma matrices are

$$
\begin{aligned}
\gamma^0 &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \\
\gamma^2 &= \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}, \quad \gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix},
\end{aligned} \tag{A.1}
$$

and the identity matrix is

$$
I_4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{A.2}
$$

and the γ^5 matrix is

$$\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}. \tag{A.3}$$

Some identities related to these Gamma matrices can be expressed as

$$(\gamma_0)^2 = \mathbb{1}, \tag{A.4}$$

$$\gamma^{0\dagger} = \gamma^0, \tag{A.5}$$

$$\gamma^0\gamma^\mu\gamma^0 = \gamma^{\mu\dagger}, \tag{A.6}$$

$$(\gamma_5)^2 = \mathbb{1}, \tag{A.7}$$

$$\gamma^{5\dagger} = \gamma^5, \tag{A.8}$$

$$(\gamma_5)^T = \gamma_5, \tag{A.9}$$

$$\{\gamma_5, \gamma_\mu\} = 0, \tag{A.10}$$

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}\mathbb{1}. \tag{A.11}$$

A.0.2 Charge Conjugation and Properties

The definition of charge conjugation and some properties related to it can be written as

$$C = i\gamma^2\gamma^0, \tag{A.12}$$

$$C^T = -C, \tag{A.13}$$

$$C^\dagger = C, \tag{A.14}$$

$$C^2 = -\mathbb{1}, \tag{A.15}$$

$$(C\gamma_5) = (\gamma_5 C), \tag{A.16}$$

$$C\gamma_5 C = -\gamma_5^T = -\gamma_5, \tag{A.17}$$

$$C\gamma_\mu C = -\gamma_\mu^T, \tag{A.18}$$

$$C\gamma_\mu^T C^T = -\gamma_\mu, \tag{A.19}$$

$$C\gamma_5 C^{-1} = +\left(\gamma_5\right)^T, \tag{A.20}$$

$$C\sigma_{\mu\nu} C^{-1} = -\left(\sigma_{\mu\nu}\right)^T, \tag{A.21}$$

$$C\gamma_5\gamma_\mu C^{-1} = +\left(\gamma_5\gamma_\mu\right)^T. \tag{A.22}$$

A.0.3 Trace Samples Identities

Some situations regarding traces are as follows:

$$\mathrm{Tr}\left(\gamma^{\mu}\right) = 0, \tag{A.23}$$

$$\mathrm{Tr}\left(\gamma^{5}\right) = 0, \tag{A.24}$$

$$\mathrm{Tr}\left(\gamma^{\mu}\gamma^{\nu}\gamma^{5}\right) = 0, \tag{A.25}$$

$$\mathrm{Tr}\left(\gamma^{\mu}\gamma^{\nu}\right) = 4g^{\mu\nu}, \tag{A.26}$$

$$\mathrm{Tr}[\text{ odd \# of gamma matrices }] = 0, \tag{A.27}$$

$$\mathrm{Tr}[\text{ odd \# of gamma matrices } \gamma^{5}] = 0, \tag{A.28}$$

$$\mathrm{Tr}\left(\gamma^{\mu}\gamma^{\nu}\gamma^{\rho}\gamma^{\sigma}\right) = 4\left(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho}\right), \tag{A.29}$$

$$\mathrm{Tr}\left(\gamma^{\mu}\gamma^{\nu}\gamma^{\rho}\gamma^{\sigma}\gamma^{5}\right) = -4i\epsilon^{\mu\nu\rho\sigma}, \tag{A.30}$$

$$\mathrm{Tr}\left(\gamma^{\mu_1}\ldots\gamma^{\mu_n}\right) = \mathrm{Tr}\left(\gamma^{\mu_n}\ldots\gamma^{\mu_1}\right), \tag{A.31}$$

$$\not{x}\gamma_{\nu} = 2x_{\nu} - \gamma_{\nu}\ \not{x}, \tag{A.32}$$

$$\not{x}\gamma_{\nu}\ \not{x}\gamma_{\mu} = 2x_{\nu}\ \not{x}\gamma_{\mu} - \gamma_{\nu}x^{2}\gamma_{\mu}, \tag{A.33}$$

$$Tr[\not{x}\gamma_{\nu}\ \not{x}\gamma_{\mu}] = 8x_{\nu}x_{\mu} - 4x^{2}g_{\mu\nu}. \tag{A.34}$$

A.0.4 Some Other Identities

Also some identities used in calculations are

$$\varepsilon_{abc}\varepsilon_{abc} = 6, \tag{A.35}$$

$$\varepsilon_{abc}\varepsilon_{abc'} = 2\delta_{cc'}, \tag{A.36}$$

$$\begin{aligned}
\varepsilon_{abc}\varepsilon_{a'b'c'} = &\begin{vmatrix} \delta_{aa'} & \delta_{ab'} & \delta_{ac'} \\ \delta_{ba'} & \delta_{bb'} & \delta_{bc'} \\ \delta_{ca'} & \delta_{cb'} & \delta_{cc'} \end{vmatrix} \\
&= \delta_{aa'}\left(\delta_{bb'}\delta_{cc'} - \delta_{bc'}\delta_{cb'}\right) - \delta_{ab'}\left(\delta_{ba'}\delta_{cc'} - \delta_{bc'}\delta_{ca'}\right) + \delta_{ac'}\left(\delta_{ba'}\delta_{cb'} - \delta_{bb'}\delta_{ca'}\right) \\
&= \delta_{bb'}\delta_{cc'} - \delta_{bc'}\delta_{cb'}.
\end{aligned} \tag{A.37}$$

For $t^N = \lambda^N/2$ where λ^N are the Gell-Mann matrices:

$$\mathrm{Tr}\left(t^N\right) = 0, \tag{A.38}$$

$$\mathrm{Tr}\left(t^N t^N\right) = 4, \tag{A.39}$$

$$\begin{aligned}
\varepsilon_{abc}\varepsilon_{a'b'c'}\delta_{aa'}t^N_{bb'}t^M_{cc'}\delta^{NM} &= \varepsilon_{abc}\varepsilon_{ab'c'}\delta_{aa'}t^N_{bb'}t^N_{cc'} \\
&= \left(\delta_{bb'}\delta_{cc'} - \delta_{bc'}\delta_{b'c}\right)t^N_{bb'}t^N_{cc'} \\
&= \left(t^N_{bb}t^N_{cc} - t^N_{bc}t^N_{cb}\right) \\
&= \mathrm{Tr}\left(t^N\right)\mathrm{Tr}\left(t^N\right) - \mathrm{Tr}\left(t^N t^N\right) = -4,
\end{aligned} \tag{A.40}$$

$$\begin{aligned}
\varepsilon_{abc}\varepsilon_{a'b'c'}\delta_{aa'}\delta_{bb'}t^N_{cd}t^N_{dc'}\delta^{NM} &= \varepsilon_{abc}\varepsilon_{a'b'c'}\delta_{aa'}\delta_{bb'}t^M_{cd}t^N_{dc'} \\
&= 2\delta_{cc'}t^N_{cd}t^N_{dc'} = 2t^N_{cd}t^N_{dc} \qquad \text{(A.41)} \\
&= 2\,\mathrm{Tr}\left(t^N t^N\right) = 8. \qquad \text{(A.42)}
\end{aligned}$$

Appendix B

SOME SPECIAL FUNCTIONS

B.0.1 Gamma Function

The integral representation of the gamma function is as follows.

$$\Gamma(n) = \int_0^\infty t^{n-1} e^{-t} dt. \tag{B.1}$$

For positive integer n, Gamma function can be written as

$$\Gamma(n) = (n-1)!. \tag{B.2}$$

Using the definition of the Gamma function, the Schwinger representation can be given as

$$\frac{1}{A^n} = \frac{1}{\Gamma(n)} \int_0^\infty dt t^{n-1} \exp(-At) \quad \text{for} \quad A > 0. \tag{B.3}$$

B.0.2 Heaviside Step Function and Dirac-Delta Function and Relationship

The Heaviside step function $\Theta(x)$ is defined as

$$\Theta(x) = \begin{cases} 1, & x > 0 \\ \frac{1}{2}, & x = 0 \\ 0, & x < 0. \end{cases} \tag{B.4}$$

Integral representation of the Heaviside step function is

$$\theta(x) = \lim_{\epsilon \to 0} \frac{1}{2\pi i} \int_{-\infty}^{+\infty} \frac{e^{izx}}{z - i\epsilon} dz. \tag{B.5}$$

Dirac-Delta function is defined as

$$\delta(x) = \begin{cases} \infty & \text{if } x = 0 \\ 0 & \text{if } x \neq 0, \end{cases} \tag{B.6}$$

such that

$$\int_{-\infty}^{\infty} f(x)\delta(x)dx = f(0). \tag{B.7}$$

The integral representation of the Dirac-Delta function is

$$\delta(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{ixt}} \, \mathrm{d}t. \tag{B.8}$$

The relationship between the Heaviside Step function and the Dirac-Delta function is given as

$$\frac{dH(x)}{dx} = \delta(x), \tag{B.9}$$

and

$$H(x)=\int_{-\infty}^{x}\delta(x)dx=\begin{cases}0 & \text{if } x<0\\ \\ 1 & \text{if } x>0.\end{cases} \tag{B.10}$$

B.0.3 Properties of Dirac-Delta Function

Some properties of Dirac-Delta function can be given as

$$\delta(x)=\delta(-x), \tag{B.11}$$

$$\frac{d}{dx}\delta(x)=-\frac{d}{dx}\delta(-x), \tag{B.12}$$

$$\int_{-\infty}^{\infty} f(x)\delta(x-a)dx=f(a), \tag{B.13}$$

$$\int_{-\infty}^{\infty} f(x)\delta'(x-a)dx=-f'(a), \tag{B.14}$$

$$\delta(ax)=\frac{1}{|a|}\delta(x), \tag{B.15}$$

$$\delta(g(x))=\sum_{i}\frac{1}{|g'(x_i)|}\delta(x-x_i), \tag{B.16}$$

where $g(x_i)=0$, and

$$\delta\left(x^2-a^2\right)=|2a|^{-1}[\delta(x-a)+\delta(x+a)]. \tag{B.17}$$

Appendix C

ANALYTICAL RESULTS OF T_{cc}^{+} TETRAQUARK

For the T_{cc}^{+} tetraquark the numerical results obtained are as follows:

$$\Pi_{\mu\nu,T_{cc}^{+}}\left(M^2,s_0\right)=\frac{m_c^8 p_\mu p_\nu}{3840\pi^6}\int_0^{s_0} dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x} dy\frac{(1-x-y)^6\delta(s-s(x,y))}{x^3y^3}$$

$$+\frac{m_c^8 g_{\mu\nu}}{7680\pi^6}\int_0^{s_0} dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x} dy\frac{(1-x-y)^6 M^2\delta(s-s(x,y))}{x^3y^3}$$

$$-\frac{2m_c^4\langle g_s^2G^2\rangle g_{\mu\nu}}{73728\pi^6}\int_0^{s_0} dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x} dy\frac{(1-x-y)^4 M^2\delta(s-s(x,y))}{x^2y^2}$$

$$-\frac{2m_c^4\langle g_s^2G^2\rangle p_\mu p_\nu}{36864\pi^6}\int_0^{s_0} dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x} dy\frac{(1-x-y)^4\delta(s-s(x,y))}{x^2y^2}$$

$$-\frac{2m_c^4\langle g_s^2G^2\rangle g_{\mu\nu}}{18432\pi^6}\int_0^{s_0} dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x} dy\frac{(1-x-y)^3 M^2\delta(s-s(x,y))}{x^2y^2}$$

$$+\frac{m_c^8\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{36\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\int_0^{1-x} dy\frac{(1-x-y)^3\delta(s_0-s(x,y))}{x^3y^3}\right]$$

$$+\frac{m_c^8\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{36M^2\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\int_0^{1-x} dy\frac{(1-x-y)^3\delta(s_0-s(x,y))}{x^3y^3}$$

$$+\frac{m_c^8\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{36M^4\pi^2}\int_0^{s_0} dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)^3\delta(s-s(x,y))}{x^3y^3}$$

$$+\frac{m_c^8\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{18\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)^3\delta(s_0-s(x,y))}{x^3y^3}\right]$$

$$+\frac{m_c^8\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{18M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)^3\delta(s_0-s(x,y))}{x^3y^3}\right]$$

$$+\frac{m_c^8\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{18M^4\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)^3\delta(s_0-s(x,y))}{x^3y^3}$$

$$+\frac{m_c^8\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{18M^6\pi^2}\int_0^{s_0} dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)^3\delta(s-s(x,y))}{x^3y^3}$$

$$-\frac{m_c^6\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{216\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s_0-s(x,y))}{x^2y^3}\right]$$

$$-\frac{m_c^6\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{216M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s_0-s(x,y))}{x^2y^3}\right]$$

$$-\frac{m_c^6\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{216M^4\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s_0-s(x,y))}{x^2y^3}$$

$$-\frac{m_c^6\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{216M^6\pi^2}\int_0^{s_0} dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s-s(x,y))}{x^2y^3}$$

$$-\frac{m_c^6\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{108\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^3}{ds_0^3}\left[\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s_0-s(x,y))}{x^2y^3}\right]$$

$$-\frac{m_c^6\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{108M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s_0-s(x,y))}{x^2y^3}\right]$$

$$-\frac{m_c^6\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{108M^4\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s_0-s(x,y))}{x^2y^3}\right]$$

$$-\frac{m_c^6\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{108M^6\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s_0-s(x,y))}{x^2y^3}$$

$$-\frac{m_c^6\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{108M^8\pi^2}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s-s(x,y))}{x^2y^3}$$

$$-\frac{m_c^6\langle g_s^2G^2\rangle g_{\mu\nu}}{9216\pi^6}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)^4\delta(s-s(x,y))}{x^2y^3}$$

$$-\frac{m_c^6\langle g_s^2G^2\rangle p_\mu p_\nu}{4608\pi^6}e^{-\frac{s_0}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)^4\delta(s_0-s(x,y))}{x^2y^3}$$

$$-\frac{m_c^6\langle g_s^2G^2\rangle p_\mu p_\nu}{4608M^2\pi^6}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)^4\delta(s-s(x,y))}{x^2y^3}$$

$$-\frac{2m_c^8m_0^2\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{48\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)^2\delta(s_0-s(x,y))}{x^3y^3}\right]$$

$$-\frac{2m_c^8m_0^2\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{48M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)^2\delta(s_0-s(x,y))}{x^3y^3}\right]$$

$$-\frac{2m_c^8m_0^2\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{48M^4\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)^2\delta(s_0-s(x,y))}{x^3y^3}$$

$$-\frac{2m_c^8m_0^2\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{48M^6\pi^2}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1dx\int_0^{1-x}dy\frac{(1-x-y)^2\delta(s-s(x,y))}{x^3y^3}$$

$$-\frac{2m_c^8m_0^2\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{24\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^3}{ds_0^3}\left[\int_0^1dx\int_0^{1-x}dy\frac{(1-x-y)^2\delta(s_0-s(x,y))}{x^3y^3}\right]$$

$$-\frac{2m_c^8m_0^2\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{24M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1dx\int_0^{1-x}dy\frac{(1-x-y)^2\delta(s_0-s(x,y))}{x^3y^3}\right]$$

$$-\frac{2m_c^8m_0^2\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{24M^4\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1dx\int_0^{1-x}dy\frac{(1-x-y)^2\delta(s_0-s(x,y))}{x^3y^3}\right]$$

$$-\frac{2m_c^8m_0^2\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{24M^6\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1dx\int_0^{1-x}dy\frac{(1-x-y)^2\delta(s_0-s(x,y))}{x^3y^3}$$

$$-\frac{2m_c^8m_0^2\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{24M^8\pi^2}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1dx\int_0^{1-x}dy\frac{(1-x-y)^2\delta(s-s(x,y))}{x^3y^3}$$

$$+\frac{2m_c^6m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{864\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^3}{ds_0^3}\left[\int_0^1dx\int_0^{1-x}dy\frac{\delta(s_0-s(x,y))}{x^2y^3}\right]$$

$$+\frac{2m_c^6m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{864M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1dx\int_0^{1-x}dy\frac{\delta(s_0-s(x,y))}{x^2y^3}\right]$$

$$+\frac{2m_c^6m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{864M^4\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1dx\int_0^{1-x}dy\frac{\delta(s_0-s(x,y))}{x^2y^3}\right]$$

$$+\frac{2m_c^6m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{864M^6\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1dx\int_0^{1-x}dy\frac{\delta(s_0-s(x,y))}{x^2y^3}$$

$$+\frac{2m_c^6m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{864M^8\pi^2}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1dx\int_0^{1-x}dy\frac{\delta(s-s(x,y))}{x^2y^3}$$

$$+\frac{2m_c^6m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{432\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^4}{ds_0^4}\left[\int_0^1dx\int_0^{1-x}dy\frac{\delta(s_0-s(x,y))}{x^2y^3}\right]$$

$$+\frac{2m_c^6m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{432M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^3}{ds_0^3}\left[\int_0^1dx\int_0^{1-x}dy\frac{\delta(s_0-s(x,y))}{x^2y^3}\right]$$

$$+\frac{2m_c^6m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{432M^4\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1dx\int_0^{1-x}dy\frac{\delta(s_0-s(x,y))}{x^2y^3}\right]$$

$$+\frac{2m_c^6m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{432M^6\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1dx\int_0^{1-x}dy\frac{\delta(s_0-s(x,y))}{x^2y^3}\right]$$

$$+\frac{2m_c^6m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{432M^8\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1dx\int_0^{1-x}dy\frac{\delta(s_0-s(x,y))}{x^2y^3}$$

$$+\frac{2m_c^6m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{432M^8\pi^2}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1dx\int_0^{1-x}dy\frac{\delta(s-s(x,y))}{x^2y^3}$$

$$+\frac{m_c^8m_0^4\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{96\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^3}{ds_0^3}\left[\int_0^1dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s_0-s(x,y))}{x^3y^3}\right]$$

$$+\frac{m_c^8m_0^4\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{96M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s_0-s(x,y))}{x^3y^3}\right]$$

$$+\frac{m_c^8m_0^4\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{96M^4\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s_0-s(x,y))}{x^3y^3}\right]$$

$$+\frac{m_c^8 m_0^4\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{96M^6\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s_0-s(x,y))}{x^3y^3}$$

$$+\frac{m_c^8 m_0^4\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{96M^8\pi^2}\int_0^{s_0} dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s-s(x,y))}{x^3y^3}$$

$$+\frac{m_c^8 m_0^4\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{48\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^4}{ds_0^4}\left[\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s_0-s(x,y))}{x^3y^3}\right]$$

$$+\frac{m_c^8 m_0^4\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{48M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^3}{ds_0^3}\left[\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s_0-s(x,y))}{x^3y^3}\right]$$

$$+\frac{m_c^8 m_0^4\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{48M^4\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s_0-s(x,y))}{x^3y^3}\right]$$

$$+\frac{m_c^8 m_0^4\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{48M^6\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s_0-s(x,y))}{x^3y^3}\right]$$

$$+\frac{m_c^8 m_0^4\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{48M^8\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s_0-s(x,y))}{x^3y^3}$$

$$+\frac{m_c^8 m_0^4\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{48M^{10}\pi^2}\int_0^{s_0} dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s-s(x,y))}{x^3y^3}$$

$$-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{3456\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^4}{ds_0^4}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x))}{(1-x)^3x^2}\right]$$

$$-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{3456M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^3}{ds_0^3}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}\right]$$

$$-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{3456M^4\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}\right]$$

$$-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{3456M^6\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}\right]$$

$$-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{3456M^8\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}$$

$$-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{3456M^{10}\pi^2}\int_0^{s_0} dse^{-\frac{s}{M^2}}\int_0^1 dx\frac{\delta(s-s(x,(1-x)))}{(1-x)^3x^2}$$

$$-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{1728\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^5}{ds_0^5}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x))}{(1-x)^3x^2}\right]$$

$$-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{1728M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^4}{ds_0^4}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x))}{(1-x)^3x^2}\right]$$

$$-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{1728M^4\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^3}{ds_0^3}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}\right]$$

$$-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{1728M^6\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}\right]$$

$$-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{1728M^8\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}\right]$$

$$-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{1728M^{10}\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}$$

$$- \frac{m_c^6 m_0^4 \langle g_s^2 G^2\rangle \langle u\bar{u}\rangle \langle d\bar{d}\rangle p_\mu p_\nu}{1728 M^{12}\pi^2} \int_0^{s_0} ds e^{-\frac{s}{M^2}} \int_0^1 dx \frac{\delta(s - s(x,(1-x)))}{(1-x)^3 x^2}$$

$$- \frac{m_c^8 g_{\mu\nu}}{1280\pi^6} \int_0^{s_0} ds e^{-\frac{s}{M^2}} \int_0^1 dx \int_0^{1-x} dy \frac{(1-x-y)^5 M^2 \delta(s - s(x,y))}{x^3 y^3}$$

$$+ \frac{m_c^6 \langle g_s^2 G^2\rangle g_{\mu\nu}}{2304\pi^6} \int_0^{s_0} ds e^{-\frac{s}{M^2}} \int_0^1 dx \int_0^{1-x} dy \frac{(1-x-y)^3 \delta(s - s(x,y))}{x^2 y^3}$$

$$- \frac{m_c^8 \langle u\bar{u}\rangle \langle d\bar{d}\rangle g_{\mu\nu}}{12\pi^2} e^{-\frac{s_0}{M^2}} \frac{d}{ds_0} \left[\int_0^1 dx \int_0^{1-x} dy \frac{(1-x-y)^2 \delta(s_0 - s(x,y))}{x^3 y^3}\right]$$

$$- \frac{m_c^8 \langle u\bar{u}\rangle \langle d\bar{d}\rangle g_{\mu\nu}}{12 M^2\pi^2} e^{-\frac{s_0}{M^2}} \int_0^1 dx \int_0^{1-x} dy \frac{(1-x-y)^2 \delta(s - s(x,y))}{x^3 y^3}$$

$$- \frac{m_c^8 \langle u\bar{u}\rangle \langle d\bar{d}\rangle g_{\mu\nu}}{12 M^4\pi^2} \int_0^{s_0} ds e^{-\frac{s}{M^2}} \int_0^1 dx \int_0^{1-x} dy \frac{(1-x-y)^2 \delta(s - s(x,y))}{x^3 y^3}$$

$$+ \frac{m_c^6 \langle g_s^2 G^2\rangle \langle u\bar{u}\rangle \langle d\bar{d}\rangle g_{\mu\nu}}{216\pi^2} e^{-\frac{s_0}{M^2}} \frac{d^2}{ds_0^2} \left[\int_0^1 dx \int_0^{1-x} dy \frac{\delta(s_0 - s(x,y))}{x^2 y^3}\right]$$

$$+ \frac{m_c^6 \langle g_s^2 G^2\rangle \langle u\bar{u}\rangle \langle d\bar{d}\rangle g_{\mu\nu}}{216 M^2\pi^2} e^{-\frac{s_0}{M^2}} \frac{d}{ds_0} \left[\int_0^1 dx \int_0^{1-x} dy \frac{\delta(s_0 - s(x,y))}{x^2 y^3}\right]$$

$$+ \frac{m_c^6 \langle g_s^2 G^2\rangle \langle u\bar{u}\rangle \langle d\bar{d}\rangle g_{\mu\nu}}{216 M^4\pi^2} e^{-\frac{s_0}{M^2}} \int_0^1 dx \int_0^{1-x} dy \frac{\delta(s_0 - s(x,y))}{x^2 y^3}$$

$$+ \frac{m_c^6 \langle g_s^2 G^2\rangle \langle u\bar{u}\rangle \langle d\bar{d}\rangle g_{\mu\nu}}{216 M^6\pi^2} \int_0^{s_0} ds e^{-\frac{s}{M^2}} \int_0^1 dx \int_0^{1-x} dy \frac{\delta(s - s(x,y))}{x^2 y^3}$$

$$+\frac{2m_c^8m_0^2\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{24\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s_0-s(x,y))}{x^3y^3}\right]$$

$$+\frac{2m_c^8m_0^2\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{24M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s_0-s(x,y))}{x^3y^3}\right]$$

$$+\frac{2m_c^8m_0^2\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{24M^4\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s_0-s(x,y))}{x^3y^3}$$

$$+\frac{2m_c^8m_0^2\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{24M^6\pi^2}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s-s(x,y))}{x^3y^3}$$

$$-\frac{2m_c^6m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{864\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^3}{ds_0^3}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}\right]$$

$$-\frac{2m_c^6m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{864M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}\right]$$

$$-\frac{2m_c^6m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{864M^4\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}\right]$$

$$-\frac{2m_c^6m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{864M^6\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}$$

$$-\frac{2m_c^6m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{864M^8\pi^2}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1 dx\frac{\delta(s-s(x,(1-x)))}{(1-x)^3x^2}$$

$$-\frac{m_c^8m_0^4\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{96\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^3}{ds_0^3}\left[\int_0^1 dx\int_0^{1-x}dy\frac{\delta(s_0-s(x,y))}{x^3y^3}\right]$$

$$-\frac{m_c^8 m_0^4\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{96M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1 dx\int_0^{1-x}dy\frac{\delta(s_0-s(x,y))}{x^3y^3}\right]$$

$$-\frac{m_c^8 m_0^4\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{96M^4\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\int_0^{1-x}dy\frac{\delta(s_0-s(x,y))}{x^3y^3}\right]$$

$$-\frac{m_c^8 m_0^4\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{96M^6\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{\delta(s_0-s(x,y))}{x^3y^3}$$

$$-\frac{m_c^8 m_0^4\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{96M^8\pi^2}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{\delta(s-s(x,y))}{x^3y^3}$$

$$-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{864\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^4}{ds_0^4}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}\right]$$

$$-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{864M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^3}{ds_0^3}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}\right]$$

$$-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{864M^4\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}\right]$$

$$-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{864M^6\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}\right]$$

$$-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{864M^8\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}$$

$$+\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{864M^{10}\pi^2}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1 dx\frac{\delta(s-s(x,(1-x)))}{(1-x)^3x^2}$$

$$+\frac{2m_c^8m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{3456\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^5}{ds_0^5}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x))}{(1-x)^4x^3}\right]$$

$$+\frac{2m_c^8m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{3456M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^4}{ds_0^4}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x))}{(1-x)^4x^3}\right]$$

$$+\frac{2m_c^8m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{3456M^4\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^3}{ds_0^3}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^4x^3}\right]$$

$$+\frac{2m_c^8m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{3456M^6\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^4x^3}\right]$$

$$+\frac{2m_c^8m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{3456M^8\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^4x^3}\right]$$

$$+\frac{2m_c^8m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{3456M^{10}\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^4x^3}$$

$$+\frac{2m_c^8m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{3456M^{12}\pi^2}\int_0^{s_0} dse^{-\frac{s}{M^2}}\int_0^1 dx\frac{\delta(s-s(x,(1-x)))}{(1-x)^4x^3}$$

$$+\frac{m_c^8\langle g_s^2G^2\rangle p_\mu p_\nu}{6144\pi^6}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\int_0^{1-x} dy\frac{(1-x-y)^4\delta(s_0-s(x,y))}{x^3y^3}\right]$$

$$+\frac{m_c^8\langle g_s^2G^2\rangle p_\mu p_\nu}{6144M^2\pi^6}e^{-\frac{s_0}{M^2}}\int_0^1 dx\int_0^{1-x} dy\frac{(1-x-y)^4\delta(s-s(x,y))}{x^3y^3}$$

$$+\frac{m_c^8\langle g_s^2G^2\rangle p_\mu p_\nu}{6144M^4\pi^6}\int_0^{s_0} dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x} dy\frac{(1-x-y)^4\delta(s-s(x,y))}{x^3y^3}$$

$$+\frac{m_c^8\langle g_s^2G^2\rangle g_{\mu\nu}}{12288\pi^6}e^{-\frac{s_0}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)^4\delta(s-s(x,y))}{x^3y^3}$$

$$+\frac{m_c^8\langle g_s^2G^2\rangle g_{\mu\nu}}{12288M^2\pi^6}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)^4\delta(s-s(x,y))}{x^3y^3}$$

$$-\frac{m_c^8\langle g_s^2G^2\rangle g_{\mu\nu}}{3072\pi^6}e^{-\frac{s_0}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)^3\delta(s-s(x,y))}{x^3y^3}$$

$$-\frac{m_c^8\langle g_s^2G^2\rangle g_{\mu\nu}}{3072M^2\pi^6}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)^3\delta(s-s(x,y))}{x^3y^3}$$

$$-\frac{m_c^8\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{6\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s-s(x,y))}{x^2y^2}$$

$$-\frac{m_c^8\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{6M^2\pi^2}\int_0^{s_0}dse^{-\frac{s_0}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s-s(x,y))}{x^2y^2}$$

$$-\frac{m_c^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{72\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\int_0^{1-x}dy\frac{\delta(s_0-s(x,y))}{x^2y^2}\right]$$

$$-\frac{m_c^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{72M^2\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{\delta(s-s(x,y))}{x^2y^2}$$

$$-\frac{m_c^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{72M^4\pi^2}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{\delta(s-s(x,y))}{x^2y^2}$$

$$+\frac{m_c^6\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{216\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1 dx\int_0^{1-x}dy\frac{\delta(s_0-s(x,y))}{x^2y^3}\right]$$

$$+\frac{m_c^6\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{216M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\int_0^{1-x}dy\frac{\delta(s_0-s(x,y))}{x^2y^3}\right]$$

$$+\frac{m_c^6\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{216M^4\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{\delta(s_0-s(x,y))}{x^2y^3}$$

$$+\frac{m_c^6\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{216M^6\pi^2}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{\delta(s-s(x,y))}{x^2y^3}$$

$$-\frac{m_c^6 g_{\mu\nu}}{256\pi^6}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)^4M^4\delta(s-s(x,y))}{x^2y^2}$$

$$-\frac{m_c^4\langle g_s^2G^2\rangle g_{\mu\nu}}{768\pi^6}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)^3M^2\delta(s-s(x,y))}{x^2y^2}$$

$$+\frac{m_c^6\langle g_s^2G^2\rangle g_{\mu\nu}}{2304\pi^6}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)^3M^2\delta(s-s(x,y))}{x^2y^3}$$

$$+\frac{m_c^6m_0^2\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{24\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\int_0^{1-x}dy\frac{\delta(s_0-s(x,y))}{x^2y^2}\right]$$

$$+\frac{m_c^6m_0^2\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{24M^2\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{\delta(s_0-s(x,y))}{x^2y^2}$$

$$+\frac{m_c^6m_0^2\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{24M^4\pi^2}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{\delta(s-s(x,y))}{x^2y^2}$$

$$+\frac{2m_c^4m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{288\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}\right]$$

$$+\frac{2m_c^4m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{288M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^2x^2}\right]$$

$$+\frac{2m_c^4m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{288M^4\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^2x^2}$$

$$+\frac{2m_c^4m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{288M^6\pi^2}\int_0^{s_0} dse^{-\frac{s}{M^2}}\int_0^1 dx\frac{\delta(s-s(x,(1-x)))}{(1-x)^2x^2}$$

$$-\frac{2m_c^6m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{864\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^3}{ds_0^3}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}\right]$$

$$-\frac{2m_c^6m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{864M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}\right]$$

$$-\frac{2m_c^6m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{864M^4\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}\right]$$

$$-\frac{2m_c^6m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{864M^6\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}$$

$$-\frac{2m_c^6m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{864M^8\pi^2}\int_0^{s_0} dse^{-\frac{s}{M^2}}\int_0^1 dx\frac{\delta(s-s(x,(1-x)))}{(1-x)^3x^2}$$

$$-\frac{m_c^6m_0^4\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{96\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}\right]$$

$$-\frac{m_c^6m_0^4\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{96M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^2x^2}\right]$$

$$- \frac{m_c^6 m_0^4 \langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{96 M^4 \pi^2} e^{-\frac{s_0}{M^2}} \int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^2 x^2}$$

$$- \frac{m_c^6 m_0^4 \langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{96 M^6 \pi^2} \int_0^{s_0} ds e^{-\frac{s_0}{M^2}} \int_0^1 dx \frac{\delta(s - s(x,(1-x)))}{(1-x)^2 x^2}$$

$$+ \frac{m_c^4 m_0^4 \langle g_s^2 G^2\rangle \langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{384 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d^3}{ds_0^3} \left[\int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^2 x^2} \right]$$

$$+ \frac{m_c^4 m_0^4 \langle g_s^2 G^2\rangle \langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{384 M^2 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d^2}{ds_0^2} \left[\int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^2 x^2} \right]$$

$$+ \frac{m_c^4 m_0^4 \langle g_s^2 G^2\rangle \langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{384 M^4 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d}{ds_0} \left[\int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^2 x^2} \right]$$

$$+ \frac{m_c^4 m_0^4 \langle g_s^2 G^2\rangle \langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{384 M^6 \pi^2} e^{-\frac{s_0}{M^2}} \int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^2 x^2}$$

$$+ \frac{m_c^4 m_0^4 \langle g_s^2 G^2\rangle \langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{384 M^8 \pi^2} \int_0^{s_0} ds e^{-\frac{s_0}{M^2}} \int_0^1 dx \frac{\delta(s - s(x,(1-x)))}{(1-x)^2 x^2}$$

$$- \frac{m_c^6 m_0^4 \langle g_s^2 G^2\rangle \langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{864 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d^4}{ds_0^4} \left[\int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^3 x^2} \right]$$

$$- \frac{m_c^6 m_0^4 \langle g_s^2 G^2\rangle \langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{864 M^2 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d^3}{ds_0^3} \left[\int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^3 x^2} \right]$$

$$- \frac{m_c^6 m_0^4 \langle g_s^2 G^2\rangle \langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{864 M^4 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d^2}{ds_0^2} \left[\int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^3 x^2} \right]$$

$$-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{864M^6\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}\right]$$

$$-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{864M^8\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}$$

$$-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{864M^{10}\pi^2}\int_0^{s_0} dse^{-\frac{s_0}{M^2}}\int_0^1 dx\frac{\delta(s-s(x,(1-x)))}{(1-x)^3x^2}$$

$$-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{1152\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^4}{ds_0^4}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^3}\right]$$

$$-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{1152M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^3}{ds_0^3}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^3}\right]$$

$$-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{1152M^4\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^3}\right]$$

$$-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{1152M^6\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^3}\right]$$

$$-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{1152M^8\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^3}$$

$$-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{1152M^{10}\pi^2}\int_0^{s_0} dse^{-\frac{s_0}{M^2}}\int_0^1 dx\frac{\delta(s-s(x,(1-x)))}{(1-x)^3x^3}$$

$$-\frac{m_c^6\langle g_s^2G^2\rangle g_{\mu\nu}}{1024\pi^6}\int_0^{s_0} dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x} dy\frac{(1-x-y)^2\delta(s-s(x,y))}{x^2y^2}. \tag{C.1}$$

Appendix D

ANALYTICAL RESULTS OF 1^{+-} TETRAQUARK

For the 1^{+-} tetraquark the analytical results obtained are as follows:

$$\Pi^{1^{+-}}_{\mu\nu}\left(M^2,s_0\right)=\frac{m_c^8 p_\mu p_\nu}{7680\pi^6}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1dx\int_0^{1-x}dy\frac{(1-x-y)^6\delta(s-s(x,y))}{x^3y^3}$$

$$+\frac{m_c^8 g_{\mu\nu}}{15360\pi^6}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1dx\int_0^{1-x}dy\frac{(1-x-y)^6M^2\delta(s-s(x,y))}{x^3y^3}$$

$$-\frac{m_c^4\langle g_s^2G^2\rangle g_{\mu\nu}}{147456\pi^6}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1dx\int_0^{1-x}dy\frac{(1-x-y)^4M^2\delta(s-s(x,y))}{x^2y^2}$$

$$-\frac{m_c^4\langle g_s^2G^2\rangle p_\mu p_\nu}{73728\pi^6}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1dx\int_0^{1-x}dy\frac{(1-x-y)^4\delta(s-s(x,y))}{x^2y^2}$$

$$-\frac{m_c^4\langle g_s^2G^2\rangle g_{\mu\nu}}{36864\pi^6}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1dx\int_0^{1-x}dy\frac{(1-x-y)^3M^2\delta(s-s(x,y))}{x^2y^2}$$

$$+\frac{m_c^8\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{72\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1dx\int_0^{1-x}dy\frac{(1-x-y)^3\delta(s_0-s(x,y))}{x^3y^3}\right]$$

$$+\frac{m_c^8\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{72M^2\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1dx\int_0^{1-x}dy\frac{(1-x-y)^3\delta(s-s(x,y))}{x^3y^3}$$

$$+\frac{m_c^8\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{72M^4\pi^2}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1dx\int_0^{1-x}dy\frac{(1-x-y)^3\delta(s-s(x,y))}{x^3y^3}$$

$$+\frac{m_c^8\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{36\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1dx\int_0^{1-x}dy\frac{(1-x-y)^3\delta(s_0-s(x,y))}{x^3y^3}\right]$$

$$+\frac{m_c^8\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{36M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1dx\int_0^{1-x}dy\frac{(1-x-y)^3\delta(s_0-s(x,y))}{x^3y^3}\right]$$

$$+\frac{m_c^8\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{36M^4\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1dx\int_0^{1-x}dy\frac{(1-x-y)^3\delta(s_0-s(x,y))}{x^3y^3}$$

$$+\frac{m_c^8\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{36M^6\pi^2}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1dx\int_0^{1-x}dy\frac{(1-x-y)^3\delta(s-s(x,y))}{x^3y^3}$$

$$-\frac{m_c^6\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{432\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s_0-s(x,y))}{x^2y^3}\right]$$

$$-\frac{m_c^6\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{432M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s_0-s(x,y))}{x^2y^3}\right]$$

$$-\frac{m_c^6\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{432M^4\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s_0-s(x,y))}{x^2y^3}$$

$$-\frac{m_c^6\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{432M^6\pi^2}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s-s(x,y))}{x^2y^3}$$

$$-\frac{m_c^6\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{216\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^3}{ds_0^3}\left[\int_0^1dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s_0-s(x,y))}{x^2y^3}\right]$$

$$-\frac{m_c^6\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{216M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s_0-s(x,y))}{x^2y^3}\right]$$

$$-\frac{m_c^6\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{216M^4\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s_0-s(x,y))}{x^2y^3}\right]$$

$$-\frac{m_c^6\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{216M^6\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s_0-s(x,y))}{x^2y^3}$$

$$-\frac{m_c^6\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{216M^8\pi^2}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s-s(x,y))}{x^2y^3}$$

$$-\frac{m_c^6\langle g_s^2G^2\rangle g_{\mu\nu}}{18432\pi^6}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)^4\delta(s-s(x,y))}{x^2y^3}$$

$$-\frac{m_c^6\langle g_s^2G^2\rangle p_\mu p_\nu}{9216\pi^6}e^{-\frac{s_0}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)^4\delta(s_0-s(x,y))}{x^2y^3}$$

$$-\frac{m_c^6\langle g_s^2G^2\rangle p_\mu p_\nu}{9216M^2\pi^6}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)^4\delta(s-s(x,y))}{x^2y^3}$$

$$-\frac{m_c^8m_0^2\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{48\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)^2\delta(s_0-s(x,y))}{x^3y^3}\right]$$

$$-\frac{m_c^8m_0^2\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{48M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)^2\delta(s_0-s(x,y))}{x^3y^3}\right]$$

$$-\frac{m_c^8m_0^2\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{48M^4\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)^2\delta(s_0-s(x,y))}{x^3y^3}$$

$$-\frac{m_c^8 m_0^2\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{48M^6\pi^2}\int_0^{s_0} dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x} dy\frac{(1-x-y)^2\delta(s-s(x,y))}{x^3y^3}$$

$$-\frac{m_c^8 m_0^2\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{24\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^3}{ds_0^3}\left[\int_0^1 dx\int_0^{1-x} dy\frac{(1-x-y)^2\delta(s_0-s(x,y))}{x^3y^3}\right]$$

$$-\frac{m_c^8 m_0^2\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{24M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1 dx\int_0^{1-x} dy\frac{(1-x-y)^2\delta(s_0-s(x,y))}{x^3y^3}\right]$$

$$-\frac{m_c^8 m_0^2\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{24M^4\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\int_0^{1-x} dy\frac{(1-x-y)^2\delta(s_0-s(x,y))}{x^3y^3}\right]$$

$$-\frac{m_c^8 m_0^2\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{24M^6\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\int_0^{1-x} dy\frac{(1-x-y)^2\delta(s_0-s(x,y))}{x^3y^3}$$

$$-\frac{m_c^8 m_0^2\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{24M^8\pi^2}\int_0^{s_0} dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x} dy\frac{(1-x-y)^2\delta(s-s(x,y))}{x^3y^3}$$

$$+\frac{m_c^6 m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{864\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^3}{ds_0^3}\left[\int_0^1 dx\int_0^{1-x} dy\frac{\delta(s_0-s(x,y))}{x^2y^3}\right]$$

$$+\frac{m_c^6 m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{864M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1 dx\int_0^{1-x} dy\frac{\delta(s_0-s(x,y))}{x^2y^3}\right]$$

$$+\frac{m_c^6 m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{864M^4\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\int_0^{1-x} dy\frac{\delta(s_0-s(x,y))}{x^2y^3}\right]$$

$$+\frac{m_c^6 m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{864M^6\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\int_0^{1-x} dy\frac{\delta(s_0-s(x,y))}{x^2y^3}$$

$$+\frac{m_c^6 m_0^2 \langle g_s^2 G^2\rangle \langle u\bar{u}\rangle \langle d\bar{d}\rangle g_{\mu\nu}}{864 M^8 \pi^2} \int_0^{s_0} ds e^{-\frac{s}{M^2}} \int_0^1 dx \int_0^{1-x} dy \frac{\delta(s-s(x,y))}{x^2 y^3}$$

$$+\frac{m_c^6 m_0^2 \langle u\bar{u}\rangle \langle d\bar{d}\rangle p_\mu p_\nu}{432 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d^3}{ds_0^3} \left[\int_0^1 dx \int_0^{1-x} dy \frac{\delta(s_0-s(x,y))}{x^2 y^3}\right]$$

$$+\frac{m_c^6 m_0^2 \langle u\bar{u}\rangle \langle d\bar{d}\rangle p_\mu p_\nu}{432 M^2 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d^2}{ds_0^2} \left[\int_0^1 dx \int_0^{1-x} dy \frac{\delta(s_0-s(x,y))}{x^2 y^3}\right]$$

$$+\frac{m_c^6 m_0^2 \langle u\bar{u}\rangle \langle d\bar{d}\rangle p_\mu p_\nu}{432 M^4 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d}{ds_0} \left[\int_0^1 dx \int_0^{1-x} dy \frac{\delta(s_0-s(x,y))}{x^2 y^3}\right]$$

$$+\frac{m_c^6 m_0^2 \langle u\bar{u}\rangle \langle d\bar{d}\rangle p_\mu p_\nu}{432 M^6 \pi^2} e^{-\frac{s_0}{M^2}} \int_0^1 dx \int_0^{1-x} dy \frac{\delta(s_0-s(x,y))}{x^2 y^3}$$

$$+\frac{m_c^6 m_0^2 \langle u\bar{u}\rangle \langle d\bar{d}\rangle p_\mu p_\nu}{432 M^8 \pi^2} \int_0^{s_0} ds e^{-\frac{s}{M^2}} \int_0^1 dx \int_0^{1-x} dy \frac{\delta(s-s(x,y))}{x^2 y^3}$$

$$+\frac{m_c^8 m_0^4 \langle u\bar{u}\rangle \langle d\bar{d}\rangle g_{\mu\nu}}{192 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d^3}{ds_0^3} \left[\int_0^1 dx \int_0^{1-x} dy \frac{(1-x-y)\delta(s_0-s(x,y))}{x^3 y^3}\right]$$

$$+\frac{m_c^8 m_0^4 \langle u\bar{u}\rangle \langle d\bar{d}\rangle g_{\mu\nu}}{192 M^2 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d^2}{ds_0^2} \left[\int_0^1 dx \int_0^{1-x} dy \frac{(1-x-y)\delta(s_0-s(x,y))}{x^3 y^3}\right]$$

$$+\frac{m_c^8 m_0^4 \langle u\bar{u}\rangle \langle d\bar{d}\rangle g_{\mu\nu}}{192 M^4 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d}{ds_0} \left[\int_0^1 dx \int_0^{1-x} dy \frac{(1-x-y)\delta(s_0-s(x,y))}{x^3 y^3}\right]$$

$$+\frac{m_c^8 m_0^4 \langle u\bar{u}\rangle \langle d\bar{d}\rangle g_{\mu\nu}}{192 M^6 \pi^2} e^{-\frac{s_0}{M^2}} \int_0^1 dx \int_0^{1-x} dy \frac{(1-x-y)\delta(s_0-s(x,y))}{x^3 y^3}$$

$$+\frac{m_c^8 m_0^4\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{192M^8\pi^2}\int_0^{s_0} dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x} dy\frac{(1-x-y)\delta(s-s(x,y))}{x^3y^3}$$

$$+\frac{m_c^8 m_0^4\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{96\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^4}{ds_0^4}\left[\int_0^1 dx\int_0^{1-x} dy\frac{(1-x-y)\delta(s_0-s(x,y))}{x^3y^3}\right]$$

$$+\frac{m_c^8 m_0^4\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{96M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^3}{ds_0^3}\left[\int_0^1 dx\int_0^{1-x} dy\frac{(1-x-y)\delta(s_0-s(x,y))}{x^3y^3}\right]$$

$$+\frac{m_c^8 m_0^4\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{96M^4\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1 dx\int_0^{1-x} dy\frac{(1-x-y)\delta(s_0-s(x,y))}{x^3y^3}\right]$$

$$+\frac{m_c^8 m_0^4\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{96M^6\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\int_0^{1-x} dy\frac{(1-x-y)\delta(s_0-s(x,y))}{x^3y^3}\right]$$

$$+\frac{m_c^8 m_0^4\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{96M^8\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\int_0^{1-x} dy\frac{(1-x-y)\delta(s_0-s(x,y))}{x^3y^3}$$

$$+\frac{m_c^8 m_0^4\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{96M^{10}\pi^2}\int_0^{s_0} dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x} dy\frac{(1-x-y)\delta(s-s(x,y))}{x^3y^3}$$

$$-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{6912\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^4}{ds_0^4}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x))}{(1-x)^3x^2}\right]$$

$$-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{6912M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^3}{ds_0^3}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}\right]$$

$$-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{6912M^4\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}\right]$$

$$-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{6912M^6\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}\right]$$

$$-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{6912M^8\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}$$

$$-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{6912M^{10}\pi^2}\int_0^{s_0} dse^{-\frac{s}{M^2}}\int_0^1 dx\frac{\delta(s-s(x,(1-x)))}{(1-x)^3x^2}$$

$$-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{3456\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^5}{ds_0^5}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x))}{(1-x)^3x^2}\right]$$

$$-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{3456M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^4}{ds_0^4}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x))}{(1-x)^3x^2}\right]$$

$$-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{3456M^4\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^3}{ds_0^3}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}\right]$$

$$-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{3456M^6\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}\right]$$

$$-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{3456M^8\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}\right]$$

$$-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{3456M^{10}\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}$$

$$-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle p_\mu p_\nu}{3456M^{12}\pi^2}\int_0^{s_0} dse^{-\frac{s}{M^2}}\int_0^1 dx\frac{\delta(s-s(x,(1-x)))}{(1-x)^3x^2}$$

$$
-\frac{m_c^8 g_{\mu\nu}}{2560\pi^6}\int_0^{s_0} dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x} dy\frac{(1-x-y)^5 M^2\delta(s-s(x,y))}{x^3y^3}
$$

$$
+\frac{m_c^6\langle g_s^2G^2\rangle g_{\mu\nu}}{4608\pi^6}\int_0^{s_0} dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x} dy\frac{(1-x-y)^3\delta(s-s(x,y))}{x^2y^3}
$$

$$
-\frac{m_c^8\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{24\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\int_0^{1-x} dy\frac{(1-x-y)^2\delta(s_0-s(x,y))}{x^3y^3}\right]
$$

$$
-\frac{m_c^8\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{24M^2\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\int_0^{1-x} dy\frac{(1-x-y)^2\delta(s-s(x,y))}{x^3y^3}
$$

$$
-\frac{m_c^8\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{24M^4\pi^2}\int_0^{s_0} dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x} dy\frac{(1-x-y)^2\delta(s-s(x,y))}{x^3y^3}
$$

$$
+\frac{m_c^6\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{432\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1 dx\int_0^{1-x} dy\frac{\delta(s_0-s(x,y))}{x^2y^3}\right]
$$

$$
+\frac{m_c^6\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{432M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\int_0^{1-x} dy\frac{\delta(s_0-s(x,y))}{x^2y^3}\right]
$$

$$
+\frac{m_c^6\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{432M^4\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\int_0^{1-x} dy\frac{\delta(s_0-s(x,y))}{x^2y^3}
$$

$$
+\frac{m_c^6\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{432M^6\pi^2}\int_0^{s_0} dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x} dy\frac{\delta(s-s(x,y))}{x^2y^3}
$$

$$
+\frac{m_c^8 m_0^2\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{24\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1 dx\int_0^{1-x} dy\frac{(1-x-y)\delta(s_0-s(x,y))}{x^3y^3}\right]
$$

$$+\frac{m_c^8 m_0^2\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{24M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s_0-s(x,y))}{x^3y^3}\right]$$

$$+\frac{m_c^8 m_0^2\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{24M^4\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s_0-s(x,y))}{x^3y^3}$$

$$+\frac{m_c^8 m_0^2\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{24M^6\pi^2}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s-s(x,y))}{x^3y^3}$$

$$-\frac{m_c^6 m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{864\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^3}{ds_0^3}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}\right]$$

$$-\frac{m_c^6 m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{864M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}\right]$$

$$-\frac{m_c^6 m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{864M^4\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}\right]$$

$$-\frac{m_c^6 m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{864M^6\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}$$

$$-\frac{m_c^6 m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{864M^8\pi^2}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1 dx\frac{\delta(s-s(x,(1-x)))}{(1-x)^3x^2}$$

$$-\frac{m_c^8 m_0^4\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{192\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^3}{ds_0^3}\left[\int_0^1 dx\int_0^{1-x}dy\frac{\delta(s_0-s(x,y))}{x^3y^3}\right]$$

$$-\frac{m_c^8 m_0^4\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{192M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1 dx\int_0^{1-x}dy\frac{\delta(s_0-s(x,y))}{x^3y^3}\right]$$

$$- \frac{m_c^8 m_0^4 \langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{192 M^4 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d}{ds_0}\left[\int_0^1 dx \int_0^{1-x} dy \frac{\delta(s_0 - s(x,y))}{x^3 y^3}\right]$$

$$- \frac{m_c^8 m_0^4 \langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{192 M^6 \pi^2} e^{-\frac{s_0}{M^2}} \int_0^1 dx \int_0^{1-x} dy \frac{\delta(s_0 - s(x,y))}{x^3 y^3}$$

$$- \frac{m_c^8 m_0^4 \langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{192 M^8 \pi^2} \int_0^{s_0} ds e^{-\frac{s}{M^2}} \int_0^1 dx \int_0^{1-x} dy \frac{\delta(s - s(x,y))}{x^3 y^3}$$

$$- \frac{m_c^6 m_0^4 \langle g_s^2 G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{1728 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d^4}{ds_0^4}\left[\int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^3 x^2}\right]$$

$$- \frac{m_c^6 m_0^4 \langle g_s^2 G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{1728 M^2 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d^3}{ds_0^3}\left[\int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^3 x^2}\right]$$

$$- \frac{m_c^6 m_0^4 \langle g_s^2 G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{1728 M^4 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d^2}{ds_0^2}\left[\int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^3 x^2}\right]$$

$$- \frac{m_c^6 m_0^4 \langle g_s^2 G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{1728 M^6 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d}{ds_0}\left[\int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^3 x^2}\right]$$

$$- \frac{m_c^6 m_0^4 \langle g_s^2 G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{1728 M^8 \pi^2} e^{-\frac{s_0}{M^2}} \int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^3 x^2}$$

$$+ \frac{m_c^6 m_0^4 \langle g_s^2 G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{1728 M^{10} \pi^2} \int_0^{s_0} ds e^{-\frac{s}{M^2}} \int_0^1 dx \frac{\delta(s - s(x,(1-x)))}{(1-x)^3 x^2}$$

$$+ \frac{m_c^8 m_0^4 \langle g_s^2 G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{3456 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d^5}{ds_0^5}\left[\int_0^1 dx \frac{\delta(s_0 - s(x,(1-x))}{(1-x)^4 x^3}\right]$$

$$+\frac{m_c^8 m_0^4 \langle g_s^2 G^2\rangle \langle u\bar{u}\rangle \langle d\bar{d}\rangle g_{\mu\nu}}{3456 M^2 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d^4}{ds_0^4}\left[\int_0^1 dx \frac{\delta(s_0 - s(x,(1-x))}{(1-x)^4 x^3}\right]$$

$$+\frac{m_c^8 m_0^4 \langle g_s^2 G^2\rangle \langle u\bar{u}\rangle \langle d\bar{d}\rangle g_{\mu\nu}}{3456 M^4 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d^3}{ds_0^3}\left[\int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^4 x^3}\right]$$

$$+\frac{m_c^8 m_0^4 \langle g_s^2 G^2\rangle \langle u\bar{u}\rangle \langle d\bar{d}\rangle g_{\mu\nu}}{3456 M^6 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d^2}{ds_0^2}\left[\int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^4 x^3}\right]$$

$$+\frac{m_c^8 m_0^4 \langle g_s^2 G^2\rangle \langle u\bar{u}\rangle \langle d\bar{d}\rangle g_{\mu\nu}}{3456 M^8 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d}{ds_0}\left[\int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^4 x^3}\right]$$

$$+\frac{m_c^8 m_0^4 \langle u\bar{u}\rangle \langle d\bar{d}\rangle g_{\mu\nu}}{3456 M^{10} \pi^2} e^{-\frac{s_0}{M^2}} \int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^4 x^3}$$

$$+\frac{m_c^8 m_0^4 \langle g_s^2 G^2\rangle \langle u\bar{u}\rangle \langle d\bar{d}\rangle g_{\mu\nu}}{3456 M^{12} \pi^2} \int_0^{s_0} ds e^{-\frac{s}{M^2}} \int_0^1 dx \frac{\delta(s - s(x,(1-x)))}{(1-x)^4 x^3}$$

$$+\frac{m_c^8 \langle g_s^2 G^2\rangle p_\mu p_\nu}{24576 \pi^6} e^{-\frac{s_0}{M^2}} \frac{d}{ds_0}\left[\int_0^1 dx \int_0^{1-x} dy \frac{(1-x-y)^4 \delta(s_0 - s(x,y))}{x^3 y^3}\right]$$

$$+\frac{m_c^8 \langle g_s^2 G^2\rangle p_\mu p_\nu}{24576 M^2 \pi^6} e^{-\frac{s_0}{M^2}} \int_0^1 dx \int_0^{1-x} dy \frac{(1-x-y)^4 \delta(s - s(x,y))}{x^3 y^3}$$

$$+\frac{m_c^8 \langle g_s^2 G^2\rangle p_\mu p_\nu}{24576 M^4 \pi^6} \int_0^{s_0} ds e^{-\frac{s}{M^2}} \int_0^1 dx \int_0^{1-x} dy \frac{(1-x-y)^4 \delta(s - s(x,y))}{x^3 y^3}$$

$$+\frac{m_c^8 \langle g_s^2 G^2\rangle g_{\mu\nu}}{49152 \pi^6} e^{-\frac{s_0}{M^2}} \int_0^1 dx \int_0^{1-x} dy \frac{(1-x-y)^4 \delta(s - s(x,y))}{x^3 y^3}$$

$$+\frac{m_c^8\langle g_s^2G^2\rangle g_{\mu\nu}}{49152M^2\pi^6}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1dx\int_0^{1-x}dy\frac{(1-x-y)^4\delta(s-s(x,y))}{x^3y^3}$$

$$-\frac{m_c^8\langle g_s^2G^2\rangle g_{\mu\nu}}{12288\pi^6}e^{-\frac{s_0}{M^2}}\int_0^1dx\int_0^{1-x}dy\frac{(1-x-y)^3\delta(s-s(x,y))}{x^3y^3}$$

$$-\frac{m_c^8\langle g_s^2G^2\rangle g_{\mu\nu}}{12288M^2\pi^6}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1dx\int_0^{1-x}dy\frac{(1-x-y)^3\delta(s-s(x,y))}{x^3y^3}$$

$$-\frac{m_c^6\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{12\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s-s(x,y))}{x^2y^2}$$

$$-\frac{m_c^6\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{12M^2\pi^2}\int_0^{s_0}dse^{-\frac{s_0}{M^2}}\int_0^1dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s-s(x,y))}{x^2y^2}$$

$$-\frac{m_c^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{144\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1dx\int_0^{1-x}dy\frac{\delta(s_0-s(x,y))}{x^2y^2}\right]$$

$$-\frac{m_c^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{144M^2\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1dx\int_0^{1-x}dy\frac{\delta(s-s(x,y))}{x^2y^2}$$

$$-\frac{m_c^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{144M^4\pi^2}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1dx\int_0^{1-x}dy\frac{\delta(s-s(x,y))}{x^2y^2}$$

$$+\frac{m_c^6\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{432\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1dx\int_0^{1-x}dy\frac{\delta(s_0-s(x,y))}{x^2y^3}\right]$$

$$+\frac{m_c^6\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{432M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1dx\int_0^{1-x}dy\frac{\delta(s_0-s(x,y))}{x^2y^3}\right]$$

$$+\frac{m_c^6\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{432M^4\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{\delta(s_0-s(x,y))}{x^2y^3}$$

$$+\frac{m_c^6\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{432M^6\pi^2}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{\delta(s-s(x,y))}{x^2y^3}$$

$$-\frac{m_c^6 g_{\mu\nu}}{512\pi^6}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)^4M^4\delta(s-s(x,y))}{x^2y^2}$$

$$-\frac{m_c^4\langle g_s^2G^2\rangle g_{\mu\nu}}{1536\pi^6}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)^3M^2\delta(s-s(x,y))}{x^2y^2}$$

$$+\frac{m_c^6\langle g_s^2G^2\rangle g_{\mu\nu}}{4608\pi^6}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)^3\delta(s-s(x,y))}{x^2y^3}$$

$$+\frac{m_c^6m_0^2\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{48\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\int_0^{1-x}dy\frac{\delta(s_0-s(x,y))}{x^2y^2}\right]$$

$$+\frac{m_c^6m_0^2\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{48M^2\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{\delta(s_0-s(x,y))}{x^2y^2}$$

$$+\frac{m_c^6m_0^2\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{48M^4\pi^2}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{\delta(s-s(x,y))}{x^2y^2}$$

$$+\frac{m_c^4m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{288\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}\right]$$

$$+\frac{m_c^4m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{288M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^2x^2}\right]$$

$$+\frac{m_c^4 m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{288M^4\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^2x^2}$$

$$+\frac{m_c^4 m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{288M^6\pi^2}\int_0^{s_0} dse^{-\frac{s}{M^2}}\int_0^1 dx\frac{\delta(s-s(x,(1-x)))}{(1-x)^2x^2}$$

$$-\frac{m_c^6 m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{864\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^3}{ds_0^3}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}\right]$$

$$-\frac{m_c^6 m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{864M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}\right]$$

$$-\frac{m_c^6 m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{864M^4\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}\right]$$

$$-\frac{m_c^6 m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{864M^6\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}$$

$$-\frac{m_c^6 m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{864M^8\pi^2}\int_0^{s_0} dse^{-\frac{s}{M^2}}\int_0^1 dx\frac{\delta(s-s(x,(1-x)))}{(1-x)^3x^2}$$

$$-\frac{m_c^6 m_0^4\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{192\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}\right]$$

$$-\frac{m_c^6 m_0^4\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{192M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^2x^2}\right]$$

$$-\frac{m_c^6 m_0^4\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{192M^4\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^2x^2}$$

$$- \frac{m_c^6 m_0^4 \langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{192M^6\pi^2}\int_0^{s_0} ds e^{-\frac{s_0}{M^2}}\int_0^1 dx \frac{\delta(s-s(x,(1-x)))}{(1-x)^2x^2}$$

$$+ \frac{m_c^4 m_0^4 \langle g_s^2 G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{768\pi^2} e^{-\frac{s_0}{M^2}} \frac{d^3}{ds_0^3}\left[\int_0^1 dx \frac{\delta(s_0-s(x,(1-x)))}{(1-x)^2x^2}\right]$$

$$+ \frac{m_c^4 m_0^4 \langle g_s^2 G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{768M^2\pi^2} e^{-\frac{s_0}{M^2}} \frac{d^2}{ds_0^2}\left[\int_0^1 dx \frac{\delta(s_0-s(x,(1-x)))}{(1-x)^2x^2}\right]$$

$$+ \frac{m_c^4 m_0^4 \langle g_s^2 G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{768M^4\pi^2} e^{-\frac{s_0}{M^2}} \frac{d}{ds_0}\left[\int_0^1 dx \frac{\delta(s_0-s(x,(1-x)))}{(1-x)^2x^2}\right]$$

$$+ \frac{m_c^4 m_0^4 \langle g_s^2 G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{768M^6\pi^2} e^{-\frac{s_0}{M^2}} \int_0^1 dx \frac{\delta(s_0-s(x,(1-x)))}{(1-x)^2x^2}$$

$$+ \frac{m_c^4 m_0^4 \langle g_s^2 G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{768M^8\pi^2} \int_0^{s_0} ds e^{-\frac{s_0}{M^2}} \int_0^1 dx \frac{\delta(s-s(x,(1-x)))}{(1-x)^2x^2}$$

$$- \frac{m_c^6 m_0^4 \langle g_s^2 G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{1728\pi^2} e^{-\frac{s_0}{M^2}} \frac{d^4}{ds_0^4}\left[\int_0^1 dx \frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}\right]$$

$$- \frac{m_c^6 m_0^4 \langle g_s^2 G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{1728M^2\pi^2} e^{-\frac{s_0}{M^2}} \frac{d^3}{ds_0^3}\left[\int_0^1 dx \frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}\right]$$

$$- \frac{m_c^6 m_0^4 \langle g_s^2 G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{1728M^4\pi^2} e^{-\frac{s_0}{M^2}} \frac{d^2}{ds_0^2}\left[\int_0^1 dx \frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}\right]$$

$$- \frac{m_c^6 m_0^4 \langle g_s^2 G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle g_{\mu\nu}}{1728M^6\pi^2} e^{-\frac{s_0}{M^2}} \frac{d}{ds_0}\left[\int_0^1 dx \frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}\right]$$

$$- \frac{m_c^6 m_0^4 \langle g_s^2 G^2\rangle \langle u\bar{u}\rangle \langle d\bar{d}\rangle g_{\mu\nu}}{1728 M^8 \pi^2} e^{-\frac{s_0}{M^2}} \int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^3 x^2}$$

$$- \frac{m_c^6 m_0^4 \langle g_s^2 G^2\rangle \langle u\bar{u}\rangle \langle d\bar{d}\rangle g_{\mu\nu}}{1728 M^{10} \pi^2} \int_0^{s_0} ds e^{-\frac{s_0}{M^2}} \int_0^1 dx \frac{\delta(s - s(x,(1-x)))}{(1-x)^3 x^2}$$

$$- \frac{m_c^6 m_0^4 \langle g_s^2 G^2\rangle \langle u\bar{u}\rangle \langle d\bar{d}\rangle g_{\mu\nu}}{2304 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d^4}{ds_0^4} \left[\int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^3 x^3} \right]$$

$$- \frac{m_c^6 m_0^4 \langle g_s^2 G^2\rangle \langle u\bar{u}\rangle \langle d\bar{d}\rangle g_{\mu\nu}}{2304 M^2 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d^3}{ds_0^3} \left[\int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^3 x^3} \right]$$

$$- \frac{m_c^6 m_0^4 \langle g_s^2 G^2\rangle \langle u\bar{u}\rangle \langle d\bar{d}\rangle g_{\mu\nu}}{2304 M^4 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d^2}{ds_0^2} \left[\int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^3 x^3} \right]$$

$$- \frac{m_c^6 m_0^4 \langle g_s^2 G^2\rangle \langle u\bar{u}\rangle \langle d\bar{d}\rangle g_{\mu\nu}}{2304 M^6 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d}{ds_0} \left[\int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^3 x^3} \right]$$

$$- \frac{m_c^6 m_0^4 \langle g_s^2 G^2\rangle \langle u\bar{u}\rangle \langle d\bar{d}\rangle g_{\mu\nu}}{2304 M^8 \pi^2} e^{-\frac{s_0}{M^2}} \int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^3 x^3}$$

$$- \frac{m_c^6 m_0^4 \langle g_s^2 G^2\rangle \langle u\bar{u}\rangle \langle d\bar{d}\rangle g_{\mu\nu}}{2304 M^{10} \pi^2} \int_0^{s_0} ds e^{-\frac{s_0}{M^2}} \int_0^1 dx \frac{\delta(s - s(x,(1-x)))}{(1-x)^3 x^3}$$

$$- \frac{m_c^6 \langle g_s^2 G^2\rangle g_{\mu\nu}}{4096 \pi^6} \int_0^{s_0} ds e^{-\frac{s}{M^2}} \int_0^1 dx \int_0^{1-x} dy \frac{(1-x-y)^2 \delta(s - s(x,y))}{x^2 y^2}. \tag{D.1}$$

Appendix E

ANALYTICAL RESULTS OF 0^{++} TETRAQUARK

For the 0^{++} tetraquark the analytical results obtained are as follows:

$$\Pi^{0^{++}}\left(M^2, s_0\right) = \frac{m_c^6\langle g_s^2 G^2\rangle}{4608\pi^6}\int_0^{s_0} dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x} dy\frac{(1-x-y)^3\delta(s-s(x,y))}{x^2y^3}$$

$$-\frac{m_c^8}{2560\pi^6}\int_0^{s_0} dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x} dy\frac{(1-x-y)^5 M^2\delta(s-s(x,y))}{x^3y^3}$$

$$-\frac{m_c^4\langle g_s^2 G^2\rangle}{36864\pi^6}\int_0^{s_0} dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x} dy\frac{(1-x-y)^3 M^2\delta(s-s(x,y))}{x^2y^2}$$

$$-\frac{m_c^8\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{24\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\int_0^{1-x} dy\frac{(1-x-y)^2\delta(s_0-s(x,y))}{x^3y^3}\right]$$

$$-\frac{m_c^8\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{24M^2\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\int_0^{1-x} dy\frac{(1-x-y)^2\delta(s-s(x,y))}{x^3y^3}$$

$$-\frac{m_c^8\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{24M^4\pi^2}\int_0^{s_0} dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x} dy\frac{(1-x-y)^2\delta(s-s(x,y))}{x^3y^3}$$

$$+\frac{m_c^6\langle g_s^2 G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{432\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1 dx\int_0^{1-x} dy\frac{\delta(s_0-s(x,y))}{x^2y^3}\right]$$

$$+\frac{m_c^6\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{432M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\int_0^{1-x}dy\frac{\delta(s_0-s(x,y))}{x^2y^3}\right]$$

$$+\frac{m_c^6\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{432M^4\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{\delta(s_0-s(x,y))}{x^2y^3}$$

$$+\frac{m_c^6\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{432M^6\pi^2}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{\delta(s-s(x,y))}{x^2y^3}$$

$$+\frac{m_c^8m_0^2\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{24\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s_0-s(x,y))}{x^3y^3}\right]$$

$$+\frac{m_c^8m_0^2\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{24M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s_0-s(x,y))}{x^3y^3}\right]$$

$$+\frac{m_c^8m_0^2\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{24M^4\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s_0-s(x,y))}{x^3y^3}$$

$$+\frac{m_c^8m_0^2\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{24M^6\pi^2}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s-s(x,y))}{x^3y^3}$$

$$-\frac{m_c^6m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{864\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^3}{ds_0^3}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}\right]$$

$$-\frac{m_c^6m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{864M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}\right]$$

$$-\frac{m_c^6m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{864M^4\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}\right]$$

$$-\frac{m_c^6 m_0^2 \langle g_s^2 G^2\rangle \langle u\bar{u}\rangle \langle d\bar{d}\rangle}{864 M^6 \pi^2} e^{-\frac{s_0}{M^2}} \int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^3 x^2}$$

$$-\frac{m_c^6 m_0^2 \langle g_s^2 G^2\rangle \langle u\bar{u}\rangle \langle d\bar{d}\rangle}{864 M^8 \pi^2} \int_0^{s_0} ds e^{-\frac{s}{M^2}} \int_0^1 dx \frac{\delta(s - s(x,(1-x)))}{(1-x)^3 x^2}$$

$$-\frac{m_c^8 m_0^4 \langle u\bar{u}\rangle \langle d\bar{d}\rangle}{192 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d^3}{ds_0^3} \left[\int_0^1 dx \int_0^{1-x} dy \frac{\delta(s_0 - s(x,y))}{x^3 y^3}\right]$$

$$-\frac{m_c^8 m_0^4 \langle u\bar{u}\rangle \langle d\bar{d}\rangle}{192 M^2 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d^2}{ds_0^2} \left[\int_0^1 dx \int_0^{1-x} dy \frac{\delta(s_0 - s(x,y))}{x^3 y^3}\right]$$

$$-\frac{m_c^8 m_0^4 \langle u\bar{u}\rangle \langle d\bar{d}\rangle}{192 M^4 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d}{ds_0} \left[\int_0^1 dx \int_0^{1-x} dy \frac{\delta(s_0 - s(x,y))}{x^3 y^3}\right]$$

$$-\frac{m_c^8 m_0^4 \langle u\bar{u}\rangle \langle d\bar{d}\rangle}{192 M^6 \pi^2} e^{-\frac{s_0}{M^2}} \int_0^1 dx \int_0^{1-x} dy \frac{\delta(s_0 - s(x,y))}{x^3 y^3}$$

$$-\frac{m_c^8 m_0^4 \langle u\bar{u}\rangle \langle d\bar{d}\rangle}{192 M^8 \pi^2} \int_0^{s_0} ds e^{-\frac{s}{M^2}} \int_0^1 dx \int_0^{1-x} dy \frac{\delta(s - s(x,y))}{x^3 y^3}$$

$$-\frac{m_c^6 m_0^4 \langle g_s^2 G^2\rangle \langle u\bar{u}\rangle \langle d\bar{d}\rangle}{1728 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d^4}{ds_0^4} \left[\int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^3 x^2}\right]$$

$$-\frac{m_c^6 m_0^4 \langle g_s^2 G^2\rangle \langle u\bar{u}\rangle \langle d\bar{d}\rangle}{1728 M^2 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d^3}{ds_0^3} \left[\int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^3 x^2}\right]$$

$$-\frac{m_c^6 m_0^4 \langle g_s^2 G^2\rangle \langle u\bar{u}\rangle \langle d\bar{d}\rangle}{1728 M^4 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d^2}{ds_0^2} \left[\int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^3 x^2}\right]$$

$$- \frac{m_c^6 m_0^4 \langle g_s^2 G^2\rangle \langle u\bar{u}\rangle \langle d\bar{d}\rangle}{1728 M^6 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d}{ds_0}\left[\int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^3 x^2}\right]$$

$$- \frac{m_c^6 m_0^4 \langle g_s^2 G^2\rangle \langle u\bar{u}\rangle \langle d\bar{d}\rangle}{1728 M^8 \pi^2} e^{-\frac{s_0}{M^2}} \int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^3 x^2}$$

$$+ \frac{m_c^6 m_0^4 \langle g_s^2 G^2\rangle \langle u\bar{u}\rangle \langle d\bar{d}\rangle}{1728 M^{10} \pi^2} \int_0^{s_0} ds e^{-\frac{s}{M^2}} \int_0^1 dx \frac{\delta(s - s(x,(1-x)))}{(1-x)^3 x^2}$$

$$+ \frac{m_c^8 m_0^4 \langle u\bar{u}\rangle \langle d\bar{d}\rangle}{3456 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d^4}{ds_0^4}\left[\int_0^1 dx \frac{\delta(s_0 - s(x,(1-x))}{(1-x)^4 x^3}\right]$$

$$+ \frac{m_c^8 m_0^4 \langle u\bar{u}\rangle \langle d\bar{d}\rangle}{3456 M^2 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d^4}{ds_0^4}\left[\int_0^1 dx \frac{\delta(s_0 - s(x,(1-x))}{(1-x)^4 x^3}\right]$$

$$+ \frac{m_c^8 m_0^4 \langle u\bar{u}\rangle \langle d\bar{d}\rangle g_{\mu\nu}}{3456 M^4 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d^3}{ds_0^3}\left[\int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^4 x^3}\right]$$

$$+ \frac{m_c^8 m_0^4 \langle u\bar{u}\rangle \langle d\bar{d}\rangle g_{\mu\nu}}{3456 M^6 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d^2}{ds_0^2}\left[\int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^4 x^3}\right]$$

$$+ \frac{m_c^8 m_0^4 \langle u\bar{u}\rangle \langle d\bar{d}\rangle g_{\mu\nu}}{3456 M^8 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d}{ds_0}\left[\int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^4 x^3}\right]$$

$$+ \frac{m_c^8 m_0^4 \langle u\bar{u}\rangle \langle d\bar{d}\rangle g_{\mu\nu}}{3456 M^{10} \pi^2} e^{-\frac{s_0}{M^2}} \int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^4 x^3}$$

$$+ \frac{m_c^8 m_0^4 \langle u\bar{u}\rangle \langle d\bar{d}\rangle g_{\mu\nu}}{3456 M^{12} \pi^2} \int_0^{s_0} ds e^{-\frac{s}{M^2}} \int_0^1 dx \frac{\delta(s - s(x,(1-x)))}{(1-x)^4 x^3}$$

$$- \frac{m_c^8\langle g_s^2G^2\rangle}{12288\pi^6}e^{-\frac{s_0}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)^3\delta(s-s(x,y))}{x^3y^3}$$

$$- \frac{m_c^8\langle g_s^2G^2\rangle}{12288M^2\pi^6}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)^3\delta(s-s(x,y))}{x^3y^3}$$

$$- \frac{m_c^6\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{12\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s-s(x,y))}{x^2y^2}$$

$$- \frac{m_c^6\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{12M^2\pi^2}\int_0^{s_0}dse^{-\frac{s_0}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)\delta(s-s(x,y))}{x^2y^2}$$

$$- \frac{m_c^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{144\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\int_0^{1-x}dy\frac{\delta(s_0-s(x,y))}{x^2y^2}\right]$$

$$- \frac{m_c^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{144M^2\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{\delta(s-s(x,y))}{x^2y^2}$$

$$- \frac{m_c^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{144M^4\pi^2}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{\delta(s-s(x,y))}{x^2y^2}$$

$$+ \frac{m_c^6\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{432\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1 dx\int_0^{1-x}dy\frac{\delta(s_0-s(x,y))}{x^2y^3}\right]$$

$$+ \frac{m_c^6\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{432M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\int_0^{1-x}dy\frac{\delta(s_0-s(x,y))}{x^2y^3}\right]$$

$$+ \frac{m_c^6\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{432M^4\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{\delta(s_0-s(x,y))}{x^2y^3}$$

$$+\frac{m_c^6\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{432M^6\pi^2}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1dx\int_0^{1-x}dy\frac{\delta(s-s(x,y))}{x^2y^3}$$

$$-\frac{m_c^6}{512\pi^6}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1dx\int_0^{1-x}dy\frac{(1-x-y)^4M^4\delta(s-s(x,y))}{x^2y^2}$$

$$-\frac{m_c^4\langle g_s^2G^2\rangle}{1536\pi^6}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1dx\int_0^{1-x}dy\frac{(1-x-y)^3M^2\delta(s-s(x,y))}{x^2y^2}$$

$$+\frac{m_c^6\langle g_s^2G^2\rangle}{4608\pi^6}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1dx\int_0^{1-x}dy\frac{(1-x-y)^3M^2\delta(s-s(x,y))}{x^2y^3}$$

$$+\frac{m_c^6m_0^2\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{48\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1dx\int_0^{1-x}dy\frac{\delta(s_0-s(x,y))}{x^2y^2}\right]$$

$$+\frac{m_c^6m_0^2\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{48M^2\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1dx\int_0^{1-x}dy\frac{\delta(s_0-s(x,y))}{x^2y^2}$$

$$+\frac{m_c^6m_0^2\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{48M^4\pi^2}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1dx\int_0^{1-x}dy\frac{\delta(s-s(x,y))}{x^2y^2}$$

$$+\frac{m_c^4m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{288\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^2}\right]$$

$$+\frac{m_c^4m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{288M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^2x^2}\right]$$

$$+\frac{m_c^4m_0^2\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{288M^4\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^2x^2}$$

$$+ \frac{m_c^4 m_0^2 \langle g_s^2 G^2 \rangle \langle u\bar{u} \rangle \langle d\bar{d} \rangle}{288 M^6 \pi^2} \int_0^{s_0} ds e^{-\frac{s}{M^2}} \int_0^1 dx \frac{\delta(s - s(x,(1-x)))}{(1-x)^2 x^2}$$

$$- \frac{m_c^6 m_0^2 \langle g_s^2 G^2 \rangle \langle u\bar{u} \rangle \langle d\bar{d} \rangle}{1728 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d^3}{ds_0^3} \left[\int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^3 x^2} \right]$$

$$- \frac{2 m_c^6 m_0^2 \langle g_s^2 G^2 \rangle \langle u\bar{u} \rangle \langle d\bar{d} \rangle}{1728 M^2 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d^2}{ds_0^2} \left[\int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^3 x^2} \right]$$

$$- \frac{2 m_c^6 m_0^2 \langle g_s^2 G^2 \rangle \langle u\bar{u} \rangle \langle d\bar{d} \rangle}{1728 M^4 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d}{ds_0} \left[\int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^3 x^2} \right]$$

$$- \frac{2 m_c^6 m_0^2 \langle g_s^2 G^2 \rangle \langle u\bar{u} \rangle \langle d\bar{d} \rangle}{1728 M^6 \pi^2} e^{-\frac{s_0}{M^2}} \int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^3 x^2}$$

$$- \frac{2 m_c^6 m_0^2 \langle g_s^2 G^2 \rangle \langle u\bar{u} \rangle \langle d\bar{d} \rangle}{1728 M^8 \pi^2} \int_0^{s_0} ds e^{-\frac{s}{M^2}} \int_0^1 dx \frac{\delta(s - s(x,(1-x)))}{(1-x)^3 x^2}$$

$$- \frac{m_c^6 m_0^4 \langle u\bar{u} \rangle \langle d\bar{d} \rangle}{192 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d^2}{ds_0^2} \left[\int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^3 x^2} \right]$$

$$- \frac{m_c^6 m_0^4 \langle u\bar{u} \rangle \langle d\bar{d} \rangle}{192 M^2 \pi^2} e^{-\frac{s_0}{M^2}} \frac{d}{ds_0} \left[\int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^2 x^2} \right]$$

$$- \frac{m_c^6 m_0^4 \langle u\bar{u} \rangle \langle d\bar{d} \rangle}{192 M^4 \pi^2} e^{-\frac{s_0}{M^2}} \int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^2 x^2}$$

$$- \frac{m_c^6 m_0^4 \langle u\bar{u} \rangle \langle d\bar{d} \rangle}{192 M^6 \pi^2} \int_0^{s_0} ds e^{-\frac{s_0}{M^2}} \int_0^1 dx \frac{\delta(s - s(x,(1-x)))}{(1-x)^2 x^2}$$

$$+\frac{m_c^4 m_0^4 \langle g_s^2 G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{768\pi^2} e^{-\frac{s_0}{M^2}} \frac{d^3}{ds_0^3}\left[\int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^2 x^2}\right]$$

$$+\frac{m_c^4 m_0^4 \langle g_s^2 G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{768 M^2\pi^2} e^{-\frac{s_0}{M^2}} \frac{d^2}{ds_0^2}\left[\int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^2 x^2}\right]$$

$$+\frac{m_c^4 m_0^4 \langle g_s^2 G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{768 M^4\pi^2} e^{-\frac{s_0}{M^2}} \frac{d}{ds_0}\left[\int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^2 x^2}\right]$$

$$+\frac{m_c^4 m_0^4 \langle g_s^2 G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{768 M^6\pi^2} e^{-\frac{s_0}{M^2}} \int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^2 x^2}$$

$$+\frac{m_c^4 m_0^4 \langle g_s^2 G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{768 M^8\pi^2} \int_0^{s_0} ds e^{-\frac{s_0}{M^2}} \int_0^1 dx \frac{\delta(s - s(x,(1-x)))}{(1-x)^2 x^2}$$

$$-\frac{m_c^6 m_0^4 \langle g_s^2 G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{1728\pi^2} e^{-\frac{s_0}{M^2}} \frac{d^4}{ds_0^4}\left[\int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^3 x^2}\right]$$

$$-\frac{m_c^6 m_0^4 \langle g_s^2 G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{1728 M^2\pi^2} e^{-\frac{s_0}{M^2}} \frac{d^3}{ds_0^3}\left[\int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^3 x^2}\right]$$

$$-\frac{m_c^6 m_0^4 \langle g_s^2 G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{1728 M^4\pi^2} e^{-\frac{s_0}{M^2}} \frac{d^2}{ds_0^2}\left[\int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^3 x^2}\right]$$

$$-\frac{m_c^6 m_0^4 \langle g_s^2 G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{1728 M^6\pi^2} e^{-\frac{s_0}{M^2}} \frac{d}{ds_0}\left[\int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^3 x^2}\right]$$

$$-\frac{m_c^6 m_0^4 \langle g_s^2 G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{1728 M^8\pi^2} e^{-\frac{s_0}{M^2}} \int_0^1 dx \frac{\delta(s_0 - s(x,(1-x)))}{(1-x)^3 x^2}$$

$$
\begin{aligned}
&-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{1728M^{10}\pi^2}\int_0^{s_0}dse^{-\frac{s_0}{M^2}}\int_0^1 dx\frac{\delta(s-s(x,(1-x)))}{(1-x)^3x^2}\\
&-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{2304\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^4}{ds_0^4}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^3}\right]\\
&-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{2304M^2\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^3}{ds_0^3}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^3}\right]\\
&-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{2304M^4\pi^2}e^{-\frac{s_0}{M^2}}\frac{d^2}{ds_0^2}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^3}\right]\\
&-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{2304M^6\pi^2}e^{-\frac{s_0}{M^2}}\frac{d}{ds_0}\left[\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^3}\right]\\
&-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{2304M^8\pi^2}e^{-\frac{s_0}{M^2}}\int_0^1 dx\frac{\delta(s_0-s(x,(1-x)))}{(1-x)^3x^3}\\
&-\frac{m_c^6 m_0^4\langle g_s^2G^2\rangle\langle u\bar{u}\rangle\langle d\bar{d}\rangle}{2304M^{10}\pi^2}\int_0^{s_0}dse^{-\frac{s_0}{M^2}}\int_0^1 dx\frac{\delta(s-s(x,(1-x)))}{(1-x)^3x^3}\\
&-\frac{m_c^6\langle g_s^2G^2\rangle}{4096\pi^6}\int_0^{s_0}dse^{-\frac{s}{M^2}}\int_0^1 dx\int_0^{1-x}dy\frac{(1-x-y)^2\delta(s-s(x,y))}{x^2y^2}.
\end{aligned}
\tag{E.1}
$$

CURRICULUM VITAE

PERSONAL INFORMATION

Surname, Name: Sarı Ferah, Semra

EDUCATION

Degree	Institution	Year of Graduation
M.S.	Karadeniz Teknik Üniversitesi Physics	2013
B.S.	AÜ Management Information Systems	2024
B.S.	AÜ Political Science and Public Administration	2020
B.S.	Karadeniz Teknik Üniversitesi Physics	2009

PROFESSIONAL EXPERIENCE

Year	Place	Enrollment
2019-	Middle East Technical University	Research Assistant
2013 - 2018	Middle East Technical University	Research Assistant
2011-2013	Karadeniz Teknik Universitesi	Research Assistant

SCHOOLS AND WORKSHOP ATTENDED

- Kuantum Renk Dinamiği Kış Okulu (Quantum chromodynamics Winter School in Turkish), Jan 16-20, 2023 (Ankara, Turkey)
- Hadron Fiziği Kış Okulu (Hadron Physics Winter School - in Turkish), Jan 25-29, 2016 (Ankara, Turkey).
- (ULUYEF13) II.Uludağ Yüksek Enerji Fiziği Kış Okulu (ULUYEF13-II.Uludağ High Energy Physics School - in Turkish), Feb 18-22, 2013 (Bursa, Turkey)
- (İZYEF) Yüksek Enerji Fiziği ve Uygulamaları Çalıştayı (Istanbul High Energy Physics Workshop - in Turkish), Jun 19-23, 2012 (İzmir, Turkey)
- Nükleer ve Parçacık Fiziğinde Monte Carlo Uygulamaları Yaz Okulu (Monte Carlo Applications in Nuclear and Particle Physics Summer School in Turkish), May 10-12, 2012 (Bitlis, Turkey)
- Hızlandırıcı ve Parçacık Fiziğinde Bilgisayar Uygulamaları Okulu (Accelerator and Computer Applications in Particle Physics School - in Turkish), Feb 25-29, 2012 (Kars, Turkey)