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Dilatonic Brans-Dicke Anisotropic Collapsing Fluid Sphere And de Broglie Quantum Wave Motion

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Abstract. Two dimensional (2D) analogue of vacuum sector of the Brans Dicke (BD) gravity [1] is studied to obtain dynamics of anisotropic spherically symmetric perfect fluid. Our obtained static solutions behave as dark matter with state equation $\gamma = \frac{p(\rho)}{\rho} = -0.25$ but in non-static regimes behave as regular perfect fluid with barotropic index $\gamma > 0$. Positivity property of total mass of the fluid causes that the BD parameter to be $\omega > \frac{2}{3}$ and/or $\omega < -1$. Locations of the event and apparent horizons of the collapsing fluid are obtained in its static regime. In case $\omega > 0$ the apparent horizon is covered by event horizon where the cosmic censorship hypothesis is still valid. According to the model [1], we obtain de Broglie pilot wave of our metric solution which describes particles ensemble which become distinguishable via different values of ω . Incident current density of particles ensemble on the horizons is evaluated which describe the ‘Hawking radiation’. The de Broglie -Bohm quantum potential effect is calculated also on the event (apparent) horizon which is independent (dependent) to values of ω .

1. Introduction

Around a collapsing star, gravity takes an ultra-strong intensity. Hence corresponding matter density and space-time curvature diverge to infinity. It is predicted from singularity theorems in general relativity [2] that end of stellar collapse leads to a visible naked or invisible covered singularity. Property of visibility of singularities are determined by the causal structure of collapsing process, satisfying the gravitational field equations. Internal dynamics of the collapse can be determine type of singularities. They will be naked and hence visible from view of external observer if collapse process delays the formation of the event horizon [3,4,5]. Penrose represented ‘cosmic censorship hypotheses’ [6], which explains properties of the final singularity of a gravitational collapse. In this conjecture, space-time singularities created from gravitational collapse should be usually covered by the event horizon hyper-surface. Hence it will be invisible from view of outside observers of collapsed object namely black hole [7]. However there is still a doubt for reality of censorship hypotheses and has been recognized as one of the most open problems in classical general theory of relativity. The censorship hypotheses may to be still valid in quantum gravity approach (see [8] and references therein). Black holes and naked singularities have different properties observationally. They have different characteristics in the gravitational lensing [9,10] and also many exact solutions of Einstein’s gravitational field equations are obtained which admit naked singularities [11-18]. In the present paper we use vacuum sector of the BD gravity [19] in 2D analogue [1] and obtain non-static solutions of linearized field equations. Stress tensor of our solutions treats as anisotropic spherically symmetric perfect fluid. Equation of state takes dark and visible regular matter regimes during



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the collapse process under particular conditions on different values of the parameters of the system. Apparent and event horizons of the collapsing object are evaluated where the apparent horizon become inside of the event horizon for $\omega > 0$ at static regime. According to results of the work [1] we obtain de Broglie-Bohm quantum wave functional of the collapsing fluid sphere. Probability amplitude of particles ensemble and their radial velocities are calculated from incident current density (ICD) of particles ensemble. Mass of the collapsing fluid must be takes some positive real values numerically reaching to conditions $\omega < -1; \omega > \frac{2}{3}$. The work is organized as follows: In section 2 we call dilatonic BD gravity in 2D analogue [1] and obtain linearized metric solution. Corresponding stress tensor is obtained to be treat as an anisotropic spherically symmetric BD perfect fluid. Location of the horizons are determined as spherically symmetric plan waves. Using some suitable conditions we obtained time independent horizons location and mass function. In section 3 we use results of the work [1] to obtain de Broglie wave functional of the model and then evaluate ICD of particles ensemble moving on the horizons. de Broglie-Bohm quantum potential effect is also considered as corrections on our results. Section 4 denotes to summery and conclusion.

2. Dilatonic Brans-Dicke collapsing sphere

One can obtain effective parts of the BD gravity [19] defined on spherically symmetric curved space times $ds^2 = g_{ab}dx^a dx^b + \psi^2(x^a)(d\theta^2 + \sin^2\theta d\varphi^2)$ in 2D analogue [1] as $I[\phi, \psi, g_{ab}] = \frac{1}{2} \int d^2x \sqrt{g} \{ \phi + \frac{1}{2}\phi\psi^2 R + \phi g^{ab} \partial_a \psi \partial_b \psi + 2\psi g^{ab} \partial_a \psi \partial_b \phi - \frac{\omega\psi^2}{2\phi} g^{ab} \partial_a \phi \partial_b \phi \}$ where ϕ and ω are BD field and parameter respectively. Experimentally ω is obtained to be as ≥ 40000 [20,21]. ψ^2 is 2-sphere conformal factor and R is Ricci scalar of 2D metric g_{ab} defined by 2D coordinates system x^a with $a \equiv 1, 2$. We solved linearized dynamical field equations and obtained metric solution as $Gds^2 \approx -(1 + \varepsilon x_0 e^{a\tau+b\rho})d\tau^2 + 2(1 + \varepsilon y_0 e^{a\tau+b\rho})d\tau d\rho + (1 + \varepsilon z_0 e^{a\tau+b\rho})d\rho^2$ where $|\varepsilon| \ll 1$ is order parameter of the perturbation series expansion, $t = \sqrt{G}\tau$ and $r = \sqrt{G}\rho$ (details of calculations are given in ref. [23]). Also we obtained that $\psi \approx \sqrt{eG}(1 + \varepsilon u_0 e^{a\tau+b\rho})$ and $G\phi \approx 1 + \varepsilon v_0 e^{a\tau+b\rho}$ in which a, b are integral constants and for moving waves with light velocity we must be set $a = \pm b$. Other parameters defined by $\omega, u_0, v_0, y_0, x_0$ and γ are obtained exactly against a, b in ref. [23]. In the next section we show ε is related to the corresponding quantum potential of de Broglie wave evaluated on the event horizon hypersurface. In case of static regime where $a = 0$ we will have $\gamma = -0.25$ describing a dark matter dominant of the gravitational system and $b = -\sqrt{\frac{6\omega-4}{\omega+1}}, \omega > \frac{2}{3}$. With these conditions our metric solutions become asymptotically flat at large scales $\rho \gg 1$ for which $\frac{u_0}{v_0} = \frac{3\omega-2}{5}, \frac{x_0}{v_0} = \frac{y_0}{v_0} = \frac{8(3\omega-2)}{5}, \frac{z_0}{v_0} = \frac{2(2-3\omega)(2\omega-3)}{5(\omega+1)}$. In case of non static regime of the metric solution we must be set $(a, b) \neq 0$. In the latter case one can obtain moving position of event horizon by solving $g_{tt}(\tau, \rho) = 0$ as $a\tau + b\rho = -\ln|\varepsilon x_0|$ which in static regime become $\rho_{EH} = \sqrt{\frac{\omega+1}{6\omega-4}} \ln \left| \frac{4\varepsilon v_0}{5} (6\omega-4) \right|$. Surface area of moving apparent horizon is $4\pi\psi^2(\tau, \rho)$. Its position is obtained by solving $g^{ab} \partial_a \psi \partial_b \psi = 0$ as $a\tau + b\rho = \ln \left| \frac{2(b^2+2ab-a^2)}{\varepsilon[(a^2+b^2)z_0+2(b^2+ab-a^2)y_0-(a-b)^2x_0]} \right|$ which is $\rho_{AH} = \sqrt{\frac{\omega+1}{6\omega-4}} \ln \left| \frac{4\varepsilon v_0}{5} \frac{(3\omega-2)(\omega+5)}{(\omega+1)} \right|$ in static regime. In the general relativistic limits $\omega \rightarrow +\infty$ we obtain $\rho_{EA} = 0.94$. One can determine mass function $m(r)$ up to the radius r by calculating $m(r) = \int_0^r 4\pi r^2 \varrho(r) dr$ which by setting $a = 0$, $r = \sqrt{G}\rho$, and $b = -\sqrt{\frac{6\omega-4}{\omega+1}}$, we will have $m^*(\rho) = \frac{m(\rho)}{8\pi\sqrt{G}v_0\varepsilon} = \sqrt{\frac{\omega+1}{6\omega-4}} \left\{ 1 - \frac{1}{2} \left[\left(\frac{6\omega-4}{\omega+1} \right)^2 - \left(\frac{6\omega-4}{\omega+1} \right) + 2 \right] e^{-\sqrt{\frac{6\omega-4}{\omega+1}}\rho} \right\}$ (see ref. [23]). Positivity and reality of the mass quantity of the fluid restrict us to choose $\omega > \frac{2}{3}$ or $\omega < -1$.

3. de Broglie quantum gravity perspective

According to results of the work [1] (see also [23]) we can obtain phase \bar{D} and amplitude \bar{R} of de Broglie pilot wave $\Psi(\psi, \phi) = \sqrt{\bar{R}}e^{i\bar{D}}$ of the spherically symmetric BD action given in the previous section as $\phi\psi^2 = \left(\frac{3+2\omega}{2+\omega}\right)\frac{\bar{R}}{4}$, and $\psi = \sqrt{G}\bar{R}^{\left(\frac{1+\omega}{3+2\omega}\right)}e^{i\bar{D}\frac{\sqrt{4+2\omega}}{3+2\omega}}$. Inserting the solutions given at the previous section the parts of amplitude \bar{R} and the phase \bar{D} read $\bar{R}(\tau, \rho) = \left(\frac{8+2\omega}{3+2\omega}\right)\exp\{1+\varepsilon(u_0+v_0)e^{a\tau+b\rho}\}$ and $\bar{D}(\tau, \rho) = \frac{1}{\sqrt{(4+2\omega)(3+2\omega)}}\left\{\frac{1}{2} - (1+\omega)\ln\left(\frac{8+4\omega}{3+2\omega}\right) + \varepsilon[(2+\omega)u_0 - (1+\omega)v_0]e^{a\tau+b\rho}\right\}$ which by setting $a = 0$ reduces to their static forme (standing waves) as $\bar{R}(\rho) = \left(\frac{8+2\omega}{3+2\omega}\right)\exp\{1 + \varepsilon(3v_0/5)(\omega+1)e^{-\sqrt{\frac{6\omega-4}{\omega+1}}\rho}\}$ and $\bar{D}(\rho) = \frac{1}{\sqrt{(4+2\omega)(3+2\omega)}}\left\{\frac{1}{2} - (1+\omega)\ln\left(\frac{8+4\omega}{3+2\omega}\right) + \varepsilon(v_0/5)(3\omega^2 - \omega - 9)e^{-\sqrt{\frac{6\omega-4}{\omega+1}}\rho}\right\}$. We calculate now ICD of particles ensemble containing the Plank mass $m_p = \frac{1}{\sqrt{G}}$ (in units $\hbar = c = 1$) via $J_{incident} = \frac{i\hbar}{2m_p}\{\Psi\partial_\rho\Psi^* - \Psi^*\partial_\rho\Psi\}$ as $J_{incident}(\rho) = \left(\frac{9-\omega-3\omega^2}{3+2\omega}\right)\sqrt{\frac{(4+\omega)(3\omega-2)}{(\omega+1)(\omega+2)}}\left(\frac{3\sqrt{2}v_0}{5}\right)\exp\left\{\frac{1}{2} - \sqrt{\frac{6\omega-4}{1+\omega}}\rho + \varepsilon\left(\frac{3v_0}{10}\right)(1+\omega)e^{-\sqrt{\frac{6\omega-4}{1+\omega}}\rho}\right\}$. Its value become $J_{incident}(\rho_{EH}) = \left(\frac{9-\omega-3\omega^2}{3+2\omega}\right)\sqrt{\frac{(4+\omega)}{(\omega+1)(\omega+2)(3\omega-2)}}\left(\frac{3\sqrt{2}}{8\varepsilon}\right)e^{\frac{5}{8}\left(\frac{3\omega-1}{3\omega-2}\right)}$ on the event horizon hypersurface and $J_{incident}(\rho_{AH}) = \left(\frac{9-\omega-3\omega^2}{3+2\omega}\right)\sqrt{\frac{(4+\omega)(1+\omega)}{(3\omega-2)(\omega+2)}}\left(\frac{3\sqrt{2}}{4\varepsilon}\right)\exp\left\{\frac{15\omega^2+58\omega-37}{8(3\omega-2)(\omega+5)}\right\}$ on the apparent horizon hypersurface respectively. Quantum potential of the de Broglie wave is defined by $Q = -\frac{\nabla_\mu\nabla^\mu\sqrt{\bar{D}}}{\sqrt{\bar{D}}}$ which for our metric solutions in static regime become $Q(\rho) = -\varepsilon^2\left(\frac{3v_0}{5}\right)(3\omega-2)e^{-\sqrt{\frac{6\omega-4}{\omega+1}}\rho}$. Its value become $Q(\rho_{EH}) = -\frac{3}{8}\varepsilon$ on the event horizon hypersurface and $Q(\rho_{AH}) = -\frac{3}{4}\left(\frac{1+\omega}{5+\omega}\right)\varepsilon$ on the apparent horizon hypersurface respectively. The former result shows relation between quantum potential and perturbation order parameter ε . Namely one can evolve physical meaning of ε which is supported by quantum potential counterpart evaluated on the event horizon as independent of ω . Obviously our results and inferences must be corrected by regarding backreaction corrections of de Broglie-Bohm quantum potential effects on moving particle ensembles (see section 3 in ref. [24]) which is not considered here. It is suitable to define relative normalized ICD of particle ensemble as $\Delta(\omega) = \frac{J_{incident}(\rho_{EH}) - J_{incident}(\rho_{AH})}{J_{incident}(\rho_{AH})} = \frac{\exp\left\{\frac{3\omega}{2(3\omega-2)(5+\omega)}\right\}}{2(1+\omega)}$ and relative normalized quantum potential as $\delta(\omega) = \frac{Q(\rho_{EH}) - Q(\rho_{AH})}{Q(\rho_{AH})} = \frac{3-\omega}{2(1+\omega)}$. We plotted diagrams of Δ and δ against ω with solid and dash lines respectively in figure 1. We see potential barrier for $-1 < \omega < 3$ and potential well for $\omega < -1$ and/or $\omega > 3$ where one can evolve quantum tunneling of particle ensemble! In other words diagram shows that for $-5 < \omega < -4.7$ and $-1.6 < \omega < -0.8$ we have $\Delta < 0$ which means that the particles ensemble moves into the event horizon.

4. Concluding remarks

Two dimensional analogue of vacuum sector of the BD gravity model is used to solve its dynamical equations. Our classical solutions describe anisotropic perfect fluid collapsing sphere behaves as dark matter gravitational source in its static regime and as regular visible matter in its dynamical non-static regime. Total mass of the fluid is calculated against the BD parameter for which we must be choose $\omega \leq -1$ and/or $\omega > \frac{2}{3}$. In general relativistic approach $\omega \rightarrow +\infty$ ratio of total mass of the fluid per the Plank mass leads to $M^* = \frac{1}{\sqrt{6}} \ll 1$. It is eliminated with $\omega = -1$ and takes infinite value with $\omega = \frac{2}{3}$. In short, end of the collapsing fluid reaches to a black

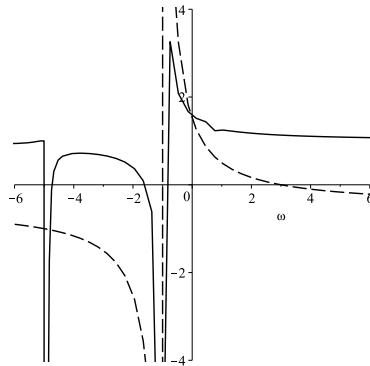


Figure 1. Diagram of Δ (the normalized relative ICD of particles ensemble) and δ (normalized relative quantum potential) is plotted against ω with solid and dash lines respectively.

hole structure and covered the causal singularity satisfying the cosmic censorship hypothesis. de Broglie-Bohm quantum pilot wave of the system is also obtained and corresponding ICD of particles ensemble is evaluated on the event and the apparent horizons. They describe statistical approach of thermal ‘Hawking radiation’. Relative ICD of particles ensemble moving on the horizons take maximum value where the quantum potential diverges to an infinite value at $\omega = -1$.

References

- [1] Ghaffarnejad H 2014 *Int. J. Theor. Phys.* **53** 2616 *Preprint* gr-qc/1309.6157
- [2] Wald R M 1984 *General Relativity*, (The University of Chicago Press, Chicago).
- [3] Joshi P S 2007 *Gravitational Collapse and Space-time Singularities* (Cambridge University Press).
- [4] Singh T P 1999 *J. Astrophys. Astr.* **20** 221
- [5] Ziaie A H, Atazadeh Kh and Tavakoli Y 2010 *Class. Quantum Grav.* **27** 075016; Erratum-*ibid.* 27, 209801 *Preprint* gr-qc/1003.1725.
- [6] Penrose R 1969 *Rev. Nuovo Cimento* **1** 252
- [7] Ori A and Piran T 1990 *Phys. Rev. D* **42** 1068
- [8] Ghaffarnejad H, Neyad H and Mojahedi M A 2013 *Astrophys. Space Sci.* **346** 497 *preprint* physics.gen-ph/1305.6914
- [9] Virbhadra K S and Keeton C R 2008 *Phys. Rev. D* **77** 124014 *preprint* gr-qc/0710.2333.
- [10] Virbhadra K S and Ellis G F R 2002 *Phys. Rev. D* **65** 103004
- [11] Mena F C Nolan B C and Tavakol R 2004 *Phys. Rev. D* **70** 084030 *preprint* gr-qc/0405041.
- [12] Martin-Garcia J M and Gundlach C 2003 *Phys. Rev. D* **68** 024011
- [13] Patil K D 2003 *Phys. Rev. D* **67** 024017
- [14] Joshi P S 1995 *Phys. Rev. D* **51** 6778
- [15] Lemos J 1992 *Phys. Rev. Lett.* **68** 1447
- [16] Joshi P S and Dwivedi I H 1992 *Commun. Math. Phys.* **146** 333
- [17] Ghosh S G and Dadhich N 2003 *Gen. Relativ. Gravit.* **35** 359
- [18] Szekeres P and Lyer V 1993 *Phys. Rev. D* **47** 4362
- [19] Brans C and Dicke R 1961 *Phys. Rev.* **124** 925
- [20] Will C M 1993 *Theory And Experiment In Gravitational Physics* (Cambridge University press) *Preprint* gr-qc/9811036.
- [21] Will C M 2006 *Living Rev. Rel.* **9**; <http://WWW.livingreviews.org/lrr-2006-3>.
- [22] Culetu H 2011 *Preprint* gr-qc/1101.2980
- [23] Ghaffarnejad H 2014 *Preprint* gr-qc/1407.7367
- [24] Ghaffarnejad H 2006 *Astrophys. and Space Sci.* **301** 145