Shape isomers and their clusterization

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Abstract. The role of the symmetries, phases and deformation is analysed in the interrelation of the cluster, shell and collective models. Based on that a symmetry-adapted method is presented for the determination of the shape isomers, and for the study of their possible fragmentation. Comparison with experimental data is discussed both for the cluster-shell competition, and for the existence and fragmentation of shape isomeres.

1. Introduction

The search for shape isomers, e.g. superdeformed (SD), or hyperdeformed (HD) states is an exciting topic of the present-day nuclear structure studies. Especially interesting is their existence in the $N = Z$ nuclei, in which the role of pairing, quartering, clustering, etc. can be studied.

Finding the possible clusterization of a specific nuclear state is important from two aspects. First it contributes to our understanding of the nuclear structure. Second it gives valuable information on the nuclear reactions, which can populate the state in question. In particular, a binary cluster configuration with the clusters in their ground intrinsic state is uniquely related to a reaction channel, like target and projectile in their ground state. Actually, one of the possible definitions (probably the best one) of a cluster state is provided by a reaction channel.

Here we discuss the relation of the collective, shell and cluster models for large deformation, and based on that we present a new method for the determination of stable nuclear shapes, as well as for finding their binary fragmentations. In doing so different kinds of symmetries are applied, and they allow us to study the phases of the shell and cluster models, too.

2. Collective, shell and cluster models

2.1. Symmetries

The connection of the shell, collective [1] and cluster models [2, 3], found in terms of the SU(3) symmetry in 1958, was based on a single-shell problem, described in spherical basis, related to simple symmetries. In a recent paper [4] the extension to multi-major shells was addressed, and the U(3) ⊗ U(3) ⊃ U(3) dynamical symmetry was found as the common intersection of the three models. Here we consider the role of more general symmetries and the case of large deformation.

Historically two kinds of the SU(3) symmetry were applied in this respect. i) The exact symmetry, and ii) the dynamical SU(3) symmetry (sometimes called dynamically broken symmetry). The latter kind of special breaking is achieved by incorporating an interaction

Table 1. Different SU(3) symmetries in nuclear stucture models. The signs '+' and '-' indicate if the Hamiltonian and its eigenvectors are symmetric or not. The operator is said to be symmetric, if it is a scalar, while a set of eigenvectors is symmetric if they transform according to an irreducible representation.

symmetry	Hamiltonian	eigenvectors	model
exact	+	+	harmonic oscillator
dynamical(ly broken)	-	+	Elliott, symplectic, IBM, vibron, SACM
quasi-dynamical	-	-	shell, quantum phase, IBM, SACM

which is expressed in terms of the invariant operator of the SO(3) subgroup, in addition to the SU(3) scalar part.

The relation between the shell and collective models, established by Elliott [1] was based on the SU(3) ⊃ SO(3) dynamical symmetry. The Wildermuth-connection between the shell and cluster models was originally based on harmonic oscillator Hamiltonians of exact SU(3) symmetry, but it turns out to be valid also for the dynamical symmetry, as discussed e.g. in [5].

In studying the case of large deformation the more general quasi-dynamical SU(3) symmetry [6] turns out to be important, too. It is a generalization of the concept of the U(3) dynamical symmetry in the following sense. The Hamiltonian breaks the symmetry in such a way that the U(3) quantum numbers are not valid for its eigenvectors (contrary to the case of the real U(3) dynamical symmetry). In other words neither the operator is symmetric, nor its eigenvectors [7]. (For comparison with the exact and dynamical symmetries see Table I.) Yet, the symmetry is present in some sense, and it may survive even for strong symmetry-breaking interactions [6].

An asymptotic Nilsson-state serves as an intrinsic state for the quasi-dynamical SU(3) representation. Thus the effective quantum numbers are determined by the Nilsson-states in the regime of large deformation [8]. When the deformation is not large enough, then we can expand the Nilsson-states in the asymptotic basis, and calculate the effective quantum numbers based on this expansion [9].

The concept of effective U(3) symmetry is applicable also for the case when the simple leading representation approximation is valid, and then the real and effective U(3) quantum numbers usually coincide [9]. The quasi-dynamical symmetry appears in the investigation of both quantum phases and the large deformation.

Some interesting extensions of Elliott's SU(3) symmetry is illustrated in Figure 1.

2.2. Phases
Symmetry-adapted models are especially suitable for the studies of the phases and phase-transitions in finite quantum systems [10].

In the limit of large particle number phase-transitions are seen in the sense that the derivative of the energy-minimum, as a function of the control-parameter is discontinuous. The order of the derivative, showing the discontinuity, gives the order of the phase-transition. A phase is defined as a region of the phase diagram between the endpoint of the dynamical symmetry and the transition point. It is also conjectured [11] that such a quantum phase is characterised by a quasi-dynamical symmetry. Therefore, although the real dynamical symmetry is valid only at a single point of the phase-diagram, the more general quasi-dynamical symmetry may survive, and in several cases does survive [11, 12], in a finite volume of the phase diagram. If this conjecture really turns out to be true, then the situation is similar to Landau's theory: different phases are determined by different (quasi-dynamical) symmetries, and phase transitions correspond to a change of the symmetry.

In the case of the finite particle number the discontinuities are smoothed out, as the
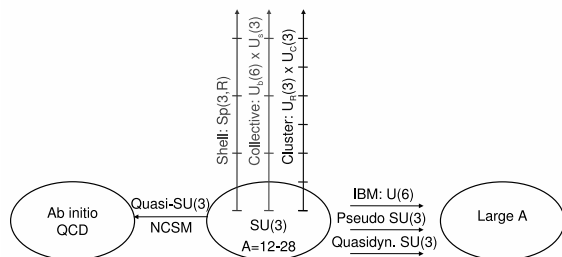
Figure 1. Extension of Elliott's SU(3) symmetry in different directions.
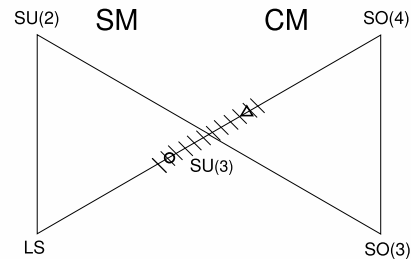

Figure 2. Joint phase diagram of the shell and cluster models, and the location of the ground-band of the ^{20}Ne nucleus, from shell (circle), cluster (triangle) and quasicluster (diffuse) model calculations.

consequence of the finite size effect, but still remarkable changes can be detected in the behaviour of the corresponding functions.

The semimicroscopic algebraic cluster model (SACM) [13] of a binary cluster system has three dynamical symmetries. Two of them come from the vibron model of the relative motion: $U_R(3)$ corresponds to shell-like clusterization, or in the language of the collective motion to a soft vibrator, while $O_R(4)$ represents a rigid molecule-like rotator. The third symmetry corresponds to a situation, when the coupling between the relative motion and internal degrees of freedom of the clusters is weak. Therefore, the phase diagram of a binary system is two-dimensional, and can be illustrated by a triangle [14].

Schematic calculations show that there is a second order phase transition (in between the shell-like ($U_R(3)$) and rigid molecule-like ($O_R(4)$) clusterizations) when interaction terms up to second order are included, but also a first order transition shows up, if third order interactions are involved [15].

A triangle-like phase diagram has been proposed for the shell model, too [16], which in addition to the SU(3) and SU(2) symmetries has the independent-particle model as the third corner. The limiting cases correspond to the situations, in which the quadrupole-quadrupole, pairing (both of them are two-body) interactions, and the single-particle energies dominate, respectively. (When a single shell calculation is performed then the spin-orbit interaction may be the most relevant part of the single-particle contribution.) The two phase diagrams match each other at the SU(3) corner, as shown in Figure 2. The real nuclear systems can be allocated to this diagram.

Here we refer to the results of three different calculations on the ground-band of the ^{20}Ne nucleus. Vargas et al [17] performed a full shell calculation for the 4 nucleons outside the core. Yepez-Martinez et al have carried out a similar calculation on the cluster-side of the phase diagram [18], by applying the semimicroscopic algebraic cluster model [13].

Itagaki et al used the antisymmetrized quasi-cluster model [19] for the description of the ground band. This model can take a direct route from the rigid molecule-like (SO(4)) clusterization via the shell-like cluster limit (SU(3)) to the jj-coupled shell model dominated by a strong LS interaction. Thus, it is especially illuminative from the viewpoint of the cluster-shell competition. It does not have, however, a well-defined algebraic structure, therefore, it does not

determine a well-defined position.

It is remarkable that three different model calculations have similar conclusion on the closeness of the ground-band of ^{20}Ne to the matching point between the shell and cluster models.

2.3. Deformation

In [20] it was shown that the symmetry algebra of the anisotropic harmonic oscillator is SU(3), whenever its frequencies are commensurate. In considering realistic interactions the Nilsson model plays an important role. Soon after the experimental discovery of the superdeformation, it was realised that the $L - S$ coupling recovers for the SD shape [21].

We also apply the Nilsson-model in order to investigate the survival (or appearance) of the SU(3) symmetry systematically in a large range of the quadrupole deformation. We obtain the shape isomers from a selfconsistent calculation concerning the quadrupole deformation. In particular: one varies systematically the parameters $(\beta_{in}, \gamma_{in})$, as an input for the Nilsson-model. The calculations provide us with the effective U(3) quantum numbers, which can be translated into the $(\beta_{out}, \gamma_{out})$ quadrupole deformation, since they are uniquely related to each other [22]. Then one can check if the selfconsistency is satisfied, as well as the question whether or not the result is stable with respect to the (small) changes of the input values. This method for the determination of the shape isomers is an alternative to the standard energy-minimum calculation and has been shown to be effective for a range of light nuclei [23, 24, 25].

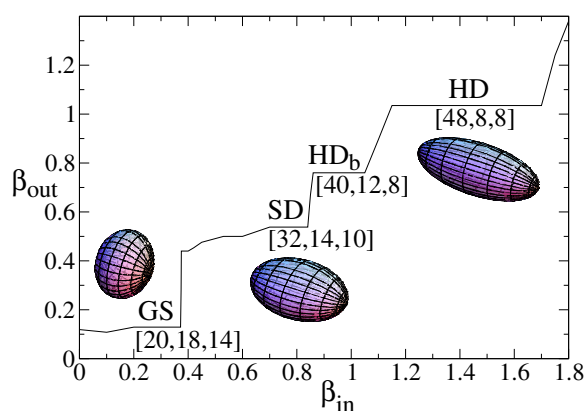The result of the Nilsson model + quasi-dynamical SU(3) calculation for ^{36}Ar is shown in Figure 3.

Figure 3. Shape isomers of the ^{36}Ar nucleus from the deformation self-consistency calculation, with effective U(3) quantum numbers and the schematic illustration of the shape at the plateaus.
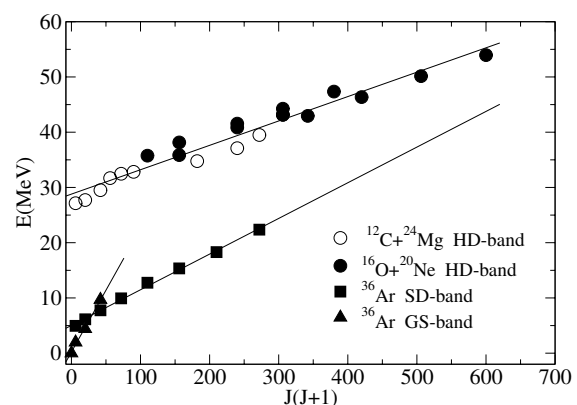

Figure 4. Excited states in ^{36}Ar, observed as resonances in the ^{12}C+^{24}Mg (open circles) and ^{16}O+^{20}Ne (full circles) reactions. The SD band observed in ^{36}Ar and the ground-state band are also included.

2.4. Fragmentation

The U(3) connection [1, 3] between the collective, shell and cluster models works well in case i) the U(3) symmetry is approximatelly valid, and ii) the relation between the cluster and shell model wavefunctions is simple. This latter condition means that the expansion of the cluster U(3) state in terms of shell basis reduces to a few terms.

As discussed in the previous subsection the U(3) symmetry recovers for the superdeformed, hyperdeformed, etc shapes, in spite of the important role of the symmetry-breaking spin-orbit

interaction. The second condition (on the simple shell model expansion) turns out to be valid also for several shape isomers. E.g. in case of the ^{20}Ne nucleus each of the 4 shape isomers (of the U(3) symmetry: [12,4,4], [16,8,0], [24,4,0], [40,0,0]) has single multiplicity in the shell model basis [26]. Therefore, if a cluster state with the same U(3) symmetry is allowed it is identical with the shell state.

In addition to the U(3) selection, there is another simple prescription by Harvey [27] for the determination of the allowed clusterizations. It also applies harmonic oscillator basis, so the two requirements are somewhat similar. Nevertheless, their physical content is not the same; in some sense they are complementary to each other. Therefore, the best way is to apply them in a combined way [28, 29, 30]. (Their relation is discussed more in detail in [31].) When a cluster configuration is forbidden, one can characterize its forbiddenness quantitatively [32].

The energetic preference represents a complementary viewpoint for the selection of clusterization. We usually incorporate it in two different ways: i) by applying simple binding-energy arguments [33], and ii) by performing double-folding calculations, according to the dinuclear system model [34, 35].

3. Application
We have applied the above methods for the determination of the shape isomers from the ground state up to the linear alpha-chain states in several light $N = Z$ nuclei: ^{20}Ne [26], ^{28}Si [25], ^{36}Ar [23], ^{56}Ni [24]. Several isomers have been found, and their possible binary fragmentations have also been determined. In our description the clusters are supposed to be in their intrinsic ground state, like the free nucleus, without any simplifying assumption. Their relative orientation is also free to change, without any constraint. Some general features of these results are as follows. The same binary cluster-configuration can be present in different shape isomers of a nucleus due to the different orientation of the deformed clusters with respect to the molecular axis. The ground state usually prefers asymmetric fragmentations, while in the hyperdeformed state more symmetric clusterizations appear. The superdeformed state represents a situation in between. This feature is a consequence of the similar shapes and volumes of close-lying nuclei in their ground state.

As a specific example we cite here the case of the ^{36}Ar nucleus. In [28] the possible binary clusterizations of the ground, the (experimentally known) SD, and the (theoretically predicted [36]) HD states was studied systematically. It was found that the HD state could be populated in the ^{24}Mg+^{12}C and ^{20}Ne+^{16}O reactions. A recent analysis of the ^{24}Mg+^{12}C elastic scattering [37] revealed the existence of resonances, as shown in the upper part of Fig. 4. The moment of inertia is in a very good agreement with that of the HD shape predicted from alpha-cluster model [36]. Nilsson-model+quasi-dynamical SU(3) calculation gave the same HD state [23], as the alpha-cluster model.

The similarity of the (predicted and observed) moments of inertia, and the fact that the resonances were seen in exactly those reactions, which define the preferred cluster-configurations of the HD shape suggest that the recently observed band in Fig. 4 is a good candidate for the hyperdeformed shape isomer of the ^{36}Ar nucleus. Thus ^{36}Ar may be a nucleus to have experimental evidence for the existence of ground, superdeformed, as well as hyperdeformed bands. (The question of the ternary clusterization for these shape isomers have been addressed in [29].)

4. Summary and conclusion
In this contribution we have considered the interrelation of the fundamental structure models of atomic nuclei for the case of general symmetries and large deformation.

The cluster-shell competition or coexistence has been interpreted in terms of the joint phase diagram of the shell and cluster models. Three different (shell, cluster and quasi-cluster)

calculations indicate the position of the ground-band of ^{20}Ne very close to the SU(3) matching point of the two models.

Concerning the large deformations the quasi-dynamical SU(3) symmetry is found to be stable for Nilsson-type interactions for several shapes with commensurable ratios of the main axes. This symmetry-oriented method allows the determination of the possible clusterizations, as well, which reveals the connection to reaction channels.

All these considerations, as well as many others, are based on the extension of Elliott's SU(3) symmetry. In Figure 1 we summarized some of its generalization along different directions.

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